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Research Article

On the Transformation Mechanism for Formulating a Multiproduct Two-Layer Supply Chain Network Design Problem as a Network Flow Model

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The multiproduct two-layer supply chain is very common in various industries. In this paper, we introduce a possible modeling and algorithms to solve a multiproduct two-layer supply chain network design problem. The decisions involved are the DCs location and capacity design decision and the initial distribution planning decision. First we describe the problem and give a mixed integer programming (MIP) model; such problem is NP-hard and it is not easy to reduce the complexity. Inspired by it, we develop a transformation mechanism of relaxing the fixed cost and adding some virtual nodes and arcs to the original network. Thus, a network flow problem (NFP) corresponding to the original problem has been formulated. Given that we could solve the NFP as a minimal cost flow problem. The solution procedures and network simplex algorithm (INS) are discussed. To verify the effectiveness and efficiency of the model and algorithms, the performance measure experimental has been conducted. The experiments and result showed that comparing with MIP model solved by genetic algorithm (GA) and Benders, decomposition algorithm (BD) the NFP model and INS are also effective and even more efficient for both small-scale and large-scale problems.

1. Introduction

The supply chain network system is the soul and backbone throughout the overall supply chain management (SCM). Many decisions have to be made over the SCM. For instance, to determine the structure of the supply chain; to determine number, size, and location of facilities in supply chain; and to make the distribution or transportation plan, sometimes, the decision of product design could also seem as an issue in SCM. Not going that far, in this paper, we focus on the supply chain network design (SCND) problem. The SCND makes the decision of the structure of a chain and affects its costs and performance, which is not only of significant importance to supply chain management but also a classic case in operation research [1].

Although in the last decades thousands of research literatures could be brought into SCND scope, it is not hard to classify the proposed works into several parts: (i) from the type of problem, SCND can be divided into two parts, the design in strategy phase and the operation phase [2]; (ii) from the type of customer demand, researches could be grouped into SCND under demand certainty, uncertainty and random problems [3, 4]; (iii) from the structure of supply chain, it may be divided into single-product multilayer (level) problem, or multiproduct multilayer problem [5]. A simple multilayer supply chain network in real life is shown in Figure 1 [1], which includes plants, distribution centers (DCs), and customers. The flow of products in such SC is distributed forward through the plants via DCs to final customer. While the products are of a variety of categories, the network is defined as a multiproduct multilayer supply chain network.

To specify our research problem in this paper, we notice that there is a very common SC network (SCN) structure in the real world. Some of manufacturers usually pay more attention to optimization of their multicategory products supply and marketing chain rather than the certain raw material supply sources, food manufacturing, or other products with single or simple source of materials for instance [1]. The SCN of this situation possesses a clear and not hard to define

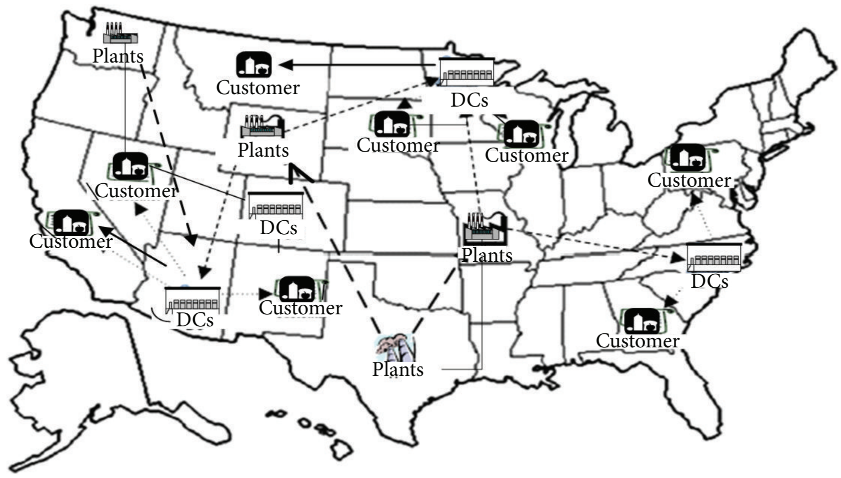

Figure 1: A realistic two-layer supply chain network.

structure, which is multiproduct, two-layer SCN (actually the one which is indicated in Figure 1 is of such a structure). Such structure of SCN is not only common in manufacture industry but also common in various industries. In addition, it could be found in many social real life cases, for example, the human blood supply system in China [6]. The "products flow" in the blood supply system is human blood, which is normally divided into types A, B, O, and AB; the "plants" can be seen as blood collection sites; the "DCs" are the facilities in charge of blood testing, processing, storage, and distribution; the "customers" are urban hospitals. Hence, it is applicable and typical to choose this kind of SCN as our research subject.

In the multiproduct two-layer SCN, decision makers often faced such a problem of, when the location of plants sites and the design capacity of various sites have been fixed, how to locate the DCs and design the DCs' capacity in the retail market (or demand area) and how to make the preliminary distribution plan so that the total cost of location-operation-distribution is minimal and customers' demands are met at the same time. In fact, customers think that the more DCs the better; in this condition, it takes less time and costs less for distribution in the SCN, but obviously it takes much to construct and operate the DCs. On the contrary, the less DCs have been set, the more time and costs will be taken for distribution in the SCN. Moreover, the costs for constructing and operating the DCs will be reduced but it may be difficult to meet the customers' demand, and the delivery time from the plants via DCs to customers becomes longer. Notice that there are many tradeoffs in the multiproduct two-layer SCND problem and we are trying to formulate an integrated optimal model and solve it in an effective way.

As mentioned before, the multiproduct two-layer SCN structure is very common in various industries. Hence, there already existed numerous literatures dealing with this kind of SCND in both application and research area. However, most of the literatures are related to modeling the problem with various characteristics or modeling the problem in different planning phases [7]. In the optimization technique parts of the literatures, the complexity of algorithm for solving SCN models has also been investigated. Such problem has been proved to be belonging to NP-hard problem; when the design capacity of DCs is considered, the complexity of solving the problem is increased. Although the optimization algorithm existing has been proved to be effective, it is still hard to reduce the complexity of the algorithm [8, 9]. Thus, this paper is trying to formulate and solve this complicated problem in a classic and simpler way. Instead of modeling SCND problem with addressing and setting more complicated characteristics, we attempt to pull in some transformation mechanisms to formulate the SCND problem as a network flow problem and solve it by a classic algorithm.

As a matter of fact, based on existing literatures, many NP-hard problems or nonlinear optimization problems can also be formulated as a network flow problem [10]. Such a problem and its solution algorithms are characterized by the fact that the complexity can be optimized to a polynomial running time; with the increasing of network scale, the performance is even more remarkable [11, 12]. Inspired by the above research attribution, we try to develop a transformation mechanism for the original multiproduct two-layer SCND problem, so that the problem may be formulated as a network flow problem through adding some virtual nodes and arcs and nonlinear costs relaxation; therefore, the problem can be solved by some classic and effective minimum cost flow algorithm.

The rest of this paper is structured as follows. The second part is literature review of relevant problems; the third part provides the transformation mechanisms and network flow models in designing multiproduct multilayer supply chain network. We propose a solution program and apply the network simplex algorithm to solve the model and discuss the algorithm's performances compared with genetic algorithm, and Benders' decomposition algorithm through numerical examples. The last part gives the conclusion and future research direction.

2. Literature Review

Broadly speaking, our research could be related to a number of literature streams. First, it pertains to the facility location research, in particular, the inventory-location problem and location-allocation problem. Secondly, it relates to SCND problem. When we just consider the details of the costs involving in the problem, the research is relevant to fixed cost transportation problem. The transformation mechanisms and algorithms are related to network flow algorithms domain. In this paper, we just review the most closely related literatures.

2.1. Facility Location and SCN. The aim of SCND is to design an efficient network structure for new chain's entities or to reengineer an existing network to increase its total value. Various traditional SCNDs aim at obtaining the optimal distribution plan and facility location in the supply chain and optimizing the original supply chain network. Thus, it can be said that SCND subordinates to the joint optimization of product distribution and facility location [13]. When the capacity of facilities is limited, the problem can be transformed into a facility location problem with certain capacity. The heuristic algorithm is mostly used to solve problems in multistage supply chain, especially when capacity constraints exist in all nodes, such as plants and DCs [14, 15].

Study of this problem can generally be expressed as mixed integer programming model whose goal is to minimize the costs of various expenses. Hanssmann [16] in 1959 first established the model for SCND problem, in which he considered, material procurement, producing and selling. However, this model can only apply to simple supply chain system for it does not take into account the different materials and products types. In 1974, Geoffrion and Graves [17] built a multiproduct single-stage mixed integer programming with improvement of the model Hanssmann proposed. Starting around 2001, the SCND problem was formally mentioned in research. Jayaraman and Pirkul [18] has developed a heuristic algorithm based on Lagrange relaxation variables for single-source, multiproduct, multilayer SCND. Handfield and Nichols Jr [19] made use of discrete variables to build model for a redesign problem on the basis of the existing supply chain network design. Again Syam [20] developed another SCND model for multisource SCN based on Lagrangian relaxation and the corresponding simulated annealing heuristic algorithm has also been proposed. Jang et al. [21] proposed a model to solve the joint production/distribution planning problem. Against formulating models for SC with different characteristics, researches have also tried to find the optimal algorithms for solving the models. Syarif et al. [22] developed a genetic algorithm based on Prufer number coding spanning tree to solve the multisource, single-product, multistage SCND problem. Jayaraman and Ross [23] presented a simulated annealing based heuristic method to solve the distribution network design problem in supply chain management. Yeh [24] developed a hybrid heuristic algorithm to solve the problem. Syarif et al. have proposed [22] the algorithm integrating the greedy algorithm, linear programming technology, and the local search method. Yeh [25] also put forward a culture genetic algorithm to solve the same problem. The representative's latest study is: Solo [26] provided a supply chain management integration model; Mak [27] built an integration SCND model with consideration of customer demand and inventory uncertainty. Pan and Nagi [28] have conducted a SCND model for agile manufacturing system. The summary of the models and solution procedures for the SCND problems is as follows: they formulate the problem as a mixed integer programming problem and most of the solution algorithms are heuristic algorithm.

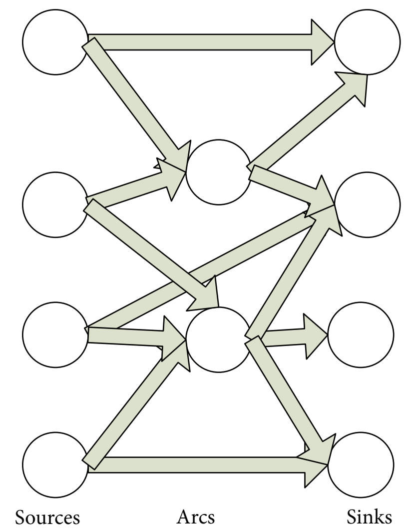

Figure 2: A general network flow diagram.

2.2. Network Flow Problem. A general network flow problem could be shown in Figure 2. To illustrate the general network flow problem in the diagram, we have a number of sources of material and a number of sinks (or demand points) for material. Between the sources and the sinks are intermediate nodes through which material can be shipped (flows) to other intermediate nodes or to the sinks. The arcs between each pair of nodes have been associated with an upper limit (or capacity) on the amount of material which can flow down the arc and a cost per unit of material sent down the arc. Hence the problem is deciding how to supply the sinks from the sources at minimum cost. This problem is known as the minimum cost network flow problem [29]. Any problem which can be represented in the form of a picture such as shown in Figure 2 can be regarded as a minimum cost network flow problem. Pioneers study on formulating complex problem as network flow problem has applied a mechanism which is adding some virtual nodes and arcs to handling the nonlinear part [30, 31]. Sheffi first shows such mechanism in his dissertation when formulating a traffic flow equilibrium model [32]. Nagurney improved Sheffi's research expanding the mechanism to many application areas in supply chain [33, 34]. Ahuja et al. tried to formulate the racial balancing problem as a multicommodity flow problem, so a minimum multicommodity flow will specify an optimal assignment of students to the schools [30]. Obviously, a network model may be solved with either special purpose network flow programming algorithms or with general purpose linear programming algorithms. Due to its special structure, researchers have developed a number of corresponding algorithms. One of the acceptable effective and efficient algorithms is network simplex algorithms, which is developed by Ahuja, Goldberg, and othres and has been proved as the most effective approach to solving this kind of problem [11]. The basic idea of network simplex method is to make every step maintain the feasible solution of the original circumstances and use dual relaxation theorem to find its prerequisites of optimal result. In network diagram, the optimal result is derived from feasible solutions spanning, and Goldberg, Tarjan, and Orlin had, respectively, promoted the algorithm [35]. The computational time of such algorithm is $O(|C|)$, while the complexity for solving the potential value and reduced cost of the node is easily set as $O(|T_2|)$, so the complexity of each iterative computing is $O(|C| + |T_2|)$. Owing to the high efficiency of the computation time, we will apply the network simplex algorithms in the algorithm and computational part.

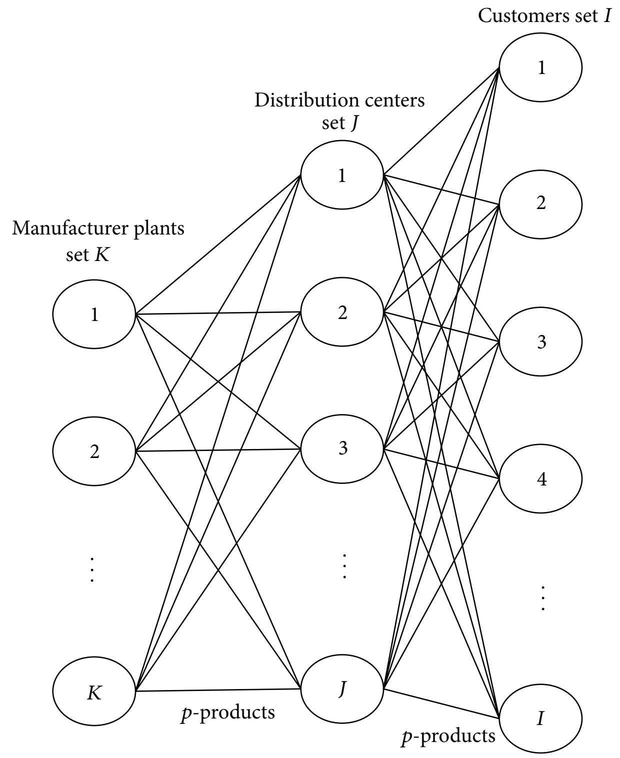

FIGURE 3: Original two-layer supply chain network.

3. Formulation

3.1. Description of the Problem and Mathematical Modeling. The multiproduct multilayer SCND in this paper can be stated asfollows. As shown in Figure 3, we consider a two-layer supply chain network which involves k manufacturer plants, j DCs, and i final market customers. The necessary instructions and assumptions are as follows.

(i) The manufacturers supply a variety of products; the plants location and production capacity are fixed; final market customer demand can be also predicted.

(ii) We only take the forward logistics flow into account.

(iii) The candidate location of the DCs can be obtained, and the fixed costs in constructing and the basic operation costs in operating the DCs can be estimated.

(iv) The costs for plants to produce each product are known; the basic costs of delivery products through the supply chain are known, regardless of the carbon tax and other additional costs accompanied with logistics.

Decisions to be made in this problem are (i) determining the location of distribution center and design capacity and (ii) determining the initial distribution plan.

The following parameters and labels are introduced.

Sets and constancies

K: Set of manufacturer plants $(1, 2, \ldots, k)$

J: Set of distribution centers $(1, 2, \ldots, j)$

I: Set of final market customers $(1, 2, \ldots, i)$

P: Set of products $(1, 2, \ldots, p)$

d_i^p: The demand of product p of nodes I in customer set I

c_{kj}^p, c_{ji}^p: Unit distribution costs of product p of (k, j) and (j, i), respectively

o_j: Fixed constructing cost during certain period of distribution center j

T_j^p: Unit operation cost of product p in distribution center j

θ_k^p: The unit manufacturing cost of product p in plants k

N_k^p: The design capacity of product p in plants k.

Variables

$$z_j = \begin{cases} 1 & \text{if the distribution center is located at node } j, \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

q_k^p: The production amount of product p in plants k

q_{kj}^p: The distribution amount of product p from plants k to distribution center j

q_{ji}^p: The distribution amount of product p from distribution center j to final market customer i

W_j^p: The design capacity of product p in distribution center j.

It is not hard to model the problem as a mixed integer programming model (MIP):

$$\min \quad f = \sum_{k\in K}\sum_{p\in P}\theta_k^p q_k^p + \sum_{k\in K}\sum_{j\in J}\sum_{p\in P} c_{kj}^p q_{kj}^p + \sum_{j} z_j o_j + \sum_{j\in J}\sum_{k\in K}\sum_{p\in P} T_j^p q_{kj}^p + \sum_{j\in J}\sum_{i\in I}\sum_{p\in P} c_{ji}^p q_{ji}^p, \tag{2}$$

$$\text{s.t.} \quad q_k^p = \sum_{j\in J} q_{kj}^p, \quad \forall k, p, \tag{3}$$

$$N_k^p \geq \sum_{j\in J} q_{kj}^p, \quad \forall k, p, \tag{4}$$

$$\sum_{k\in K}\sum_{p\in P} q_{kj}^p = \sum_{i\in I}\sum_{p\in P} q_{ji}^p, \tag{5}$$

$$\sum_{j\in J}\sum_{p\in P} q_{ji}^p = \sum_{p\in P} d_i^p, \tag{6}$$

$$\sum_{p\in P} w_j^p \leq \sum_{i\in I}\sum_{p\in P} q_{ji}^p, \tag{7}$$

$$q_k^p \geq 0, \quad q_{ji}^p \geq 0, \quad q_{kj}^p \geq 0, \quad z_j = 0, 1. \tag{8}$$

The objective function (2) relates to the overall costs of supply chain network. From the left side of (2), the cost components successively stand for manufacturing costs, the costs of delivery for the product from the plants to the DCs, the basic fixed construction costs of DCs, the operations costs of DCs, and the distribution costs from the DCs to the customers.

Constraints (3) limits the production capacity of each manufacturing plants must equal to the number of products sent to DCs; (4) is the plant's capacities constraint; (5) is the conservation of products sent and received in each DCs; (6) guarantees the customers demand can be satisfied. Inequality (7) provides a constraint of DCs designing capacity. Inequalities (8) give the nonnegative constraints and the binary constraint.

Such model has already been classified to a NP-hard problem. In the next section, we will show the transformation mechanisms of how to formulate a lower bound network flow model of the proposed MIP model; therefore, we can solve the MIP model through solving the lower bound problem in polynomial time.

3.2. Network Flow Problem Modeling. Observation shows that the costs involved in objective function and constraints almost linear over the flow in supply chain network (production, operation, and distribution amount), and the only fixed cost is the facility's construction costs. In this section we give the transformation mechanisms for costs, network structures, and flows and how we formulate the proposed problem as a network flow model.

3.2.1. Processing of Fixed Cost and Lower Bound Effectiveness. Palekar et al. and others developed a new relaxation algorithm that can be used to solve the transportation problem with fixed costs, and they had proved this relaxation method a more effective one [36] than Driebeek Punishment and Lagrangian relaxation in solving this kind of problems. Jawahar and Balaji promoted such relaxation approach and applied it to the fixed cost problems in two-stage supply chain; models and experiments have proved that the lower bound problems raised by using appropriate genetic algorithm to get relaxation are of strong robustness and effectiveness as in [37]. After application of the specific relaxation methods, the fixed cost model is converted into a linear programming model via replacing the fixed cost and transportation cost with a lower bound cost. For illustrating the method, we introduce some index. The lower bound costs are symbolized by CF_{ij} and CF_{jk}; the production capacity of the plants is S_i; the total amount of products operation in distribution center is A_j, while the customer's demand is D_k.

The transform function could be written as

$$CF_{jk} = C_{jk} + \left\{ \frac{F_{jk}}{\text{Min}\left(A_j, D_k\right)} \right\}, \qquad CF_{ij} = C_{ij} + \left\{ \frac{F_{ij}}{\text{Min}\left(S_i, A_j\right)} \right\}. \tag{9a}$$

To avoid the nonlinear cost part of the objective function, Let

$$A_j = \sum_{k=1} D_k, \quad \forall j. \tag{9b}$$

After the relaxation, the lower bound model can be formulated as minimal cost flow problem so that it is easy to find the solution.

Although the relaxation functions ((9a) and (9b)), in general, yield a loose lower bound, Jawahar et al. have employed an example to illustrate that the technique is still effective, which could be found in detail in their literatures [37]. In the future study, we will try to find a tighter lower bound for the proposed problem.

3.2.2. Network Flow Modeling. First, we build a network $G = [(O, D, K, J, J', I), A]$ for the problem raised in Section 3.1 and take A as the arc set of the network G.

(i) Transformation Technique to Network Structure. Now we develop a method to transfer the original supply chain network structure into a network flow structure by adding some virtual sources, sink nodes, and virtual arcs, respectively, based real network structure. The real network structure for MTSN problem has already been abstracted and displayed in Figure 3. The conversion programs are as follows.

Step 1. Add virtual decision nodes for distribution centers location for each distribution center. Thus, as illustrated in Figure 4, each DCs node J has been expanded to node pairs (J, J'); the virtual flow through DCs J to virtual DCs J' represents the logistics operations flow in each DC. Once the flow equals zero, it stands for the corresponding DCs site which has not been selected.

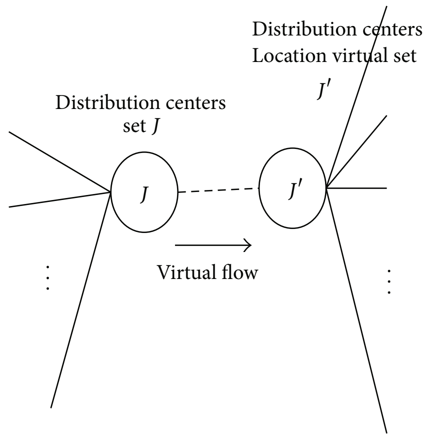

Figure 4: Transformation process of the DCs nodes.

Step 2. Add virtual source node and its corresponding virtual arcs. As shown in Figure 5, the virtual flows represent the logistics flow in each manufacturer plant. Since the plants capacity has already been set, the capacity of virtual arcs equals the designing capacity of the corresponding plant. (e.g., the capacity of arc(O, K) equals the capacity of plants K).

Step 3. Add virtual sink node and its corresponding virtual arcs. Figure 6 gives a direct illustration of how to add the sink node and arcs. In our research, the preliminary customer demand is assumed to be fixed and can be calculated in the designing phase. We set the virtual arc to convert the node demand into arc flow, so that the sink node can give the whole supply chain a pulling force. It is clear that the capacity of the corresponding virtual arcs equals the demand of the connected customers node (e.g., the capacity of arc(i, D) equals the demand of customer i).

So far, the network flow supply chain network structure has been obtained. We provide the framework in Figure 7.

(ii) Analysis and Conversation of Constants for Modeling the Network Flow Model. Observing Figure 7, the virtual and physical flow is associated with the network, entering and leaving the nodes and passing through the arcs; the upper bound of the flow is the arcs' capacity; the lower bound of the physical flow can be simply treated as 0. We will discuss the lower bound of the virtual flow lately. In this case, if the cost associated with each arc is linear to unit flow, the multiproduct, two-layer SCND problem can be entirely modeled as a network flow model.

We first introduce some additional notations:

E^p_{ok}: The flow capacity of arc(o, k)

E^p_{kj}: The flow capacity of arc(k, j)

$E^p_{jj'}$: The flow capacity of arc(j, j')

$E^p_{j'i}$: The flow capacity of arc(j', i)

E^p_{iD}: The flow capacity of arc(i, D)

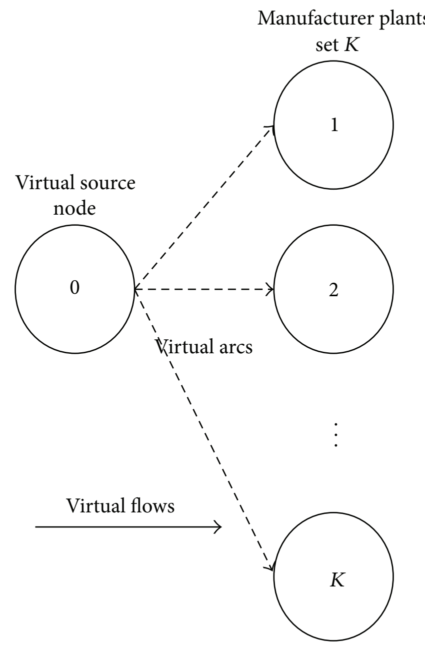

Figure 5: Transformation process of the plants nodes.

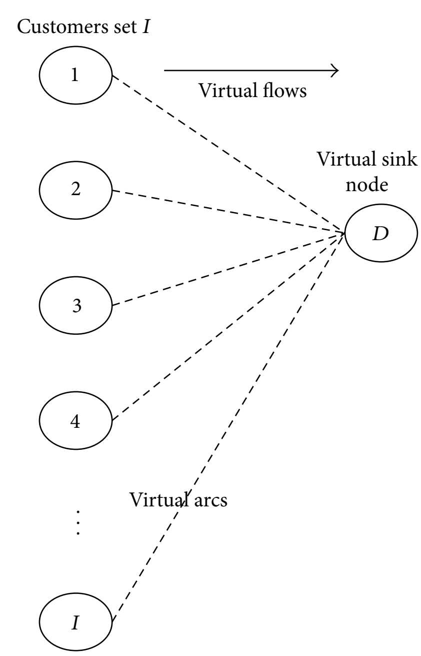

Figure 6: Transformation process of the customers nodes.

b^p_{ok}: The cost associated with arc(o, k)

b^p_{iD}: The cost associated with arc(k, j)

$b^p_{jj'}$: The cost associated with arc(j, j')

$b^p_{j'i}$: The cost associated with arc(j', i)

b^p_{iD}: The cost associated with arc(i, D)

f^p_{ok}: The flow of arc(o, k)

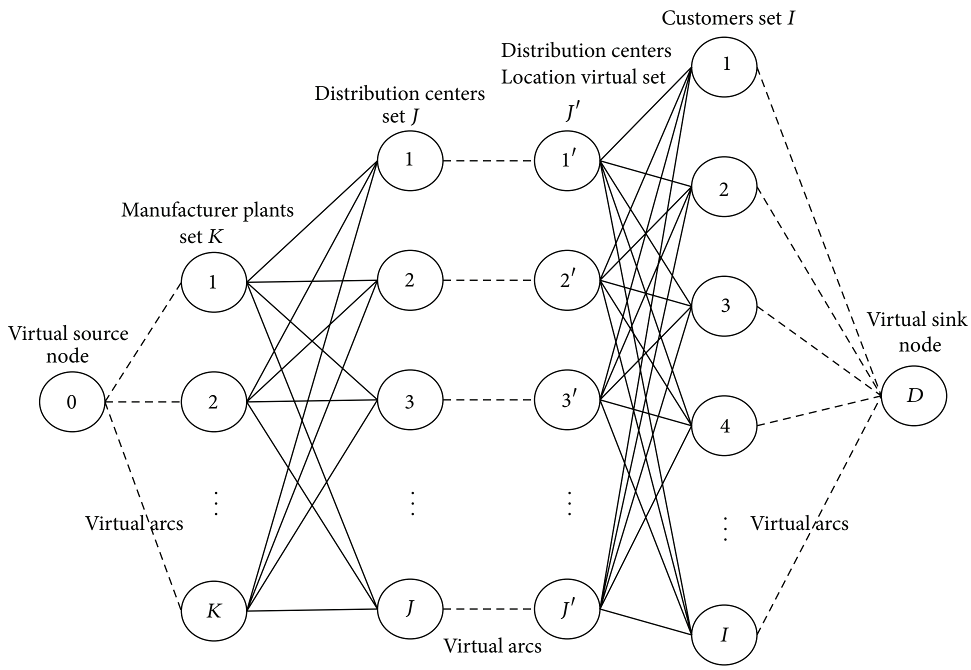

Figure 7: Framework of transformed supply chain network structure.

f_{kj}^{p}: The flow of arc(k, j)

$f_{jj'}^{p}$: The flow of arc(j, j')

$f_{j'i}^{p}$: The flow of arc(j', i)

f_{iD}^{p}: The flow of arc(i, D).

Capacity analysis of each arc is as follows.

(a) As investigated in transformation techniques in Steps 2, 1, and 3, $E_{ok}^{p} = \sum_{p\in P} q_k^p$, $E_{jj'}^{p} = \sum_{p\in P} w_j^p$, and $E_{iD}^{p} = \sum_{p\in P} d_i^p$.

(b) Since the flow associated with the arc(k, j) and arc (j', i) represents the physical product flow distribution from plants set to DCs candidate sets and from DCs sets to final customer, we do not take the roadway construction pass-through capacity into account and set the capacity of such arcs as infinite. Then, $E_{kj}^{p} = M$ and $E_{j'i}^{p} = M$, where M is a constant big enough.

(c) Now we investigate the upper bound and lower bound of the flows. To verify if there is always a flow entering the customer demand node, we set that the upper bound and lower bound of f_{iD} are E_{iD} and the upper bound and lower bound of f_{ok} are E_{ok}. Since the flow of virtual arcs(j, j') refers to the operation process of products in DCs, the upper bound of the $f_{jj'}$ equals $E_{jj'}$, and the lower bound is 0.

Cost analysis of each arc is as follows.

(a) The cost associated with virtual source is the unit manufacturing cost of product; then $b_{ok}^{p} = \theta_k^p$.

(b) It is clear that cost associated with virtual sink is 0; then $b_{ok}^{p} = b_{id}^{p} = 0$.

(c) The cost associated with virtual arc(j, j') is the operation cost of DCs j. Then $b_{jj'}^{p} = T_j^p$.

(d) The cost associated with the arc(j', i) is the distribution cost c_{ji}^{p}.

(e) It is hard to define the cost associated with the arc(k, j). The cost to represent whether there is a flow in arc(k, j) is the transportation/distribution cost c_{kj}^{p} and the fixed construction cost o_j. Notice that there existed a nonlinear cost. We now applied the relaxation functions (9a) and (9b) to deal with the nonlinear cost.

Substituting the cost function into (9a), we obtain the lower bound cost function of arc(k, j):

$$b_{kj}^{p} = c_{kj}^{p} + \left\{ \frac{o_j}{\operatorname{Min}\left(\sum_{p\in P} q_k^p, \sum_{p\in P} w_j^p\right)} \right\}. \tag{10a}$$

However, it is easy to notice that the $\sum_{p\in P} w_j^p$ is a decision variable of our problem. It is obviously there is an ideal case for our problem: the designing capacity of each DC can satisfy the total customer demand. Thus we apply the relaxation (9b) for the extreme case and substitute the total customer demand variables of DCs' capacity variables:

$$\underline{b_{kj}^{p}} = c_{kj}^{p} + \left\{ \frac{o_j}{\operatorname{Min}\left(\sum_{p\in P} q_k^p, \sum_{i\in I}\sum_{p\in P} d_i^p\right)} \right\}. \tag{10b}$$

Now we give the derivation of (10b) as the lower bound of (10a).

Proof. Since the total customer's demand is the upper bound of each DCs capacity, we can obtain $\sum_{p\in P} w_j^p \le \sum_{i\in I}\sum_{p\in P} d_i^p$.

If

$$\sum_{p\in P} q_k^p \ge \sum_{i\in I}\sum_{p\in P} d_i^p, \tag{11}$$

then,

$$\begin{gathered}\operatorname{Min}\left(\sum_{p\in P} q_k^p, \sum_{p\in P} w_j^p\right) = \sum_{p\in P} w_j^p,\\ \operatorname{Min}\left(\sum_{p\in P} q_k^p, \sum_{i\in I}\sum_{p\in P} d_i^p\right) = \sum_{i\in I}\sum_{p\in P} d_i^p.\end{gathered} \tag{12}$$

It is obvious that

$$\frac{o_j}{\sum_{p\in P} w_j^p} \ge \frac{o_j}{\sum_{i\in I}\sum_{p\in P} d_i^p} \Longrightarrow \underline{b}_{kj}^p \le b_{kj}^p. \tag{13}$$

When $\sum_{p\in P} q_k^p \le \sum_{i\in I}\sum_{p\in P} d_i^p$, it is easy to obtain $\Rightarrow \underline{b}_{kj}^p = b_{kj}^p = c_{kj}^p + \{o_j / \sum_{p\in P} q_k^p\}$.

So far, we deduce that (10b) is the lower bound of (10a).

Based on the above analysis and derivation, we formulate the network flow problem of multiproduct multilayer SCND problem (NFP) as

$$\min \quad b(f) = \sum_{[(O,k),(k,j),(j,j'),(j',i),(i,D)\in A]} \sum_{p\in P} \left(b_{ok}^p f_{ok}^p + \underline{b}_{kj}^p f_{kj}^p + b_{jj'}^p f_{jj'}^p + b_{j'i}^p f_{j'i}^p + b_{iD}^p f_{iD}^p\right), \tag{14}$$

$$\text{s.t.} \quad \sum_{p\in P} f_{ok}^p \le \sum_{p\in P} q_k^p, \quad \forall k, \tag{15}$$

$$\sum_{p\in P} f_{kj}^p \le M, \quad \forall k, j, \tag{16}$$

$$\sum_{p\in P} f_{jj'}^p \le \sum_{p\in P} w_j^p, \quad \forall j, j', \tag{17}$$

$$\sum_{p\in P} f_{j'i}^p \le M, \quad \forall j', i, \tag{18}$$

$$\sum_{p\in P} f_{id}^p = \sum_{p\in P} d_i^p, \quad \forall i, \tag{19}$$

$$\sum_{k\in K}\sum_{p\in P} f_{ok}^p = \sum_{i\in I}\sum_{p\in P} f_{id}^p, \tag{20}$$

$$\sum_{k\in K}\sum_{p\in P} f_{kj}^p = \sum_{j\in J}\sum_{p\in P} f_{jj'}^p, \tag{21}$$

$$\sum_{j\in J}\sum_{p\in P} f_{j'i}^p = \sum_{p\in P} f_{id}^p, \quad \forall i, \tag{22}$$

$$f_{ok}^p, f_{kj}^p, f_{jj'}^p, f_{j'i}^p, f_{id}^p \ge 0, \quad \forall k, j, j', i, p. \tag{23}$$

The objective function (14) is the sum of cost flow of network G omitting those whose cost is zero.

Constraints (15)–(19) mean that the flow in the network flow model is less than the capacity constraints, and (20)–(22) show that there is a conservation between the flow of the source and that of the sink, and the flow on both ends of each node is equivalent. Inequality (23) is a nonnegative constraint of flow. □

4. Approach and Numerical Experience

4.1. Solution Technique. The structure of algorithms to solve the model we proposed in Section 3 is given in Figure 8. We first apply the transformation techniques illustrated in Section 3 to convert the problem as a ready formulated network flow problem, and then we solve the NFP model by network simplex algorithms. The details of such algorithms procedure are given in the next section.

4.2. Network Simplex Algorithms Part. After the application of transformation techniques to the original problem, the multiproduct two-layer SCND problem has been transformed into a network flow problem with some virtual nodes and arcs. In this condition, the network simplex algorithm could be employed to solving the NFP model. The steps of solve algorithms are as follows.

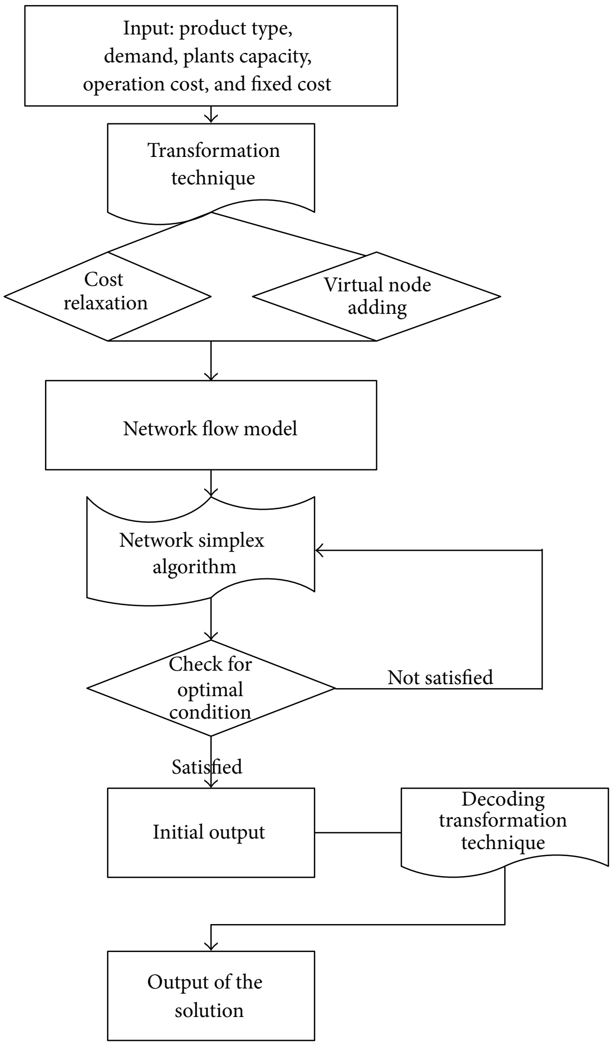

Figure 8: Solution technique.

Step 1. Initializing viable forest structure, order $N(i) = \{o, K, J, J', I, d\}$ and q indicates that the initial flow y means the potential value of the node. Take O as a starting point, D is the end, and generate an initial feasible flow. Generate an initial feasible solution of the tree on the basis of initial feasible flow. Finally, choosing an arc from the tree solution randomly and adding the arc to the entering tree structure.

Adding a corresponding supernode $N'(i)$ for each node in $N(i)$ (note: the supernode set here for solving the problem is different from the virtual node we have added when formulating the problem). If the node is a supply point, order $\mu_{ii} = 0.5$; if it is the demand point, order $\mu_{ii} = 2$. Order the cost from $N(i)$ to its supercharge corresponding one $N(i)$ a constant M which is sufficiently large.

Step 2 (test the optimal conditions and fix the enter arc). Define (F, L, U) as a viable augmented forest structure, where F is feasible spanning tree, L is an upper bound network consisting of the upper bounds of flows, and U represents the upper bound network of flows made up of capacity values. The optimal result should satisfy the following new optimal conditions:

$$\begin{aligned} &\text{If } (i, j) \in F, \quad C'_{ij} = 0; \\ &\text{If } (i, j) \in L, \quad C'_{ij} \geq 0; \\ &\text{If } (i, j) \in U, \quad C'_{ij} \leq 0. \end{aligned} \tag{24}$$

Provide the rule of enter arc, which is to find a variable which does not meet the optimal conditions; that is, if there exist the following conditions, the solution to the problem is not optimal. If $(i, j) \in U$, there is a negative reduced cost;

$$\text{If } (i, j) \in L, \quad \text{there is a positive reduced cost.} \tag{25}$$

Generally speaking, any arc fulfilling the conditions above can be taken as an enter arc to go through the iteration in network map. In order to increase the efficiency of the algorithm, this paper compared the reduced cost C'_{ij} of the enter arcs to be selected and selected the arc of maximum reduced cost to carry out the iterating.

Step 3 (select the leaving arc). Endow the enter arc with flow until the flow of a basic arc reaches the upper or lower bound of the capacity. Assuming the enter arc in Step 1 is $(s, t) \in L \cup U$, then the leaving arc$(i, j) \in F$. Obviously, the benefit which increased to every unit of flow in (s, t) is related to the loss of every unit of flow on leaving arc. The loss of every unit of flow on leaving arc is symbolized as loss_{ij}, and the corresponding benefits on leaving arc can be expressed as follows:

$$\text{gain}_{ij} = \begin{cases} \dfrac{\left(f_{ij} - q_{ij}\right)}{\text{loss}_{ij}} & \text{if } \text{loss}_{ij} > 0, \\ \dfrac{q_{ij}}{\left(-\text{loss}_{ij}\right)} & \text{if } \text{loss}_{ij} < 0, \\ \infty & \text{if } \text{loss}_{ij} = 0. \end{cases} \tag{26}$$

Thus the available mechanisms for leaving arc are

$$\text{leaving arc} = \min\left[\text{gain}_{ij} : (i, j) \in F \cup \{(s, t)\}\right]. \tag{27}$$

Step 4 (renew the augmented forest structure). Renew the augmented forest structure through selecting the leaving arc and enter arc in viable base solutions. It is known that this kind of iterative method iterates from a relatively augmented forest to another one, until the optimal solution is reached.

Step 5 (update the potential values of node). The function m described below is programmed to get the potential value of the node of forest structure.

```
Function [p] = potential(L, p_i, g, U, n)
m = length (L);
For i = 1 : m
e = edge(L(i), n)
p(i) = g(e(1), e(2))+p_i(e(1))−U(e(1), e(2) * p_i(e(2)));
End
End
```

Table 1: The limitation of data scale in experiments.

Experiment	1	2	3	4	5	6	7	8	9	10
Category of products	2	4	6	8	10	12	14	16	18	20
Manufacturer	1	4	6	8	10	12	14	16	18	20
Distribution centers	1	4	6	8	10	12	14	16	18	20
Customers	2	4	6	8	10	12	14	16	18	20
Number of actual network arcs	18	144	486	1152	2250	3888	6174	9216	13122	18000
Number of solving network arcs	27	178	561	1284	2455	4182	6573	9736	13779	18810

Table 2: Range of randomly generated constant data for experiment.

Experiment data	Name of constant	Range of data generated	Unit of measurement
1	The scale of customers' demand	100–10,000	Piece
2	The capacity of plants	100–10,000	Piece
3	Manufacturing costs per unit of product	5–18	RMB/piece
4	Operation costs per unit of product in DCs	6–12	RMB/piece
5	Transportation costs per unit of product	0.8–2.2	RMB/piece
6	Fixed costs of facilities construction	3,000–10,000	RMB/period

Step 6 (terminate the algorithm). When all arcs meet the optimum conditions, the minimum cost flow network problems get the optimal solution, and the optimal solution is q^*. If $q^*_{dg} > 0$, it means that manufacturing plant b is to be selected, and the design capacity is $q^*_{bb'}$; if $q^*_{cc'} > 0$, it means that manufacturing plant c is to be selected, and the design capacity is $q^*_{cc'}$; if $q^*_{b'f} > 0$, it means that the path between b and f is to be expanded. If $q^*_{c'f'} > 0$, it means that the path between c and f is to be expanded, while the flow rate g from point to point is set only to help cover all requirements in the process of searching for the optimal result which has no practical meaning.

For small-scale problem, we could directly program "m" function by Matlab2010 (b) thus to gain the result. While for problems of large scale, a docking of Ilog Cplex 12.2 and Matlab programming can be applied to solve the problem. The complexity of calculating the basic solution, the potential value of the node and the determination of the leaving arc and the enter arc is: $O(n)$, and the computational complexity of re-determining the reduced costs of every arc is $O(1)$; therefore, the algorithm is of high solution efficiency.

4.3. Numerical Experience. In this section, we will solve some numerical problems by MIP and corresponding algorithm, and by NFP and network simplex algorithm we have proposed in this paper to verify the effective and efficiency of NFP models.

In recent years, the related algorithms to MIP problem are heuristic algorithm Benders' decomposition algorithm, and so forth. We choose the genetic algorithm (GA) and Benders' decomposition algorithm (BD), such two-type well-known effective algorithm for MIP problem, to measure the MIP model of this paper and algorithm performance. We then apply the NFP model and network simplex algorithm (INS) with the same data to compare the optimal solution and algorithm running time with MIP model solved by GA and BD, respectively. The scale of experimental data is given in Table 1, and the generated range of other constants is shown in Table 2. The algorithm running experiment environment is Personal Computer, windows 7 professional system, 2.10 Ghz Cpu, 4 GBRM. During the numerical experience, we applied the GA and BD software tools package of Ilog Cplex 12.0 directly. And programming a "m" file of the INS by Matlab language, then calling Ilog Cplex 12.0 to run the Matlab file. As for the scale of experiments, the number of arcs in the first set of 5 experiments does not exceed 3000, which is considered to be small-scale experiments, and experiments in groups 5–10 can be seen as large-scale network experiment.

The comparison of result of experimenting and performance is shown in Table 3. The deviation comparison result has been illustrated in Table 4.

The deviation of the INS from GA and BD is tiny, which indicates that the NFP model is almost equivalent to the MIP model. From the point of time, as for small-scale experiments, all the performance of algorithm is shown in Figure 9, while the performance of the large-scale experiments is represented in Figure 10.

Comparing the algorithms performance for small-scale problem, it is interesting that, for the problem with the arc scale less than 1200, the INS performs much better than GA and BD. For the problem with the arc scale more than 1500, although the INS also performs better than others, its running time yield is greater than others with the expanding of network scales. More numerical studies should be conducted to analyze the efficiency of NFP and INS for small-scale SCND problem.

While for the algorithms performance for large-scale problem, it is noteworthy that the benefits of the INS appear to be somewhat remarkable and stable. The experimental result indicates that the NFP and INS are particularly suitable for solving large-scale SCND problem.

5. Conclusions

For a manufacturer-center multiproduct two-layer supply chain, the SCND problem is involved with the DCs location

Table 3: Comparison between running time and the optimal solution.

Problem	GA		BD		INS	
	Minimum cost	Running time	Minimum cost	Running time	Minimum cost	Running time
1	1025,640	3	1025,640	2	1025,580	2
2	3,710,342	6	3,710,342	5	3,709,573	4
3	10,855,375	189	10,855,375	141	10,835,485	92
4	24,779,022	463	24,779,022	399	24,778,267	351
5	46,520,863	655	47,526,986	592	47,513,097	521
6	678,362,757	1255	678,362,922	1202	678,354,052	978
7	138,225,390	2231	138,226,758	1931	138,210,352	1429
8	177,462,355	3732	177,462,355	3211	176,677,827	2534
9	402,100,473	5002	402,100,473	4887	402,080,659	3997
10	482,395,459	6974	482,395,459	6499	482,211,954	6240

Table 4: Deviation comparison of INS with GA and BD.

Problem number	Percentage deviation from GA	Percentage deviation from BD	Minimal and maximal deviation from GA	Minimal and maximal deviation from BD
1	−0.00058	−0.00058		
2	−0.00207	−0.00207		
3	−0.18000	−0.18000		
4	−0.00030	−0.00030	−0.00030	−0.00030
5	−0.00169	−0.00292		
6	−0.00128	−0.00014		
7	−0.00109	−0.00118		
8	−0.3900	−0.3900	−0.3900	−0.3900
9	−0.00049	−0.00049		
10	−0.00380	−0.00380		

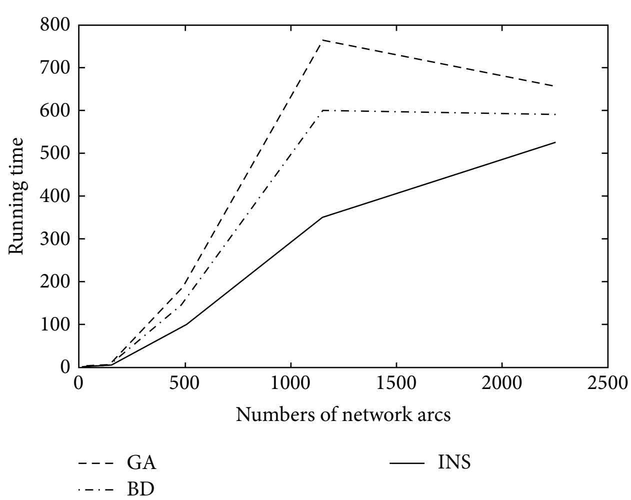

Figure 9: Algorithms performance comparison for small-scale network problem.

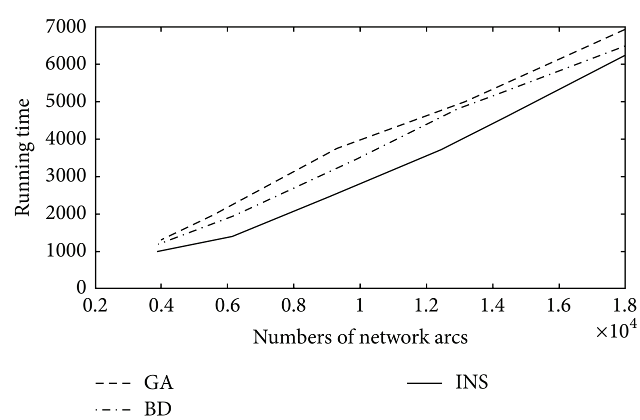

Figure 10: Algorithms performance comparison for large-scale network problem.

and capacity design decision and the initial distribution planning decision. In this paper, we have first proposed a classic MIP (mixed integer programming) model to solve such problem, and then we developed a transformation mechanism for modeling such problem as a network flow problem. After that, the NFP (network flow problem) model has been formulated. Since the NFP model could be solved as a minimal cost flow problem. The solution procedures and corresponding network simplex algorithm (INS) are designed. To verify the effectiveness and efficiency of the NFP model and algorithms, the performance measure experimental has been conducted for 10 various-scale problems, in which the plant's capacity, the customer demand, and the relevant costs data are randomly generated in a reasonable region. The experiments and result showed that, comparing with MIP model solved by GA and and BD, the NFP model and INS are even more efficient for both small-scale and large-scale problem; furthermore, the advantage of NFP model and INS is stable for solving large-scale SCND problem.

However, there are still many works that have to be accomplished in the future. In this paper, we have just initiated work on the former. Although this paper affords a new solving approach to SCND problem. It only dealt with certain customer demand and a two-layer supply chain. The uncertainty and random demand scenario and more complicated competitive SCND should be considered in the future. Besides, as mentioned previously, a tight relaxation mechanism for the DCs construction cost should be developed and more numerical experimental should be investigated in the future study.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] F. Altiparmak, M. Gen, L. Lin, and I. Karaoglan, "A steady-state genetic algorithm for multi-product supply chain network design," *Computers and Industrial Engineering*, vol. 56, no. 2, pp. 521–537, 2009.

[2] W. Wilhelm, D. Liang, B. Rao, D. Warrier, X. Zhu, and S. Bulusu, "Design of international assembly systems and their supply chains under NAFTA," *Transportation Research E*, vol. 41, no. 6, pp. 467–493, 2005.

[3] M. S. Daskin, A. P. Hurter, and M. G. VanBuer, "Toward an intergrated model of facility location and transportation network design," Working Paper, The Transportation Center, Northwestern University, 1993.

[4] S. Melkote and M. S. Daskin, "An integrated model of facility location and transportation network design," *Transportation Research A*, vol. 35, no. 6, pp. 515–538, 2001.

[5] A. Nagurney, T. Woolley, and Q. Qiang, "Multiproduct supply chain horizontal network integration: models, theory, and computational results," *International Transactions in Operational Research*, vol. 17, no. 3, pp. 333–349, 2010.

[6] A. Nagurney, A. H. Masoumi, and M. Yu, "Supply chain network operations management of a blood banking system with cost and risk minimization," *Computational Management Science*, vol. 9, no. 2, pp. 205–231, 2012.

[7] R. Z. Farahani, S. Rezapour, T. Drezner, and S. Fallah, "Competitive supply chain network design: an overview of classifications, models, solution techniques and applications," *Omega*, 2013.

[8] K. E. Caggiano, P. L. Jackson, J. A. Muckstadt, and J. A. Rappold, "Efficient computation of time-based customer service levels in a multi-item, multi-echelon supply chain: a practical approach for inventory optimization," *European Journal of Operational Research*, vol. 199, no. 3, pp. 744–749, 2009.

[9] P. Tsiakis, N. Shah, and C. C. Pantelides, "Design of multi-echelon supply chain networks under demand uncertainty," *Industrial and Engineering Chemistry Research*, vol. 40, no. 16, pp. 3585–3604, 2001.

[10] R. K. Ahuja, A. V. Goldberg, J. B. Orlin, and R. E. Tarjan, "Finding minimum-cost flows by double scaling," *Mathematical Programming*, vol. 53, no. 1–3, pp. 243–266, 1992.

[11] R. K. Ahuja and J. B. Orlin, "Equivalence of the primal and dual simplex algorithms for the maximum flow problem," *Operations Research Letters*, vol. 20, no. 3, pp. 101–108, 1997.

[12] A. V. Goldberg and R. E. Tarjan, "Finding minimum-cost circulations by successive approximation," *Mathematics of Operations Research*, vol. 15, no. 3, pp. 430–466, 1990.

[13] A. Jabbarzadeh, S. G. J. Naini, H. Davoudpour, and N. Azad, "Designing a supply chain network under the risk of disruptions," *Mathematical Problems in Engineering*, vol. 2012, Article ID 234324, 23 pages, 2012.

[14] M. Abbasi, R. Hosnavi, and M. Mohammadi, "Designing a fuzzy strategic integrated multiechelon agile supply chain network," *Journal of Optimization*, vol. 2013, Article ID 543604, 16 pages, 2013.

[15] G. Zhou, Y. Yang, and J. Cao, "Competition and integration in closed-loop supply chain network with variational inequality," *Mathematical Problems in Engineering*, vol. 2012, Article ID 524809, 21 pages, 2012.

[16] F. Hanssmann, "Optimal inventory location and control in production and distribution networks," *Operations Research*, vol. 7, pp. 483–498, 1959.

[17] A. M. Geoffrion and G. W. Graves, "Multicommodity distribution system design by Benders decomposition," *Management Science*, vol. 20, no. 5, pp. 822–844, 1974.

[18] V. Jayaraman and H. Pirkul, "Planning and coordination of production and distribution facilities for multiple commodities," *European Journal of Operational Research*, vol. 133, no. 2, pp. 394–408, 2001.

[19] R. B. Handfield and E. L. Nichols Jr., *Supply Chain Redesign: Transforming Supply Chains into Integrated Value Systems*, Financial Times Prentice Hall Books, FT Press, Prentice Hall, Upper Saddle River, NJ, USA, 2002.

[20] S. S. Syam, "A model and methodologies for the location problem with logistical components," *Computers and Operations Research*, vol. 29, no. 9, pp. 1173–1193, 2002.

[21] Y. J. Jang, S. Y. Jang, B. M. Chang, and J. Park, "A combined model of network design and production/distribution planning for a supply network," *Computers and Industrial Engineering*, vol. 43, no. 1-2, pp. 263–281, 2002.

[22] A. Syarif, Y. Yun, and M. Gen, "Study on multi-stage logistic chain network: a spanning tree-based genetic algorithm approach," *Computers and Industrial Engineering*, vol. 43, no. 1-2, pp. 299–314, 2002.

[23] V. Jayaraman and A. Ross, "A simulated annealing methodology to distribution network design and management," *European Journal of Operational Research*, vol. 144, no. 3, pp. 629–645, 2003.

[24] W. C. Yeh, "A hybrid heuristic algorithm for the multistage supply chain network problem," *International Journal of Advanced Manufacturing Technology*, vol. 26, no. 5-6, pp. 675–685, 2005.

[25] W. C. Yeh, "An efficient memetic algorithm for the multi-stage supply chain network problem," *International Journal of Advanced Manufacturing Technology*, vol. 29, no. 7-8, pp. 803–813, 2006.

[26] C. J. Solo, *Multi-objective, integrated supply chain design and operation under uncertainty [Ph.D. thesis]*, The Pennsylvania State University, 2009.

[27] H. Y. Mak, *Integrated supply chain design under uncertainty [Ph.D. thesis]*, University of California, Berkeley, Berkeley, Calif, USA, 2009.

[28] F. Pan and R. Nagi, "Multi-echelon supply chain network design in agile manufacturing," *Omega*, vol. 41, no. 6, pp. 969–983, 2013.

[29] R. K. Ahuja and H. W. Hamacher, "A network flow algorithm to minimize beam-on time for unconstrained multileaf collimator problems in cancer radiation therapy," *Networks*, vol. 45, no. 1, pp. 36–41, 2005.

[30] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows—Theory, Algorithms and Applications*, Prentice Hall, Englewood Cliffs, NJ, USA, 1993.

[31] P. T. Sokkalingam and P. Sharma, "A combinatorial arc tolerance analysis for network flow problems," *Journal of Applied Mathematics and Decision Sciences*, vol. 2005, no. 2, pp. 83–94, 2005.

[32] Y. Sheffi, *Transportation network equilibrium with discrete choice models [Ph.D. thesis]*, Civil Engineering Department, Massachusetts Institute of Technology, Cambridge, Mass, USA, 1978.

[33] A. Nagurney and J. F. Smith, *Supernetworks: Decision-Making for the Information Age*, Edward Elgar Publishing, 2002.

[34] A. Nagurney, "Optimal supply chain network design and redesign at minimal total cost and with demand satisfaction," *International Journal of Production Economics*, vol. 128, no. 1, pp. 200–208, 2010.

[35] P. T. Sokkalingam and P. Sharma, "A combinatorial arc tolerance analysis for network flow problems," *Journal of Applied Mathematics and Decision Sciences*, vol. 2005, no. 2, pp. 83–94, 2005.

[36] U. S. Palekar, M. H. Karwan, and S. Zionts, "A branch-and-bound method for the fixed charge transportation problem," *Management Science*, vol. 36, no. 9, pp. 1092–1105, 1990.

[37] N. Jawahar and A. N. Balaji, "A genetic algorithm for the two-stage supply chain distribution problem associated with a fixed charge," *European Journal of Operational Research*, vol. 194, no. 2, pp. 496–537, 2009.