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*Research Article*

# Formation Control for Unmanned Aerial Vehicles with Directed and Switching Topologies

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Formation control problems for unmanned aerial vehicle (UAV) swarm systems with directed and switching topologies are investigated. A general formation control protocol is proposed firstly. Then, by variable transformation, the formation problem is transformed into a consensus problem, which can be solved by a novel matrix decomposition method. Sufficient conditions to achieve formation with directed and switching topologies are provided and an explicit expression of the formation reference function is given. Furthermore, an algorithm to design the gain matrices of the protocol is presented. Finally, numerical simulations are provided to illustrate the effectiveness of the theoretical results.

## 1. Introduction

In the past decades, unmanned aerial vehicles (UAVs) have been widely used in civilian and military areas, such as surveillance and reconnaissance [1, 2] and target search and localization [3]. Since the performance of a team of UAVs working cooperatively exceeds the performance of individual UAVs, formation control of UAVs is of importance and has received a lot of attention.

The formation control of UAVs has been studied with many different methods, such as leader–follower [4], behavior [5], and virtual structure-based [6] approaches. Recently, with the development of consensus theory [7–15], some related methods are also used to deal with the formation control problems of UAVs. Consensus means that all agents reach a common state. The results in [16] show that consensus approaches can be used to deal with formation control problems, and leader–follower, behavior, and virtual structure-based formation control approaches are special cases of consensus-based approaches.

Based on consensus method, Abdessameud and Tayebi [17] proposed controllers for UAV swarm systems to achieve formation in the presence of communication delays. A consensus protocol together with an output feedback linearization method is presented in [18] such that the UAV swarm systems can achieve partially time-varying formation. Besides, indoor and outdoor flight experiments for quadrotor swarm systems to achieve formation by consensus approaches are carried out in [19] and [20], respectively. Based on consensus theory, we know that the achievement of formation depends on not only the individual UAV dynamics but also the structure of the networks between UAVs which can be modeled by directed and undirected graphs. However, the interaction topologies between UAVs in [19, 20] are assumed to be fixed. In practical applications, the interaction topologies of UAV swarm systems may be switching due to the fact that the communication channel may fail and new channels may be created during flight. Time-varying formation control for UAV swarm systems and high-order LTI systems with switching interaction topologies are studied by Dong [20, 21], but the topologies are assumed to be undirected. To the best of our knowledge, there is still work to do on formation control of UAV swarm systems with directed and switching topologies.

In this paper, we aim to solve the formation problem of UAV swarm systems with directed and switching topologies. Compared with the existing results, the assumptions of the communication topology are quite general. The remainder of this paper is organized as follows. In Section 2, some necessary concepts and useful results on graph theory are summarized and the problem formulation is given. Main theoretical results are proposed in Section 3. In Section 4, a numerical simulation is presented. Section 5 is the conclusion.

## 2. Preliminaries and Problem Description

*2.1. Notations and Graph Theory.* In this paper, the following notations will be used. $R^{n\times n}$ and $C^{n\times n}$ denote the set of $n\times n$ real and complex matrices, respectively. For $\mu \in C$, the real part is $\mathrm{Re}(\mu)$. $\otimes$ denotes the Kronecker product. $I_n$ is the identity matrix of order $n$. For a square matrix $A$, $\lambda(A)$ denotes the eigenvalues of matrix $A$. $A>0$ ($A\geq 0$) means that $A$ is positive definite (positive semidefinite). $\max\{\lambda(A)\}$ ($\min\{\lambda(A)\}$) denotes the largest (smallest) eigenvalue of the matrix $A$.

A directed graph $G=(\mathscr{V},\mathscr{E},\mathscr{A})$ contains the vertex set $\mathscr{V}=\{1,2,\ldots,N\}$, the directed edges set $\mathscr{E}\subseteq\mathscr{V}\times\mathscr{V}$, and the adjacency matrix $\mathscr{A}=[a_{ij}]_{N\times N}$ with nonnegative elements $a_{ij}$. $a_{ij}=1$ if there is a directed edge from vertex $j$ to $i$; $a_{ij}=0$, otherwise. The Laplacian matrix of the graph $G$ is defined as $L=[L_{ij}]_{N\times N}$, where $L_{ii}=\sum_{j\neq i}a_{ij}$ and $L_{ij}=-a_{ij}$ ($i\neq j$). Zero is an eigenvalue of $L$ with the eigenvector $\mathbf{1}_N$. A directed graph is said to have a spanning tree if there is a vertex such that there is a directed path from this vertex to every other vertex.

**Lemma 1** (see [8]). *Zero is a simple eigenvalue of L and all the other nonzero eigenvalues have positive real parts if and only if the graph has a directed spanning tree.*

*2.2. Problem Description.* Consider UAV swarm systems with $N$ UAVs. The interaction topology of the UAV swarm systems can be described by a directed graph $G$, in which UAV $i$ can be denoted by a vertex and the interaction channel from UAV $i$ to UAV $j$ can be denoted by an edge. Compared with the attitude dynamics, the trajectory dynamics of each UAV have much larger time constants, which means the attitude controller and trajectory controller can be designed separately. On the formation level, only trajectory control needs to be considered. Therefore, in this brief, the dynamics of each UAV can be described by the following double integrator [18, 21, 22]:

$$\begin{aligned}\dot{x}_i(t)&=v_i(t),\\ \dot{v}_i(t)&=u_i(t),\end{aligned}\tag{1}$$

where $i=1,2,\ldots,N$, $x_i(t)\in R^n$ and $v_i(t)\in R^n$ denote the position and velocity vectors of UAV $i$, respectively, and $u_i(t)\in R^n$ are the control inputs. In the following, for simplicity of description, it is assumed that $n=1$, if not otherwise specified.

Therefore, UAV swarm systems (1) can be rewritten as

$$\dot{\xi}_i(t)=A\xi_i(t)+Bu_i(t),\tag{2}$$

where $\xi_i(t)=[x_i(t),v_i(t)]^T$, $A=\left[\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right]$, $B=\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$.

A formation is specified by a vector $h(t)=[h_1^T(t),h_2^T(t),\ldots,h_N^T(t)]^T\in R^{2N}$ with $h_i(t)=[h_{ix}(t),h_{iv}(t)]^T$ ($i=1,2,\ldots,N$) continuously differentiable and $h_{iv}(t)$ being the derivative of $h_{ix}(t)$. Let $h_x(t)=[h_{x1}^T(t),h_{x2}^T(t),\ldots,h_{xN}^T(t)]^T$ and let $h_v(t)=[h_{v1}^T(t),h_{v2}^T(t),\ldots,h_{vN}^T(t)]^T$; then one has that if $h_v(t)$ are not equal to zeros, the formation is time-varying.

*Definition 2* (see [21]). UAV swarm systems (2) are said to achieve formation $h(t)$ if there exists a function $r(t)=[r_x(t),r_v(t)]\in R^2$ with $r_v(t)$ being the derivative of $r_x(t)$ such that

$$\lim_{t\to\infty}\left(\xi_i(t)-h_i(t)-r(t)\right)=0,\quad i=1,2,\ldots,N,\tag{3}$$

where $r(t)$ is called a formation center function.

In this paper, the communication topology is molded by a directed graph and we assume that the communication topology is time-varying. Let $\widehat{G}=\{G^1,G^2,\ldots,G^p\}$, $p\geq 1$, be the set of all possible directed topologies. We define the switching signal $\sigma(t)$, where $\sigma(t):[0,+\infty)\to\mathscr{P}=\{1,2,\ldots,p\}$. $0=t_0<t_1<t_2<\cdots$ denote the switching instants of $\sigma(t)$. Let $G^{\sigma(t)}\in\widehat{G}$ be the communication topology at time $t$. $L^{\sigma(t)}$ stands for the corresponding Laplacian matrix of $G^{\sigma(t)}$.

*Assumption 3.* Each possible graph $G^{\sigma(t)}\in\widehat{G}$ is fixed and contains a directed spanning tree.

Let $\lambda_i^{\sigma(t)}$ ($i=1,2,\ldots,N$) be the eigenvalues of the Laplacian matrix $L^{\sigma(t)}$. Without loss of generality, it is assumed that $\mathrm{Re}(\lambda_1^{\sigma(t)})\leq\mathrm{Re}(\lambda_2^{\sigma(t)})\leq\cdots\leq\mathrm{Re}(\lambda_N^{\sigma(t)})$. Furthermore, from Lemma 1, one can obtain that $\lambda_1^{\sigma(t)}=0$ and $0\leq\mathrm{Re}(\lambda_2^{\sigma(t)})\leq\cdots\leq\mathrm{Re}(\lambda_N^{\sigma(t)})$. Let $\lambda_{\min}=\min\{\lambda_i^m\ (\forall m\in\mathscr{P};\ i=2,3,\ldots,N)\}$.

Consider the following formation protocol:

$$\begin{aligned}u_i(t)={}&K_1\left(\xi_i(t)-h_i(t)\right)\\&+K_2\sum_{j=1}^{N}a_{ij}\left(\left(\xi_j(t)-h_j(t)\right)-\left(\xi_i(t)-h_i(t)\right)\right)\\&+\dot{h}_{iv}(t),\quad i=1,2,\ldots,N,\end{aligned}\tag{4}$$

where $i=1,2,\ldots,N$, $K_1\in R^{1\times 2}$ and $K_2\in R^{1\times 2}$ are constant gain matrices, and $a_{ij}$ is defined as in Section 2.1. Let $\xi(t)=[\xi_1^T(t),\xi_2^T(t),\ldots,\xi_N^T(t)]^T$, let $h_x(t)=[h_{1x}^T(t),h_{2x}^T(t),\ldots,h_{Nx}^T(t)]^T$, and let $h_v(t)=[h_{1v}^T(t),h_{2v}^T(t),\ldots,h_{Nv}^T(t)]^T$. Under protocol (4), the UAV swarm systems (2) can be written in a compact closed-loop form as follows:

$$\begin{aligned}\dot{\xi}(t)={}&\left(I_N\otimes\left(A+BK_1\right)-L^{\sigma(t)}\otimes\left(BK_2\right)\right)\xi(t)\\&-\left(I_N\otimes BK_1-L^{\sigma(t)}\otimes\left(BK_2\right)\right)h(t)\\&+\left(I_N\otimes B\right)\dot{h}_v(t).\end{aligned}\tag{5}$$

This brief mainly investigates how to design the gain matrices in protocol (4) for the UAV swarm systems (5) to achieve the formation $h(t)$.

## 3. Main Results

Let $z_i(t) = \xi_i(t) - h_i(t)$, $z(t) = [z_1^T(t), z_2^T(t), \ldots, z_N^T(t)]^T$. Then the UAV swarm systems can be rewritten as follows:

$$\begin{aligned}\dot{z}(t) &= \left(I_N \otimes (A + BK_1) - L^{\sigma(t)} \otimes BK_2\right) z(t) \\ &\quad + (I_N \otimes A) h(t) - (I_N \otimes I_n) \dot{h}(t) \\ &\quad + (I_N \otimes B) \dot{h}_v(t).\end{aligned} \quad (6)$$

As for $\dot{h}_{ix}(t) = h_{iv}(t)$, one can obtain that

$$(I_N \otimes A) h(t) - (I_N \otimes I_n) \dot{h}(t) + (I_N \otimes B) \dot{h}_v(t) = 0. \quad (7)$$

Thus (6) can be further rewritten as

$$\dot{z}(t) = \left(I_N \otimes (A + BK_1) - L^{\sigma(t)} \otimes BK_2\right) z(t). \quad (8)$$

It holds directly that UAV swarm systems (2) with directed and switching topologies achieve formation $h(t)$ if and only if system (8) achieves consensus.

Before the consensus analysis of system (8), the following lemmas and definition are introduced.

**Lemma 4** (see [23]). *For a Laplacian matrix $L$ of graph $G$ and a full row rank matrix $E$ defined as*

$$E = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}, \quad (9)$$

*there exists a matrix $M$ such that $L = ME$. Further, if the graph has a directed spanning tree, $M$ is of full column rank and the eigenvalues of $EM$ are equal to the nonzero eigenvalues of $L$.*

**Lemma 5** (see [24]). *Suppose that the eigenvalues of $A \in R^{N\times N}$ have positive real parts; then there exists a positive definite matrix $Q > 0$ such that*

$$A^T Q + QA > 0. \quad (10)$$

*Definition 6.* For a switching signal $\sigma(t)$ over time interval $[0, t)$, the average dwell time of the switching signal is defined as $\tau_a = t/(N_\sigma(t) + 1)$, where $N_\sigma(t)$ denotes the number of the switches.

*Remark 7.* In [11, 25], the definition of the average dwell time of a switching signal $\sigma(t)$ over time interval $[0, t)$ can be described as follows. If there exist two positive numbers $N_0$ and $\tau_a$ such that $N_\sigma(t) \le N_0 + t/\tau_a$, where $N_\sigma(t)$ denotes the number of the switches, $\tau_a$ is called the average dwell time. It is inaccurate to give the definition by an inequality, but, according to Definition 6, it can be seen that $N_\sigma(t) \le t/\tau_a$.

From Lemma 4, one can obtain that, for each $L^{(i)}$, $i \in \mathscr{P}$, there exists a matrix $M^{(i)}$ such that $L^{(i)} = M^{(i)}E$. Given Assumption 3 and Lemma 1, it can be known that the eigenvalues of each $EM^{(i)}$ ($i \in \mathscr{P}$) have positive real parts. Based on Lemma 5, one can obtain that there exist positive definite matrices $Q^{(i)}$ such that

$$\left(EM^{(i)} - \alpha I\right)^T Q^{(i)} + Q^{(i)} \left(EM^{(i)} - \alpha I\right) > 0, \quad (11)$$

where $\alpha < \lambda_{\min}$. Further, one can obtain that

$$\left(EM^{(i)}\right)^T Q^{(i)} + Q^{(i)} EM^{(i)} > 2\alpha Q^{(i)}, \quad i \in \mathscr{P}. \quad (12)$$

Let $\theta_i(t) = z_{i+1}(t) - z_i(t)$, $i = 1, 2, \ldots, N$, and let $\theta(t) = [\theta_1^T(t), \theta_2^T(t), \ldots, \theta_N^T(t)]^T$. One can obtain that $\theta(t) = (E \otimes I_n) z(t)$, where $E$ is defined as in Lemma 4.

Premultiplying both sides of (8) by $(E \otimes I_n)$ leads to

$$\dot{\theta}(t) = \left(I_{N-1} \otimes (A + BK_1) - EM^{\sigma(t)} \otimes BK_2\right) \theta(t), \quad (13)$$

where $M^{\sigma(t)} = L^{\sigma(t)} E^T (EE^T)^{-1}$.

According to the definition of $\theta(t)$, it is obvious that $z_1(t) = z_2(t) = \cdots = z_N(t)$ if and only if $\theta(t) = 0$. So if system (13) converges to zero, system (8) achieves consensus and UAV swarm systems (2) with directed and switching topologies achieve formation $h(t)$.

**Theorem 8.** *Suppose that Assumption 3 holds. The formation problem of UAV swarm systems (2) with directed and switching topologies can be solved by controller (4) if there exists a positive definite matrix $P$ such that*

$$(A + BK_1)^T P + P (A + BK_1) - 2\alpha PBB^T P + \beta P \le 0, \quad (14)$$

*where $\alpha < \lambda_{\min}$, $\beta > \ln h/\tau_a$, $h = \varphi_1/\varphi_2$, $\varphi_1 = \max_{i\in\mathscr{P}}\{\lambda(Q^{(i)})\}$, $\varphi_2 = \min_{i\in\mathscr{P}}\{\lambda(Q^{(i)})\}$, and $Q^{(i)}$ satisfies (12). The feedback matrix is designed as $K_2 = B^T P$.*

*Proof.* Consider the following piecewise Lyapunov candidate of system (13):

$$V(t) = \theta(t)^T \left(Q^{\sigma(t)} \otimes P\right) \theta(t), \quad (15)$$

where $P$ is a solution of inequality (14) and $Q^{\sigma(t)}$ are feasible solutions of (12). □

Note that the communication topology is fixed for $t \in [t_i, t_{i+1})$, $i = 0, 1, \ldots$. Then, the derivation of this Lyapunov candidate along the trajectory of system (13) within the interval is

$$\begin{aligned}\dot{V}(t) &= \theta(t)^T \left(I_{N-1} \otimes (A + BK_1) - EM^{\sigma(t)} \otimes BK_2\right)^T \\ &\quad \cdot \left(Q^{\sigma(t)} \otimes P\right) \theta(t) + \theta(t)^T \left(Q^{\sigma(t)} \otimes P\right) \\ &\quad \cdot \left(I_{N-1} \otimes (A + BK_1) - EM^{\sigma(t)} \otimes BK_2\right) \\ &\quad \cdot \theta(t).\end{aligned} \quad (16)$$

Substituting $K_2 = B^T P$ into (16) yields

$$\begin{aligned}\dot{V}(t) &= \theta(t)^T \Big(Q^{\sigma(t)} \otimes \left((A + BK_1)^T P + P (A + BK_1)\right) \\ &\quad - \left(\left(EM^{\sigma(t)}\right)^T Q^{\sigma(t)} + Q^{\sigma(t)} EM^{\sigma(t)}\right) \otimes PBB^T P\Big) \\ &\quad \cdot \theta(t).\end{aligned} \quad (17)$$

It then follows from (12) that

$$\dot{V}(t) \leq \theta(t)^T \left(Q^{\sigma(t)} \otimes \left((A+BK_1)^T P + P(A+BK_1) - 2\alpha PBB^T P\right)\right) \cdot \theta(t). \tag{18}$$

Based on (14), one has

$$\dot{V}(t) < -\beta\theta(t)^T \left(Q^{\sigma(t)} \otimes P\right)\theta(t). \tag{19}$$

Thus, from (15), one can obtain that

$$V(t) < e^{-\beta(t-t_i)} V(t_i). \tag{20}$$

Note that the communication topology switches at $t = t_i$; then one can get

$$V(t_i) \leq hV(t_i^-), \tag{21}$$

where $h = \varphi_1/\varphi_2$, $\varphi_1 = \max_{i\in\mathcal{P}}\{\lambda(Q^{(i)})\}$, and $\varphi_2 = \min_{i\in\mathcal{P}}\{\lambda(Q^{(i)})\}$.

Thus, when $t \in [t_i, t_{i+1})$, from (20) and (21), one has

$$\begin{aligned} V(t) &< e^{-\beta(t-t_i)} hV(t_i^-) < e^{-\beta(t-t_i)} h e^{-\beta(t_i-t_{i-1})} V(t_{i-1}) \\ &< e^{-\beta t} h^i V(0). \end{aligned} \tag{22}$$

Since $i \leq N_\sigma(t) \leq t/\tau_a$,

$$V(t) < e^{-(\beta - \ln h/\tau_a)t} V(0). \tag{23}$$

From (15), one can obtain that

$$\begin{aligned} V(0) &\leq \phi_1 \|\theta(0)\|^2, \\ \phi_2 \|\theta(t)\|^2 &\leq V(t), \end{aligned} \tag{24}$$

where $\phi_1 = \varphi_1 \max\{\lambda(P)\}$ and $\phi_2 = \varphi_2 \min\{\lambda(P)\}$.

According to (23) and (24), one has

$$\|\theta(t)\|^2 \leq \frac{\phi_1}{\phi_2} e^{-(\beta-\ln h/\tau_a)t} \|\theta(0)\|^2. \tag{25}$$

Note that $\beta > \ln h/\tau_a$; one has $\theta(t) \to 0$ as $t \to \infty$. This means that the consensus problem of system (8) is solved. Furthermore, formation for UAV swarm systems (2) with directed and switching topologies is achieved.

**Corollary 9.** *If UAV swarm systems (2) achieve formation $h(t)$, the formation center function $r(t)$ can be determined as follows:*

$$r(t) = e^{(A+BK_1)(t-t_i)} \left(p^{\sigma(t)T} \otimes I\right)\left(x(t_i) - h(t_i)\right), \quad t \in [t_i, t_{i+1}), \tag{26}$$

*where $p^{\sigma(t)}$ is the left eigenvector of $L^{\sigma(t)}$ associated with eigenvalue* 0 *and $(p^{\sigma(t)})^T \mathbf{1}_N = 1$.*

*Proof.* From [7], there exists a left eigenvector $p^{\sigma(t)}$ of $L^{\sigma(t)}$ associated with eigenvalue 0 and $(p^{\sigma(t)})^T\mathbf{1}_N = 1$. For $t \in [t_i, t_{i+1})$, $L^{\sigma(t)}$ is fixed and so is $p^{\sigma(t)}$. □

Premultiplying both sides of (8) by $((p^{\sigma(t)})^T \otimes I)$ results in

$$\begin{aligned} &\left(\left(p^{\sigma(t)}\right)^T \otimes I\right)\dot{z}(t) \\ &= (A+BK_1)\left(\left(p^{\sigma(t)}\right)^T \otimes I\right) z(t). \end{aligned} \tag{27}$$

Based on Definition 2, one can obtain that

$$\lim_{t\to\infty} \left(z(t) - (\mathbf{1}_N \otimes I) r(t)\right) = 0. \tag{28}$$

Premultiplying both sides by $((p^{\sigma(t)})^T \otimes I)$, one has

$$\lim_{t\to\infty} \left(\left(\left(p^{\sigma(t)}\right)^T \otimes I\right) z(t) - r(t)\right) = 0. \tag{29}$$

Therefore, a formation center function can be

$$r(t) = \left(\left(p^{\sigma(t)}\right)^T \otimes I\right) z(t). \tag{30}$$

It follows from (27) and (30) that

$$\dot{r}(t) = (A+BK_1) r(t). \tag{31}$$

Thus, (26) can be obtained.

*Remark 10.* As can be seen, the formation center is discontinuous due to the switching of the communication topology. In addition, $K_1$ can be used to design the motion modes of the formation center function. If $K_1 = 0$, protocol (4) becomes a totally distributed controller. $K_2$ has no effect on the formation center function.

*Remark 11.* Compared with [21, 22], the interaction topologies are more common. Formation for UAV swarm systems with directed and switching topologies is solved. Furthermore, the gain matrix was designed by solving an LMI, which is simpler than solving an algebraic Riccati equation in [22]. In fact, undirected topologies are just special cases of directed topologies. So the algorithms presented in this paper are applicable to those cases in [21, 22].

Based on the above results, a design procedure of protocol (4) can be summarized as follows. First, choose $K_1$ to design the motion modes of the formation center by assigning the eigenvalues of $(A+BK_1)$. Then design $K_2$ using the conclusion of Theorem 8.

## 4. Examples

In this section, we provide an example to illustrate the effectiveness of the above theoretical results. UAV swarm

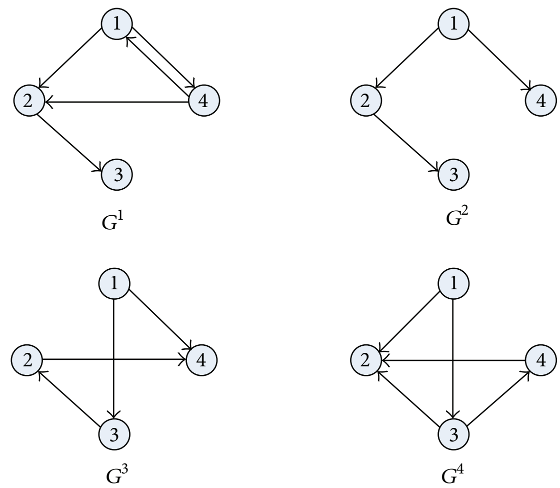



Figure 1: Communication topologies.

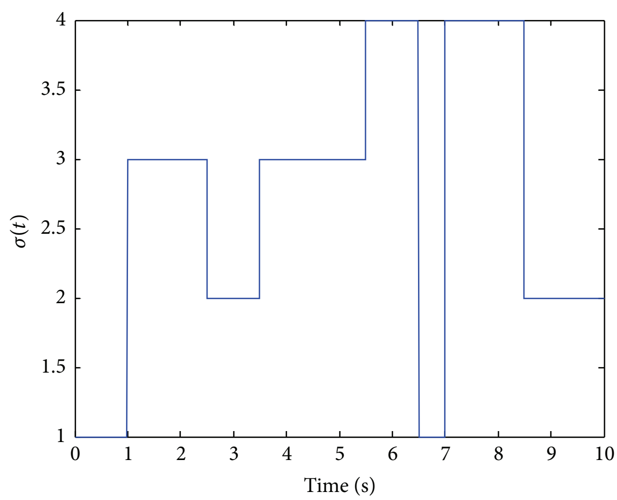



Figure 2: Switching signal $\sigma(t)$.

systems consisting of four agents are considered. The system matrices are defined as

$$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \\ x_{i4} \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (32)$$

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

where $x_{i1}$, $x_{i2}$, $x_{i3}$, and $x_{i4}$ stand for east position, east velocity, north position, and north velocity. The directed communication topologies are given in Figure 1. Clearly, each topology contains a directed spanning tree. The switching signal is shown in Figure 2.

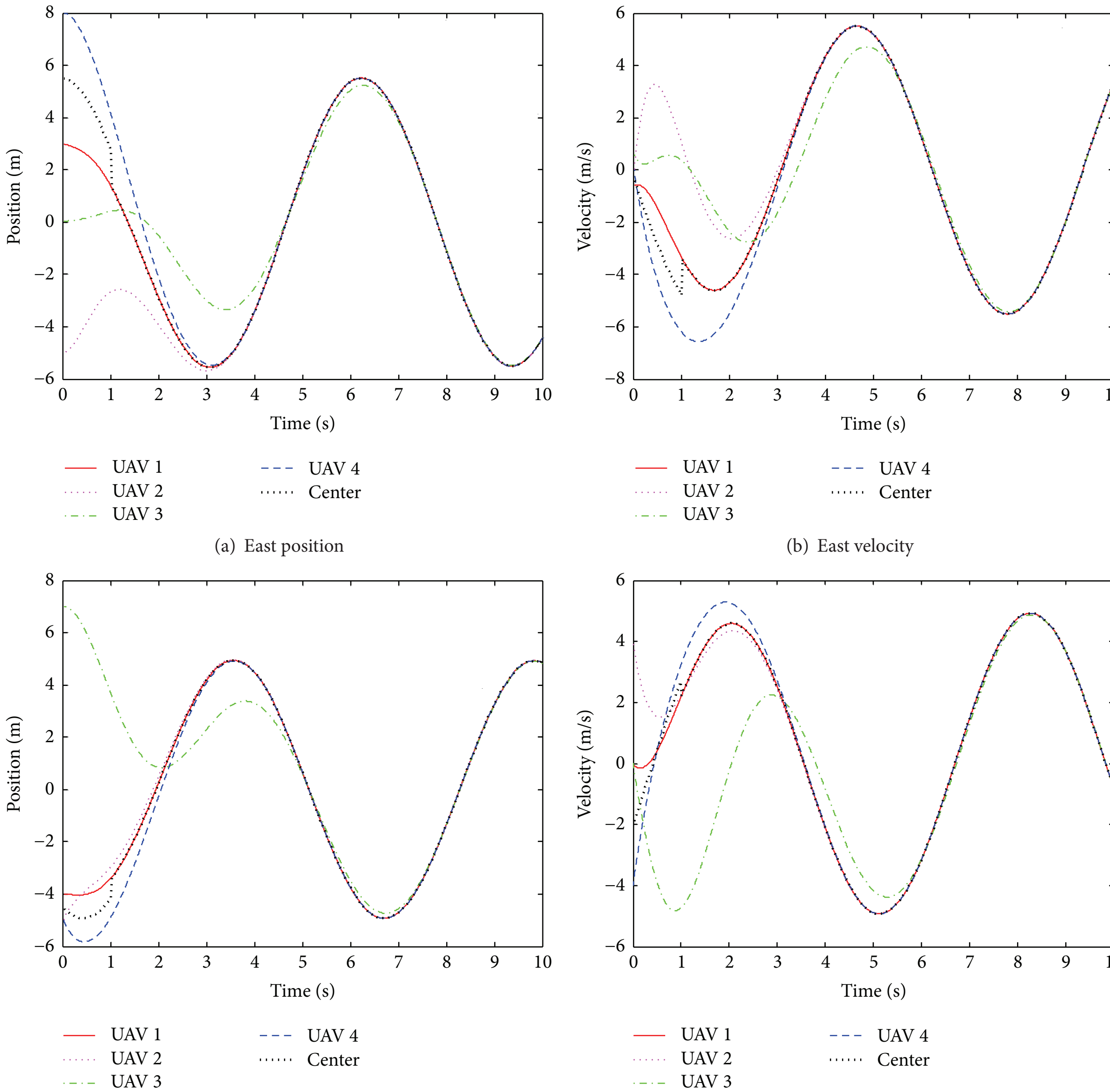
Thus, we can obtain $\lambda_{\min} = 1$ and then choose $\alpha = 0.9$. Further, we can get that $\varphi_1 = 3.4175$, $\varphi_2 = 0.2009$, and $h = 17.0075$. From Figure 2, we can get that the average dwell time is 1.25 s and then choose $\beta = 5$.

Assign the eigenvalues of $(A + BK_1)$ at $(\pm i, \pm i)$; we get

$$K_1 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}. \quad (33)$$

Solve LMI (14) with $\alpha = 0.9$ and $\beta = 5$; a feasible solution can be obtained. Accordingly, we can get

$$K_2 = \begin{bmatrix} 0.6565 & 1.3431 & 0 & 0 \\ 0 & 0 & 0.6565 & 1.3431 \end{bmatrix}. \quad (34)$$




(a) East position (b) East velocity (c) North position (d) North velocity

Figure 3: Difference of UAV state and time-varying formation.

Choose the following time-varying formation:

$$h_i(t) = \begin{bmatrix} 3\sin\left(0.2t + \frac{(i-1)\pi}{2}\right) \\ 0.6\cos\left(0.2t + \frac{(i-1)\pi}{2}\right) \\ 8\cos\left(0.5t + \frac{(i-1)\pi}{2}\right) \\ -4\sin\left(0.5t + \frac{(i-1)\pi}{2}\right) \end{bmatrix} \quad (i = 1, 2, 3, 4). \tag{35}$$

If $h(t)$ is achieved, both the positions and velocities of the four UAVs locate at the vertexes of a rotating parallelogram, respectively. Choose initial states of four UAVs as $x_1(0) = [3\ \ 0\ \ 4\ \ 0]^T$, $x_2(0) = [-2\ \ 0\ \ -5\ \ 0]^T$, $x_3(0) = [0\ \ 0\ \ -1\ \ 0]^T$, and $x_4(0) = [5\ \ 0\ \ -5\ \ 0]^T$.

Figure 3 shows the trajectories of the difference of UAV states and time-varying formation, which are denoted by solid line, dotted line, dash-dotted line, and dashed line. And the bold dotted line denotes the formation center trajectory. It is obvious that the differences achieve consensus after about $t = 7$ s and converge to the formation center. From Definition 2, one can obtain that the time-varying formation problem is solved. Figure 4 shows the snapshots of four UAV

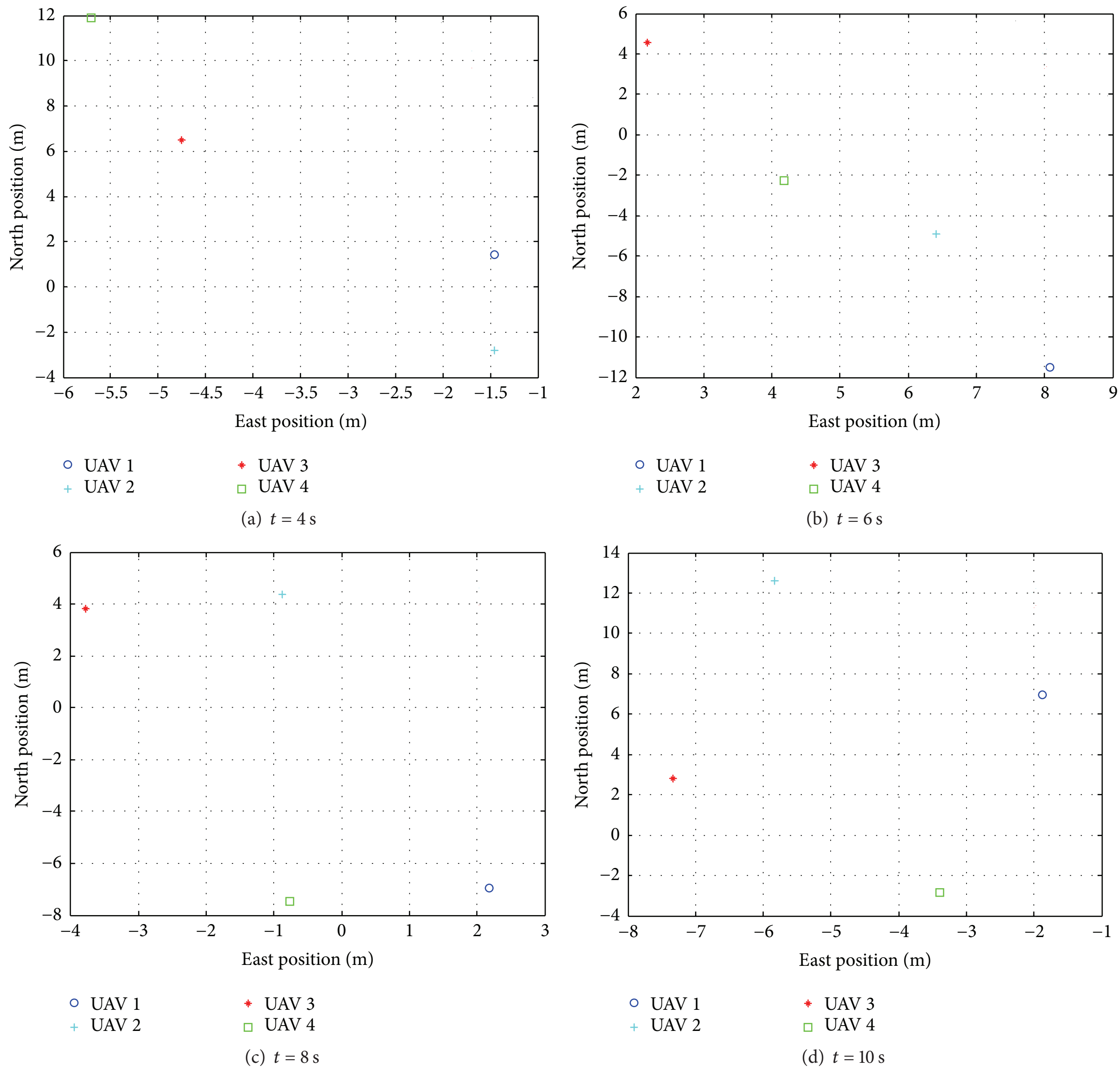



Figure 4: Snapshots of UAV positions.

positions at different time. It can be seen that, after $t = 6$ s, the UAV swarm systems achieve a time-varying parallelogram formation. Therefore, the time-varying formation is achieved under the directed and switching topologies.

## 5. Conclusions

Formation problems for UAV swarm systems with directed and switching topologies are studied. The average dwell time of the switching topologies is introduced, based on which an LMI-based method to design the protocol is proposed. Though the UAV swarm systems can achieve the specified formation with the presented method, there are still problems in real application. As mentioned in Assumption 3, each of the switching topologies is supposed to have a spanning tree, which may not be applicable, so there is still work to do in our future work.

## Competing Interests

The authors declare that they have no competing interests.

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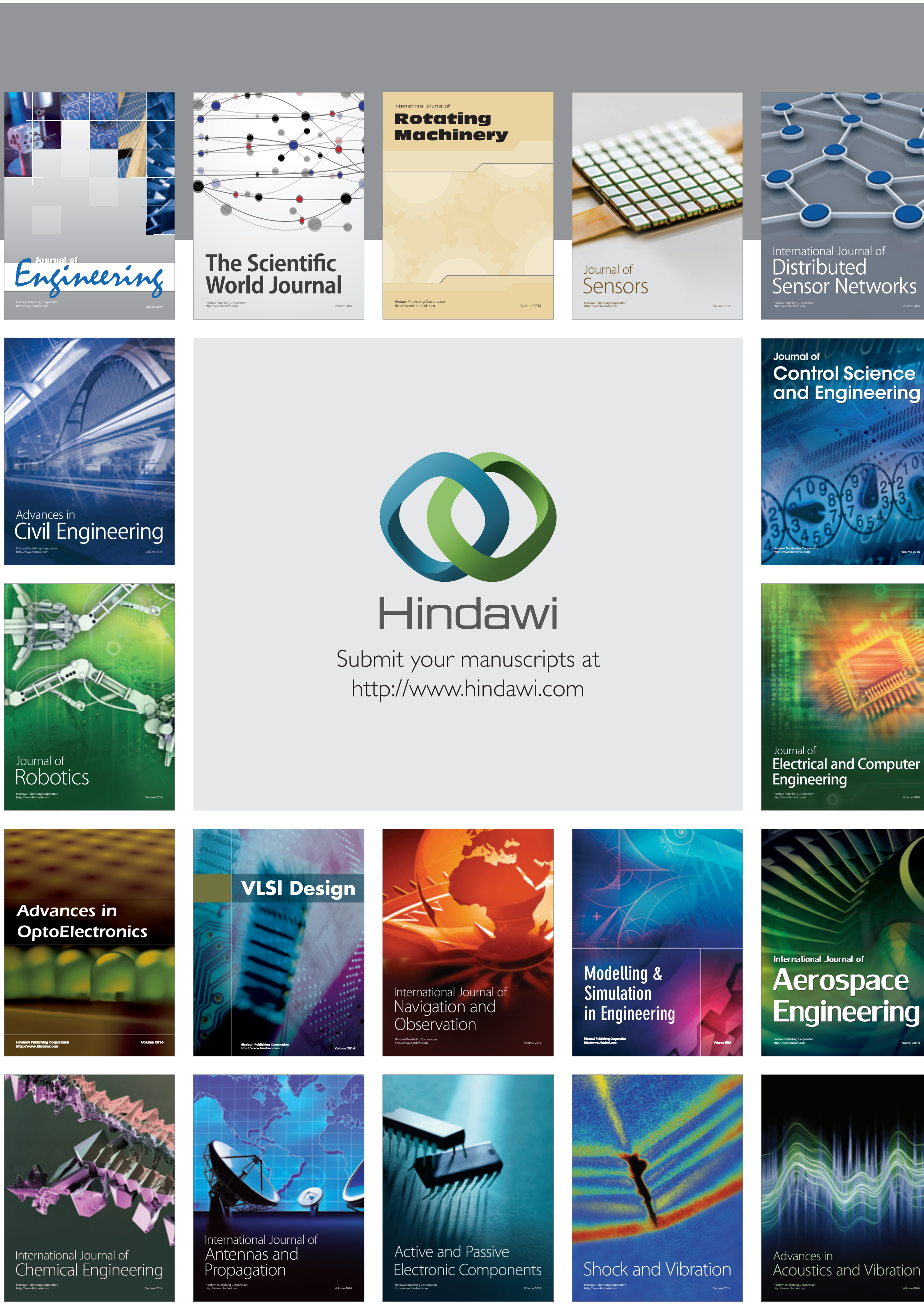
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