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Research Article

Decentralized H_∞ Control for Large-Scale Systems with Uncertain Missing Measurements Probabilities

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For large-scale systems which are modeled as interconnection of N networked control systems with uncertain missing measurements probabilities, a decentralized state feedback H_∞ controller design is considered in this paper. The occurrence of missing measurements is assumed to be a Bernoulli random binary switching sequence with an unknown conditional probability distribution in an interval. A state feedback H_∞ controller is designed in terms of linear matrix inequalities to make closed-loop system exponentially mean square stable and a prescribed H_∞ performance is guaranteed. Sufficient conditions are derived for the existence of such controller. A numerical example is also provided to demonstrate the validity of the proposed design approach.

1. Introduction

With the advances in network technology, more and more control systems have appeared whose feedback control loop is based on a network. This kind of control systems are called networked control systems (NCSs) [1–4]. Owing to the data communication errors in network and the temporarily disabled sensor, missing measurements and transmission time delay usually occur, which can degrade the system performance and even make the system unstable. There have been significant research efforts on the design of controllers and filters for system with missing measurements. There are two main approaches to handle missing measurements. One approach is to replace the missing measurements with an estimated value [5], and the other approach is to view missing measurements as "zero" [6], such as Markov chains [7] and Bernoulli binary switching sequence [8–13]. Fault detection is considered for NCS with missing measurements probabilities being known in [8]. Furthermore, still fault detection is considered for NCS with delays and missing measurements in [9]. In [10], the robust H_∞ control problem is investigated for stochastic uncertain discrete time-delay systems with missing measurements. In [11], an observer-based H_∞ controller is designed for NCS with missing measurements, where the missing measurements are assumed to obey the Bernoulli random binary distribution. The controlled systems in references [8–11] are linear discrete systems and the missing measurements probabilities are known constants. A robust fault detection method is proposed for NCS with uncertain missing measurements probabilities in [12].

In most existing results, the controlled NCS is usually treated as isolated one and the missing measurement probability is known [13–18]. However, on one hand, in practice the missing measurements probability usually keeps varying and cannot be measured exactly. On the other hand, in many practical applications, controlled systems are large-scale systems which are composed of discrete-time NCSs. Each discrete-time NCS is influenced not only by missing measurements, but also by interconnection terms generated by the other NCSs. At the same time, due to the dispersion of some large-scale systems such as power systems, it is impossible to feed back all states of whole large-scale systems to design the controller. So the decentralized controller that only feed back local information is more practical. In [19], for large-scale systems composed by N discrete-time NCSs with missing measurements, where the missing measurements are modeled as Bernoulli distribution with a known conditional probability, the H_∞ control problem is considered using linear matrix inequality (LMI) method. In summary, to study the decentralized control for large-scale systems composed by discrete-time NCSs with uncertain missing measurements probability is of important significance. But as far as the authors know, such research is seldom to be found.

In this paper, the decentralized H_∞ control problem is studied for linear discrete-time large-scale systems composed of N discrete-time NCSs with missing measurements, where the occurrence of missing measurements is assumed to be a Bernoulli random binary switching sequence with an unknown conditional probability distribution that is assumed to be in an interval. Decentralized stabilization H_∞ controller design is proposed for such systems. Sufficient conditions are established by means of LMI, which can be solved conveniently by MATLAB LMI toolbox.

2. Problem Formulation

Consider the linear large-scale systems composed of N discrete-time NCSs with missing measurements. The ith NCSs are assumed to be of the form

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i u_i(k) + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) \\ &\quad + E_i w_i(k), \\ y_i(k) &= x_i(k), \\ z_i(k) &= C_i(k) x_i(k), \quad i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_i(k) \in R^{n_i}$, $u_i(k) \in R^{m_i}$, $z_i(k) \in R^{P_i}$, $y_i(k) \in R^{q_i}$, and $w_i(k) \in R^{r_i}$ denote the state vector, the control input, the controlled output, the measuring output, and the disturbance of ith subsystem, respectively; $w_i(k) \in l_2[0,\infty)$; A_i, B_i, C_i, and E_i are known real matrices with appropriate dimensions; $G_{ij} \in R^{n_i \times n_j}$ is the interconnection between the jth subsystem and ith subsystem.

The measurements with packet loss are described by

$$\hat{x}_i(k) = r_i(k) x_i(k), \tag{2}$$

where $\hat{x}_i(k) \in R^{ni}$ is the actual measured states, $r_i(k) \in R$ is a Bernoulli distributed white sequence taking the values of 0 and 1 with certain probability

$$\begin{aligned} \text{Prob}\{r_i(k) = 1\} &= E\{r_i(k)\} = \bar{r}_i, \\ \text{Prob}\{r_i(k) = 0\} &= 1 - E\{r_i(k)\} = 1 - \bar{r}_i, \end{aligned} \tag{3}$$

and the unknown positive scalar $\bar{r}_i$: $0 < \bar{r}_i < 1$ means the occurrence probability of the missing measurements. Without loss of generality, we assume

$$\bar{r}_i \in [r_{i\min} \quad r_{i\max}], \tag{4}$$

where $r_{i\max}$ and $r_{i\min}$ are the upper limit and lower limit of the probability, respectively, and satisfy

$$0 < r_{i\min} \leq r_{i\max} \leq 1. \tag{5}$$

Choose $r_{0i} = (r_{i\min} + r_{i\max})/2$ and $r_{1i} = (r_{i\max} - r_{i\min})/2$; we can obtain another expression about $\bar{r}_i$ as follows:

$$\begin{aligned} \bar{r}_i &= r_{0i} + r_{1i}\Delta r_i, \\ |\Delta r_i| &\leq 1. \end{aligned} \tag{6}$$

Remark 1. The missing measurements probability usually keeps varying and cannot be measured exactly. However, it can be estimated by a value region shown as (4), which is much more practical. In (5), $r_{i\max} = 1$ means that no measurement is lost and $r_{i\min} = 0$ means that measurements are lost completely.

For system (1), the control input can be chosen as

$$u_i(k) = -K_i \hat{x}_i(k) = -r_i(k) K_i x_i(k), \tag{7}$$

where K_i, $i = 1, \ldots, N$, are gain matrices to be designed. Submit (7) into (1); we can get the following closed-loop system:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) - (r_i(k) - \bar{r}_i) B_i K_i x_i(k) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij} x_j(k) - \bar{r}_i B_i K_i x_i(k) \\ &\quad + E_i w_i(k), \\ y_i(k) &= x_i(k), \\ z_i(k) &= C_i(k) x_i(k). \end{aligned} \tag{8}$$

Definition 2 (see [11]). Closed-loop system (8) with $w(k) = 0$ is said to be exponentially mean-square stable if there exist constants $\kappa > 0$ and $0 < \tau < 1$ such that

$$E\left\{\|x(k)\|^2\right\} < \kappa\tau^\kappa E\left\{\|x(0)\|^2\right\}, \quad \forall x(0) \neq 0, \tag{9}$$

where $x(k) = [x_1^T(k), \ldots, x_N^T(k)]$.

The objective of this paper is to design the state feedback controller (7) for system (1), such that closed-loop system (8) satisfies following requirements:

(1) When $w(k) = 0$, closed-loop system (8) is exponentially mean-square stable.

(2) Under the zero-initial condition, the controlled output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} E\left\{\|z(k)\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} E\left\{\|w(k)\|^2\right\}, \tag{10}$$

where $z(k) = [z_1^T(k), \ldots, z_N^T(k)]^T$, $w(k) = [w_1^T(k), \ldots, w_N^T(k)]^T$, and $\gamma > 0$ is a prescribed scalar.

We first give following useful two lemmas.

Lemma 3 (see [20]). *Let $V(x(k))$ be a Lyapunov functional. If there exist real scalars $\lambda \geq 0$, $\mu > 0$, $\nu > 0$, and $0 < \psi < 1$ such that*

$$\begin{aligned} &\mu \|x(k)\|^2 \leq V(x(k)) \leq \nu \|x(k)\|^2, \\ &E\{V(x(k+1)) \mid x(k)\} - V(x(k)) \\ &\quad \leq \lambda - \psi V(x(k))^2, \end{aligned} \tag{11}$$

then sequence $x(k)$ satisfies

$$E\left\{\|x(k)\|^2\right\} \le \frac{\nu}{\mu}\|x(0)\|^2(1-\psi)^k + \frac{\lambda}{\mu\psi}. \tag{12}$$

Lemma 4 (see [21]). *For any parameter $\xi > 0$ and matrices G, F, and E with appropriate dimensions, if $EE^T \le I$, then*

$$GEF + F^TE^TG^T \le \xi GG^T + \xi^{-1}F^TF. \tag{13}$$

3. Main Results

At first, for the case of system (1) without disturbance, that is, $w(k) = 0$, we have the following two theorems.

Theorem 5. *Closed-loop system (8) with $w(k) = 0$ is exponentially mean-square stable if there exist positive definite matrices $P \in R^{nN\times nN}$ and the controller gain matrices $K \in R^{nN\times nN}$ satisfying*

$$\begin{bmatrix} -P & * & * & * & * \\ PA & -P & * & * & * \\ aPB & 0 & -aP & * & * \\ \xi r_1 PA_0 & 0 & 0 & -\xi P & * \\ PB & 0 & 0 & 0 & -\xi P \end{bmatrix} < 0, \tag{14}$$

where $\xi > 0$ is an arbitrary given constant,

$$\begin{aligned}
a_i &= (1-r_{0i})\,r_{0i},\\
a &= \operatorname{diag}\{a_1, a_2, \ldots, a_N\},\\
r_1 &= \operatorname{diag}\{r_{11}, r_{12}, \ldots, r_{1N}\},\\
P &= \operatorname{diag}\{P_1, P_2, \ldots, P_N\},\\
B &= \operatorname{diag}\{B_1, B_2, \ldots, B_N\},\\
K &= \operatorname{diag}\{K_1, K_2, \ldots, K_N\},\\
A &= \begin{bmatrix} (A_1 - r_{01}B_1K_1) & G_{12} & \cdots & G_{1N} \\ G_{21} & (A_2 - r_{02}B_2K_2) & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & (A_N - r_{0N}B_NK_N) \end{bmatrix},\\
A_0 &= \begin{bmatrix} \left(A_1 - \frac{1}{2}B_1K_1\right) & G_{12} & \cdots & G_{1N} \\ G_{21} & \left(A_2 - \frac{1}{2}B_2K_2\right) & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & \left(A_N - \frac{1}{2}B_NK_N\right) \end{bmatrix}.
\end{aligned} \tag{15}$$

Proof. Consider the following Lyapunov functional:

$$V(x(k)) = \sum_{i=1}^{N} x_i(k)^T P_i x_i(k); \tag{16}$$

when $w(k) = 0$, we have

$$\begin{aligned}
V(x(k+1)) - V(x(k)) &= \sum_{i=1}^{N}\left(A_ix_i(k) - (r_i(k) - \overline{r}_i)B_iK_ix_i(k) + \sum_{\substack{j=1\\ j\ne i}}^{N} G_{ij}x_j(k) - \overline{r}_iB_iK_ix_i(k)\right)^T P_i\left(A_ix_i(k)\right.\\
&\quad \left. - (r_i(k) - \overline{r}_i)B_iK_ix_i(k) + \sum_{\substack{j=1\\ j\ne i}}^{N} G_{ij}x_j(k) - \overline{r}_iB_iK_ix_i(k)\right) - \sum x_i(k)^T P_i x_i(k).
\end{aligned} \tag{17}$$

By virtue of Lemma 4 and $E\{r_i(k) - \overline{r}_i\} = 0$ and $E\{(r_i(k) - \overline{r}_i)^2\} = \beta_i^2 = (1-\overline{r}_i)\overline{r}_i$, we have

$$\begin{aligned}
&E\{V(x(k+1)) \mid V(x(k))\} - V(x(k))\\
&= \sum_{i=1}^{N}\left(A_ix_i(k) + \sum_{\substack{j=1\\ j\ne i}}^{N} G_{ij}x_j(k) - \overline{r}_iB_iK_ix_i(k)\right)^T\\
&\quad \cdot P_i\left(A_ix_i(k) + \sum_{\substack{j=1\\ j\ne i}}^{N} G_{ij}x_j(k) - \overline{r}_iB_iK_ix_i(k)\right)\\
&\quad + \sum_{i=1}^{N}\beta_i^2(B_iK_ix_i(k))^T P_i(B_iK_ix_i(k)) - \sum_{i=1}^{N} x_i(k)^T P_i x_i(k)\\
&= \sum_{i=1}^{N}\left((A_i - r_{0i}B_iK_i)x_i(k) + \sum_{\substack{j=1\\ j\ne i}}^{N} G_{ij}x_j(k)\right)^T\\
&\quad \cdot P_i\left((A_i - r_{0i}B_iK_i)x_i(k) + \sum_{\substack{j=1\\ j\ne i}}^{N} G_{ij}x_j(k)\right)
\end{aligned}$$

$$
\begin{aligned}
&-\sum_{i=1}^{N} r_{1i}\Delta r_i \left(B_iK_ix_i(k)\right)^T \\
&\cdot P_i\left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right) \\
&-\sum_{i=1}^{N} r_{1i}\Delta r_i \left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right)^T \\
&\cdot P_i\left(B_iK_ix_i(k)\right) + \sum_{i=1}^{N}\left(\left(r_{1i}\Delta r_i\right)^2 + \beta_i^2\right) \\
&\cdot \left(B_iK_ix_i(k)\right)^T P_i\left(B_iK_ix_i(k)\right) - \sum_{i=1}^{N} x_i(k)^T P_i x_i(k) \\
&= \sum_{i=1}^{N}\left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right)^T \\
&\cdot P_i\left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right) \\
&-\sum_{i=1}^{N} r_{1i}\Delta r_i \left(B_iK_ix_i(k)\right)^T \\
&\cdot P_i\left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right) \\
&-\sum_{i=1}^{N} r_{1i}\Delta r_i \left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right)^T \\
&\cdot P_i\left(B_iK_ix_i(k)\right) + \sum_{i=1}^{N}\left(a_i + \left(1 - 2r_{0i}\right)r_{1i}\Delta r_i\right) \\
&\cdot \left(B_iK_ix_i(k)\right)^T P_i\left(B_iK_ix_i(k)\right) - \sum_{i=1}^{N} x_i(k)^T P_i x_i(k) \\
&\leq \sum_{i=1}^{N}\left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right)^T \\
&\cdot P_i\left(\left(A_i - r_{0i}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right) \\
&+\xi\sum_{i=1}^{N} {r_{1i}}^2\left(\left(A_i - \frac{1}{2}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right)^T \\
&\cdot P_i\left(\left(A_i - \frac{1}{2}B_iK_i\right)x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k)\right) \\
&+\sum_{i=1}^{N}\left(a_i + \xi^{-1}\right)\left(B_iK_ix_i(k)\right)^T P_i\left(B_iK_ix_i(k)\right) \\
&-\sum_{i=1}^{N} x_i(k)^T P_i x_i(k) \triangleq x(k)^T \theta_1 x(k),
\end{aligned} \tag{18}
$$

where $x(k) = \left[x_1(k)^T \;\; x_2(k)^T \;\; \cdots \;\; x_N(k)^T\right]^T$. By Schur complement, (14) implies $\theta_1 < 0$ and we obtain

$$
\begin{aligned}
&E\left\{V\left(x(k+1)\right) \mid V\left(x(k)\right)\right\} - V\left(x(k)\right) \\
&\quad \leq x(k)^T \theta_1 x(k) \leq -\lambda_{\min}\left(-\theta_1\right)x(k)^T x(k) \\
&\quad < -\alpha x(k)^T x(k),
\end{aligned} \tag{19}
$$

where $0 < \alpha < \min\{\lambda_{\min}(-\theta_1), \lambda_{\max}(P)\}$. Definite $\sigma = \lambda_{\max}(P)$; we get

$$
\begin{aligned}
&E\left\{V\left(x(k+1)\right) \mid V\left(x(k)\right)\right\} - V\left(x(k)\right) \\
&\quad \leq -\alpha x(k)^T x(k) \leq -\frac{\alpha}{\sigma}V\left(x(k)\right) \triangleq -\psi V\left(x(k)\right),
\end{aligned} \tag{20}
$$

where $\psi = \alpha/\sigma \in (0, 1)$.

By Definition 2 and Lemma 3, closed-loop system (8) is exponentially mean-square stable. This completed the proof. □

It should be noted that matrix inequality (14) is not a linear matrix inequality and difficult to be solved. For this, we have following Theorem 6.

Theorem 6. *Closed-loop system (8) with $w(k) = 0$ is exponentially mean-square stable if there exist positive definite matrix M and gain matrix N satisfying the following linear matrix inequality:*

$$
\begin{bmatrix}
-M & * & * & * & * \\
AM & -M & * & * & * \\
aBM & 0 & -aM & * & * \\
\xi r_1 A_0 M & 0 & 0 & -\xi M & * \\
BM & 0 & 0 & 0 & -\xi M
\end{bmatrix} < 0, \tag{21}
$$

where $\xi > 0$ is an arbitrary given constant,

$$M = \operatorname{diag}\{M_1, M_2, \ldots, M_N\} = P^{-1},$$
$$N = \operatorname{diag}\{N_1, N_2, \ldots, N_N\} = KP^{-1},$$
$$AM = \begin{bmatrix} A_1M_1 - r_{01}B_1N_1 & G_{12}M_2 & \cdots & G_{1N}M_N \\ G_{21}M_1 & A_2M_2 - r_{02}B_2N_2 & \cdots & G_{21}M_N \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1}M_1 & G_{N2}M_2 & \cdots & A_NM_N - r_{0N}B_NN_N \end{bmatrix},$$
$$A_0M = \begin{bmatrix} A_1M_1 - \frac{1}{2}B_1N_1 & G_{12}M_2 & \cdots & G_{1N}M_N \\ G_{21}M_1 & A_2M_2 - \frac{1}{2}B_2N_2 & \cdots & G_{21}M_N \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1}M_1 & G_{N2}M_2 & \cdots & A_NM_N - \frac{1}{2}B_NN_N \end{bmatrix}. \tag{22}$$

Proof. Through left-and-right multiplication of (14) by

$$\operatorname{diag}\left\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}\right\}, \tag{23}$$

we can get

$$\begin{bmatrix} -P^{-1} & * & * & * & * \\ AP^{-1} & -P^{-1} & * & * & * \\ aBP^{-1} & 0 & -aP^{-1} & * & * \\ \xi r_1A_0P^{-1} & 0 & 0 & -\xi P^{-1} & * \\ BP^{-1} & 0 & 0 & 0 & -\xi P^{-1} \end{bmatrix} < 0 \tag{24}$$

which is equivalent to LMI (21). By solving (21), we can obtain matrices M and N. Furthermore, from (21), we can get matrices P and K. This completed the proof. □

For the case of system (1) with disturbance, that is, $w(k) \neq 0$, we have the following two theorems.

Theorem 7. *Closed-loop system (8) is exponentially mean-square stable and achieves the prescribed H_∞ performance ifthere exist positive definite matrix P and gain matrix K satisfying the following LMI:*

$$\begin{bmatrix} -P & * & * & * & * & * & * \\ 0 & -\gamma^2 I & * & * & * & * & * \\ PA & PE & -P & * & * & * & * \\ aPB & 0 & 0 & -aP & * & * & * \\ \xi r_1PA_0 & \xi r_1PE & 0 & 0 & -\xi P & * & * \\ PB & 0 & 0 & 0 & 0 & -\xi P & * \\ C & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{25}$$

where $\gamma > 0$ is a given parameter and $\xi > 0$ is an arbitrary given constant, $C = \operatorname{diag}\{C_1, C_2, \ldots, C_N\}$, $E = \operatorname{diag}\{E_1, E_2, \ldots, E_N\}$, and a, P, B, K, A, and A_0 are the same as in (14).

Proof. When $w(k) = 0$, inequality (25) is equivalent to (14). From Theorem 5, closed-loop system (8) is exponentially mean-square stable.

When $w(k) \neq 0$, choose the Lyapunov functional as

$$V(x(k)) = x_i(k)^T P_i x_i(k); \tag{26}$$

then, we have

$$\begin{aligned} &E\{V(x(k+1)) \mid x(k)\} - V(x(k)) + E\left\{z_i(k)^T\right. \\ &\quad\left.\cdot z_i(k)\right\} - \gamma^2 E\left\{w_i(k)^T w_i(k)\right\} \\ &= E\left\{\sum_{i=1}^{N}\left(A_ix_i(k) - \left(r_i(k) - \bar{r}_i\right)B_iK_ix_i(k)\right.\right. \\ &\quad\left. + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}x_j(k) + E_iw_i(k) - \bar{r}_iB_iK_ix_i(k)\right)^T \\ &\quad\cdot P_i\left(A_ix_i(k) - \left(r_i(k) - \bar{r}_i\right)B_iK_ix_i(k)\right. \\ &\quad\left.\left. + \sum_{\substack{j=1 \\ j\neq i}}^{N} G_{ij}x_j(k) + E_iw_i(k) - \bar{r}_iB_iK_ix_i(k)\right)\right\} \\ &\quad - \sum_{i=1}^{N} x_i(k)^T P_ix_i(k) + \sum_{i=1}^{N}\left(x_i(k)^T C_i^T C_ix_i(k)\right) \end{aligned}$$

$$
\begin{aligned}
&-\gamma^2\sum_{i=1}^{N} w_i(k)^T w_i(k) \le \sum_{i=1}^{N}\Bigg(\left(A_i - r_{0i}B_iK_i\right) x_i(k) \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k) + E_i w_i(k) \Bigg)^T P_i \Bigg(\left(A_i - r_{0i}B_iK_i\right) \\
&\cdot x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k) + E_i w_i(k) \Bigg) \\
&+ \xi {r_1}^2 \sum_{i=1}^{N}\Bigg(\left(A_i - \frac{1}{2}B_iK_i\right) x_i(k) + \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k) \\
&+ E_i w_i(k) \Bigg)^T P_i \Bigg(\left(A_i - \frac{1}{2}B_iK_i\right) x_i(k) \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{N} G_{ij}x_j(k) + E_i w_i(k) \Bigg) + \sum_{i=1}^{N}\left(a_i + \zeta^{-1}\right) \\
&\cdot \left(B_iK_ix_i(k)\right)^T P_i \left(B_iK_ix_i(k)\right) - \sum_{i=1}^{N} x_i(k)^T P_i x_i(k) \\
&+ \sum_{i=1}^{N}\left(x_i(k)^T {C_i}^T C_i x_i(k)\right) - \gamma^2 \sum_{i=1}^{N} w_i(k)^T w_i(k) \\
&\triangleq \eta(k)^T \theta_2 \eta(k),
\end{aligned}
\tag{27}
$$

where

$$
\begin{aligned}
\eta(k) &= \left[x(k)^T, w(k)^T\right]^T, \\
x(k) &= \left[x_1(k)^T, x_2(k)^T, \ldots, x_N(k)^T\right]^T, \\
w(k) &= \left[w_1(k)^T, w_2(k)^T, \ldots, w_N(k)^T\right]^T.
\end{aligned}
\tag{28}
$$

Based on the Schur complement, inequality (25) implies $\theta_2 < 0$, and then we get

$$
\begin{aligned}
&E\{V(x(k+1)) \mid x(k)\} - V(x(k)) \\
&\quad + E\left\{z_i(k)^T z_i(k)\right\} - \gamma^2 E\left\{w_i(k)^T w_i(k)\right\} < 0.
\end{aligned}
\tag{29}
$$

Now summing (29) from 0 to ∞ with respect to k yields

$$
\begin{aligned}
\sum_{k=0}^{\infty} E\left\{z^T(k) z(k)\right\} &< \gamma^2 \sum_{k=0}^{\infty} E\left\{w^T(k) w(k)\right\} \\
&\quad + E\{V(0)\} - E\{V(\infty)\}.
\end{aligned}
\tag{30}
$$

Since system (8) is exponentially mean-square stable. Under the zero-initial condition, it is straightforward to see that

$$
\sum_{k=0}^{\infty} E\left\{\|z(k)\|^2\right\} < \sum_{k=0}^{\infty} \gamma^2 E\left\{\|w(k)\|^2\right\}. \tag{31}
$$

This completed the proof. □

Theorem 8. *Closed-loop system (8) is exponentially mean-square stable and achieves the prescribed H_∞ performance if there exist positive definite matrix M and gain matrix N satisfying the following LMI:*

$$
\begin{bmatrix}
-M & * & * & * & * & * & * \\
0 & -\gamma^2 I & * & * & * & * & * \\
AM & EM & -M & * & * & * & * \\
aBM & 0 & 0 & -aM & * & * & * \\
\xi r_1 A_0 M & \xi r_1 EM & 0 & 0 & -\xi M & * & * \\
BM & 0 & 0 & 0 & 0 & -\xi M & * \\
C & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix}
< 0, \tag{32}
$$

where $\gamma > 0$ is a given parameter and a, ξ, P, M, N, E, AM, BM, and A_0M are the same as in (21).

Proof. Through left-and-right multiplication (25) by

$$
\operatorname{diag}\left\{P^{-1}, I, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I\right\}, \tag{33}
$$

we have

$$
\begin{bmatrix}
-P^{-1} & * & * & * & * & * & * \\
0 & -\gamma^2 I & * & * & * & * & * \\
AP^{-1} & EP^{-1} & -P^{-1} & * & * & * & * \\
aBP^{-1} & 0 & 0 & -aP^{-1} & * & * & * \\
\xi r_1 A_0 P^{-1} & \xi r_1 EP^{-1} & 0 & 0 & -\xi P^{-1} & * & * \\
BP^{-1} & 0 & 0 & 0 & 0 & -\xi P^{-1} & * \\
C & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix}
< 0. \tag{34}
$$

Then matrix inequality (32) is equivalent to (25). From Theorem 7, we can conclude that closed-loop system (8) is exponentially mean-square stable and achieves the prescribed H_∞ performance. This completed the proof. □

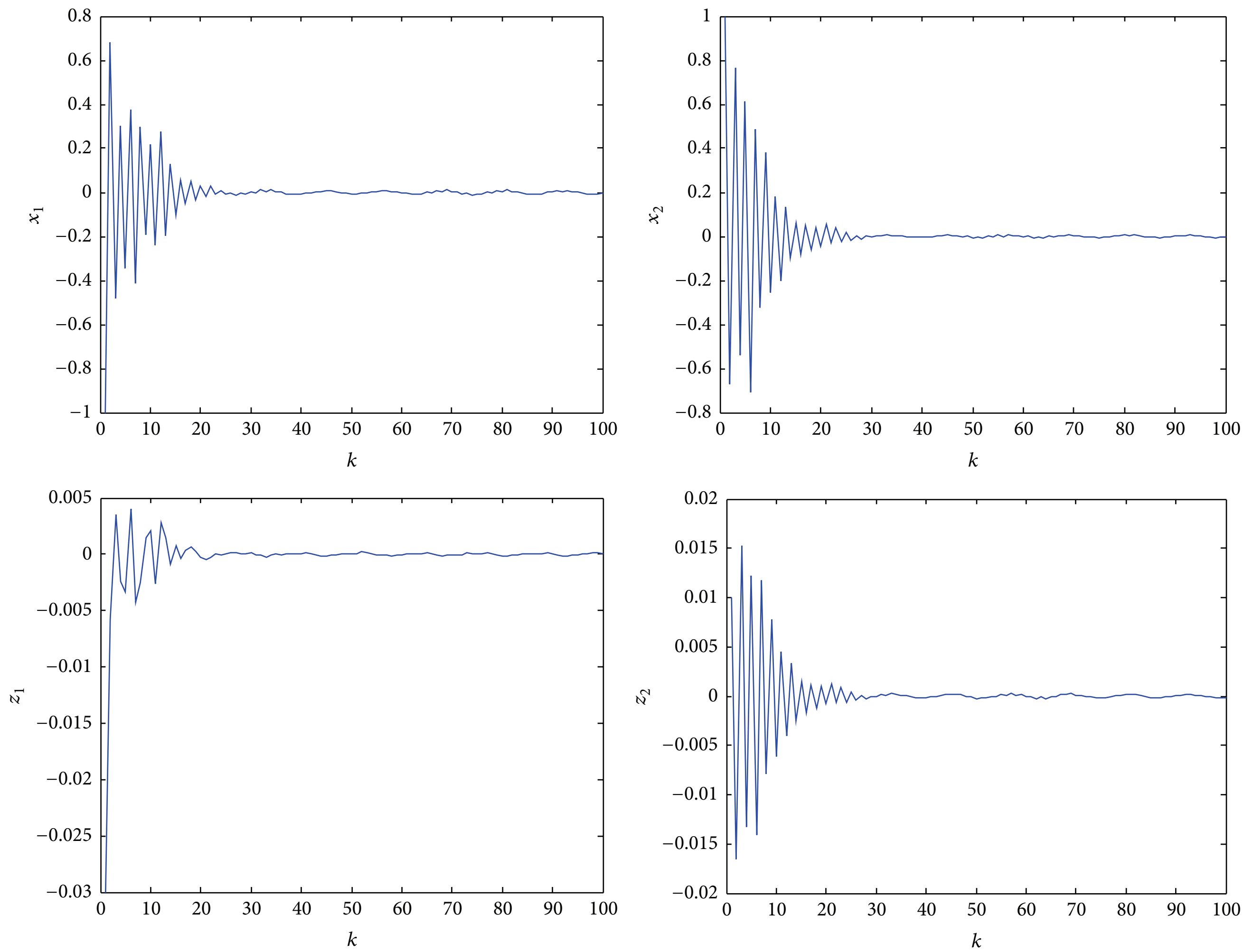

Figure 1: Closed-loop system with certain missing measurements probabilities ($\bar{r}_1 = \bar{r}_2 = 0.6$).

4. Simulation Example

Consider a linear discrete-time large-scale system which is composed of two NCSs as follows:

$$\begin{aligned}
x_1(k+1) &= \begin{bmatrix} -1.16 & 0.03 \\ 0.04 & 0.02 \end{bmatrix} x_1(k) + \begin{bmatrix} -0.02 \\ 0.03 \end{bmatrix} u_1(k) \\
&\quad + \begin{bmatrix} -0.03 & 0.02 \\ 0.01 & 0.03 \end{bmatrix} x_2(k) \\
&\quad + \begin{bmatrix} 0.3 & 0.2 \\ 0.3 & 0.1 \end{bmatrix} w_1(k), \\
y_1(k) &= x_1(k), \\
z_1(k) &= \begin{bmatrix} 0.01 & -0.02 \end{bmatrix} x_1(k), \\
x_2(k+1) &= \begin{bmatrix} -1.15 & 0.01 \\ 0.03 & 0.01 \end{bmatrix} x_2(k) + \begin{bmatrix} -0.03 \\ 0.02 \end{bmatrix} u_2(k) \\
&\quad + \begin{bmatrix} -0.01 & 0.02 \\ 0.03 & 0.01 \end{bmatrix} x_1(k) \\
&\quad + \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix} w_2(k), \\
y_2(k) &= x_2(k), \\
z_2(k) &= \begin{bmatrix} 0.02 & 0.01 \end{bmatrix} x_2(k).
\end{aligned} \tag{35}$$

Assume that $E\{r_1(k) \mid r_1(k) = 1\} = E\{r_2(k) \mid r_2(k) = 1\} = 0.6$. We can obtain the Lyapunov function solution matrices and controller parameters as follows:

$$\begin{aligned}
K_1 &= \begin{bmatrix} 21.8285 & -0.7578 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} 15.3237 & -0.1161 \end{bmatrix}.
\end{aligned} \tag{36}$$

Choose the disturbance input $w_1(k) = w_2(k) = 0.01 \begin{bmatrix} \sin(100k) \\ \sin(100k) \end{bmatrix}$. The initial state values are $x_1(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $x_2(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. The simulation results are shown in Figure 1 and the closed-loop systems are stable.

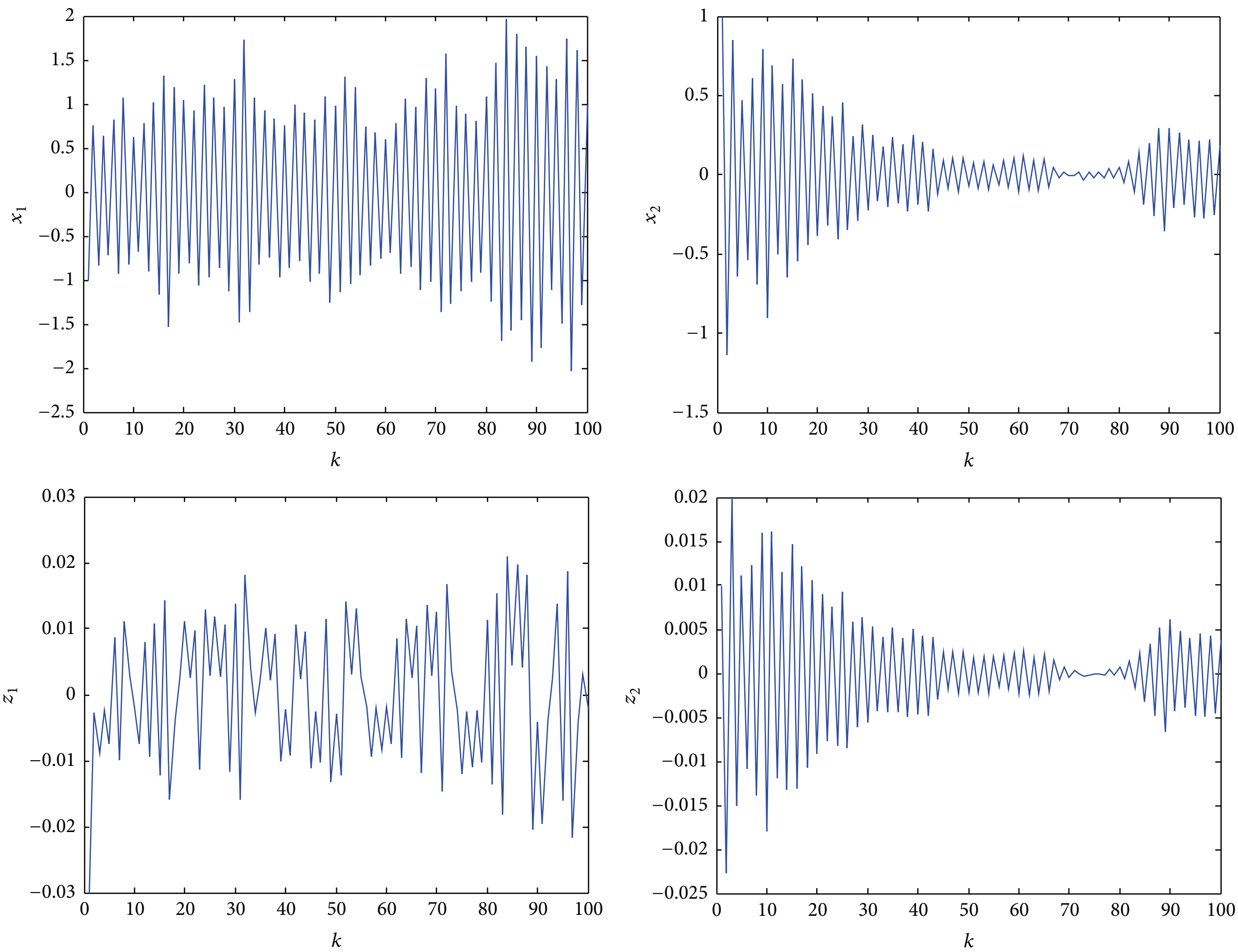

FIGURE 2: Closed-loop system with certain missing measurements probabilities ($\bar{r}_1 = \bar{r}_2 = 0.4$).

When $\bar{r}_1 = \bar{r}_2 = 0.4$, the simulation results are shown in Figure 2 and the closed-loop systems are unstable. From Figures 1 and 2, we can conclude that the closed-loop systems cannot be guaranteed to be stable when the missing measurements probabilities are large enough. For the limit of space, the detailed design procedure is omitted here.

When $\bar{r}_1, \bar{r}_2$ are uncertain and $\bar{r}_1 = \bar{r}_2 \in [0.4 \ \ 1]$, we can get the following parameters in Theorem 8 by using the YALMIP toolbox in MATLAB:

$$
\begin{gathered}
M_1 = \begin{bmatrix} 0.0038 & 0.0078 \\ 0.0078 & 0.2073 \end{bmatrix}, \\
M_2 = \begin{bmatrix} 0.0024 & 0.0020 \\ 0.0020 & 0.1486 \end{bmatrix}, \\
N_1 = \begin{bmatrix} 0.0782 & 0.0164 \end{bmatrix}, \\
N_2 = \begin{bmatrix} 0.0376 & 0.0142 \end{bmatrix}.
\end{gathered} \tag{37}
$$

According to $P = M^{-1}$ and $K = NP$, we have the Lyapunov function solution matrices and controller parameters as follows:

$$
\begin{gathered}
P_1 = M_1^{-1} = \begin{bmatrix} 283.9157 & -10.6226 \\ -10.6226 & 5.2206 \end{bmatrix}, \\
P_2 = M_2^{-1} = \begin{bmatrix} 416.7692 & -5.7418 \\ -5.7418 & 6.8073 \end{bmatrix}, \\
K_1 = \begin{bmatrix} 22.0245 & -0.7449 \end{bmatrix}, \\
K_2 = \begin{bmatrix} 15.6096 & -0.1193 \end{bmatrix}.
\end{gathered} \tag{38}
$$

The simulation results are shown in Figure 3 and the closed-loop systems are stable. It can be verified that $\sum_{k=0}^{\infty} E\{\|z(k)\|^2\} < \gamma^2 \sum_{k=0}^{\infty}\{\|w(k)\|^2\}$.

In summary, the closed-loop stability cannot be guaranteed using the method where probability is known to deal with the missing measurements. However, when the probability varies within a given interval, the closed-loop stability can be guaranteed through the controller designed by the method proposed in this paper.

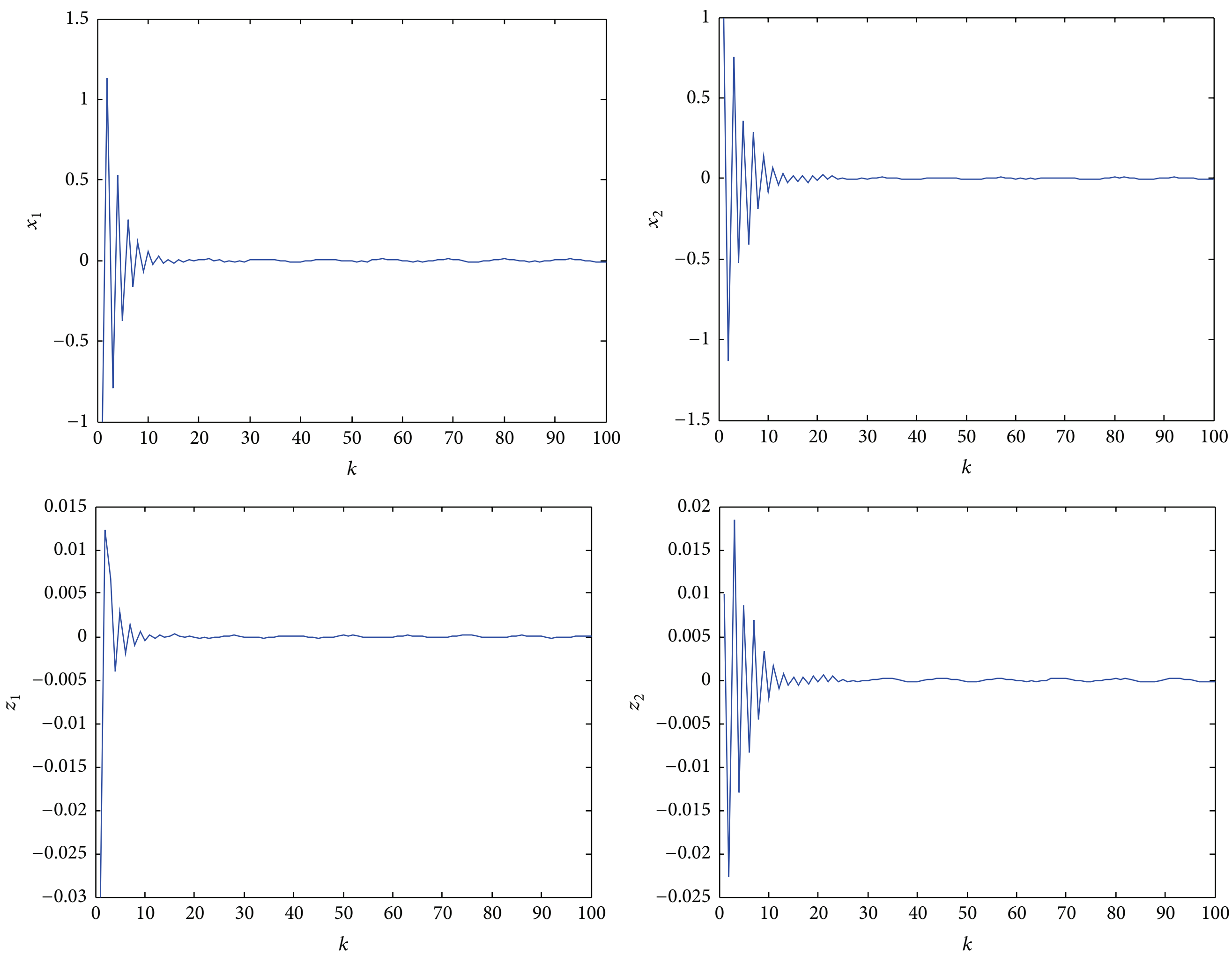

FIGURE 3: Closed-loop system with uncertain missing measurements probabilities.

5. Conclusions

In this paper, the decentralized H_∞ controller has been designed for a class of large-scale systems with uncertain missing measurements probabilities. The random missing measurements are modeled as a stochastic variable satisfying Bernoulli distribution with uncertain probabilities. Sufficient conditions for the existence of a stable H_∞ controller are presented via LMI, and the designed controller enables the closed-loop system to be exponentially mean-square stable and achieve the prescribed H_∞ performance.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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