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Research Article

Decentralized Control for Large-Scale Interconnected Nonlinear Systems Based on Barrier Lyapunov Function

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We present a novel decentralized tracking control scheme for a class of large-scale nonlinear systems with partial state constraints. For the first time, backstepping design with the newly proposed BLF is incorporated to effectively deal with the control problem of nonlinear systems with interconnected constraints. To prevent the states of each subsystem from violating the constraints, we employ a special barrier Lyapunov function (BLF), which grows to infinity whenever its argument approaches some finite limits. By ensuring boundedness of the barrier Lyapunov function in the closed loop, we ensure that those limits are not transgressed. Asymptotic tracking is achieved without violation of the constraints, and all closed-loop signals remain bounded. In the end, an illustrative example is presented to demonstrate the performance of the proposed control.

1. Introduction

The control problem of constrained systems is by far one of the most common challenges faced by control engineers. In practical physical systems, constraints are ubiquitous, such as physical stoppages, saturation, and performance and safety specifications. Violation of the constraints during operation may result in performance degradation, hazards, or system damage. Driven by practical needs and theoretical challenges, the rigorous handling of constraints in control design has become an important research topic in recent decades. Various techniques have been developed to solve the constrained control problems, namely, override control [1], linear model predictive control [2] for linear systems, invariance control [3], nonlinear reference governor [4], and nonlinear model predictive control [5] for nonlinear systems.

Integrator backstepping design was developed in [6] for nonlinear systems with triangular structures and has become a very popular nonlinear control design. This control design can overcome some restrictions that traditional Lyapunov-based design faced, such as matching conditions, extended matching conditions, or growth conditions, and has been applied to a large class of systems. The concept of control Lyapunov functions (CLFs) was used to construct stable controllers, and for simplicity, quadratic Lyapunov functions (QLFs) of the form $V(z) = (1/2)z^T Pz$ were often proposed as CLF candidates. However, it is not until recently that insights into structural properties of stabilizable constrained systems were provided. In [7], backstepping design was introduced to stabilize a class of pure strict feedback nonlinear systems. Full states constraints but system output were considered and systematic control method was developed based on choosing appropriate symmetrical BLF. The reason why this method can really work lies in the fact that the BLF will approach infinity whenever error signals z_i approach k_{b_i} and thus the constraint violation is avoided. Following this result, the control problem of output constrained system with state and output feedback was investigated in [8], where not symmetrical BLF but asymmetrical one was used to add flexibility in control design and relaxes the restriction on initial conditions; then system with time-varying output constrains was considered in [9, 10], as well as multiple BLFs under a switching scheme [11]. Other works [12–16] extended this systematic design to systems with full state constrains, adaptive neural control, indirect adaptive fuzzy control, the adaptive control for output-feedback constraint systems, and the nonlinear switched systems. A practical electrostatic microactuator system control was presented in [15, 17] within this framework.

However, despite the maturity of BLF in dealing with SISO systems, the more challenging control problem of constrained large-scale systems has received little attention, for the reason that the constrained states are distributed in the subsystems. In this paper, we tackle the tracking problem of large-scale nonlinear system with partial states constrains, motivated by the fact that full state constrains systems and output constrained systems mentioned before are subset of it. By using a BLF, new decentralized tracking control design is presented based on backstepping methodology, but more efforts are made to deal with the constrained states of the subsystems. The stability analysis shows that all closed-loop signals are ensured to be bounded, and the output tracking errors can converge to zero asymptotically. Simulation results demonstrate the effectiveness of the proposed approach.

2. Problem Statement

Consider a partial constrained large-scale system comprised of N subsystems interconnected by their outputs. The ith subsystem Σ_i $(i = 1, \dots, N)$ is given by

$$\begin{aligned}
\dot{x}_{i,1} &= x_{i,2} + f_{i,1}(x_{i,1}) + g_{i,1}(y) \\
&\vdots \\
\dot{x}_{i,p_i-1} &= x_{i,p_i} + f_{i,p_i-1}(\overline{x}_{i,p_i-1}) + g_{i,p_i-1}(y) \\
\dot{x}_{i,p_i} &= x_{i,p_i+1} + f_{i,p_i}(\overline{x}_{i,p_i}) + g_{i,p_i}(y) \\
&\vdots \\
\dot{x}_{i,q_i} &= x_{i,q_i+1} + f_{i,q_i}(\overline{x}_{i,q_i}) + g_{i,q_i}(y) \\
\dot{x}_{i,q_i+1} &= x_{i,q_i+2} + f_{i,q_i+1}(\overline{x}_{i,q_i+1}) + g_{i,q_i+1}(y) \\
&\vdots \\
\dot{x}_{i,n_i} &= u_i + f_{i,n_i}(x_i) + g_{i,n_i}(y) \\
y_i &= x_{i,1},
\end{aligned} \tag{1}$$

where $x_i = [x_{i,1}, \dots, x_{i,n_i}]^T \in R^{n_i}$, $u_i \in R$, and $y_i \in R$ are the state vector, control input, and system output, respectively, $\overline{x}_{i,j} = [x_{i,1}, \dots, x_{i,j}]^T$, $y = [y_1, \dots, y_N]^T$, $f_{i,j}$ are smooth nonlinear functions, and $g_{i,j}$ are smooth nonlinear interconnections, where $j = 1, \dots, n_i$; $x_{i,p_i}, \dots, x_{i,q_i}$ are constrained states of ith subsystem, where $1 \le p_i \le q_i \le n_i$ are constants. The control objective is to design decentralized controller u_i such that the system outputs $y_{i,1}$ can track a desired trajectory $y_{i,r}$ while ensuring that all closed-loop signals are bounded and that the state constrains of $x_{i,p_i}, \dots, x_{i,q_i}$ are not violated.

For $j = p_i, \dots, q_i$, the constraints $k_{i,j}$ are specified so that $x_{i,j}$ is not driven out of the interval $|x_{i,j}| < k_{i,j}$. However, when $p_i \ne 1$, then, for $j = 1, \dots, p_i - 1$, the constraints $k_{i,j}$ are not explicitly specified as problem requirement. To keep the real constraints $k_{i,j}$, $j = p_i, \dots, q_i$, never violated, the constraints $k_{i,j}$, $j = 1, \dots, p_i - 1$, are artificially imposed as part of the design procedure.

Assumption 1. The reference signals $y_{i,r}(t)$ and their first n_i derivatives are piecewise continuous and bounded in the interval $(-\infty, \infty)$, and the bounds of $y_{i,r}(t)$ and $y_{i,r}^{(j)}$ are specified as

$$\left|y_{i,r}(t)\right| \le A_{i,0} < k_{i,1}, \qquad \left|y_{i,r}^{(j)}(t)\right| \le Y_{i,j} \tag{2}$$

for all $t \ge 0$, where $j = 1, 2, \dots, q_i - 1$.

Remark 2. The large-scale nonlinear systems considered in this paper are more complicated than output constrains [8] and full state constrains [12] because of the interconnections. Moreover, when $p_i = q_i = 1$ with $N = 1$, system (1) is equivalent to [8], and when $p_i = 1, q_i = n_i$ with $N = 1$, system (1) is equivalent to [12], so the systems discussed in [8, 12] are just subset of system (1).

3. Controller Design

In this section, adaptive decentralized controller design for system (1) is presented. Instead of QLF used in [6], BLF is introduced to tackle the constrain states.

Define the error variables $z_i = [z_{i,1}, \dots, z_{i,n_i}]^T$ and a change of coordinates:

$$z_{i,j} = x_{i,j} - \alpha_{i,j-1}, \quad 1 \le j \le n_i, \tag{3}$$

where $\alpha_{i,j-1}$ is the stabilizing function to be designed and $\alpha_{i,0} = y_{i,r}$.

Step 1. To keep the constraint $k_{i,1}$ not violated, we employ the following BLF in this design procedure:

$$V_{i,1} = \frac{1}{2} \log \frac{\kappa_{i,1}^2}{\kappa_{i,1}^2 - z_{i,1}^2}, \tag{4}$$

where

$$\kappa_{i,1} = k_{i,1} - A_{i,0}. \tag{5}$$

It can be shown that $V_{i,1}$ is positive definite and continuously differentiable in the open set $|z_{i,1}| < \kappa_{i,1}$, and thus it is a valid Lyapunov function candidate. The derivative of $V_{i,1}$ along the closed-loop trajectories is

$$\begin{aligned}
\dot{V}_{i,1} &= \frac{z_{i,1}\dot{z}_{i,1}}{\kappa_{i,1}^2 - z_{i,1}^2} \\
&= \frac{z_{i,1}(x_{i,2} + f_{i,1} + g_{i,1} - \dot{\alpha}_{i,0})}{\kappa_{i,1}^2 - z_{i,1}^2} \\
&= \frac{z_{i,1}(z_{i,2} + \alpha_{i,1} + f_{i,1} + g_{i,1} - \dot{\alpha}_{i,0})}{\kappa_{i,1}^2 - z_{i,1}^2}.
\end{aligned} \tag{6}$$

Design the stabilizing function $\alpha_{i,1}$ as

$$\alpha_{i,1} = -\left(\kappa_{i,1}^2 - z_{i,1}^2\right) c_{i,1} z_{i,1} - f_{i,1} - g_{i,1} + \dot{\alpha}_{i,0}, \quad (7)$$

where $c_{i,1} > 0$ is constant, so that

$$\dot{z}_{i,1} = z_{i,2} - \left(\kappa_{i,1}^2 - z_{i,1}^2\right) c_{i,1} z_{i,1}. \quad (8)$$

The derivative of $V_{i,1}$ along (8) is

$$\dot{V}_{i,1} = -c_{i,1} z_{i,1}^2 + \frac{1}{\kappa_{i,1}^2 - z_{i,1}^2} z_{i,1} z_{i,2}. \quad (9)$$

Step j $(2 \le j \le q_i)$. To design a control that does not drive $x_{i,j}$ out of the interval $|x_{i,j}| < k_{i,j}$, we choose the following BLF candidate:

$$V_{i,j} = V_{i,j-1} + \frac{1}{2} \log \frac{\kappa_{i,j}^2}{\kappa_{i,j}^2 - z_{i,j}^2}. \quad (10)$$

The derivative of $V_{i,j}$ along the closed-loop trajectories is

$$\begin{aligned} \dot{V}_{i,j} &= \dot{V}_{i,j-1} + \frac{z_{i,j} \dot{z}_{i,j}}{\kappa_{i,j}^2 - z_{i,j}^2} \\ &= \dot{V}_{i,j-1} + \frac{z_{i,j} \left(z_{i,j+1} + \alpha_{i,j} + f_{i,j} + g_{i,j} - \dot{\alpha}_{i,j-1}\right)}{\kappa_{i,j}^2 - z_{i,j}^2}. \end{aligned} \quad (11)$$

Design the stabilizing function $\alpha_{i,j}$ as

$$\begin{aligned} \alpha_{i,j} = &- \frac{\kappa_{i,j}^2 - z_{i,j}^2}{\kappa_{i,j-1}^2 - z_{i,j-1}^2} z_{i,j-1} - \left(\kappa_{i,j}^2 - z_{i,j}^2\right) c_{i,j} z_{i,j} \\ &- f_{i,j} - g_{i,j} + \dot{\alpha}_{i,j-1}, \end{aligned} \quad (12)$$

where $c_{i,j} > 0$ is constant, so that

$$\begin{aligned} \dot{z}_{i,j} = &- \frac{\kappa_{i,j}^2 - z_{i,j}^2}{\kappa_{i,j-1}^2 - z_{i,j-1}^2} z_{i,j-1} \\ &- \left(\kappa_{i,j}^2 - z_{i,j}^2\right) c_{i,j} z_{i,j} + z_{i,j+1}. \end{aligned} \quad (13)$$

The derivative of $V_{i,j}$ along (13) is

$$\dot{V}_{i,j} = -\sum_{l=1}^{j} c_{i,l} z_{i,l}^2 + \frac{1}{\kappa_{i,j}^2 - z_{i,j}^2} z_{i,j} z_{i,j+1}. \quad (14)$$

Step $q_i + 1$. To eliminate the residual coupling term of the previous step, a_{i,q_i+1} must be designed alone. The following Lyapunov candidate was chosen:

$$V_{i,q_i+1} = V_{i,q_i} + \frac{1}{2} z_{i,q_i+1}^2. \quad (15)$$

The derivative of V_{i,q_i+1} along the closed trajectories is

$$\begin{aligned} \dot{V}_{i,q_i+1} &= \dot{V}_{i,q_i} + z_{i,q_i+1} \dot{z}_{i,q_i+1} \\ &= -\sum_{l=1}^{q_i} c_{i,l} z_{i,l}^2 + \frac{1}{\kappa_{i,q_i}^2 - z_{i,q_i}^2} z_{i,q_i} z_{i,q_i+1} \\ &\quad + z_{i,q_i+1} \left(z_{i,q_i+2} + \alpha_{i,q_i+1} \right. \\ &\quad \left. + f_{i,q_i+1} + g_{i,q_i+1} - \dot{\alpha}_{i,q_i}\right). \end{aligned} \quad (16)$$

Design the stabilizing function α_{i,q_i+1} as

$$\begin{aligned} \alpha_{i,q_i+1} = &- \frac{1}{\left(\kappa_{i,q_i}^2 - z_{i,q_i}^2\right)} z_{i,q_i} - c_{i,q_i+1} z_{i,q_i+1} \\ &- f_{i,q_i+1} - g_{i,q_i+1} + \dot{\alpha}_{i,q_i}, \end{aligned} \quad (17)$$

where $c_{i,q_i+1} > 0$ is constant, so that

$$\begin{aligned} \dot{z}_{i,q_i+1} = &- \frac{z_{i,q_i}}{\left(\kappa_{i,q_i}^2 - z_{i,q_i}^2\right)} \\ &- c_{i,q_i+1} z_{i,q_i+1} + z_{i,q_i+2}. \end{aligned} \quad (18)$$

The derivative of V_{i,q_i+1} along (18) is

$$\dot{V}_{i,q_i+1} = -\sum_{j=1}^{q_i+1} c_{i,j} z_{i,j}^2 + z_{i,q_i+1} z_{i,q_i+2}. \quad (19)$$

Step ι $(q_i + 2 \le \iota \le n_i)$. The design procedure is similar to traditional backstepping design procedure, and we give the results directly:

$$V_{i,\iota} = V_{i,\iota-1} + \frac{1}{2} z_{i,\iota}^2, \quad (20)$$

$$\alpha_{i,\iota} = -z_{i,\iota-1} - c_{i,\iota} z_{i,\iota} - f_{i,\iota} - g_{i,\iota} + \dot{\alpha}_{i,\iota-1}, \quad (21)$$

$$\dot{z}_{i,\iota} = -z_{i,\iota-1} - c_{i,\iota} z_{i,\iota} + z_{i,\iota+1}, \quad (22)$$

$$\dot{V}_{i,\iota} = -\sum_{l=1}^{\iota} c_{i,l} z_{i,l}^2 + z_{i,\iota} z_{i,\iota+1}, \quad (23)$$

where $c_{i,\iota} > 0$ is constant, $z_{i,n_i+1} := 0$, and $u := \alpha_{i,n_i}$. Then we have

$$\dot{z}_{i,n_i} = -z_{i,n_i-1} - c_{i,n_i} z_{i,n_i}, \quad (24)$$

$$u = -z_{i,n_i-1} - c_{i,n_i} z_{i,n_i} - f_{i,n_i} - g_{i,n_i} + \dot{\alpha}_{i,n_i-1}, \quad (25)$$

$$\dot{V}_{i,n_i} = -\sum_{j=1}^{n_i} c_{i,j} z_{i,j}^2. \quad (26)$$

4. Stability Analysis

Theorem 3. *Consider the closed-loop systems (1), (8), (13), (18), (22), (24), and (25) under Assumption 1. Denote by* $A_{i,j}$ *an upper bound for* $\alpha_{i,j}$ *in the compact set* $\Omega_{i,j}$*, where* $1 \le j \le q_i$*; that is,*

$$A_{i,j} \ge \sup \left| \alpha_{i,j} \left(\overline{x}_{i,j}, \overline{z}_{i,j}, \overline{y}_{i,r}^{(j)} : \overline{c}_{i,j}, \overline{\kappa}_{i,j}\right) \right|, \quad (27)$$

where $\overline{\kappa}_{i,j} = [\kappa_{i,1}, \ldots, \kappa_{i,j}]^T$*,* $\overline{c}_{i,j} = [c_{i,1}, \ldots, c_{i,j}]^T$*, and* $\overline{y}_{i,r}^{(j)} = [y_{i,r}^{(0)}, \ldots, y_{i,r}^{(j)}]^T$*. So* $\alpha_{i,j}$ *is parameterized by* $\overline{c}_{i,j}, \overline{\kappa}_{i,j}$*. The compact set* $\Omega_{i,j}$ *is defined by*

$$\begin{aligned} \Omega_{i,j} := \Big\{ &\overline{x}_{i,j}, \overline{z}_{i,j}, \overline{y}_{i,j}^{(j)} : \\ &\left|x_{i,\varrho}\right| \le D_{z_{i,\varrho}} + A_{i,\varrho-1}, \\ &\left|z_{i,\varrho}\right| \le D_{z_{i,\varrho}}, \left|y_{i,r}^{(\varrho)}\right| \le Y_{i,\varrho} \Big\} \end{aligned} \quad (28)$$

with $1 \le \varrho \le j$, *where*

$$D_{z_{i,\varrho}} := \kappa_\varrho \sqrt{1 - e^{-2V(0)}}. \tag{29}$$

Define $z_i = [z_{i,j}]^T$ *and* $j = 1, \ldots, N_i$. *Given the constraints* $\kappa_{i,1}, \ldots, \kappa_{i,q_i}$, *and* $k_{i,j+1} > A_{i,j} + \kappa_{i,j+1}$, $j = 1, \ldots, q_i - 1$, *when*

$$z_i(0) \in \Omega_{z_i(0)} := \left\{ z_i : \left| z_{i,j} \right| < \kappa_{i,j},\ 1 \le j \le q_i \right\}, \tag{30}$$

then the following properties hold.

*P*1. *The signals* $z_{i,j}(t)$ *remain in the set* $\Omega_{z_i} := \{z_i : |z_{i,j}| \le D_{z_{i,j}},\ \|z_{i,q_i+1:n_i}\| \le \sqrt{2V(0)}\}$ *for* $\forall t > 0$, *where* $j = 1, 2, \ldots, q_i$ *and* $z_{i,q_i+1:n_i} := [z_{i,q_i+1}, \ldots, z_{i,n_i}]^T$.

*P*2. *The states* $x_{i,j}$ *remain in the set* $\Omega_{x_i} := \{x_i : |x_{i,j}| \le D_{z_{i,j}} + A_{i,j-1} \le k_{i,j},\ j = p_i, \ldots, q_i\}$, *for* $\forall t > 0$; *that is, the state constraints are never violated.*

*P*3. *All closed-loop signals are bounded.*

*P*4. *The output tracking errors* $z_{i,1}(t)$ *converge to zero asymptotically; that is,* $y_i(t) \to y_{i,r}(t)$ *as* $t \to \infty$.

Proof. The properties $P1 \sim P4$ will be proved in sequence as follows.

$P1$. Define Lyapunov function as

$$V = \sum_{i=1}^{N} V_{i,n_i}. \tag{31}$$

The time derivative of V is

$$\dot{V} = -\sum_{i=1}^{N} \sum_{j=1}^{n_i} c_{i,j} z_{i,j}^2. \tag{32}$$

From (32), it is clear that $V(t) \le V(0)$, so

$$\sum_{i=1}^{N} \left\{ \sum_{j=1}^{q_i} \frac{1}{2} \log \frac{\kappa_{i,j}^2}{\kappa_{i,j}^2 - z_{i,j}^2(t)} + \sum_{j=q_i+1}^{n_i} \frac{1}{2} z_{i,j}^2(t) \right\} \le V(0). \tag{33}$$

When $j = 1, \ldots, q_i$, we have that

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{j=1}^{q_i} \frac{1}{2} \log \frac{\kappa_i^2}{\kappa_i^2 - z_{i,j}^2(t)} \le V(0) \\ &\Longrightarrow \frac{1}{2} \log \frac{\kappa_i^2}{\kappa_i^2 - z_{i,j}^2(t)} \le V(0) \\ &\Longrightarrow z_{i,j} \le \kappa_i \sqrt{1 - e^{-2V(0)}} = D_{z_{i,j}}. \end{aligned} \tag{34}$$

On the other hand, when $j = q_i + 1, \ldots, n_i$, we have that

$$\begin{aligned} &\sum_{i=1}^{N} \sum_{j=q_i+1}^{n_i} \frac{1}{2} z_{i,j}^2(t) \le V(0) \\ &\Longrightarrow \left\| z_{i,q_i+1:n_i} \right\| \le \sqrt{2V(0)}. \end{aligned} \tag{35}$$

Hence, $z_{i,j}(t)$ remains in the compact set Ω_z for $\forall t > 0$.

$P2$. From (3), we have that

$$x_{i,1} = z_{i,1} + \alpha_{i,0} \tag{36}$$

with $a_{i,0} = y_{i,r}$. From (28), (29), and Assumption 1, it is clear that

$$x_{i,1} \le D_{z_{i,1}} + A_{i,0} \le \kappa_{i,1} + A_{i,0} \le k_{i,1}. \tag{37}$$

From (28), we know that $|x_{i,1}| \le D_{z_{i,1}} + A_{i,0}$, $|z_{i,1}| \le D_{z_{i,1}}$, and $|y_{i,r}^{(1)}| \le Y_{i,1}$, so that the stabilizing function $\alpha_{i,1}$ is bounded. Then we can conclude that an upper bound $A_{i,1}$ can be found from (27). Similar to (36) and (37) and from (3), (28), and (29) and $k_{i,j+1} > A_{i,j} + \kappa_{i,j+1}$, we have

$$\left| x_{i,2} \right| = z_{i,2} + \alpha_{i,1} \le k_{i,2} \tag{38}$$

Progressively, after verifying $|x_{i,j-1}| \le D_{z_{i,j-1}} + A_{i,j-2}$, $|z_{i,j-1}| \le D_{z_{i,j-1}}$, and $|y_{i,r}^{j-1}| \le Y_{i,j}$ with $j = 2, \ldots, q_i$, we can conclude that the stabilizing function $|\alpha_{i,j-1}|$ is bounded by $A_{i,j-1}$ from (27). Then it is clear that $|x_{i,j}| = z_{i,j} + \alpha_{i,j-1} \le k_{i,j}$.

$P3$. For $\alpha_{i,j}, x_{i,j}, z_{i,j}$ $(j = 1, \ldots, q_i)$ are all bounded from $P1$ and $P2$, and together with the fact that $\|z_{i,q_i+1:n_i}\| \le \sqrt{2V(0)}$, we can progressively show that the remaining $\alpha_{i,j}$ and $x_{i,j}$ are also bounded for $j = q_i + 1, \ldots, n_i$. Then it is straightforward to show that the control u is bounded. Thus, all closed-loop signals are bounded.

$P4$. From (32), we have

$$\dot{V} \le -c \sum_{i=1}^{N} z_i^T z_i, \tag{39}$$

where $c = \min\{c_{i,j}\}$, $i = 1, \ldots, N$, $j = 1, \ldots, n_i$. By LaSalle-Yoshizawa theorem [6, p.24 Theorem 2.1], we know that $\log(\kappa_{i,j}^2 / (\kappa_{i,j}^2 - z_{i,j}^2)) \to 0$, $j = 1, \ldots, q_i$, and $z_{i,j} \to 0$, $j = q_i + 1, \ldots, n_i$. Then we can directly get that $z_{i,1}(t) \to 0$; that is, $y_i(t) \to y_{i,r}(t)$ as $t \to \infty$. □

5. Simulation Results

In this section, we consider the following large-scale system consisting of two second-order subsystems:

$$\begin{aligned} \dot{x}_{1,1} &= x_{1,2} + \sin(x_{1,1}) + y_2 \cos(y_1), \\ \dot{x}_{1,2} &= u_1 + x_{1,1} + x_{1,2} + y_1 + y_2, \\ y_1 &= x_{1,1}, \\ \dot{x}_{2,1} &= x_{2,2} + 0.1 x_{2,1}^3 + \sin(y_1 y_2), \\ \dot{x}_{2,2} &= u_2 + x_{2,1} x_{2,2} + \tanh(y_1 y_2), \\ y_2 &= x_{2,1}, \end{aligned} \tag{40}$$

where $y_1 = x_{1,1}$, $y_2 = x_{2,1}$, and $x_{2,2}$ are required that $|y_1| \le k_{1,1} = 1.2$, $|y_2| \le k_{2,1} = 1.2$, and $|x_{2,2}| \le k_{2,2} = 1.6$. The reference signals are $y_{1,r}(t) = 0.5(\sin(t) + \sin(0.5t))$ and $y_{2,r}(t) = \sin(0.5t)$. As far as we know, the existing decentralized control approaches cannot be applied to the system $\Sigma_1 - \Sigma_2$ because of the existing output and state constraints.

Based on the control scheme proposed in this paper, we have

$$\begin{aligned}\alpha_{1,1} &= -\left(\kappa_{1,1}^2 - z_{1,1}^2\right) c_{1,1} z_{1,1} \\ &\quad - \sin\left(x_{1,1}\right) - y_2 \cos\left(y_1\right) + \dot{y}_{1,r}, \\ \alpha_{2,1} &= -\left(\kappa_{2,1}^2 - z_{2,1}^2\right) c_{2,1} z_{2,1} \\ &\quad - 0.1 x_{2,1}^3 - \tanh\left(y_1 y_2\right) + \dot{y}_{2,r},\end{aligned} \tag{41}$$

and the control input u can be designed as

$$\begin{aligned}u_1 &= -\frac{1}{\left(\kappa_{1,1}^2 - z_{1,1}^2\right)} z_{1,1} - c_{1,2} z_{1,2} \\ &\quad - \left(x_{1,1} + x_{1,2}\right) - \left(y_1 + y_2\right) + \dot{\alpha}_{1,1}, \\ u_2 &= -\frac{\left(\kappa_{2,2}^2 - z_{2,2}^2\right)}{\left(\kappa_{2,1}^2 - z_{2,1}^2\right)} z_{2,1} - \left(\kappa_{2,2}^2 - z_{2,2}^2\right) c_{2,2} z_{2,2} \\ &\quad - \left(x_{2,1} x_{2,2}\right) - \tanh\left(y_1 y_2\right) + \dot{\alpha}_{2,1},\end{aligned} \tag{42}$$

where the design parameters are chosen as $\kappa_{1,1} = 0.3$, $\kappa_{2,1} = 0.2$, $\kappa_{2,2} = 0.8$, and $c_{1,1} = c_{1,2} = c_{2,1} = c_{2,2} = 2$.

The tracking performances are shown in Figures 1 and 2, the control performances of constrained state $x_{2,2}$ and the unconstrained state $x_{1,2}$ are plotted in Figure 3, and the control input signals are shown in Figure 4.

From Figures 1 and 2, we can see that the interconnected output signals y_1 and y_2 can track the reference signals $y_{1,r}$ and $y_{2,r}$ asymptotically, while the constraints $|y_1| \leq k_{1,1} = 1.2$ and $|y_2| \leq k_{2,1} = 1.2$ are not violated. From Figure 3, we can see that the constrained state $x_{2,2}$, which has never broken the constraint $|x_{2,2}| \leq k_{2,2} = 1.6$, together with the unconstrained state $x_{1,2}$, is bounded in the closed loop. The simulation results have shown that although the partial constraint systems Σ_1 and Σ_2 interconnected with each other through their constrained outputs, the decentralized controller proposed in this paper can achieve good control performance, which further verifies the feasibility of our control scheme.

6. Conclusion

The problem of tracking control for a class of interconnected large-scale systems with partial state constraints has been considered. Such systems are very common in practice due to physical/performance limitations. The main contribution of this paper is the first extension of the BLF-based backstepping control methodology to interconnected large-scale systems with distributed constrained states. Future research will focus on extending the proposed approach to a more general class of nonlinear systems.

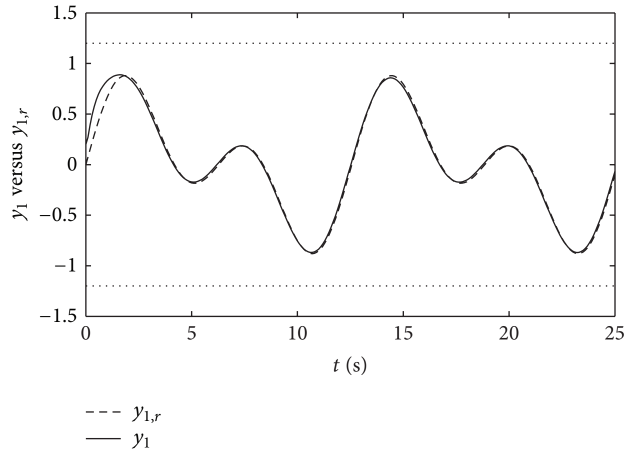

Figure 1: Tracking performance of subsystem 1.

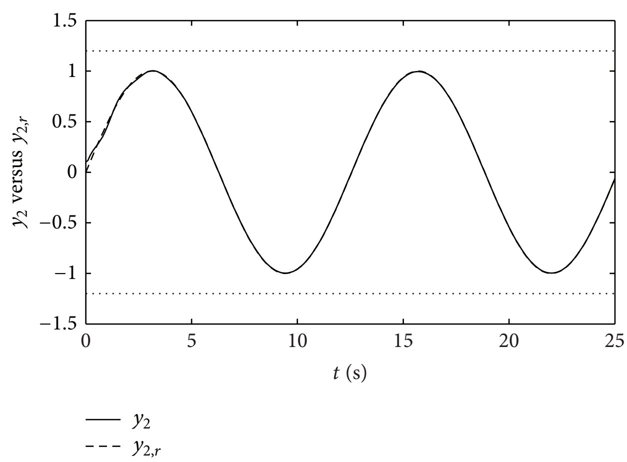

Figure 2: Tracking performance of subsystem 2.

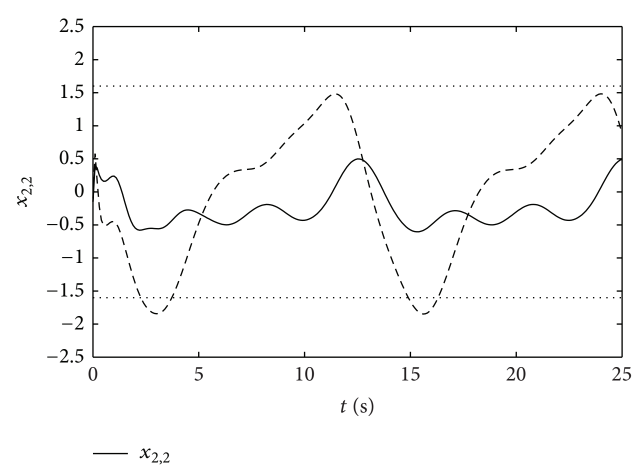

Figure 3: State $x_{1,2}$ and constrained state $x_{2,2}$.

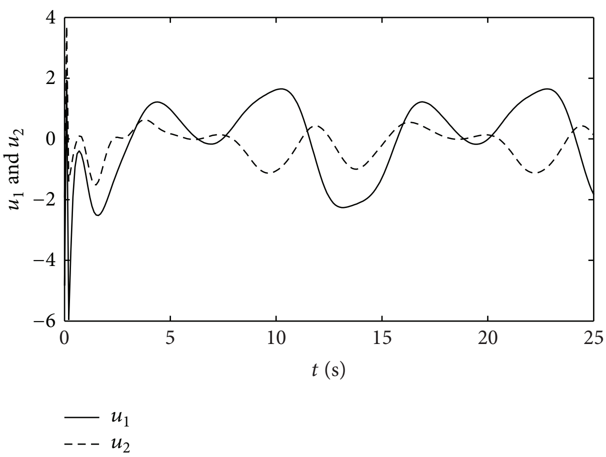

Figure 4: Control efforts.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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