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Individual Differences in Children's Paths to Arithmetical Development

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Abstract

Cross-sectional and longitudinal data consistently indicate that mathematical difficulties are more prevalent in older than in younger children (e.g. Department of Education, 2011). Children's trajectories can take a variety of shapes such as linear, flat, curvilinear and uneven, and shape has been found to vary within children and across tasks (J Jordan et al. 2009). There has been an increase in the use of statistical methods which are specifically designed to study development, and this has greatly improved our understanding of children's mathematical development. However, the effects of many cognitive and social variables (e.g. working memory and verbal ability) on mathematical development are unclear. It is likely that greater consistency between studies will be achieved by adopting a componential approach to study mathematics, rather than treating mathematics as a unitary concept.

Key words: Individual differences; children; development; componential

Individual differences in children's paths to arithmetical development.

1. Introduction

At any point in time during the formal schooling period, a significant number of children are considered to have inadequate mathematical skills. In a UK context, this is evident from the key stage 1-3 teacher assessments for mathematics, which are designed to indicate how well children are progressing at ages 7, 11 and 14. The 2011 results of these assessments showed that, one in ten children aged 7 in England failed to meet the expected level, and at ages 11 and 14 the percentages of children failing to meet the expected level were 18% and 19% (Department for Education 2011). The percentage point difference between the proportion of children who failed to meet the expected level at ages 7 (13%) and 14 (25%), is even greater in Wales (Statistical Directorate Welsh Assembly Government 2010), and at age 8 (5%) and age 14 (23%) in Northern Ireland in 2009/10 (Department of Education 2011). These figures suggest that many children who initially have no apparent difficulty, start to struggle as the curriculum becomes more demanding. The proportion of children failing to achieve the expected level varies across curriculum area. For example, the Welsh 2009 data (Statistical Directorate Welsh Assembly Government 2009) indicate that *Using and Applying Mathematics* at ages 7, 11 and 14 is the area in which most children fail to achieve the expected level, while the largest

percentage point decrease over time in children achieving the expected level was in *Shape Space and Measures*.

To some extent the differences between proportions of children meeting the expected level at ages 7 and 14 can be explained by cohort effects. For example, relative to the 2007 cohort the proportion of children meeting the expected level at ages 11 and 14 was 2-4 % points greater for the 2011 cohort, although no change was found for 7 year olds. Additionally, as the data are cross-sectional, they do not tell us what proportions of children: 1) have persistent difficulties; 2) are initially impaired but outgrow their difficulty; 3) initially meet expected levels in mathematics but then develop difficulty. On the other hand, longitudinal data can provide a much more detailed and precise picture of children's development. The Department for Education (2011) tracked English children from ages 7 to 11 and found that 18% failed to make the expected progress in mathematics. A comparable study which tracked children from 11 to 16 (Department for Education, 2011) found that an even higher proportion (38%) of students from England failed to achieve their predicted score in the General Certificate of Secondary Education (GCSE) mathematics using predictions made based on mathematics performance at the end of primary school. However, as these studies only focus on two time points, they provide a one dimensional picture of development, and tell us nothing about the shape (e.g. linear, curvilinear) of children's trajectories. Information on the shape of growth could be obtained by linking the datasets for key stage one, two and three. Tracking students over even longer periods would be possible if the recommendation by The Royal Society (2011) to allow unique pupil numbers to be carried over from school and college into higher education is implemented. This would allow school and higher education datasets to be linked, thus enabling the association between children's trajectories in mathematics and their choice of subject in higher education to be studied.

Employers have raised concern over the numeracy levels of school leavers. For example, a survey by the Confederation of British Industry (CBI) and Education Development International (2011) revealed that two fifths of employers have needed to provide remedial numeracy support to new employees who were school leavers. A more numerate workforce is likely to have a considerable economic impact, as those leaving school with inadequate numeracy skills are estimated to cost the UK taxpayer an estimated 2.4 billion per year (KPMG 2009), and those with poor numeracy skills are five times more likely to be unemployed (OECD 2006). The CBI (2010) proposed that more young people should complete some form of mathematics qualification post-16 to meet the need emphasised by employers for a more numerate workforce. These economic issues highlight that there is a strong need to understand what influences mathematics achievement, and, given that research indicates that children can develop mathematical difficulty at any stage during their schooling, this should be done in a developmental context. This chapter outlines research which has characterised children's

trajectories on a range of mathematical tasks and identified several trajectory groups including those with persistent difficulties from when they enter school, those who initially perform well but fall behind, and those who outgrow their difficulty. Furthermore, those factors which influence the characteristics of children's trajectories are considered.

2. Characteristics of trajectories

Children's trajectories are characterised by their initial and final status, steepness, and shape (e.g. linear, uneven, and curvilinear). J Jordan et al. (2009) examined the degree of heterogeneity amongst typically achieving children aged 5-7 years in terms of trajectory characteristics on seven mathematical tasks: exact calculation, story problems, approximate arithmetic, place value, calculation principles, forced retrieval and written problems. Each child's development over four time points on the seven mathematical tasks was classified, using regression, as one of four trajectory types: linear, quadratic, s-curve or flat. Considerable heterogeneity in trajectory characteristics between children was evident on all mathematical tasks. Specifically, linear, quadratic, and s-curve trajectories were found on all tasks, with flat trajectories also found on most tasks. Interestingly, although a linear trajectory best fitted the group data on most tasks, individual level analysis indicated that s-shaped trajectories were most prevalent on the majority of mathematical tasks. This shows that caution must be shown when documenting the shape of a group's trajectory, as the average trajectory may not adequately reflect the dominant trajectory type. This indicates that individual as opposed to average trajectory analysis may be a more reliable method for drawing conclusions about the shape of children's trajectories. Heterogeneity within children across tasks was evident; for example, linear development was the most common shape for place value and forced retrieval, while s-shaped was most common for the other five tasks. Furthermore, some children exhibited low performance and flat growth on some of the mathematical tasks (e.g. place value and calculation principles) despite achieving a standardised mathematics score above the 35th percentile.

Another study by N Jordan et al. (2006) focused on other characteristics of children's trajectories from ages 5 to 6; namely, steepness and performance at age 6. The children were examined at four time points on a battery of number sense measures: counting, enumeration, count sequence, counting principles, number knowledge, nonverbal calculation, story problems, estimation, number patterns and number recognition. Using growth mixture modelling, they identified three trajectory types and labelled these according to the average growth rate and average performance at age 6 on the number sense battery: 1) low/flat - lower than average performance at age 6 and flat growth; 2) average/moderate – average performance at age 6 and moderate growth; 3) high/moderate – better than average

performance at age 6 and moderate growth. Growth mixture modelling was also carried out on three of the number sense tasks, because there was evidence of divergent growth trajectories based on gender and income. When these number sense tasks, namely, story problems, non-verbal calculation and number combinations, were modelled, three trajectory types were found for each task. However, the nature of the three trajectory subtypes varied across task. For example, steep growth was evident for nonverbal calculation and number combinations but not for story problems. Individual differences within children and across these three tasks are evident from the variation in numbers of children falling into each trajectory classification. For example, while a large proportion of children had low/flat growth for story problems (70%) and number combinations (60%), a relatively smaller proportion of children had this type of trajectory for non-verbal combinations (44%). In contrast to the findings of J Jordan et al. (2009), no curvilinear trajectory types were identified. It is possible that a greater number of trajectory classifications would have been found had the tasks captured more growth. Indeed, very little growth was captured on the tasks as evidenced by the lack of significant growth rate and variation in growth rate.

Given that there is considerable variability amongst children on cognitive and social variables (e.g. N Jordan et al. 2006; TIMSS 2007), and that mathematical tasks vary considerably in terms of their cognitive demands and how they are perceived (e.g. Holmes et al. 2008; Gregory et al. 1999), the degree to which cognitive and social variables explain trajectory characteristics should vary across mathematical tasks. It is therefore likely that a componential developmental approach which looks at development on a range of mathematical tasks is a more appropriate approach than non-componential methods.

This chapter documents many of the factors identified by longitudinal research as being associated with individual differences in development including foundational numerical skills; reading and language; cognitive abilities; teaching, curriculum and classroom experiences; social factors, socio-economic status and attitudes; and factors highlighted by neuroscience. Longitudinal studies employing a componential approach are also detailed; however, unfortunately, this type of approach is relatively uncommon and therefore in some sections the discussion is mainly limited to non-componential evidence.

3. Foundational numerical skills

N Jordan et al. (2008, pp.46-58) define number sense as abilities that involve numbers and operations during the 3- to 6- year old period. On the whole, number sense tasks tend to require foundational number skills. For example, the number sense battery used by N Jordan et al. (2006, 2007) included tasks such as counting and number knowledge (e.g. which number is

bigger: 3 or 4). Non-verbal number sense skills are diverse and while there is evidence to suggest that some non-verbal number sense skills are present at infancy (e.g. Mix et al. 2002), there are debates about which abilities are present in that early time (e.g. Cohen and Marks 2002) On the other hand, some number sense skills are generally considered to be more sensitive to pre-school instruction (Dowker 2005). N Jordan et al. (2007) tested children on a number sense battery at six time points from ages 5-6. Number sense performance and growth in number sense were found to explain 66% of the variation in mathematics performance as measured by the Woodcock Johnson test at age 7, even after controlling for gender, income, age and reading ability. Given the similarities between the number sense battery and the items on the Woodcock Johnson test, the argument that number sense predicts later maths achievement could be viewed as circular. Arguably the real value of this research is that potentially children who will have weak mathematical development could be detected very early, as number sense tasks can be administered at the start of formal schooling. Furthermore, some non-verbal tasks (e.g. non-verbal magnitude comparison) could potentially be administered before formal schooling and used to predict later mathematics achievement.

Some aspects of number sense have emerged as better predictors of later mathematics achievement than others. Locuniak and N Jordan (2008) looked at how different aspects of number sense as measured at age 5 were related to later calculation fluency approximately two years later. Their number sense battery included the following tasks: counting, number knowledge, nonverbal calculation, story problems, and number combinations. When other variables, such as memory, reading and spatial ability were controlled for, number knowledge and to a greater extent, number combinations emerged as the best predictors of later maths performance (calculation fluency), whereas counting, non verbal calculation and story problems did not.

4. Reading and language

Verbal weaknesses can have a negative influence on maths development, as mathematics makes numerous demands on verbal ability; for example, the processing of speech sounds (Bull & Johnston 1997; Geary 1993; Hecht et al. 2001; Rourke & Conway 1997), and retrieval and retention of verbal number codes (Robinson et al. 2002). Additionally, there is neuropsychological evidence to suggest that the effect of language on mathematics varies across mathematical task, thus supporting the need for a componential approach. For example, exact calculation produces greater activation than approximate arithmetic in the angular gyrus, an area of the brain associated with language (Dehaene et al. 1999). There is also evidence to suggest that the effects of linguistic ability on addition are indirect, with linguistic effects mediated by skill on symbolic quantity tasks (Cirino 2011).

In a longitudinal componential study by N Jordan et al. (2006) examining the performance of 5-6 year old children, in which income status, gender and age were controlled for, reading ability was positively associated with performance on a wide range of mathematics tasks. The results indicated that even those tasks that are considered to be relatively non-verbal, such as estimation and non-verbal calculation (e.g. Dehaene et al. 1999), have some language requirements; for example, understanding the examiner's instructions. When the analysis focused on growth rates rather than performance levels, better readers did not have stronger growth than poorer readers on any of the tasks including story problems which is considered to be verbally demanding (N Jordan & Montani 1997; N Jordan & Hanich 2000). These results suggest that the poor readers started school with a disadvantage in many areas of mathematics, and they did not catch up with or fall further behind their typically achieving peers. However, as discussed previously, the lack of differences in growth rate may be attributed to the tasks being too difficult to capture sufficient growth and thus allow for meaningful statistical comparisons.

When sufficient levels of growth in mathematics are captured, the influence of phonological ability on mathematics achievement is evident for certain mathematical tasks (J Jordan et al. 2010). Children aged 5 years were classified by J Jordan et al. (2010) as one of the following subtypes: phonological difficulty (PD), co-morbid phonological difficulties and mathematical difficulties (PDMD) or typically achieving (TA). Children with phonological difficulty exhibited weaker mathematical development than typically achieving children, with approximately half of the PD subgroup meeting the criteria for PDMD at age 7 years. Further exploration indicated that those with phonological difficulties tended not to progress as well as typically achieving children on three out of four formal tasks - number facts, formal calculation and concepts. On the other hand, similar development was found for most informal tasks - numbering, number comparison and concepts. Informal mathematics is considered to be acquired through a child's interaction with their environment and some informal mathematical abilities are considered innate (Ginsburg & Baroody 2003). In contrast, formal mathematical knowledge refers to the mathematics taught at school and tends to have greater language requirements than informal mathematics (Dowker 2005). The weaker mathematical development of children with phonological difficulty in this study appears to be associated with the shift from informal to formal mathematics in the curriculum, which was reflected in the items on the standardised Test of Early Mathematical Ability 3 (Ginsburg & Baroody, 2003) from age entry points five to seven. Interestingly, not all PD children struggled to cope with the increasing verbal demands of the mathematics test, as many showed similar growth to TA children. Although not explicitly investigated, J Jordan et al. (2010) speculated that these children may have been able to compensate for their weaknesses. For example, they may have been motivated to do extra work at home, or to employ alternative cognitive strategies.

Verbal weaknesses also appear to have a negative impact on mathematics performance in older children aged 7-9 years. Specifically, N Jordan et al. (2002) found in a study using standardised tests of achievement that children with specific mathematical difficulties were more likely than those with comorbid mathematics and reading difficulties to outgrow their mathematical difficulties. Yet in another study (N Jordan et al. 2001, 2003) using the same sample and design but a separate battery of mathematical tasks, no effects of reading ability on growth rate were found for any of the following mathematical subtasks: exact calculation, story problems, approximate arithmetic, place, value, calculation principles, forced retrieval and written computation. Unlike the studies of N Jordan et al. (2002) and J Jordan et al (2010), in this study children were given the exact same items on each occasion. An effect of verbal ability on development may be absent simply because the actual verbal requirements of the test were static over time. None of these studies controlled for other cognitive variables such as working memory, and therefore caution must be shown when interpreting differences in performance at a single time point and in growth rate. In fact, in a study that did control for working memory (Locuniak and N Jordan 2008), it was found that reading ability did not predict calculation fluency. As this study only examined one mathematical task, it is unclear if this finding would generalise to other mathematical tasks.

5. Cognitive abilities

A case study from the Leverhulme study (Brown et al. 2008, pp.85-108) highlights how memory problems, in general, can impact negatively on a child's mathematical development. The authors present a series of case studies, one of which describes a child with weak mathematical development in place value and fractions. Closer inspection revealed incorrect answers to questions which had been answered correctly at an earlier age. In addition, they made errors on simple questions yet performed well in more difficult areas. Brown et al. suggest that this child's memory problems made it difficult for them to remember the procedural steps that they had been taught to answer these questions. In fact, they often tried to answer questions using procedural methods, but appeared to get answers correct only when they could remember the steps. This finding suggests that children with weak memory may have weaker growth or very uneven trajectories across some mathematical tasks compared to those with typical memory.

Much of the research looking at the influence of memory on mathematics has focused on the three components of the Baddeley and Hitch (1974) model of working memory: central executive, phonological loop and visuo-spatial sketchpad. The visuo-spatial sketchpad (VSSP) is considered to be important in areas of maths such as aligning columns of digits (Dowker, 2005), borrowing and carrying (Bull et al. 2008; Venneri et al. 2003); number magnitude and estimation (Dehaene et al. 1999), and subtraction and written calculation (Venneri et al. 2003). On the other hand, neuroimaging and evidence from children with visuospatial learning difficulty indicates that spatial skills in general are less important on tasks such as exact calculation (Dehaene et al. 1999) and addition (Venneri et al. 2003). The effects of spatial ability may be indirect; for example, Cirino (2011) found that the effect of spatial ability on addition skill is mediated by performance on symbolic quantity measures. The central executive (CE) function, storage and manipulation of information in long term memory, plays an important role in arithmetic. For example, a child could use this function to solve an arithmetic problem such as 5 + 2, by recalling the answer to another fact such as 4 + 3 from long term memory, then inferring the answer through the process of establishing equivalence (McLean & Hitch 1999). There is also some evidence to suggest that the executive function of inhibition of irrelevant information is linked to mathematics performance (e.g. Espy et al. 2004; Bull and Scerif 2001), although not all studies have found conclusive evidence of a relationship (e.g. van der Sluis et al. 2004). Geary (1993) argued that the phonological loop (PL) is important for solving arithmetic problems, as it is used to temporarily store both the problem and the solution, and the ability to do so will ultimately lead to the commitment of the problem and the solution to long term memory. Furthermore, the better the ability to store a problem and solution in the phonological loop, the stronger the long term memory representation will

become. In contrast, the phonological loop is unlikely to be heavily involved in tasks such as estimation which tend not to produce strong activation in language areas of the brain (Dehaene et al. 1999).

Cross-sectional and longitudinal studies which have looked at how the influence of working memory on mathematics achievement varies over time have produced conflicting findings. For each component there have been reports of weakening (e.g. Holmes & Adams 2006; Bull et al. 2008), stable (e.g. Geary in press) and increasing (e.g. Bull et al. 2008; De Smedt et al. 2009; Geary, in press) importance over time. There are a number of possible reasons for these conflicting findings such as not controlling for the effects of other components of working memory (e.g. Holmes et al. 2008), use of cross-sectional design (e.g. Holmes et al. 2008), and not controlling for other key variables such as reading ability. For example, when Bull et al. (2008) controlled for reading ability, weaker relationships were found between mathematics and working memory.

In many studies the composite mathematics test given to each age group was comprised of different mathematical tasks. Yet detailed item or subtask analysis was not performed to see if changes in the influence of working memory over time were related to changes in the test. In some cases, each age group was given the same type of mathematical task; however, more difficult items were presented to the older children. For example, when testing number knowledge De Smedt et al. (2009) gave younger children items requiring numbers from 1-10 but older children items with numbers 1-20. They acknowledged that the increasing importance of the phonological loop that they observed in older children may in part be due to this task involving more knowledge of the number system and therefore greater language demands. Other studies have tested children on completely different mathematical subtasks at each time point. For example, Bull et al. (2008) tested children on graphical representation at age 7 (not at age 4 years), which may explain why the visuo-spatial sketchpad was significantly related to mathematics performance for this age group only. The use of different tasks at each time point makes it difficult to know if the relationship between working memory and mathematics achievement varies over time because the items/tasks used have different cognitive requirements or if children are solving the tasks via different cognitive routes as they get older. There is considerable evidence to suggest that mathematics is not a unitary ability, but rather it is composed of numerous mathematics abilities, and therefore greater insights can gained by adopting a componential approach (Russell & Ginsburg, 1984; Denvir & Brown, 1986; Dowker, 1998, 2005; Gifford & Rockcliffe, 2008). A study employing a componential approach suggests that the role of the visuo-spatial sketchpad varies across task, and over time, when subcomponents of the visuo-spatial sketchpad are considered separately. Specifically, Holmes et al. (2008) found that the visuo-spatial sketchpad explained similar amounts of variation in number and algebra skills at ages 7 ½ and 9 ½ years, yet did not explain significant variation in

performance on shape, space and measures, handling data or mental arithmetic for either age group. Interestingly, they found that the spatial subcomponent of the VSSP was more important for younger children, whereas the visual subcomponent was more important for older children's mathematics. These results indicate the importance of considering both mathematical subtask and type of working memory subcomponent separately.

6. Teaching, Curriculum & classroom experiences

Before children begin formal schooling, heterogeneity in mathematical abilities is already evident, and while some of this variability can be attributed to individual differences in aptitude, other factors such as learning experiences prior to attending school also explain unique variation (Crosnoe 2006; Sadowski 2006). Teaching has been highlighted as a limiting factor on children's mathematics development by Ofsted (2009) which found that many teachers needed help to enhance their own numeracy skills, and needed more guidance on teaching children the value of mathematics in everyday life.

Crosnoe et al. (2010) proposed that if a child has poor maths ability when they enter school, their trajectory will in part be influenced by the type of curriculum to which they are exposed and by their teacher-student relationships. Crosnoe et al. (2010) outlined how three different types of curriculum could influence a low achieving child's development in mathematics. 1) *Basic skills curriculum* - if those with weak maths skills are consistently exposed to a less challenging maths curriculum than their high ability peers they will not be able to catch up. 2) *Common curriculum* - if given the same curriculum as their average/high achieving peers, although the material is more challenging, they have the potential to catch up. However, as many of these children enter school with poor foundation skills, they may find a more challenging curriculum too overwhelming. 3) *Common curriculum with basic skills supplements* - to give low achieving children the best chance of narrowing the achievement gap, they should be exposed to the same curriculum as their high achieving peers, but should receive extra support from teachers (e.g. help with their basic skills). Crosnoe et al. (2010) found evidence to support their prediction that a *common curriculum with basic skills supplements* is more developmentally appropriate for those with weak mathematical ability. Their study revealed that children with low maths skills upon entry to school exposed to this curriculum were able to narrow their achievement gap as long as they had good relations with their teachers. However, the gap was only narrowed to a small extent, suggesting that the effect of this type of curriculum in comparison to other developmental influences on mathematics is relatively small.

The longitudinal Leverhulme study (Brown et al. 2008, pp.85-108) of children aged 5-11 provides insights into classroom effects on development. Using data gained from researchers' observations, and interviews with teachers, mathematics co-ordinators and head teachers, inferences were made about the effects of classroom practice on mathematical development. Uneven trajectories were associated with changes in classroom practice, and these effects on trajectory depended on the ability level of the child. For example, in one year group, a small proportion of children struggled to understand their year 5 teacher's explanations of place value, yet made very rapid progress when exposed to the more supportive and relaxed approach of their year 6 teacher. In contrast, most children did not appear to find this teaching style challenging enough, because the class as a whole began to fall behind the rest of the sample during year 6. Poor teaching in general, rather than a teaching style that favours some children, can lead to the whole class displaying uneven growth in mathematics. In one classroom, the observational data indicated that the year 1 teacher had unclear objectives and undemanding teaching, yet the children progressed well under a different teacher in year 2.

7. Social factors: socio-economic status and attitudes

Higher income levels have been associated with better performance on a wide range of mathematical tasks in young children (N Jordan et al. 2006). National and international cross sectional data from the Trends in International Mathematics and Science Study (TIMSS) 2007 provide some indication as to why children from low income backgrounds do not perform as well in mathematics. At ages 10 and 14 years, those who have more books in their home tend to have better maths achievement. A computer at home with an internet connection was also associated with better achievement. Growth curve modelling provides evidence to suggest that those from lower income backgrounds do not progress as well as those from higher income backgrounds on story problems (N Jordan et al. 2006) but progressed at a similar rate on seven other number sense tasks and number sense as whole. As the overall levels of growth on all tasks in the N Jordan et al. (2006) study were small, it is possible that the other mathematical tasks were not sensitive enough to allow the detection of growth rate differences.

N Jordan et al. (2006) suggested that these growth rate differences on the story problems task can be attributed to the high language and auditory attention requirements of the task (e.g. Levine et al. 1992). However, this seems unlikely given that reading ability did not predict growth in story problems. A more likely explanation provided by N Jordan et al. (2006) is that children are acquiring these skills outside the classroom.

Number sense as a whole was later modelled using 6 time points in the same group of children aged 5-6 (N Jordan et al. 2007). In contrast to when only four time points were used (N Jordan et al. 2006), income was found to be a significant predictor of growth rate in number sense. This may have been due to the greater accuracy in growth rate measurement achieved by using more time points or to greater growth at the last two time points.

The TIMSS (2007) data show that higher achievers tend to have more positive attitudes towards mathematics and greater self confidence in learning mathematics at ages 10 and 14. Internationally, high levels of self-confidence in maths were more prevalent in 10 year olds (57%) than in 14 year olds (43%). This pattern was also found for high levels of positive affect towards maths in 10 (72%) and 14 (54%) year olds. Declining levels of self-confidence throughout school towards mathematics were also found by the Scottish Survey of Achievement (2008), as well as lower levels of enjoyment and interest in older children. The lower levels of positive attitudes in older children mirror the lower levels of older children achieving the expected level. However, it is not clear if poor attitudes lead to lower performance, if lower performance leads to lower attitudes or if the relationship is bi-directional (e.g. Dowker 2005, pp 250). Measuring attitudes alongside mathematics performance longitudinally rather than relying on cross-sectional data, will help to untangle cause and effect.

8) Neuroscience

Dehaene et al. (2003) proposed that performance on complex arithmetical tasks depends on interactions between extra-parietal areas and three regions within the parietal lobe. Each parietal region is considered to be involved in a different aspect of mathematics, namely, numerical magnitude processing, language and phonologically mediated processing, and attentional and spatial orientation on the number line. LeFevre et al. (2010) have developed a model comprising three pathways (quantitative, linguistic and spatial) based on the structure outlined by Dehaene et al. In the LeFevre et al. study, behavioural data indicated that at age 4, linguistic ability was related to number naming but not to non-linguistic arithmetic, while quantitative ability was related to non-linguistic arithmetic but not to number naming, indicating that quantitative and linguistic ability are two separate pathways. On the other hand spatial ability was related to both non-linguistic arithmetic and number naming tasks, and this was interpreted as evidence to support a third spatial pathway. Similar to J Jordan et al. and N Jordan et al. the study found evidence to suggest that the linguistic pathway is more important in older children's mathematics. Specifically, linguistic ability uniquely predicted the

performance of 7 year olds on a wide variety of mathematical tasks including word reading, numeration, calculation, geometry and measurement.

Unlike many of the other studies of mathematical development reviewed in this chapter, neuroscience studies tend to focus on mathematical subtasks separately. This approach is adopted for methodological reasons and because evidence suggests that different mathematical tasks engage different regions of the brain (e.g. Dehaene et al. 1999). Furthermore, the tasks tend to be very simple in nature, in order to permit researchers to develop an understanding of the basic skills that are required to perform complex tasks (Kaufmann 2008, pp.1-12). However, even studies of basic mathematical skills adopting a neuroscientific approach to development are uncommon, with most studies focusing solely on adults (Kaufmann 2008, pp.1-12).

Studies focusing on the same task have produced mixed evidence regarding developmental patterns of brain activation. Age-related shifts in brain activation from prefrontal to parietal areas were found for both symbolic (Ansari et al. 2005) and non-symbolic number processing (Ansari & Dhital 2006). This has been interpreted as reflecting a developmental shift from using areas associated with working memory and attentional resources, towards more automatic processing. On the other hand, similar patterns of brain activation on tasks involving both symbolic and non-symbolic number processing were reported in children and adults by Temple and Posner (1998) and Cantlon et al. (2006). Considering that Temple and Posner (1998) used ERP source localisation, a method with limited spatial resolution, overly strong conclusions cannot be drawn about the spatial activation patterns of both children and adults. Ansari et al. (2008, pp.13-43) have suggested that the passive fMRI paradigm used by Cantlon et al. (2006) assessed low level numerical processing rather than the ability to translate representations, and this may explain why age-related differences similar to those found by Ansari et al. (2005, 2006) were not observed.

Addition, subtraction and multiplication have also been studied in both children and adults. When Kawashima et al. (2004) compared children aged 9-14 years with adults on these tasks they found broadly similar functional activation in both groups. By contrast, Rivera et al. (2005) found that between the ages of 8 and 19 when performing addition and subtraction, there was decreased activation in areas of the pre-frontal and anterior cingulate cortex, areas that have been linked to working memory and attentional resources. The opposite pattern, increased activation with age, was observed for regions in the left parietal cortex

which is associated with arithmetic processing.

The inconsistencies between developmental imaging studies of mathematical abilities may be partly due to use of a cross-sectional design. With such a design, it is difficult to distinguish between group-specific differences and developmental differences (Ansari 2008, pp.13-43). The use of a cross-sectional design does not allow questions about individual differences in brain activation to be addressed, whereas a longitudinal approach would indicate if changes in activation are linear in some children, and curvilinear for others, as was found with behavioural data (J Jordan et al. 2009).

9. Advances in research methods and future directions

The growing popularity of longitudinal statistical techniques has helped to build up a richer picture of children's mathematical development. Growth curve analysis is preferable to other techniques such as anova for examining development, as it accounts for measurement error. This is clear when we compare the results from the same study when analysis was by anova (Hanich et al. 2001) and when growth curve modelling was used (N Jordan et al. 2003). The anova results indicate that at age 7, children with mathematical difficulty outperformed those with comorbid mathematics and reading difficulty on exact calculation and story problems. However, the growth curve analysis shows that at age 9 MD had an advantage over MDRD on story problems but not exact calculation, and that they performed better on calculation principles. As there were no significant differences in growth rate in this study this indicates that this pattern did not change between ages 7 and 9. Therefore the difference detected by the anova at age 7 on exact calculation may have reflected measurement error rather than a true difference.

Another advantage of growth curve modelling is that it allows growth rate and shape to be compared statistically. For example, N Jordan and colleagues used growth curve modelling to compare growth rate and shape in mathematics of different maths and reading ability groups. However, as mentioned previously, even within a group of children with typical maths development, the mean trajectory for a group may bear little resemblance to the trajectories of the individuals within the group. Additionally, within mathematical difficulty subtypes there may be too much variation in mathematics performance within each subtype for meaningful comparisons to be made, as many factors other than reading are associated with mathematical difficulty (e.g. working memory, number sense and attitudes). More recent studies have moved away from subtyping and have used growth curve modelling techniques which are able to explain greater amounts of variation in growth rate. For example, N Jordan et al. (2006) used growth curve modelling with multiple cognitive and demographic predictors to explain heterogeneity in development of number sense. The growth curve modelling methods being used to study mathematics development and individual differences in mathematical development are becoming progressively more advanced and more appropriate for examining development and heterogeneity. For example, rather than using one measurement of a variable to predict growth in mathematics, N Jordan et al. (2009) examined if growth in number sense predicted growth in mathematics. This is a more appropriate way of modelling predictors of maths development, such as reading ability, because they are not static in nature and individuals vary considerably in terms of rate and shape of growth on them. While growth curve modelling has been used to show how predictors affect the overall rate of development ,

multilevel modelling has been shown to be particularly useful at addressing questions about if and when the influence of predictors on mathematical development changes (e.g. Geary In Press).

Growth mixture modelling and regression have been used to characterise the shape and/or steepness of each individual child's trajectory (e.g. N Jordan et al. 2006; J Jordan et al. 2009). Using regression to fit individual trajectories for each child to mathematical tasks can provide educationists with a very rich picture of a child's needs. This type of information would be particularly useful to those delivering interventions that are individually tailored to meet the needs of the child (e.g. Dowker & Sigley 2010).

In cases where individually tailored intervention is not feasible, growth mixture modelling can provide a more simplified picture by grouping children with similar trajectories together. Furthermore, predictors can be added to these models to reveal the different factors associated with different patterns of development. Being able to identify which trajectory classification a child is likely to fall into enables predictions to be made about whether they are likely to outgrow their difficulty with little assistance or if intervention is likely to be needed (N Jordan et al. 2006).

In addition to using developmentally appropriate methods, future studies would benefit from a componential approach. In light of the findings of Holmes et al. (2008), it is quite likely that many of the findings from non-componential studies will not generalise to all mathematical abilities.

Using neuroscience in conjunction with other methods may provide further insights into the developmental issues discussed in this chapter. For example, such a combined approach may reveal if changes in strategy use as measured by behavioural data are mirrored by changes in activation patterns. Also as many of the studies of the influence of working memory over time are correlational, the neuroimaging evidence could indicate if changing relationships correspond to changes in brain activation. In addition, neuroimaging evidence may reveal if different trajectory types, or even the same trajectory type, are associated with different cognitive strategies.

Table 1 Key findings and concepts by section

Section	Key findings and concepts
Characteristics of trajectories	A child's trajectory can be described in terms of its initial and final status, growth rate and shape of growth (e.g. linear, or curvilinear). Componential and individual differences evidence indicates that there is considerable heterogeneity across mathematical tasks and within children in terms of trajectory characteristics.
Foundational numerical skills	Foundational numerical skills are required in number sense tasks such as counting and number knowledge. Number sense development is strongly associated with later mathematical performance.
Reading and language	Verbal skills are relatively more important in the formal mathematics taught at school than in informal mathematics. In studies using standardised mathematical tests, verbal skills play a greater role in older children's mathematics performance. This increase in importance has been associated with the shift from informal to formal mathematics in the curriculum and in some standardised mathematics tests.
Cognitive abilities	Evidence regarding the influence of working memory on mathematical development is inconsistent. Future studies may yield more consistent findings if a componential approach is adopted and variations in the composition of test administered to different age groups are considered.
Teaching, curriculum & classroom experiences	Type of curriculum and teaching approach can influence a child's trajectory; for example, uneven trajectories may be associated with changes in teacher and teaching approach throughout schooling. Whether the effect of teaching approach on mathematics development is positive or negative depends on a child's learning style.
Social factors: socio-economic status and attitudes	Income has been associated with growth on some tasks such as number sense and story problems. Younger children report more enjoyment in mathematics and are more likely to meet the expected level compared to older children; however, the direction of this relationship is unclear.
Neuroscience	Methodological issues such as not adopting a longitudinal approach may explain why evidence regarding age-related shifts in brain activation when performing mathematical tasks is mixed.
Advances in research methods & future directions	Growth curve modelling has identified some of the factors associated with individual differences in growth on mathematical tasks. Individual regression analyses identify the shape of each child's development on mathematical tasks, while growth mixture modelling groups similar trajectories together, thus providing useful information for individually tailored or group-based interventions.

11. Conclusion

The key findings and concepts outlined in each section are presented in Table 1. Cross-sectional and longitudinal data consistently indicate that mathematical difficulties are more prevalent in older than in younger children (e.g. Department of Education, 2011). Assessing children's mathematical performance longitudinally over two time points indicates if children perform at a satisfactory level or above, outgrow their difficulty, have persistent difficulty, or develop difficulties. On the other hand, by following children over a greater number of time points, it is possible to detect uneven developmental trajectories (J Jordan et al. 2009). The effects of a range of cognitive and social factors on mathematical development have been studied. However, for some of these variables (e.g. working memory and verbal ability) the effects on mathematical development are unclear. Such inconsistencies may be explained by the non-componential approach adopted by many studies. For example, Holmes et al. (2008) found that the influence of the visuo-spatial sketchpad on mathematical development varied across task. Studies examining individual differences in the shape of children's trajectories across mathematical tasks also indicate that there is a need to adopt a componential approach when studying children's mathematical development. This evidence suggests that interventions should be tailored to each child's needs rather than treating all children's mathematical difficulties as homogenous. Indeed, there is now evidence to suggest that the most successful interventions are those that focus on the specific weaknesses of each individual child. For example, Dowker & Sigley (2010) compared the effectiveness of three levels of intervention: 1) individually administered and tailored to strengths and weaknesses; 2) individually administered but not tailored to strengths and weaknesses; 3) no intervention. The individually tailored approach produced the greatest improvements in children's arithmetical development. Dowker & Sigley (2010) emphasised the need for more research which evaluates the relative effectiveness of individually tailored interventions for children with different levels/types of arithmetical difficulty. Furthermore, as the effectiveness of the intervention was only assessed at one point in time by Dowker & Sigley (2010), it is not known how the intervention affected the children's trajectories on the arithmetical tasks. For example, would the intervention have led to typical growth or even steeper than typical growth, allowing children with difficulty to catch up with their typically achieving peers? Future intervention and experimental studies adopting a componential approach and employing statistical methods appropriate for studying development are needed to build a clearer picture of how cognitive and social variables influence children's mathematical abilities over time.

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