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Research Article

Finite-Time Boundedness and Stabilization of Networked Control Systems with Time Delay

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The finite-time control problem of a class of networked control systems (NCSs) with time delay is investigated. The main results provided in the paper are sufficient conditions for finite-time stability via state feedback. An augmentation approach is proposed to model NCSs with time delay as linear systems. Based on finite time stability theory, the sufficient conditions for finite-time boundedness and stabilization of the underlying systems are derived via linear matrix inequalities (LMIs) formulation. Finally, an illustrative example is given to demonstrate the effectiveness of the proposed results.

1. Introduction

Networked control systems (NCSs) are feedback control systems with control loops closed via digital communication channels. Compared with the traditional point-to-point wiring, the use of the communication channels can reduce the costs of cables and power, simplify the installation and maintenance of the whole system, and increase the reliability. NCSs have many industrial applications in automobiles, manufacturing plants, aircrafts, and HVAC systems [1]. However, the insertion of communication networks in feedback control loops makes the NCSs analysis and synthesis complex; see [2–8] and the references therein.

One issue inherent to NCSs, however, is the network-induced delay that occurs while exchanging date among devices connected to the shared medium. This delay, either constant or time varying, can degrade the performance of control systems designed without considering it and even destabilize the system. Thus the issues of stability analysis for NCSs have received considerable attention for decades [9–15]. In [9, 10], NCSs with random delays are modelled as jump linear systems with two modes; the necessary and sufficient conditions on the existence of stabilizing controllers are given. By introducing indicator functions, mean-square asymptotic stability is derived for the closed-loop networked control system in [11].

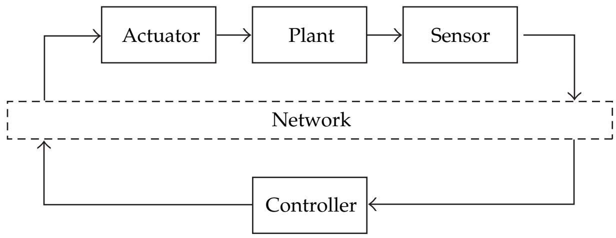

Figure 1: Illustration of NCSs over communication network.

Based on a discrete system model with time-varying input delays, stability analysis and control design are carried out in [12, 13]. In [14], an observer-based stabilizing controller has been designed for networked systems involving both random measurement and actuation delays. In [15], a novel state feedback H_∞ control with the compensator for the effects of network delays in both forward and feedback channels is proposed by introducing an augmented state variable.

On the other hand, finite-time boundedness and stability can be used in all those applications where large values of the state should not be attained, for instance, in the presence of saturations. However, most of the results in the literature are focused on Lyapunov stability. Some early results on finite-time stability (FTS) can be found in [16], more recently the concept of FTS has been revisited in the light of recent results coming from linear matrix inequalities (LMIs) theory, which has made it possible to find less conservative conditions for guaranteeing FTS and finite time stabilization of discrete-time and continuous-time systems [17–26]. In [27, 28], sufficient conditions for finite-time stability of networked control systems with packet dropout are provided; however, controller design methods are not given.

To the best of our knowledge, the finite-time stabilization problems for NCSs with delay have not been fully investigated to date. Especially for the case where the plant subjects to external interference, very few results related to NCSs are available in the existing literature, which motivates the study of this paper. The main contributions of this paper are definitions of finite-time boundedness and stabilization are extended to NCSs. Furthermore, sufficient conditions for finite-time boundedness and stabilization linear matrix inequalities formulation are given.

In this paper, the finite-time stabilization and boundedness problems of a class of NCSs with time delay are studied. The sufficient conditions for finite-time stabilization and boundedness of the underlying systems are derived via LMIs formulation. Lastly, an illustrative example is given to demonstrate the effectiveness of the proposed methods.

This paper is organized as follows. An augmentation approach is proposed to model NCSs with time delay as linear system in Section 2. The finite-time stabilization and boundedness conditions for NCSs with time delay are derived via LMIs in Section 3. Section 4 provides a numerical example to illustrate the effectiveness of our results. Finally, Section 5 gives some concluding remarks.

2. Problem Formulation and Preliminaries

Consider NCS depicted in Figure 1 consists of three components: a plant to be controlled, a network such as the Internet, and a controller.

In this paper, it is assumed that the plant is described by

$$\dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \tag{2.1}$$

and time-invariant controller

$$u(kh) \ = -Kx(Kh), \quad k = 0, 1, 2, \ldots, \tag{2.2}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, and $w(t) \in \mathbb{R}^q$ is the exogenous input. A, B, and G are known real constant matrices with appropriate dimensions. The sampling period h is fixed and known. There are two sources of delays from the network: the sensor-to-controller delay τ_{sc} and the controller-to-actuator delay τ_{ca}. For the fixed control law, the sensor-to-controller delay and the controller-to-actuator delay can be lumped together as $\tau = \tau_{\mathrm{sc}} + \tau_{\mathrm{ca}}$ for analysis purpose. We make the following assumptions about NCSs.

Assumption 2.1. The sensors are clock-driven sensors, and controllers and actuators are event-driven.

Assumption 2.2. The network-induced delay is constant and less than one sampling period.

Assumption 2.3. During the finite time T, there exists a positive constant d, such that the exogenous input $w(t)$ satisfies

$$\int_0^T w^T(t)w(t)dt \le d^2. \tag{2.3}$$

Then the system equation can be written as

$$\begin{gathered}
\dot{x}(t) = Ax(t) + Bu(t) + Gw(t), \quad t \in [kh+\tau, (k+1)h+\tau), \\
y(t) = Cx(t), \\
u(t^+) = -Kx(t-\tau), \quad t \in \{kh+\tau, k = 1, 2, \ldots\}.
\end{gathered} \tag{2.4}$$

Sampling the system with period h, we obtain

$$\begin{gathered}
x(k+1) = \Phi x(k) + \Gamma_0(\tau)u(k) + \Gamma_1(\tau)u(k-1) + \Psi w(k), \\
y(t) = Cx(t),
\end{gathered} \tag{2.5}$$

where

$$\Phi = e^{Ah}, \qquad \Psi = \int_0^h e^{As}G\,ds. \tag{2.6}$$

$\Gamma_0(\tau)$ and $\Gamma_1(\tau)$ are defined as follows:

$$\Gamma_0(\tau) = \int_0^{h-\tau} e^{As} B\, ds, \qquad \Gamma_1(\tau) = \int_{h-\tau}^{h} e^{As} B\, ds. \tag{2.7}$$

Define the augmented state vector $\tilde{x}(k)$ as follows:

$$\tilde{x}(k) = [x(k), u(k-1)]^T \tag{2.8}$$

and the augmented exogenous input vector $\tilde{w}(k)$ as

$$\tilde{w}(k) = [w(k), 0]^T. \tag{2.9}$$

Then we have the augmented closed-loop system

$$\tilde{x}(k+1) = \left(\tilde{A} + \tilde{B}\tilde{K}\right)\tilde{x}(k) + \tilde{G}\tilde{w}(k), \tag{2.10}$$

where

$$\tilde{A} = \begin{bmatrix} \Phi & \Gamma_1(\tau) \\ 0 & 0 \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} -\Gamma_0(\tau) \\ -I \end{bmatrix}, \qquad \tilde{G} = \begin{bmatrix} \Psi & 0 \\ 0 & 0 \end{bmatrix} \tag{2.11}$$

and $\tilde{K}$ is defined as follows

$$\tilde{K} = \begin{bmatrix} K & 0 \end{bmatrix}. \tag{2.12}$$

Remark 2.4. According to Assumption 2.3, we can derive that there exists a positive constant d, such that the condition

$$\sum_{k=1}^{N} \tilde{w}^T(k)\tilde{w}(k) \le d^2 \tag{2.13}$$

is satisfied, for finite positive integer N.

Remark 2.5. When the delay is longer than one sampling period, that is to say, $h < \tau < lh$, where $l > 1$, the augmented state vector $\tilde{x}(k)$ is defined as

$$\tilde{x}(k) = [x(k), u(k-l), \ldots, u(k-1)]^T \tag{2.14}$$

and the corresponding augmented closed-loop system can be derived.

The main aim of this paper is to find some sufficient conditions which guarantee that the system given by (2.10) is bounded over a finite-time interval. The general idea of finite-time stability concerns the boundedness of the state of a system over a finite time interval for given initial conditions; this concept can be formalized through the following definitions.

Definition 2.6. System (2.10) with (2.13) is said to be finite-time bounded with respect to (α, d, β, R, N), where R is a positive-definite matrix, $0 < \alpha < \beta$, if

$$x^T(0)Rx(0) \le \alpha^2 \Longrightarrow x^T(k)Rx(k) \le \beta^2, \quad k \in \{1, \dots, N\}. \tag{2.15}$$

Definition 2.7. System (2.10) with $w(k) = 0$ is said to be finite-time stable with respect to (α, β, R, N), where R is a positive-definite matrix, $0 < \alpha < \beta$, if

$$x^T(0)Rx(0) \le \alpha^2 \Longrightarrow x^T(k)Rx(k) \le \beta^2, \quad k \in \{1, \dots, N\}. \tag{2.16}$$

To this end, the following lemma will be essential for the proofs in the next section and its proof can be found in the cited references.

Lemma 2.8 (Schur complement lemma, see [29]). *For a given symmetric matrix* $W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12}^T & W_{22} \end{bmatrix}$, *where* $W_{11} \in \mathbb{R}^{p\times p}, W_{22} \in \mathbb{R}^{q\times q}$, *and* $W_{12} \in \mathbb{R}^{p\times q}$, *the following three conditions are mutually equivalent:*

(1) $W < 0$,

(2) $W_{11} < 0$, $W_{22} - W_{12}^T W_{11}^{-1} W_{12} < 0$,

(3) $W_{22} < 0$, $W_{11} - W_{12} W_{22}^{-1} W_{12}^T < 0$.

3. Main Results

In this section, we will find a state feedback control matrix K, such that system (2.10) is finite-time bounded with respect to (α, d, β, R, N). In order to solve the problem, the following theorem will be essential.

Theorem 3.1. *For given state feedback control matrix K, system* (2.10) *is finite-time bounded with respect to (α, d, β, R, N), if there exist symmetric positive definite matrices P_1 and P_2 and a scalar $\gamma \ge 1$, such that the following conditions hold:*

$$\begin{bmatrix} \left(\widetilde{A} + \widetilde{B}\widetilde{K}\right)^T P_1 \left(\widetilde{A} + \widetilde{B}\widetilde{K}\right) - \gamma P_1 & \left(\widetilde{A} + \widetilde{B}\widetilde{K}\right)^T P_1 \widetilde{G} \\ \widetilde{G}^T P_1 \left(\widetilde{A} + \widetilde{B}\widetilde{K}\right) & \widetilde{G}^T P_1 \widetilde{G} - \gamma P_2 \end{bmatrix} < 0, \tag{3.1}$$

$$\frac{\lambda_2}{\lambda_1}\gamma^N \alpha^2 + \frac{\lambda_3}{\lambda_1}\gamma^N d^2 < \beta^2, \tag{3.2}$$

where

$$\begin{aligned} \lambda_1 &= \lambda_{\min}\left(\widetilde{P}_1\right), \\ \lambda_2 &= \lambda_{\max}\left(\widetilde{P}_1\right), \\ \lambda_3 &= \lambda_{\max}(P_2), \\ \widetilde{P}_1 &= R^{-1/2} P_1 R^{1/2}. \end{aligned} \tag{3.3}$$

Proof. Choose the Lyapunov function as

$$V(\tilde{x}(k)) = \tilde{x}^T(k)P_1\tilde{x}(k). \tag{3.4}$$

Then we have

$$\begin{aligned} V(\tilde{x}(k+1)) &= \tilde{x}^T(k+1)P_1\tilde{x}(k+1) \\ &= \left(\left(\tilde{A}+\tilde{B}\tilde{K}\right)\tilde{x}(k)+\tilde{G}w(k)\right)^T P_1\left(\left(\tilde{A}+\tilde{B}\tilde{K}\right)\tilde{x}(k)+\tilde{G}w(k)\right) \\ &= \begin{bmatrix}\tilde{x}(k)\\ w(k)\end{bmatrix}^T \begin{bmatrix}\left(\tilde{A}+\tilde{B}\tilde{K}\right)^T P_1\left(\tilde{A}+\tilde{B}\tilde{K}\right) & \left(\tilde{A}+\tilde{B}\tilde{K}\right)^T P_1\tilde{G} \\ \tilde{G}^T P_1\left(\tilde{A}+\tilde{B}\tilde{K}\right) & \tilde{G}^T P_1\tilde{G}\end{bmatrix} \begin{bmatrix}\tilde{x}(k)\\ w(k)\end{bmatrix}. \end{aligned} \tag{3.5}$$

It follows from (3.1) that

$$V(\tilde{x}(k+1)) \le \gamma V(\tilde{x}(k)) + \gamma w^T(k)P_2w(k). \tag{3.6}$$

Applying iteratively (3.6), we obtain

$$\begin{aligned} V(\tilde{x}(k)) &\le \gamma^k V(\tilde{x}(0)) + \sum_{j=1}^{k}\gamma^j w^T(k-j)P_2w(k-j) \\ &= \gamma^k\left(V(\tilde{x}(0)) + \sum_{j=1}^{k}\gamma^{j-k}w^T(k-j)P_2w(k-j)\right) \\ &\le \gamma^k\left(V(\tilde{x}(0)) + \lambda_3\sum_{j=1}^{k}\gamma^{j-k}w^T(k-j)w(k-j)\right). \end{aligned} \tag{3.7}$$

Using the fact that $\gamma \ge 1$, we have

$$\begin{aligned} V(\tilde{x}(k)) &\le \gamma^k\left(V(\tilde{x}(0)) + \lambda_3\sum_{j=1}^{k}w^T(k-j)w(k-j)\right) \\ &\le \gamma^N\left(\lambda_2\alpha^2 + \lambda_3 d^2\right). \end{aligned} \tag{3.8}$$

On the other hand,

$$V(\tilde{x}(k)) = \tilde{x}^T(k)P_1\tilde{x}(k) \ge \lambda_1\tilde{x}^T(k)R\tilde{x}(k). \tag{3.9}$$

From (3.8) and (3.19), it can be seen that

$$\tilde{x}^T(k)R\tilde{x}(k) \le \frac{\lambda_2}{\lambda_1}\gamma^N\alpha^2 + \frac{\lambda_3}{\lambda_1}\gamma^N d^2 < \beta^2 \tag{3.10}$$

which means that

$$\tilde{x}^T(k)R\tilde{x}(k) \le \beta^2, \quad k = 1, \ldots, N. \tag{3.11}$$

This completes the proof. □

Corollary 3.2. *For given state feedback control matrix K, system* (2.10) *with the disturbance $\tilde{w}(k) = 0$ is finite-time stable with respect to (α, β, R, N), if there exist symmetric positive definite matrix P and a scalar $\gamma \ge 1$, such that the following conditions hold:*

$$\begin{gathered}\left(\tilde{A} + \tilde{B}\tilde{K}\right)^T P\left(\tilde{A} + \tilde{B}\tilde{K}\right) - \gamma P < 0, \\ \operatorname{cond}\left(\tilde{P}\right) < \frac{1}{\gamma^N}\frac{\beta^2}{\alpha^2},\end{gathered} \tag{3.12}$$

where

$$\tilde{P} = R^{-1/2}PR^{1/2}, \quad \operatorname{cond}\left(\tilde{P}\right) = \frac{\lambda_{\max}\left(\tilde{P}\right)}{\lambda_{\min}\left(\tilde{P}\right)}. \tag{3.13}$$

Now we turn back to our original problem, that is, to find sufficient conditions which guarantee that the system (2.4) with the controller (2.2) is finite-time bounded with respect to (α, d, β, R, N). The solution of this problem is given by the following theorem.

Theorem 3.3. *System* (2.10) *is finite-time bounded with respect to (α, d, β, R, N) if there exist symmetric positive definite matrices Q_{11}, Q_{12}, and Q_2, a matrix L, and a scalar $\gamma \ge 1$, such that the following conditions hold:*

$$\begin{bmatrix} -\gamma Q_1 & 0 & \left(\tilde{A}Q_1 + \tilde{B}LS\right)^T \\ 0 & -\gamma Q_2 & \tilde{G}^T \\ \tilde{A}Q_1 + \tilde{B}LS & \tilde{G} & -Q_1 \end{bmatrix} < 0, \tag{3.14}$$

$$\frac{\lambda_5}{\lambda_4}\gamma^N\alpha^2 + \lambda_5\lambda_6\gamma^N d^2 < \beta^2, \tag{3.15}$$

where

$$\begin{gathered}\lambda_4 = \lambda_{\min}\left(\tilde{Q}_1\right), \\ \lambda_5 = \lambda_{\max}\left(\tilde{Q}_1\right), \\ \lambda_6 = \lambda_{\max}(Q_2), \\ \tilde{Q}_1 = R^{-1/2}Q_1R^{1/2}\end{gathered} \tag{3.16}$$

S and Q_1 are defined as follows

$$S = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad Q_1 = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{12} \end{bmatrix}. \tag{3.17}$$

Then the controller K is given by the first p columns of $\widetilde{K} = LSQ_1^{-1}$, which is in the form (2.12).

Proof. Let us consider Theorem 3.1 with $Q_1 = P_1^{-1}$ and $Q_2 = P_2$. Condition (3.2) can be rewritten as in (3.15) recalling that for a positive definite matrix Q

$$\lambda_{\max}(Q) = \frac{1}{\lambda_{\min}(Q^{-1})}. \tag{3.18}$$

Denote $\widehat{A} = \widetilde{A} + \widetilde{B}\widetilde{K}$. Then condition (3.1) can be rewritten as

$$\begin{bmatrix} \widehat{A}^T Q_1^{-1} \widehat{A} - \gamma Q_1^{-1} & \widehat{A}^T Q_1^{-1} \widetilde{G} \\ \widetilde{G}^T Q_1^{-1} \widehat{A} & \widetilde{G}^T Q_1^{-1} \widetilde{G} - \gamma Q_2 \end{bmatrix} < 0. \tag{3.19}$$

Pre- and postmultiplying (3.19) by the symmetric matrix

$$\begin{bmatrix} Q_1 & 0 \\ 0 & I \end{bmatrix}, \tag{3.20}$$

the following equivalent condition is obtained

$$\begin{bmatrix} Q_1 \widehat{A}^T Q_1^{-1} \widehat{A} Q_1 - \gamma Q_1 & Q_1 \widehat{A}^T Q_1^{-1} \widetilde{G} \\ \widetilde{G}^T Q_1^{-1} \widehat{A} Q_1 & \widetilde{G}^T Q_1^{-1} \widetilde{G} - \gamma Q_2 \end{bmatrix} < 0. \tag{3.21}$$

By using Lemma 2.8, (3.21) is equivalent to the following:

$$\begin{bmatrix} Q_1 \widehat{A}^T Q_1^{-1} \widehat{A} Q_1 - \gamma Q_1 & Q_1 \widehat{A}^T Q_1^{-1} \widetilde{G} & 0 \\ \widetilde{G}^T Q_1^{-1} \widehat{A} Q_1 & -\gamma Q_2 & \widetilde{G}^T \\ 0 & \widetilde{G} & -Q_1 \end{bmatrix} < 0. \tag{3.22}$$

Premultiply (3.22) by

$$\begin{bmatrix} I & 0 & -Q_1 \widehat{A}^T Q_1^{-1} \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{3.23}$$

and postmultiply it by the transpose of (3.23). In this way, we obtain the following equivalent condition:

$$\begin{bmatrix} -\gamma Q_1 & 0 & Q_1\hat{A}^T \\ 0 & -\gamma Q_2 & \tilde{G}^T \\ \hat{A}Q_1 & \tilde{G} & -Q_1 \end{bmatrix} < 0. \tag{3.24}$$

Recalling that $\hat{A} = \tilde{A} + \tilde{B}\tilde{K}$ and letting $\tilde{K}Q_1 = LS$, we obtain that condition (3.1) is equivalent to (3.14). This completes the proof. □

Remark 3.4. The chosen structures for matrices S and Q_1 guarantee that $\tilde{K}$ is in the form (2.12). In fact

$$\tilde{K} = LSQ_1^{-1} = L\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{12} \end{bmatrix} = L\begin{bmatrix} Q_{11}^{-1} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} K & 0 \end{bmatrix}. \tag{3.25}$$

Remark 3.5. Once we have fixed γ, the feasibility of the conditions stated in (3.14) can be turned into LMI feasibility problems. On the other hand, for $\theta_1 > 0, \theta_2 > 0$, it is easy to check that condition (3.15) can be guaranteed by

$$\begin{gathered} \theta_1 R^{-1} < Q_1 < R^{-1}, \\ 0 < Q_2 < \theta_2 I, \\ \begin{bmatrix} \beta^2 - \theta_2 d^2 \gamma^N & \alpha\sqrt{\gamma^N} \\ \alpha\sqrt{\gamma^N} & \theta_1 \end{bmatrix} > 0. \end{gathered} \tag{3.26}$$

Corollary 3.6. *System* (2.10) *with the disturbance* $\tilde{w}(k) = 0$ *is finite-time stable with respect to* (α, β, R, N), *if there exist symmetric positive definite matrices* Q_1, Q_2, *a matrix* L, *and a scalar* $\gamma \geq 1$, *such that the following conditions hold:*

$$\begin{gathered} \begin{bmatrix} -\gamma Q & \left(\tilde{A}Q + \tilde{B}LS\right)^T \\ \tilde{A}Q + \tilde{B}LS & -Q \end{bmatrix} < 0, \\ R^{-1} < Q < \frac{1}{\gamma^N}\frac{\beta^2}{\alpha^2}R^{-1}, \end{gathered} \tag{3.27}$$

where

$$S = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}. \tag{3.28}$$

Then the controller K *is given by the first* p *columns of* $\tilde{K} = LSQ^{-1}$.

4. Numerical Example

Consider the following system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w(t),$$
$$y(t) = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix} x(t). \tag{4.1}$$

Choose the sampling $h = 0.3s$. Suppose $\tau = 0.1s$. The corresponding matrices are given by

$$\Phi = \begin{bmatrix} 1.0000 & 0.2955 \\ 0 & 0.9704 \end{bmatrix}, \qquad \Psi = \begin{bmatrix} 0.3000 & 0.0446 \\ 0 & 0.2955 \end{bmatrix}, \tag{4.2}$$

$$\Gamma_0(\tau) = \begin{bmatrix} 0.0020 \\ 0.0198 \end{bmatrix}, \qquad \Gamma_1(\tau) = \begin{bmatrix} 0.0025 \\ 0.0098 \end{bmatrix} \tag{4.3}$$

which yields

$$\tilde{A} = \begin{bmatrix} 1.0000 & 0.2955 & 0.0025 \\ 0 & 0.9704 & 0.0098 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} -0.0020 \\ -0.0198 \\ -1 \end{bmatrix}, \qquad \tilde{G} = \begin{bmatrix} 0.3000 & 0.0446 & 0 \\ 0 & 0.2955 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{4.4}$$

It is assumed that $\alpha = 1$, $d = 3$, $\beta = 20$, $R = I$, $N = 10$. Applying Theorem 3.3 with $\gamma = 1.5$, it is found that

$$Q_1 = \begin{bmatrix} 0.9472 & 0.0318 & 0 \\ 0.0318 & 0.7947 & 0 \\ 0 & 0 & 0.8616 \end{bmatrix},$$
$$L = \begin{bmatrix} 0.0030 & 0.0187 & 0 \end{bmatrix}. \tag{4.5}$$

Therefore, the desired controller gain is given by

$$\tilde{K} = LSQ_1^{-1} = \begin{bmatrix} K & 0 \end{bmatrix} = \begin{bmatrix} 0.0024 & 0.0235 & 0 \end{bmatrix}. \tag{4.6}$$

5. Conclusions

In this paper, we have considered the finite-time boundedness problems of a class of networked control systems (NCSs) subject to disturbances. Based on the augmentation approach, the NCSs with time delay as linear systems. The sufficient conditions for finite-time boundedness of the underlying systems are derived via linear matrix inequalities (LMIs) formulation. Lastly, an illustrative example is given to demonstrate the effectiveness of the proposed results.

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