Hindawi Publishing Corporation
Applied Computational Intelligence and Soft Computing
Volume 2014, Article ID 971894, 7 pages
http://dx.doi.org/10.1155/2014/971894

Research Article

Application of DEO Method to Solving Fuzzy Multiobjective Optimal Control Problem

Latafat A. Gardashova

Azerbaijan State Oil Academy, Azadlyg avenue, 20, Baku Az1010, Azerbaijan

Correspondence should be addressed to Latafat A. Gardashova; latsham@yandex.ru

Received 29 November 2013; Revised 27 January 2014; Accepted 27 January 2014; Published 27 February 2014

Academic Editor: Francesco Carlo Morabito

In the present paper a problem of optimal control for a single-product dynamical macroeconomic model is considered. In this model gross domestic product is divided into productive consumption, gross investment, and nonproductive consumption. The model is described by a fuzzy differential equation (FDE) to take into account imprecision inherent in the dynamics that may be naturally conditioned by influence of various external factors, unforeseen contingencies of future, and so forth. The considered problems are characterized by four criteria and by several important aspects. On one hand, the problem is complicated by the presence of fuzzy uncertainty as a result of a natural imprecision inherent in information about dynamics of real-world systems. On the other hand, the number of the criteria is not small and most of them are integral criteria. Due to the above mentioned aspects, solving the considered problem by using convolution of criteria into one criterion would lead to loss of information and also would be counterintuitive and complex. We applied DEO (differential evolution optimization) method to solve the considered problem.

1. Introduction

The studies devoted to solving optimal control problems for dynamic economic models have a long history. The models of economic growth attracted a large interest in the area of mathematical economics [1–3]. Recently, intensive revisiting of the growth models took place [4, 5] as a result of the improvements in existing models and progress in development of the associated analytical techniques [6–8].

The intensive research in this direction was also conditioned by the support of the developed countries as the latter were interested in construction of accurate economic growth models to improve their economic development [9–11]. A lot of various mathematical methods of measuring the effectiveness of economic growth were suggested. In [12] for describing the change of production and accumulated *R&D* investment in a firm, a nonlinear control model is considered and analyzed. They provide a solution of an optimal control problem with *R&D* investment rate as a control parameter. They also analyze an optimal dynamics of economic growth of a firm versus the current cost of innovation. The results of analytical investigation showed that the optimal control can be of the two types depending on the model parameters: (a) piecewise constant with at most two switchings and (b) piecewise constant with two switchings and containing a singular arc.

In [13] the authors provide results of comparison of solution methods for dynamic equilibrium economies in terms of computing time, implementation complexity, and accuracy. The considered methods are the stochastic neoclassical growth model, the finite elements method, Chebyshev polynomials, and value function iteration. Authors conduct analysis of the obtained results.

Modeling and stability analysis in fuzzy economics is discussed in [14]. In this paper, the author considers different problems of fuzzy economics, mainly time path and stability of fuzzy dynamics of economic systems, linguistic modelling of economic agents, and neurofuzzy time-series forecasting of macroeconomic processes. One of the main problems considered in [14] is stability of various macroeconomic dynamical systems described by fuzzy differential equations which are used to take into account uncertainty inherent in economic dynamics.

In [15] a fuzzy multiobjective linear programming for determination of the optimal land use of regions by taking into account economic, social, and environmental objectives and implications.

Reference [16] is devoted to the low-carbonized adjustment of energy structure in the area of world natural and cultural heritage. The authors suggest a fuzzy multiobjective optimization to investigate the relationship between energy and social economics and environment. The use of multiobjective approach is justified by the fact that sustainable development is obtained as a tradeoff between economic growth and environmental preservation and, therefore, is multidimensional concept that encompasses economic, social, ecological, technological, and ethical factors. As a result, achievement of sustainable development requires dealing with highly conflicting criteria.

In [17] some important economic and environmental criteria for the problems of multiobjective programming and their conflicts are defined.

In [18] some of the key insights of the field of international trade and economic growth are considered within a unified and tractable framework. The authors study a series of theoretical models which have a common description of technology and preferences but are of different assumptions concerning trade frictions. It is shown that on the base of comparison of the predictions of these models one can identify a variety of channels through which trade influences the dynamics and geographical distribution of world income.

The combination of fuzzy logic tools with multiobjective optimization and multicriteria decision making has a great relevance in the literature [19–23]; for example, fuzzy sets are considered for interactive multiobjective optimization in [22, 23].

A single-product dynamical macroeconomic model was first suggested by Kantorovich and Zhiyanov in [24]. They also suggested a principle of differential optimization for the developed model. Leontyev [25] suggested a single-product dynamic macroeconomic model in which gross domestic product is divided into productive consumption, gross investment, and nonproductive consumption.

In the existing works, a qualitative analysis of economic systems and development of analytic algorithms of optimization are based on sufficient conditions of optimality [26] or the maximum principle [27]. Nowadays, an analysis of optimal control problems on the base of the perturbation theory attracted a high interest. Especially, as it is shown in [28, 29], they consider application of asymptotic estimates to models to cope with nonlinearity, computational instability, high orders, and so forth. In [26] they study an optimal control problem by using a small parameter method and conduct comparative analysis with the other existing approaches. The existing approaches to solving optimal control problems suffer from a series of disadvantages. From one side, the existing approaches are not developed to deal with uncertainty inherent in economic dynamics but are based classical mathematical formalism. Mainly, there exist two types of uncertainty related to economic dynamics: probabilistic and possibilistic (fuzzy). Modelling of probabilistic uncertainty requires the use of very restrictive assumptions including availability of good statistical data. Modelling of fuzzy uncertainty is a more realistic task and allows us to describe imprecise evaluation of variables of interest coming from human expertise [30].

In the existing works, economic uncertainty is mainly captured by using stochastic techniques. However, the use of stochastic techniques is based on crucial assumptions which notably constrain its use for adequate modelling of real-world economic uncertainty. These methods can be successively applied when sufficient knowledge is available on the probability density functions of all the variables involved in the computations, which is a rather rare case. In contrast, very often prediction or estimation of variables of interest is subjective, for example, expert-opinion based. The main sources of possibilistic uncertainties include [31].

In order to take into account real-world uncertainty in formal models, they usually use sensitivity analyses. However, this is not sufficient to investigate interaction of uncertainties and their dynamics. In order to deal with possibilistic Zadeh suggested the fuzzy set theory and fuzzy logic [32].

These theories have passed a long way and provided a lot of successful applications. In particular, control systems based on fuzzy logic were successfully applied for control of various complex and uncertain objects and provided better results than their classical counterparts. Fundamental approach to modelling behaviour of a dynamical system under possibilistic uncertainty is fuzzy differential equations (FDEs) [33].

In the present paper we suggest an approach to decision making in a single-product dynamic economic model. The considered problem is represented as a multiobjective optimal control problem of dynamics of capital under uncertain relevant information. The optimal control is based on four objective functions and two control variables. The application of statistical approaches to handle uncertainty for this problem requires imposing too restrictive assumptions which are unlikely to match real environment especially in the light of the recent unsustainable and hardly predictable behaviour of the present economic reality. In order to model possibilistic uncertainty in dynamics of capital a linear FDE is used.

The use of FDE allows us to take into account imprecision inherent in the dynamics that may be naturally conditioned by influence of various external factors, unforeseen contingencies of future, and so forth.

2. Statement of the Problem

Let us consider a single-product dynamic macroeconomic model which reflects interaction between factors of production when a gross domestic product (GDP) is divided into productive consumption, gross investment, and nonproductive consumption as the performance of production activity. In its turn productive consumption is assumed to be completely consumed on capital formation and depreciation. These processes are complicated by a presence of possibilistic, that is, fuzzy uncertainty which is conditioned by imprecise evaluation of future trends, unforeseen contingencies and other vagueness and impreciseness inherent in economical processes. Under the above mentioned assumptions the considered dynamic macroeconomic model can be described by the following FDE:

$$\frac{d\tilde{x}}{dt} = \frac{1}{q}\left((1-a)\tilde{u}_1 - \mu\tilde{x} - \tilde{u}_2\right). \tag{1}$$

Here $\tilde{x}$ is a fuzzy variable describing imprecise information on capital, that is, fuzzy value of capital, $\tilde{u}_1$ is a fuzzy value of GDP (the first control variable), $\tilde{u}_2$ (the second control variable) is a fuzzy value of a nonproductive consumption, and $a, \mu, q > 0$ are coefficients related to the productive consumption, net capital formation, and depreciation, respectively.

Let us consider a multiobjective optimal control problem of (1) within the period of planning $[t_0, T]$ with four objective functions (criteria): $\tilde{J}_1$—profit, $\tilde{J}_2$—reduction of production expenditures of GDP, $\tilde{J}_3$—a value of capital at the end of period $[t_0, T]$, and $\tilde{J}_4$—a discount sum of a direct consumption over $[t_0, T]$. The considered fuzzy multiobjective optimal control problem is formulated below [34]:

$$\sup_{\tilde{\mathbf{u}}\in\mathcal{U}}\Bigg(\tilde{J}_1(\tilde{\mathbf{u}}) = \int_{t_0}^{T} p(t)\,\tilde{u}_2(t)\,dt, \qquad \tilde{J}_2(\tilde{\mathbf{u}}) = -c\int_{t_0}^{T}\left|\tilde{u}_1(t)\right|dt,$$
$$\tilde{J}_3(\tilde{\mathbf{u}}) = \tilde{K}(T), \quad \tilde{J}_4(\tilde{\mathbf{u}}) = \int_{t_0}^{T}\theta(t)\,\tilde{u}_2(t)\,dt\Bigg) \tag{2}$$

subject to

$$\begin{gathered}
\frac{d\tilde{x}}{dt} = \frac{1}{q}\left((1-a)\tilde{u}_1 - \mu\tilde{x} - \tilde{u}_2\right),\\
\tilde{x}(t) \in \mathscr{E}^1, \quad t \in [t_0, T], \quad \tilde{x}(t_0) = \tilde{x}_0,\\
\tilde{x}(T) \in \mathscr{K}(T),\\
\mathscr{K}(T) = \left\{\tilde{x} \in \mathscr{E}^1 : \tilde{x} \le \tilde{x}(T) \le \tilde{x}\right\},\\
\tilde{\mathbf{u}} = (\tilde{u}_1, \tilde{u}_2)^T \in \mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \subset \mathscr{E}^2,\\
\mathcal{U}_1 = \left\{\tilde{u}_1 \in \mathscr{E}^1 : \tilde{u}_1 \le \tilde{u}_1(t) \le \tilde{u}\right\},\\
\mathcal{U}_2 = \left\{\tilde{u}_2 \in \mathscr{E}^1 : \tilde{u}_{2*} \le \tilde{u}_2(t) \le \tilde{u}_2^*\right\}.
\end{gathered} \tag{3}$$

Here $p(t)$ is a price of production unit produced at the time t, $\theta(t)$ is the discount function, $[t_0, T]$ is the optimization period, and $c = \text{const} > 0$.

In this work we have decided to solve a problem of optimal control for a single-product dynamical macroeconomic model with the help of a method of differential evolution.

Recently many evolutionary algorithms have been proposed for global optimization of nonlinear, nonconvex, and nondifferential functions. These methods are more flexible than classical as they do not require differentiability, continuity, or other properties to hold for optimizing functions. Some of such methods are genetic algorithm (GA), particle swarm optimization (PSO) [35, 36], and differential Evolution (DE) optimization [37–39].

Also all of them have their modified versions, such as Group Principle GA with multipoint crossover and mutation, DE + Clustering [40], and Multifitness function GA. Among many other evolutionary algorithms DE is one of the highest performance. DE is a faster and improved version of GA. As opposite to GA, which uses binary coding, DE uses real coding of float point numbers. The advantage of DE over standard GA is higher performance, efficiencies for numerical problems, not using time-consuming "coding/decoding" transformations, chromosome/gen structures, and other manipulations with binary structures.

In [38] a heuristic approach for minimizing possibly nonlinear and nondifferentiable continuous space functions is presented. DEO method is a simple and efficient heuristic for global optimization over continuous spaces. By means of an extensive test bed it is demonstrated that DEO method converges faster and with more certainty than many other acclaimed global optimization methods. The DEO method requires few control variables, is robust, easy to use, and lends itself very well to parallel computation. It should be noted that dynamic programming is very computationally intensive and even more for fuzzy problem. Therefore, here we use DEO method.

3. Method of Solution

As a stochastic method, DE algorithm uses initial population randomly generated by uniform distribution, differential mutation, probability crossover, and selection operators. The population with *ps* individuals is maintained with each generation. A new vector is generated by mutation which in this case is randomly selecting from the population 3 individuals: vector indexes $r_1 \ne r_2 \ne r_3$ and adding a weighted difference vector between two individuals to a third individual (population member).

The mutated vector then undergoes crossover operation with another vector generating new offspring vector.

The selection process is done as follows. If the resulting vector yields a lower objective function value than a predetermined population member, the newly generated vector will replace the vector with which it was compared with in the following generation.

Extracting distance and direction information from the population to generate random deviations results in an adaptive scheme with excellent convergence properties. DE has been successfully applied to solve a wide range of problems such as image classification, clustering, optimization, and so forth.

Figure 1 shows the process of generation new trial solution X_{new} vector from randomly selected population members X_{r1}, X_{r2}, X_{r3} (Vector X_4 is then the candidate for replacement by the new vector, if the former is better or lower in terms of the DE cost function). Here we assume that the solution vectors are of dimension 1 (i.e., 2 optimization parameters).

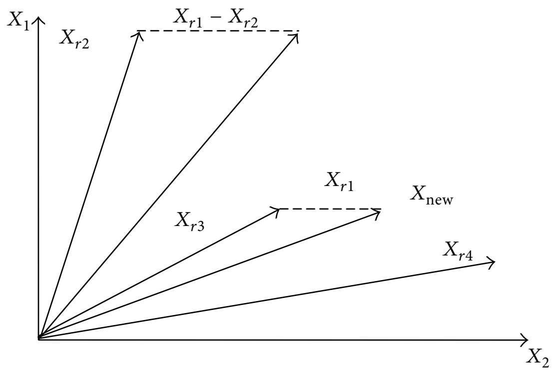


Figure 1: The process of generation new trial solution.

4. Simulation Results

Let us consider solving of the problem (1)–(3) on the base of the method suggested in Section 4 under the following data:

$$\widetilde{J}_1 = \Delta \sum_{n=0}^{N} p \left(a_2 \left(\frac{b_1 (1-a) - b_2}{(1-a) a_1 - (\mu + a_2)} + \left(\widetilde{K}_0 + \frac{b_2 - b_1 (1-a)}{(1-a) a_1 - (\mu + a_2)} \right) \times e^{((1-a)a_1 - (\mu + a_2)) \cdot n \cdot \Delta} \right) + b_2 \right) \longrightarrow \max,$$

$$\widetilde{J}_2 = -c\Delta \sum \left| a_1 \left(\frac{b_1 (1-a) - b_2}{(1-a) a_1 - (\mu + a_2)} + \left(\widetilde{K}_0 + \frac{b_2 - b_1 (1-a)}{(1-a) a_1 - (\mu + a_2)} \right) \times e^{((1-a)a_1 - (\mu + a_2)) \cdot n \cdot \Delta} \right) + b_1 \right| \longrightarrow \max,$$

$$\widetilde{J}_3 = \frac{b_1 (1-a) - b_2}{(1-a) a_1 - (\mu + a_2)} + \left(\widetilde{K}_0 + \frac{b_2 - b_1 (1-a)}{(1-a) a_1 - (\mu + a_2)} \right) \times e^{((1-a)a_1 - (\mu + a_2)) \cdot N \cdot \Delta} \longrightarrow \max,$$

$$\widetilde{J}_4 = \Delta \sum_{n=0}^{N-1} e^{rn\Delta} \left(a_2 \left(\frac{b_1 (1-a) - b_2}{(1-a) a_1 - (\mu + a_2)} + \left(\widetilde{K}_0 + \frac{b_2 - b_1 (1-a)}{(1-a) a_1 - (\mu + a_2)} \right) \times e^{((1-a)a_1 - (\mu + a_2)) \cdot n \cdot \Delta} \right) + b_2 \right) \longrightarrow \max,$$

Table 1: Crisp solution.

a_1	a_2	b_1	b_2	$J_i, i = 1 \ldots 4$
0,108092	1,37E − 05	578,5196	549,9741	j_1 = 1815005,64
2,35E − 10	2,48E − 07	600	549,9995	j_2 = −659,9999
0,048176	2,48E − 19	503,6475	550,0001	j_3 = 5908,70970
0,100707	1,63E − 09	578,9494	550	j_4 = 1151,10534

$$\widetilde{600} \le a_1 \left(\frac{b_1 (1-a) - b_2}{(1-a) a_1 - (\mu + a_2)} + \left(\widetilde{K}_0 + \frac{b_2 - b_1 (1-a)}{(1-a) a_1 - (\mu + a_2)} \right) \times e^{((1-a)a_1 - (\mu + a_2)) \cdot n \cdot \Delta} \right) + b_1 \le \widetilde{800},$$

$$\widetilde{450} \le a_2 \left(\frac{b_1 (1-a) - b_2}{(1-a) a_1 - (\mu + a_2)} + \left(\widetilde{K}_0 + \frac{b_2 - b_1 (1-a)}{(1-a) a_1 - (\mu + a_2)} \right) \times e^{((1-a)a_1 - (\mu + a_2)) \cdot n \cdot \Delta} \right) + b_2 \le \widetilde{550},$$

$$n = 0, \ldots, 9 \quad c = 0.5 \quad \Delta = 0.2 \quad N = 10$$

$$a = 0.05 \quad p = 1500 \quad \mu = 0.08 \quad q = 0.95$$

$$r = 0.05 \quad \widetilde{K}_0 = (1900, 2000, 2100)$$

$$\widetilde{600} = (580, 600, 620) \quad \widetilde{800} = (780, 800, 820)$$

$$\widetilde{450} = (430, 450, 470) \quad \widetilde{550} = (530, 550, 570)$$

$$a_1 \ge 0, \ a_2 \ge 0. \tag{4}$$

In addition, the following parameters are used in DE: the population size (NP), the crossover constant (CR), and the weighting factor (F) [38].

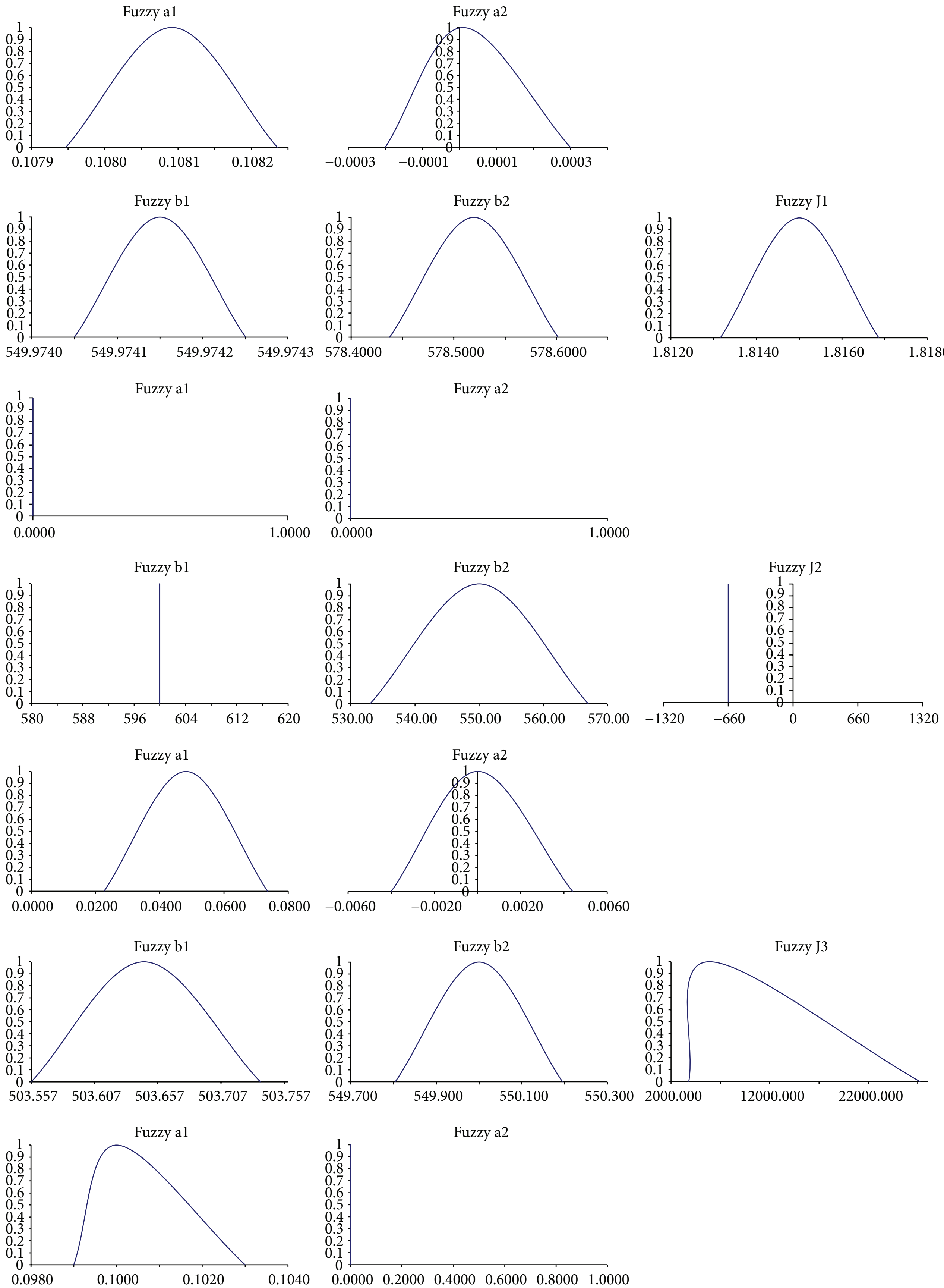
The program written on C# was used for computer simulation of the problem. The DE's parameters used were NP = 100, CR = 0.9, and F = 1.0.

The time used for solving the problem was 15–23 minutes. The problem was run for the crisp and fuzzy cases.

The results of crisp and fuzzy solutions are shown in Table 1 and Figure 2. Solution of the problem by using the DEO method ended up with the results in Table 1 and Figure 2.

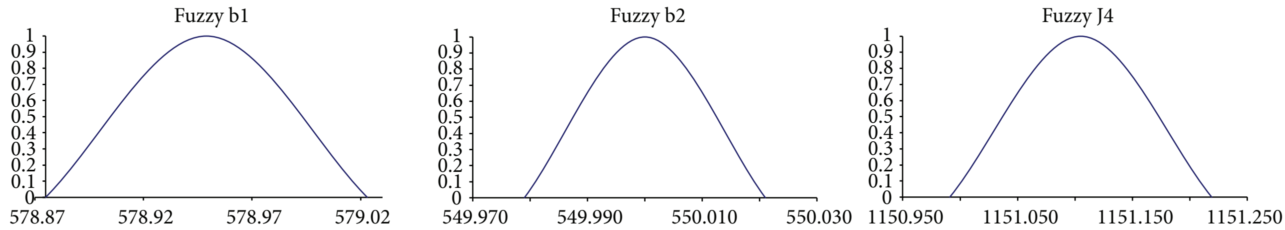
5. Conclusions

We applied DEO method to solving fuzzy multiobjective optimal control problem for a single-product dynamical macroeconomic model. For such problems, application of the well-known existing approaches would be complex both form intuitive and computational points of view. Application



(a)

Figure 2: Continued.



(b)

Figure 2: Fuzzy solution.

of DEO method allowed obtaining intuitively meaningful solution for the considered problem complicated by four conflicting criteria and nonstochastic uncertainty intrinsic to real-world economic problems. The fuzzy approach allows us to consider many internal and external effects. The solution obtained in fuzzy model is more sustainable and the objective functions in this case are less sensitive for changes. Besides the experience has shown that in DE, the processing time is relatively small. The results obtained in the paper show validity of the applied approach. The applied approach is characterized by a low computational complexity as compared with the existing methods for solving multiobjective optimal control problems.

Conflict of Interests

The author declares that there is no conflict of interests.

Acknowledgment

The author would like to thank Professor R. A. Aliyev for providing useful advice and a great support in implementing this research work.

References

[1] K. J. Arrow, "Application of control theory to economic growth," in *Mathematics of the Decision Sciences*, vol. 12 of *Lecture on Applied Mathematics*, part 2, pp. 83–119, American Mathematical Society, Providence, RI, USA, 1968.

[2] M. Intrilligator, *Mathematical Optimization and Economics Theory*, Prentice Hall, Englenwood Clirs, NJ, USA, 1981.

[3] G. M. Grossman and E. Helpman, *Innovation and Growth in the Global Economy*, MIT Press, Cambridge, UK, 1991.

[4] C. Watanabe, "Factors goverining a firm's R&D investment. A comparison between individual and aggregate data," in *IIASA, TIT, OECD Technical Meeting*, Paris, France, 1997.

[5] C. Watanabe, "Systems factors goverining afirm's R&D investment. A systems perspective of inter-sectorial technology spillover," in *IIASA, TIT, OECD Technical Meeeting*, Laxenburg, Austria, 1998.

[6] A. Kryazhimskii, C. Watanabe, and Yu. Tou, "The reachability of techno-labor homeostasis via regulation of investments in labor R&D: a model-based analysis," Interium Report IR-02-026, IIASA, Laxenburg, Austria, 2002.

[7] A. M. Tarasyev and C. Watanabe, "Optimal dynamics of innovation in models of economic growth," *Journal of Optimization Theory and Applications*, vol. 108, no. 1, pp. 175–203, 2001.

[8] S. A. Reshmin, A. M. Tarasyev, and C. Watanabe, "Optimal trajectories of the innovation process and their matching with econometric data," *Journal of Optimization Theory and Applications*, vol. 112, no. 3, pp. 639–685, 2002.

[9] B. Kwintiana, C. Watanabe, and A. M. Tarasyev, "Dynamic optimization of R&D intensity under the effect of technology assimilation : econometric identification for Japan's Automotive industry," Interium Report IR-04-058, IIASA, Laxenburg, Austria, 2004.

[10] N. Kaldor, "A model of economic growth," *The Economic Journal*, vol. 67, no. 268, pp. 591–624, 1957.

[11] A. Kryazhimskii and C. Watanabe, *Optimization of Technological Growth*, GENDAITOSHO, Kanagawa, Japan, 2004.

[12] E. V. Grigorieva and E. N. Khailov, "Optimal control of a nonlinear model of Economic growth," *Journal of Discrete and Continuous Dynamical Systems Supplement*, pp. 456–466, 2007.

[13] S. B. Aruoba, J. Fernández-Villaverde, and J. F. Rubio Ramírez, "Comparing solution methods for dynamic equilibrium economies," *Journal of Economic Dynamics and Control*, vol. 30, no. 12, pp. 2477–2508, 2006.

[14] R. A. Aliev, "Modelling and stability analysis in fuzzy economics," *Applied and Computational Mathematics*, vol. 7, no. 1, pp. 31–53, 2008.

[15] S. A. Mohaddes, M. Ghazali, K. A. Rahim, M. Nasir, and A. V. Kamaid, "Fuzzy environmental-economic model for land use planning," *American-Eurasian Journal of Agricultural & Environmental Sciences*, vol. 3, no. 6, pp. 850–854, 2008.

[16] Y. Deng and J. Xu, "A fuzzy multi-objective optimization model for energy structure and its application to the energy structure adjustment—in world natural and cultural heritage area," *World Journal of Modelling and Simulation*, vol. 7, no. 2, pp. 101–112, 2011.

[17] M. Pešić, S. Radukić, and J. Stanković, "Economic and environmental criteria in Multi-objective programming problems," *Facta Universitatis*, vol. 8, no. 4, pp. 389–400, 2011.

[18] J. Ventura, "A global view of economic growth," in *Handbook of Economic Growth*, P. Aghion and S. N. Durlauf, Eds., vol. 1, pp. 1420–1492, Elsevier, 2005.

[19] J. Kacprzyk and S. A. Orlovski, Eds., *Optimization Models Using Fuzzy Sets and Possibility Theory*, Reidel Publishing Company, Dordrecht, The Netherlands, 1987.

[20] C. Carlsson and R. Fuller, "Multiobjective optimization with linguistic variables," in *Proceedings of the 6th European Congress on Intelligent Techniques and Soft Computing (EUFIT '98)*, vol. 2, pp. 1038–1042, Veralg Mainz, Aachen, Germany, 1998.

[21] C. Carlsson and R. Fullér, "Fuzzy multiple criteria decision making: recent developments," *Fuzzy Sets and Systems*, vol. 78, no. 2, pp. 139–153, 1996.

[22] R. Fuller, C. Carlsson, and S. Giove, "Optimization under fuzzy linguistic rule constraints," in *Proceedings of Eurofuse-SIC '99*, pp. 184–187, Budapest, Hungary, 1999.

[23] M. Sakawa, *Fuzzy Sets and Interactive Multiobjective Optimization*, Plenum Press, London, UK, 1993.

[24] L. V. Kantorovich and V. I. Zhiyanov, "The one-product dynamic model of economy considering structure of funds in the presence of technical progress," *Reports of Academy of Sciences of the USSR*, vol. 211, no. 6, pp. 1280–1283, 1973.

[25] V. Leontyev, *Input-Output Economies*, Oxford University Press, New York, NY, USA, 1966.

[26] V. F. Krotov, Ed., *Bases of the Theory of Optimum Control*, The Higher School, 1990.

[27] L. S. Pontryagin, V. G. Boltyansky, R. V. Gamkrelidze, and E. F. Mishchenko, *Mathematical Theory of Optimum Processes*, Hayka, 1969.

[28] A. B. Vasilyeva and V. F. Butuzov, *Asymptotic Decomposition of Decisions Singulyarno Indignant Equations*, Science, 1973.

[29] A. B. Vasilyeva and M. G. Dmitriev, "Singulyarnye of indignation in problems of optimum control//science and equipment Results. It is gray," Matem. analysis., T. 20. VINITI, pp. 3–77, 1982.

[30] R. Östermark, "A fuzzy control model (FCM) for dynamic portfolio management," *Fuzzy Sets and Systems*, vol. 78, no. 3, pp. 243–254, 1996.

[31] B. Bede and S. G. Gal, "Generalizations of the differentiability of fuzzy-number-valued functions with applications to fuzzy differential equations," *Fuzzy Sets and Systems*, vol. 151, no. 3, pp. 581–599, 2005.

[32] L. A. Zadeh, "Toward extended fuzzy logic-a first step," *Fuzzy Sets and Systems*, vol. 160, no. 21, pp. 3175–3181, 2009.

[33] P. Diamond and P. Kloeden, *Metric Spaces of Fuzzy Sets*, Theory and applications, World Scientific, Singapoure, 1994.

[34] R. S. Gurbanov, L. A. Gardashova, O. H. Huseynov, and K. R. Aliyeva, "Decision making problem of a one-product dynamic economic model," in *Proceedings of the of the 10th International Conference on Application of Fuzzy Systems and Soft Computing (ICAFS '12)*, pp. 161–174, Lisbon, Portugal, 2012.

[35] W. Pedrycz and K. Hirota, "Fuzzy vector quantization with the particle swarm optimization: a study in fuzzy granulation-degranulation information processing," *Signal Processing*, vol. 87, no. 9, pp. 2061–2074, 2007.

[36] Y.-J. Wang, J.-S. Zhang, and G.-Y. Zhang, "A dynamic clustering based differential evolution algorithm for global optimization," *European Journal of Operational Research*, vol. 183, no. 1, pp. 56–73, 2007.

[37] R. Storn and K. Price, "Differential evolution—a simple and efficient adaptive scheme for global optimization over continuous spaces," Tech. Rep. TR-95-012, ICSI, 1995.

[38] R. Storn and K. Price, "Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces," *Journal of Global Optimization*, vol. 11, no. 4, pp. 341–359, 1997.

[39] K. Price, R. Storn, and J. Lampinen, *Differential Evolution—A Practical Approach To Global Optimization*, Natural Computing Series, Springer, Berlin, Germany, 2005.

[40] L. A. Gardashova, "Fuzzy neural network and DEO based forecasting," *Journal of Automatics, Telemechanics and Communication*, no. 29, pp. 36–44, 2012.