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Fault tolerant vibration-attenuation controller design for uncertain linear structural systems with input time-delay and saturation

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Abstract. The problem of fault tolerant vibration-attenuation controller design for uncertain linear structural systems with control input time-delay and saturation is investigated in this paper. The objective of designing controllers is to guarantee the asymptotic stability of closed-loop systems while attenuate disturbance from earthquake excitation. Firstly, based on matrix transformation, the structural system is described as state-space model, which contains actuator fault, input signal time-delay and saturation at the same time. Based on the obtained model, an LMIs-based condition for the system to be stabilizable is deduced. By solving these LMIs, the controller is established for the closed-loop system to be stable with a prescribed level of disturbance attenuation. The condition is also extended to the uncertain case. Finally, an example is included to demonstrate the effectiveness of the proposed theorems.

Keywords: Actuator saturation, structural systems, vibration, fault tolerant control, time-delay

1. Introduction

In recent years, because earthquake and tsunamis happen frequently, vibration control for buildings structure has received considerable attention. However, as buildings become higher and higher, structural stability and solidity of seismic-excited and wind-excited buildings are challenged and cannot be guaranteed only by architectural materials of high quality or those passive control methods. Therefore, research on the active vibration control of linear structural systems has received increasing attention in the recent years. Many scholars have applied themselves to the research of active vibration control strategies and many control techniques have been utilized, such as, classical H_∞ theories [1,2], Finite frequency H_∞ control [3], sliding mode control [4,5], neural networks [6], optimal control [7], bang-bang control [8,9], Semiactive – passive control [10], Semi-decentralized Control [11], mixed H_2/H_∞ output-feedback control [12], etc., have been developed with the goal of protecting structures subjected to external disturbance excitation. Accompanied with the development of structural control strategies, some active control devices were designed for applying those control algorithms, for example, active brace system (ABS) [13,14], active mass damper (AMD) [15], etc. have been widely studied and used for vibration attenuation.

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On the other hand, since unexpected faults or failures may result in substantial damage, and can even be hazardous to human and environmental security, much effort has been devoted to the fault tolerant control or reliable control during the past decades. A high degree of fault tolerance for the structural control systems is an essential and integrated part of the overall control system design. Thus, the problem of fault tolerant control in buildings structural systems has received considerable attention and various techniques have been developed. For example, by a DC-motor based active mass driving device, [16] investigated fault tolerant control system design for structure based on two experiments. [17] presented a direct adaptive fault-tolerant neural control scheme for the active control of nonlinear hysteretic base-isolated buildings using the recently developed extended minimal resource allocation network (EMRAN). Based on LMI technique, [18] dealt with the problem of robust reliable energy-to-peak controller design for seismic-excited buildings with actuator faults and parameter uncertainties.

In practice, time delay or transportation lag is commonly encountered when the control forces are applied to the practical systems. Thus another important issue of structural control is the time delay problem when the control forces are applied to the structures. The unavoidable time delays appearing in the control channel, manly include data acquisition from sensors, filtering, processing of data, calculating control forces and transmitting the control force signals from computer to the actuator, actuator response delay, A/D, and D/A conversion, etc. The time delay may be short, which can nevertheless limit the control performance or even cause the instability of the system [19]. During the last decades, the study of structural systems with control input time-delay has received increasing attention. For example, in terms of the feasibility of certain delay-dependent linear matrix inequalities (LMIs), the robust H_∞ disturbance attenuation problem for uncertain structural systems with control input time-delay was researched by [19]. [20] addressed a convex optimization approach to the problem of state-feedback H_∞ control design for vibration reduction of base-isolated building structures with delayed measurements. By combining the random search of genetic algorithms and the solvability of LMIs, [2] investigated the H_∞ controller design approach for vibration attenuation of seismic-excited building structures with time delay in control input channel. Based on LMI technique, the problem of robust active vibration control for a class of electro-hydraulic actuated structural systems with time delay in the control input channel and parameter uncertainties appearing in all the mass, damping and stiffness matrices is investigated in [21]. By considering the actuator saturation and control input time-delay, the active vibration control for a class of earthquake-excited structural systems was presented in [22]. Furthermore, most of the actuation devices are subject to amplitude saturation, that for the physical inputs such as force, torque, thrust, stroke, voltage, current, and flow rate of all conceivable applications of current technology are ultimately limited, and unexpected large amplitude disturbances can also push a system's actuators into saturation, thus forcing the system to operate in a nonlinear mode, for which it was not designed and in which it may be unstable [23]. Thus controller design for buildings structural systems, which involves actuator saturation, is also needed. However, to the best of the authors' knowledge, the fault tolerant vibration-attenuation controller design for uncertain linear structural systems with control input time-delay and saturation is still not fully investigated.

This paper is concerned with the problem of fault tolerant vibration-attenuation controller design for uncertain linear structural systems with input time-delay and saturation. The main contribution of this paper consists in two aspects. First, based on matrix transformation, an improved actuator fault and saturation description is achieved. Then, the state-space model of buildings with parameter uncertainties and input time-delay and saturation is established. Second, a novel Lyapunov functional, which includes some non-positive items, is introduced to obtain the sufficient conditions for stabilizability of the structural systems. By solving the LMIs, the controllers against actuator failures and saturation are established for the closed-loop system to be stable with the performance $\|z\|_2 < \gamma\|\omega\|_2$.

The organization of this paper is as follows. Section 2 formulates the problem and presents the dynamic models. The main results are given in Section 3. The illustrative examples are given in Section 4 to show the applicability and improvement of the presented approaches. Finally, the paper is concluded in Section 5.

Notation: Throughout this paper, for real matrices X and Y, the notation $X \geqslant Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is semi-positive definite (respectively, positive definite). I is the identity matrix with appropriate dimension, and a superscript "T" represents transpose. For a symmetric matrix, $*$ denotes the symmetric terms. The symbol R^n stands for the n-dimensional Euclidean space, and $R^{n\times m}$ is the set of $n \times m$ real matrices.

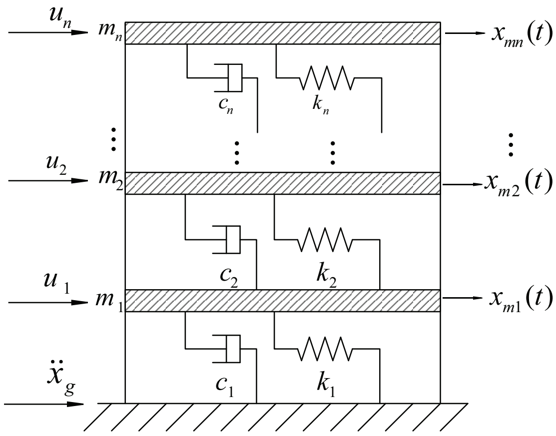

Fig. 1. n-d.o.f. structural system.

2. Problem formulation and dynamic models

Consider an n degree-of-freedom structural system. The system under consideration is depicted in Fig. 1. The linear structural model equation can be written with [1,2,17–19,21,22]

$$M\ddot{x}_m(t) + C\dot{x}_m(t) + Kx_m(t) = H_0 u(t-\tau) + H_\omega \ddot{x}_g(t), \tag{1}$$

where $x_m(t) = [x_{m1}(t), x_{m2}(t), \ldots, x_{mn}(t)]^T$, $x_{mn}(t)$ is the relative displacement of the nth storey to ground; $u(t-\tau)$ is the control force input, and τ is the control forces input time-delay; $\ddot{x}_g(t)$ is the input disturbance belongs to $L_2[0,\infty)$, $H_0 \in R^{n\times m}$ gives the locations of these controllers, $H_\omega \in R^{n\times 1}$ is an vector denoting the influence of disturbance excitation, and $M, C, K \in R^{n\times n}$ are the mass, damping and stiffness matrices of the system, respectively. From Fig. 1, we can obtain

$$M = diag\{m_1, m_2, \ldots, m_n\}, H_\omega = -[m_1, m_2, \ldots, m_n]^T,$$

$$C = \begin{bmatrix} c_1+c_2 & -c_2 & \ldots & 0 \\ -c_2 & c_2+c_3 & \ldots & \vdots \\ \vdots & \vdots & \ldots & -c_n \\ 0 & 0 & \ldots & c_n \end{bmatrix}, K = \begin{bmatrix} k_1+k_2 & -k_2 & \ldots & 0 \\ -k_2 & k_2+k_3 & \ldots & \vdots \\ \vdots & \vdots & \ldots & -k_n \\ 0 & 0 & \ldots & k_n \end{bmatrix}.$$

Defining the state variables as $x(t) = \left[x_m(t)^T, \dot{x}_m(t)^T\right]^T$, the system Eq. (1) can be written in the following state-space form:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t-\tau) + B_\omega \omega(t), \\ z(t) &= C_z x(t), \\ x(t) &= \Phi(t), \forall t \in [-\tau, 0], \end{aligned} \tag{2}$$

where $z(t)$ is the control output, C_z is a real constant matrix with appropriate dimensions, $\Phi(t)$ is the initial condition on the segment $[-\tau, 0]$, and

$$E = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}, A = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix}, B = \begin{bmatrix} 0 \\ H_0 \end{bmatrix}, B_\omega = \begin{bmatrix} 0 \\ H_\omega \end{bmatrix}, \omega(t) = \ddot{x}_g(t).$$

When considering possible actuator fault and control force saturation, we introduce a state-feedback controller in the form of

$$u(t) = \sigma\left(\upsilon(t)Fx(t)\right), \tag{3}$$

where F is the actuator fault-tolerant controller gain to be design later. Actuator failures are described by fault matrix $\upsilon(t) = \textit{diag}\left\{\upsilon_1(t), \upsilon_2(t), \ldots \upsilon_m(t)\right\}$, $0 \leqslant \underline{\upsilon}_i \leqslant \upsilon_i(t) \leqslant \bar{\upsilon}_i < \infty$. $\underline{\upsilon}_i$ and $\bar{\upsilon}_i$ represent the lower and upper bounds of $\upsilon_i(t)$, respectively. If $\underline{\upsilon}_i = \bar{\upsilon}_i = 1$, then there is no failure for the ith actuator. When $\underline{\upsilon}_i = \bar{\upsilon}_i = 0$, the ith actuator is in outage. Otherwise, if $0 < \underline{\upsilon}_i \leqslant \bar{\upsilon}_i$ and $\upsilon_i(t) \neq 1$, it corresponds to the case of partial degradation of the ith actuator. The function $\sigma(\cdot)$: $R^m \to R^m$ is a standard saturation function with the limit of $u_{\lim_i}$ for the ith actuator. $u(t)$ can be expressed as

$$u(t) = \left[\sigma\left(u_1(t)\right), \sigma\left(u_2(t)\right), \ldots, \sigma\left(u_m(t)\right)\right]^T$$

where $\sigma\left(u_i(t)\right) = \textit{sign}\left(u_i(t)\right)\min\left\{\left|u_i(t)\right|, u_{\lim_i}\right\}$.

Remark 1: It is worth to point out that $\upsilon(t)$ and $\sigma(\cdot)$ in Eq. (3) denote the possible failure and saturation existing in a practical actuator. We need to obtain a state-feedback controller, which has a fixed gain F, such that the corresponding closed-loop system can tolerate the possible actuator failure and saturation. Thus, how to deal with $\upsilon(t)$ and $\sigma(\cdot)$ is the key to achieve a satisfying controller.

Using the transform $\sigma\left(\upsilon(t)Fx(t)\right) = \psi(t)\upsilon(t)Fx(t)$ [24–26], where $\psi(t) = \textit{diag}\left\{\psi_1(t), \psi_2(t), \ldots \psi_m(t)\right\}$, $\psi_i(t) = \sigma\left(u_i(t)\right)/u_i(t)$ with $\psi_i(t) = 1$ if $u_i(t) = 0$. To obtain the high gain controller as that in [24], the command to the ith actuator is allowed to be $\varepsilon_i u_{\lim_i}$ for an arbitrary scalar $\varepsilon_i > 1$. Therefore, the resulting $\psi_i(t)$ will be bounded by $1/\varepsilon_i$ and 1, that is, $1/\varepsilon_i \leqslant \psi_i(t) \leqslant 1$, $i = 1, 2, \ldots, m$. Thus, we have

$$\psi(t)\upsilon(t) = \textit{diag}\left\{\psi_1(t)\upsilon_1(t), \psi_2(t)\upsilon_2(t), \ldots, \psi_m(t)\upsilon_m(t)\right\}, \text{ and } \underline{\upsilon}_i/\varepsilon_i \leqslant \psi_i(t)\upsilon_i(t) \leqslant \bar{\upsilon}_i, i = 1, 2, \ldots, m.$$

For notational simplicity, in the sequel, the matrix $\psi(t)\upsilon(t)$ will be denoted by $\Gamma(t)$. Then, we have $\Gamma(t) = \textit{diag}\left\{\Gamma_1(t), \Gamma_2(t), \ldots, \Gamma_m(t)\right\}$, satisfying $\underline{\Gamma}_i \leqslant \Gamma_i(t) \leqslant \bar{\Gamma}_i$, where $\Gamma_i(t) = \psi_i(t)\upsilon_i(t)$, $\underline{\Gamma}_i = \frac{\underline{\upsilon}_i}{\varepsilon_i}$, and $\bar{\Gamma}_i = \bar{\upsilon}_i$. By defining $\Gamma_{0i} = \left(\underline{\Gamma}_i + \bar{\Gamma}_i\right)/2$, $\Delta\Gamma_i = \delta_i(t)\Gamma_{0i}$, $\left|\delta_i(t)\right| \leqslant \bar{\delta}_i \leqslant 1$, $\bar{\delta}_i = \left(\bar{\Gamma}_i - \underline{\Gamma}_i\right)/\left(\underline{\Gamma}_i + \bar{\Gamma}_i\right)$, $i = 1, 2, \ldots m$, we can depict the controller Eq. (3) as $u(t) = \left(\Gamma_0 + \Delta\Gamma\right)Fx(t)$, where $\Gamma_0 = \textit{diag}\left\{\Gamma_{01}, \Gamma_{02}, \ldots, \Gamma_{0m}\right\}$, $\Delta\Gamma = \textit{diag}\left\{\Delta\Gamma_1, \Delta\Gamma_2, \ldots, \Delta\Gamma_m\right\} = \sum_{i=1}^{m} \bar{\delta}_i \Gamma_0 e_{0i} \frac{\delta_i(t)}{\bar{\delta}_i} f_{0i}^T$. e_{0i} and f_{0i} are all column vectors with the ith items to be 1, and others to be 0. Obviously, there has $\delta_i(t)/\bar{\delta}_i \leqslant 1$.

In practice, the mass, damping and stiffness are usually subjected to possible perturbations, such as measurement error, the changes in environmental temperature and plastic deformation, etc. By assuming that the uncertain $m_j \in \left[\underline{m}_j, \bar{m}_j\right]$, $k_j \in \left[\underline{k}_j, \bar{k}_j\right]$, $c_j \in \left[\underline{c}_j, \bar{c}_j\right]$, $j = 1, 2, \ldots n$, where $\underline{m}_j, \underline{k}_j, \underline{c}_j$ $\left(\bar{m}_j, \bar{k}_j, \bar{c}_j\right)$ are the lower (upper) bounds of the mass, stiffness and damping respectively, and denoting

$$\begin{aligned}
\hat{m}_j &= \frac{1}{2}\left(\underline{m}_j + \bar{m}_j\right), \Delta\hat{m}_j = \theta_{1j}\hat{m}_j, \left|\theta_{1j}\right| \leqslant \bar{\theta}_{1j} < 1, \bar{\theta}_{1j} = \left(\bar{m}_j - \underline{m}_j\right)/\left(\bar{m}_j + \underline{m}_j\right), \quad j = 1, 2, \ldots n,\\
\hat{k}_j &= \frac{1}{2}\left(\underline{k}_j + \bar{k}_j\right), \Delta\hat{k}_j = \theta_{2j}\hat{k}_j, \left|\theta_{2j}\right| \leqslant \bar{\theta}_{2j} < 1, \bar{\theta}_{2j} = \left(\bar{k} - \underline{k}\right)/\left(\bar{k} + \underline{k}\right), \quad j = 1, 2, \ldots n,\\
\hat{c}_j &= \frac{1}{2}\left(\underline{c}_j + \bar{c}_j\right), \Delta\hat{c}_j = \theta_{2(n+j)}\hat{c}_j, \left|\theta_{2(n+j)}\right| \leqslant \bar{\theta}_{2(n+j)} < 1, \bar{\theta}_{2(n+j)} = (\bar{c} - \underline{c})/(\bar{c} + \underline{c}), \quad j = 1, 2, \ldots n,
\end{aligned}$$

we can describe the uncertain system by state space equation of the form:

$$\begin{aligned}
E(\theta_1)\dot{x}(t) &= A(\theta_2)x(t) + Bu(t-\tau) + B_\omega(\theta_1)\omega(t),\\
u(t) &= \left(\Gamma_0 + \Delta\Gamma\right)Fx(t)\\
z(t) &= C_z x(t),\\
x(t) &= \Phi(t), \forall t \in [-\tau, 0],
\end{aligned} \tag{4}$$

where uncertain matrices $E(\theta_1)$, $A(\theta_2)$, $B_\omega(\theta_1)$ satisfying

$$E(\theta_1) = E_0 + \sum_{j=1}^{n} \theta_{1j} E_j, A(\theta_2) = A_0 + \sum_{j=1}^{2n} \theta_{2j} A_j, B_\omega(\theta_1) = B_{\omega 0} + \sum_{j=1}^{n} \theta_{1j} B_{\omega j},$$

$$E_0 = diag\left\{ \underbrace{1, 1, \ldots 1}_{n}, \hat{m}_1, \hat{m}_2, \ldots \hat{m}_n \right\}, B_{\omega 0} = \left[\underbrace{0, 0, \ldots, 0}_{n}, -\hat{m}_1, -\hat{m}_2, \ldots, -\hat{m}_n \right]^T,$$

$$E_j = \hat{m}_j e_{1j} f_{1j}^T, B_{\omega j} = -\hat{m}_j e_{3j} f_{3j}^T,$$

$$A_0 = \begin{bmatrix} 0 & I \\ -\hat{K} & -\hat{C} \end{bmatrix}, \hat{K} = \begin{bmatrix} \hat{k}_1 + \hat{k}_2 & -\hat{k}_2 & \ldots & 0 \\ -\hat{k}_2 & \hat{k}_2 + \hat{k}_3 & \ldots & \vdots \\ \vdots & \vdots & \ldots & -\hat{k}_n \\ 0 & 0 & \ldots & \hat{k}_n \end{bmatrix}, \hat{C} = \begin{bmatrix} \hat{c}_1 + \hat{c}_2 & -\hat{c}_2 & \ldots & 0 \\ -\hat{c}_2 & \hat{c}_2 + \hat{c}_3 & \ldots & \vdots \\ \vdots & \vdots & \ldots & -\hat{c}_n \\ 0 & 0 & \ldots & \hat{c}_n \end{bmatrix},$$

$$A_j = \hat{k}_j e_{2j} f_{2j}^T, A_{n+j} = \hat{c}_j e_{2(n+j)} f_{2(n+j)}^T, (j = 1, 2, \ldots, n).$$

$e_{1j} \in R^{2n}(j = 1, 2, \ldots, n), f_{1j} \in R^{2n}(j = 1, 2, \ldots, n), e_{2j} \in R^{2n}(j = 1, 2, \ldots, 2n), f_{2j} \in R^{2n}(j = 1, 2, \ldots, 2n), e_{3j} \in R^{2n}(j = 1, 2, \ldots, n), f_{3j} \in R^1(j = 1, 2, \ldots, n)$ are all column vectors. In this paper, the aim is to find a controller gain F such that the closed-loop system has the following properties: (i) asymptotically stable; (ii) for all non-zero $\omega \in L_2[0, \infty)$ and the prescribed constant $\gamma > 0$, it has the performance $\|z\|_2 < \gamma \|\omega\|_2$.

Lemma 1 [27]: Given any matrices X, V and U with appropriate dimensions such that $U > 0$. Then we have

$$-XUX^T \leqslant XV^T + VX^T + VU^{-1}V^T. \tag{5}$$

Lemma 2 [28]: Given matrices χ, μ and ν with appropriate dimensions and with χ symmetrical, then

$$\chi + \mu F(t)\nu + \nu^T F(t)^T \mu^T < 0 \tag{6}$$

holds for any $F(t)$ satisfying $F(t)^T F(t) \leqslant I$, if and only if there exists a scalar $\lambda > 0$ such that

$$\chi + \lambda \mu \mu^T + \lambda^{-1} \nu^T \nu < 0. \tag{7}$$

3. Main results

Theorem 1: The system Eq. (4) without uncertainties is asymptotically stabilizable with constant time-delay τ and performance $\|z\|_2 < \gamma \|\omega\|_2$ for all non-zero $\omega \in L_2[0, \infty)$, and constant $\gamma > 0$, if there exist positive definite symmetric matrices P, U, V_1, V_2, Q, matrices Z_1, Z_2, Z_3, Z_4, Z_5, S, Y_i $(i = 1, 2, 3, 4, 5)$, H_i $(i = 1, 2, 3, 4, 5)$, positive scalars $r_{01}, r_{02}, \ldots, r_{0m}$ and scalars β_1, β_2, β_3 satisfying the following LMIs

$$\Xi_1 = \begin{bmatrix} \Pi & \tau H & \tau Y & B_\omega & \Xi_{19} \\ * & -\tau V_2 & 0 & 0 & 0 \\ * & * & -\tau V_1 & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \Xi_{99} \end{bmatrix} < 0, \tag{8}$$

$$\Xi_2 = \begin{bmatrix} P + V_1 + \tau Z_3 + \tau Z_3^T + \tau^2 Z_5 + 2V_2 & Z_2 - Z_3 + \tau Z_4^T - \tau Z_5 - \frac{2}{\tau} V_2 \\ * & \frac{1}{\tau} U + \frac{1}{\tau} V_1 + Z_1 - Z_4 - Z_4^T + Z_5 + \frac{2}{\tau^2} V_2 \end{bmatrix} > 0, \tag{9}$$

where

$$\Pi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & SC_z^T + H_5^T \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \beta_1 SC_z^T - H_5^T \\ * & * & \Xi_{33} & \Xi_{34} & \beta_2 SC_z^T \\ * & * & * & Y_4 + Y_4^T - Q & \beta_3 SC_z^T + Y_5^T \\ * & * & * & * & -\gamma^2 I \end{bmatrix} + \sum_{i=1}^{m} r_{0i} \Upsilon_{0i} \Upsilon_{0i}^T,$$

$$\Xi_{11} = U + \tau V_1 + Z_2 + Z_2^T - Z_3 - Z_3^T + \tau Z_4 + \tau Z_4^T - \tau Z_5 - \tau Z_5^T - \tau^2 Q + SA_0^T + A_0 S^T + H_1 + H_1^T,$$

$$\Xi_{12} = -Z_2 + Z_3 - \tau Z_4 + \tau Z_5^T + T^T \Gamma_0^T B^T + \beta_1 A_0 S^T - H_1 + H_2^T,$$

$$\Xi_{13} = P + \tau Z_3^T + \tau Z_3 + \tau^2 Z_5^T + \tau Z_5^T - SE_0^T + \beta_2 A_0 S^T + H_3^T,$$

$$\Xi_{14} = Y_1 + Z_1 - Z_4 - Z_4^T + Z_5 + \tau Q + \beta_3 A_0 S^T + H_4^T,$$

$$\Xi_{22} = -U + \beta_1 T^T \Gamma_0^T B^T + \beta_1 B \Gamma_0 T - H_2 - H_2^T, \quad \Xi_{23} = -\beta_1 SE_0^T + \beta_2 B \Gamma_0 T - H_3^T,$$

$$\Xi_{24} = Y_2 - Z_1 + Z_4 + Z_4^T - Z_5 + \beta_3 B \Gamma_0 T - H_4^T, \quad \Xi_{33} = \frac{1}{4}\tau^4 Q + \tau V_2 - \beta_2 SE_0^T - \beta_2 E_0 S^T,$$

$$\Xi_{34} = Y_3 + Z_2 - Z_3 + \tau Z_4^T - \tau Z_5^T - \beta_3 E_0 S^T,$$

$$H = \begin{bmatrix} H_1^T & H_2^T & H_3^T & H_4^T & H_5^T \end{bmatrix}^T, Y = \begin{bmatrix} Y_1^T & Y_2^T & Y_3^T & Y_4^T & Y_5^T \end{bmatrix}^T,$$

$$\Xi_{19} = [\Lambda_{01}, \Lambda_{02}, \ldots, \Lambda_{0m}], \quad \Xi_{99} = diag\{-r_{01}, -r_{02}, \ldots, -r_{0m}\},$$

$$\Upsilon_{0i} = \begin{bmatrix} 0 & \bar{\delta}_i e_{0i}^T \Gamma_0^T B^T & 0 & 0 & 0 \end{bmatrix}^T, \quad \Lambda_{0i} = \begin{bmatrix} f_{0i}^T T & \beta_1 f_{0i}^T T & \beta_2 f_{0i}^T T & \beta_3 f_{0i}^T T & 0 & 0 & 0 & 0 \end{bmatrix}^T.$$

Furthermore, the state-feedback controller is described as $F = TS^{-T}$.

Proof: We first consider the system Eq. (4) without uncertainties, that is, $\theta_{1i} = 0 (i = 1, 2, \ldots n), \theta_{2i} = 0 (i = 1, 2, \ldots 2n)$. Substituting the control law $u(t) = (\Gamma_0 + \Delta\Gamma) Fx(t)$ into the system Eq. (4) results in the following closed-loop system

$$\begin{aligned} E_0 \dot{x}(t) &= A_0 x(t) + B(\Gamma_0 + \Delta\Gamma) Fx(t - \tau) + B_{\omega 0}\omega(t), \\ z(t) &= C_z x(t). \end{aligned} \tag{10}$$

Noting that the solutions to $\det(SE - A - BFe^{-s\tau}) = 0$ are the same as those of $\det(SE^T - A^T - F^T B^T e^{-s\tau}) = 0$, and $\|T_{z\omega}\|_\infty = \|T_{z\omega}^T\|_\infty$, as long as the H_∞ performance and stability are concerned, we can consider the following system instead of Eq. (10)

$$\begin{aligned} E_0^T \dot{x}(t) &= A_0^T x(t) + F^T (\Gamma_0 + \Delta\Gamma)^T B^T x(t - \tau) + C_z^T \omega(t), \\ z(t) &= B_{\omega 0}^T x(t). \end{aligned} \tag{11}$$

Choose a Lyapunov-Krasovskii functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{12}$$

where

$$V_1(t) = x(t)^T P x(t) + \int_{t-\tau}^{t} x^T(s) U x(s) ds + \int_{-\tau}^{0}\int_{t+\varepsilon}^{t} x_{(s)}^T V_1 x_{(s)} ds d\varepsilon + \int_{-\tau}^{0}\int_{t+\varepsilon}^{t} \dot{x}^T(s) V_2 \dot{x}(s) ds d\varepsilon,$$

$$V_2(t) = \int_{t-\tau}^{t} x_{(s)}^T ds Z_1 \int_{t-\tau}^{t} x_{(s)} ds + 2x_{(t)}^T Z_2 \int_{t-\tau}^{t} x_{(s)} ds + 2x_{(t)}^T Z_3 \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)} ds d\theta$$

$$+ 2\int_{t-\tau}^{t} x_{(s)}^T ds Z_4 \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)} ds d\theta + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)}^T ds d\theta Z_5 \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)} ds d\theta,$$

$$V_3(t) = \frac{1}{2}\tau^2 \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\varepsilon}^{t} \dot{x}_{(s)}^T Q \dot{x}_{(s)} ds d\varepsilon d\theta,$$

and $P > 0$, $U > 0$, $V_1 > 0$, $V_2 > 0$, $Q > 0$, Z_1, Z_5, are symmetric matrices to be determined. Obviously, from Eq. (9), we have

$$\begin{aligned}
V(t) &\geqslant x(t)^T P x(t) + \int_{t-\tau}^{t} x^T(s) U x(s) ds + \int_{-\tau}^{0}\int_{t+\varepsilon}^{t} x_{(s)}^T V_1 x_{(s)} ds d\varepsilon \\
&\quad + \frac{2}{\tau^2}\left(\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\varepsilon}^{t} \dot{x}^T(s) V_2 \dot{x}(s) ds d\varepsilon\right) + V_2(t) \\
&\geqslant x(t)^T (P + V_1) x(t) + \int_{t-\tau}^{t} x_{(s)}^T ds \left(\frac{1}{\tau}U + \frac{1}{\tau}V_1\right)\int_{t-\tau}^{t} x_{(s)}^T ds \\
&\quad + \frac{2}{\tau^2}\left(\tau x_{(t)}^T - \int_{t-\tau}^{t} x_{(s)}^T ds\right) V_2 \left(\tau x_{(t)} - \int_{t-\tau}^{t} x_{(s)} ds\right) + V_2(t) \\
&= \zeta(t)^T \Xi_2 \zeta(t) > 0,
\end{aligned}$$

where $\zeta(t) = \left[\begin{array}{cc} x(t)^T & \int_{t-\tau}^{t} x_{(s)}^T ds \end{array}\right]^T$. Then, the derivative of $V(t)$ along the solution of system Eq. (11) is given by

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) + x^T(t) U x(t) - x^T(t-\tau) U x(t-\tau) \\
&\quad + \tau x_{(t)}^T V_1 x_{(t)} - \int_{t+\tau}^{t} x_{(s)}^T V_1 x_{(s)} ds + \tau \dot{x}^T(t) V_2 \dot{x}(t) - \int_{t-\tau}^{t} \dot{x}^T(\varepsilon) V_2 \dot{x}(\varepsilon) d\varepsilon,
\end{aligned} \tag{13}$$

$$\begin{aligned}
\dot{V}_2(t) &= 2\left(x_{(t)} - x_{(t-\tau)}\right)^T Z_1 \int_{t-\tau}^{t} x_{(s)} ds + 2\dot{x}_{(t)}^T Z_2 \int_{t-\tau}^{t} x_{(s)} ds + 2x_{(t)}^T Z_2 \left(x_{(t)} - x_{(t-\tau)}\right) \\
&\quad + 2\dot{x}_{(t)}^T Z_3 \left(\tau x_{(t)} - \int_{t-\tau}^{t} x_{(s)} ds\right) + 2x_{(t)}^T Z_3 \left(\tau \dot{x}^T(t) - \left(x_{(t)} - x_{(t-\tau)}\right)\right) \\
&\quad + 2\left(x_{(t)} - x_{(t-\tau)}\right) Z_4 \left(\tau x_{(t)} - \int_{t-\tau}^{t} x_{(s)} ds\right) + 2\int_{t-\tau}^{t} x_{(s)}^T ds Z_4 \left(\tau \dot{x}(t) - \left(x_{(t)} - x_{(t-\tau)}\right)\right) \\
&\quad + 2\left(\tau \dot{x}^T(t) - x_{(t)} + x_{(t-\tau)}\right) Z_5 \left(\tau x_{(t)} - \int_{t-\tau}^{t} x_{(s)} ds\right),
\end{aligned} \tag{14}$$

$$\begin{aligned}
\dot{V}_3(t) &= \frac{1}{4}\tau^4 \dot{x}_{(t)}^T Q \dot{x}_{(t)} - \frac{1}{2}\tau^2 \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)}^T Q \dot{x}_{(s)} ds d\theta \\
&\leqslant \frac{1}{4}\tau^4 \dot{x}_{(t)}^T Q \dot{x}_{(t)} - \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)}^T ds d\theta Q \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}_{(s)}^T ds d\theta \\
&= \frac{1}{4}\tau^4 \dot{x}_{(t)}^T Q \dot{x}_{(t)} - \left(\tau x_{(t)}^T - \int_{t-\tau}^{t} x_{(s)}^T ds\right) Q \left(\tau x_{(t)} - \int_{t-\tau}^{t} x_{(s)} ds\right).
\end{aligned} \tag{15}$$

Then for any matrices S and scalars β_1, β_2, β_3, we obtain

$$\begin{aligned}
&\left(x^T(t) + \beta_1 x^T(t-\tau) + \beta_2 \dot{x}^T(t) + \beta_3 \int_{t-\tau}^{t} x^T(s) ds\right) \\
&S\left(A_0^T x(t) + F^T (\Gamma_0 + \Delta\Gamma)^T B^T x(t-\tau) - E_0^T \dot{x}(t) + C_z^T \omega(t)\right) = 0.
\end{aligned} \tag{16}$$

For any matrices H_i $(i = 1, 2, \dots 5)$, there holds

$$\begin{aligned} &2\left(x^T(t)H_1 + x^T(t-\tau)H_2 + \dot{x}^T(t)H_3 + \int_{t-\tau}^{t} x^T(s)dsH_4 + \omega^T(t)H_5\right) \\ &\quad\left(x(t) - x(t-\tau) - \int_{t-\tau}^{t} \dot{x}(\varepsilon)d\varepsilon\right) = 0. \end{aligned} \tag{17}$$

For any $V_2 > 0$, we can obtain

$$\begin{aligned} &-2\left(x^T(t)H_1 + x^T(t-\tau)H_2 + \dot{x}^T(t)H_3 + \int_{t-\tau}^{t} x^T(s)dsH_4 + \omega^T(t)H_5\right)\int_{t-\tau}^{t} \dot{x}(\varepsilon)d\varepsilon \\ &\quad\leqslant \tau\xi(t)^T H V_2^{-1} H^T \xi(t) + \int_{t-\tau}^{t} \dot{x}(\varepsilon)^T V_2 \dot{x}(\varepsilon)d\varepsilon, \end{aligned} \tag{18}$$

where $\xi(t) = \left[\begin{array}{ccccc} x^T(t) & x^T(t-\tau) & \dot{x}^T(t) & \int_{t-\tau}^{t} x(s)^T ds & \omega^T(t) \end{array}\right]^T$. According to Lemma 1, for any matrices $Y = \left[\begin{array}{ccccc} Y_1^T & Y_2^T & Y_3^T & Y_4^T & Y_5^T \end{array}\right]^T$, there holds

$$-\int_{t-\tau}^{t} x_{(s)}^T V_1 x_{(s)} ds \leqslant 2\xi_{(t)}^T Y \int_{t-\tau}^{t} x_{(s)} ds + \tau\xi_{(t)}^T Y V_1^{-1} Y^T \xi_{(t)}. \tag{19}$$

Next, we will establish the $\|z\|_2 < \gamma\|\omega\|_2$ performance of the system under zero initial condition, that is, $\Phi(t) = 0, \forall t \in [-\tau, 0]$, and $V(t)\,|_{t=0} = 0$. Consider the following index:

$$J = \int_0^{\infty} \left[Z^T(t)Z(t) - \gamma^2\omega(t)^T\omega(t)\right]dt. \tag{20}$$

Then, for any non-zero $\omega(t) \in L_2[0, \infty)$, there holds

$$\begin{aligned} J &\leqslant \int_0^{\infty} \left(Z^T(t)Z(t) - \gamma^2\omega(t)^T\omega(t)\right)dt + V(t)\,|_{t=\infty} - V(t)\,|_{t=0} \\ &= \int_0^{\infty} \left(Z^T(t)Z(t) - \gamma^2\omega(t)^T\omega(t) + \dot{V}(t)\right)dt. \end{aligned} \tag{21}$$

Choosing $FS^T = T$, and Noting Eqs (13)–(21), after some algebraic manipulations, we obtain

$$Z^T(t)Z(t) - \gamma^2\omega(t)^T\omega(t) + \dot{V}(t) \leqslant \xi(t)^T\tilde{\Xi}_1\xi(t), \tag{22}$$

where

$$\tilde{\Xi}_1 = \Pi + \tau H V_2^{-1} H^T + \tau Y V_1^{-1} Y^T + \Omega\Omega^T + \sum_{i=1}^{m}\left(\Upsilon_{0i}\frac{\delta_i(t)}{\bar{\delta}_i}\Lambda_{0i}^T + \Lambda_{0i}\frac{\delta_i(t)}{\bar{\delta}_i}\Upsilon_{0i}^T\right), \Omega = \left[\begin{array}{ccccc} B_\omega^T & 0 & 0 & 0 & 0 \end{array}\right]^T.$$

Then, if $\tilde{\Xi}_1 < 0$, we have $\int_0^{\infty}\left[Z^T(t)Z(t) - \gamma^2\omega(t)^T\omega(t) + \dot{V}(t)\right]dt < 0$. Thus $J < 0$, and $\|z\|_2 < \gamma\|\omega\|_2$ is satisfied for any non-zero $\omega \in L_2[0, \infty)$. Assuming the zero disturbance input, i.e. $\omega(t) \equiv 0$, if $\tilde{\Xi}_1 < 0$, we can easily obtain $\dot{V}(t) < 0$, and the asymptotic stabilizability of system Eq. (4) is established. According to the Schur compliment and Lemma 2, we can obtain $\tilde{\Xi}_1 < 0$ from Eq. (8). This completes the proof.

Theorem 2: The system Eq. (4) is robustly stabilizable with constant time-delay τ and performance $\|z\|_2 < \gamma\|\omega\|_2$ for all non-zero $\omega \in L_2[0, \infty)$, and constant $\gamma > 0$, if there exist positive definite symmetric matrices P, U, V_1,

V_2, Q, matrices Z_1, Z_2, Z_3, Z_4, Z_5, S, Y_i $(i = 1, 2, 3, 4, 5)$, H_i $(i = 1, 2, 3, 4, 5)$, scalars β_1, β_2, β_3, positive scalars $r_{01}, r_{02}, \ldots, r_{0m}$, $r_{11}, r_{12}, \ldots, r_{1n}$, $r_{21}, r_{22}, \ldots, r_{2(2n)}$, $r_{31}, r_{32}, \ldots, r_{3n}$ satisfying Eq. (9) and the following LMI

$$\begin{bmatrix} \Upsilon & M_{12} \\ * & M_{22} \end{bmatrix} < 0, \tag{23}$$

where

$$\begin{aligned}
\Upsilon &= \Xi_1 + \sum_{j=1}^{n} \Big(r_{1j}\hat{m}_j^2\bar{\theta}_{1j}^2\Upsilon_{1j}\Upsilon_{1j}^T + r_{2j}\hat{k}_j^2\bar{\theta}_{2j}^2\Upsilon_{2j}\Upsilon_{2j}^T \\
&\quad + r_{2(n+j)}\hat{c}_j^2\bar{\theta}_{2(n+j)}^2\Upsilon_{2(n+j)}\Upsilon_{2(n+j)}^T + r_{3j}\hat{m}_j^2\bar{\theta}_{1j}^2\Upsilon_{3j}\Upsilon_{3j}^T \Big), \\
\Upsilon_{1j} &= \left[0\;0\;e_{1j}^T\;0\;0\;0\;0\;0\;0\;0_{1\times m} \right]^T, \Upsilon_{2j} = \left[e_{2j}^T\;0\;0\;0\;0\;0\;0\;0\;0\;0_{1\times m} \right]^T, \Upsilon_{3j} = \left[e_{3j}^T\;0\;0\;0\;0\;0\;0\;0\;0\;0_{1\times m} \right]^T, \\
\mathrm{M}_{12} &= \left[\Lambda_{11}, \Lambda_{12}, \ldots, \Lambda_{1n}, \Lambda_{21}, \Lambda_{22}, \ldots, \Lambda_{2(2n)}, \Lambda_{31}, \Lambda_{32}, \ldots, \Lambda_{3n} \right], \\
\Lambda_{1j} &= \left[f_{1j}^T S^T \;\; \beta_1 f_{1j}^T S^T \;\; \beta_2 f_{1j}^T S^T \;\; \beta_3 f_{1j}^T S^T \;\; 0\;0\;0\;0\;0_{1\times m} \right]^T, \\
\Lambda_{2j} &= \left[f_{2j}^T S^T \;\; \beta_1 f_{2j}^T S^T \;\; \beta_2 f_{2j}^T S^T \;\; \beta_3 f_{2j}^T S^T \;\; 0\;0\;0\;0\;0_{1\times m} \right]^T, \Lambda_{3j} = \left[0\;0\;0\;0\;0\;0\;0\; f_{3j}^T \; 0_{1\times m} \right]^T, \\
M_{22} &= \mathit{diag}\left\{ -r_{11}, -r_{12}, \ldots - r_{1n}, -r_{21},\; -r_{22}, \ldots - r_{2(2n)}, -r_{31}, -r_{32}, \ldots - r_{3n} \right\}.
\end{aligned}$$

Furthermore, the state-feedback controller is described as $F = TS^{-T}$.

Proof: Replacing E_0, A_0 and $B_{\omega 0}$ with $E_0 + \sum_{j=1}^{n} \theta_{1j}E_j$, $A_0 + \sum_{j=1}^{2n} \theta_{2j}A_j$ and $B_{\omega 0} + \sum_{j=1}^{n} \theta_{1j}B_{\omega j}$, respectively, Eq. (8) can be expressed as

$$\begin{aligned}
&\Xi_1 + \sum_{j=1}^{n} \Big(\theta_{1j}\hat{m}_j\Upsilon_{1j}\Lambda_{1j}^T + \theta_{2j}\hat{k}_j\Upsilon_{2j}\Lambda_{2j}^T + \theta_{2(n+j)}\hat{c}_j\Upsilon_{2(n+j)}\Lambda_{2(n+j)}^T + \theta_{1j}\hat{m}_j\Upsilon_{3j}\Lambda_{3j}^T \\
&\quad + \theta_{1j}\hat{m}_j\Lambda_{1j}\Upsilon_{1j}^T + \theta_{2j}\hat{k}_j\Lambda_{2j}\Upsilon_{2j}^T + \theta_{2(n+j)}\hat{c}_j\Lambda_{2(n+j)}\Upsilon_{2(n+j)}^T + \theta_{1j}\hat{m}_j\Lambda_{3j}\Upsilon_{3j}^T \Big) < 0.
\end{aligned} \tag{24}$$

By Lemma 2, Eq. (24) holds if and only if there exist positive scalars $r_{11}, r_{12}, \ldots, r_{1n}, r_{21}, r_{22}, \ldots, r_{2(2n)}$, r_{31}, $r_{32}, \ldots, r_{3n}$ such that

$$\begin{aligned}
&\Xi_1 + \sum_{j=1}^{n} \big(r_{1j}\bar{\theta}_{1j}^2\hat{m}_j^2\Upsilon_{1j}\Upsilon_{1j}^T + r_{1j}^{-1}\Lambda_{1j}\Lambda_{1j}^T + r_{2j}\bar{\theta}_{2j}^2\hat{k}_j^2\Upsilon_{2j}\Upsilon_{2j}^T + r_{2j}^{-1}\Lambda_{2j}\Lambda_{2j}^T \\
&\quad + r_{2(n+j)}\bar{\theta}_{2(n+j)}^2\hat{c}_j^2\Upsilon_{2(n+j)}\Upsilon_{2(n+j)}^T + r_{2(n+j)}^{-1}\Lambda_{2(n+j)}\Lambda_{2(n+j)}^T + r_{3j}\bar{\theta}_{1j}^2\hat{m}_j^2\Upsilon_{3j}\Upsilon_{3j}^T + r_{3j}^{-1}\Lambda_{3j}\Lambda_{3j}^T \Big) < 0.
\end{aligned} \tag{25}$$

Applying the Schur complement, LMI Eq. (25) is equivalent to LMI Eq. (23). This completes the proof.

4. Illustrative example

Consider the structural system with $n = 3$. The structural parameters are $\hat{m}_i = 1000$ kg, $\hat{k}_i = 980$ kN/m, and $\hat{c}_i = 1.407$ kNs/m $(i = 1, 2, 3)$. Then the state space Eq. (4) has the following parameters [19]:

$$H_0 = \mathit{diag}\{1, 1, 1\}, x = \left[x_1 \;\; x_2 \;\; x_3 \;\; \dot{x}_1 \;\; \dot{x}_2 \;\; \dot{x}_3 \right]^T, E(\theta_1) = E_0 + \sum_{j=1}^{3} \theta_{1j}\hat{m}_j e_{1j} f_{1j}^T,$$

$$A(\theta_2) = A_0 + \sum_{j=1}^{3} \Big(\theta_{2j}\hat{k}_j e_{2j} f_{2j}^T + \theta_{2(3+j)}\hat{c}_j e_{2(3+j)} f_{2(3+j)}^T \Big), B_\omega(\theta_1) = B_{\omega 0} - \sum_{j=1}^{3} \theta_{1j}\hat{m}_j e_{3j} f_{3j}^T,$$

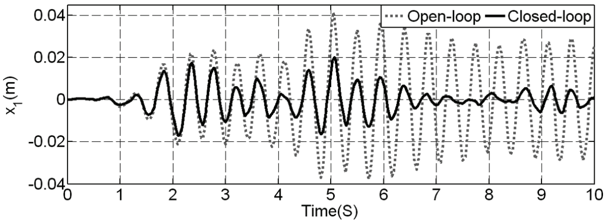

Fig. 2. Displacements of the first storey in case A1 ($\tau = 25$ ms).

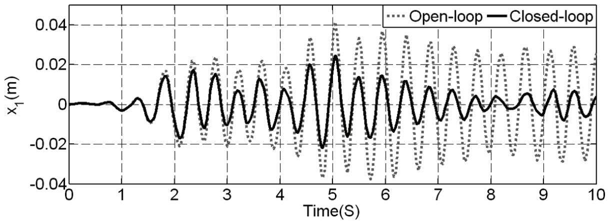

Fig. 3. Displacements of the first storey in case A2 ($\tau = 25$ ms).

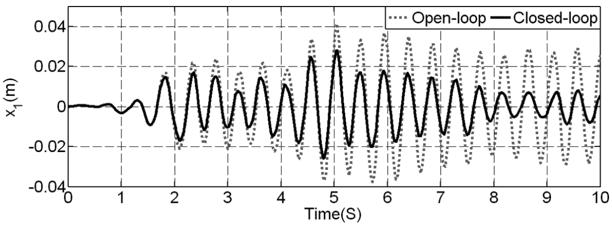

Fig. 4. Displacements of the first storey in case A3 ($\tau = 25$ ms).

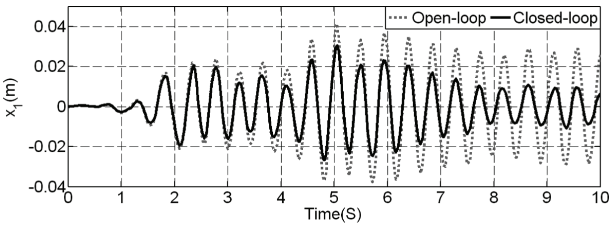

Fig. 5. Displacements of the first storey in case A4 ($\tau = 25$ ms).

where

$$
\begin{aligned}
&E_0 = \mathit{diag}\left\{1, 1, 1, \hat{m}_1, \hat{m}_2, \hat{m}_3\right\},\\
&e_{11} = f_{11} = \left[0\,0\,0\,1\,0\,0\right]^T, e_{12} = f_{12} = \left[0\,0\,0\,0\,1\,0\right]^T, e_{13} = f_{13} = \left[0\,0\,0\,0\,0\,1\right]^T,\\
&A_0 = \begin{bmatrix} 0 & I \\ \hat{K} & \hat{C} \end{bmatrix}, \hat{K} = \begin{bmatrix} \hat{k}_1 + \hat{k}_2 & -\hat{k}_2 & 0 \\ -\hat{k}_2 & \hat{k}_2 + \hat{k}_3 & -\hat{k}_3 \\ 0 & -\hat{k}_3 & \hat{k}_3 \end{bmatrix}, \hat{C} = \begin{bmatrix} \hat{c}_1 + \hat{c}_2 & -\hat{c}_2 & 0 \\ -\hat{c}_2 & \hat{c}_2 + \hat{c}_3 & -\hat{c}_3 \\ 0 & -\hat{c}_3 & \hat{c}_3 \end{bmatrix}.\\
&e_{21} = e_{24} = [0, 0, 0, 1, 0, 0]^T, e_{22} = e_{25} = [0, 0, 0, -1, 1, 0]^T, e_{23} = e_{26} = [0, 0, 0, 0, -1, 1]^T,\\
&f_{21} = [1, 0, 0, 0, 0, 0]^T, f_{22} = [-1, 1, 0, 0, 0, 0]^T, f_{23} = [0, -1, 1, 0, 0, 0], f_{24} = [0, 0, 0, 1, 0, 0]^T,\\
&f_{25} = [0, 0, 0, -1, 1, 0]^T, f_{26} = [0, 0, 0, 0, -1, 1], B_{\omega 0} = [0, 0, 0, -\hat{m}_1, -\hat{m}_2, -\hat{m}_3]^T,\\
&e_{31} = [0, 0, 0, 1, 0, 0]^T, e_{32} = [0, 0, 0, 0, 1, 0]^T, e_{33} = [0, 0, 0, 0, 0, 1]^T, f_{31} = f_{32} = f_{33} = 1.
\end{aligned}
$$

Assume that the displacements and velocities of the three storeys are all measurable for feedback in this case. The controlled output is chosen to be the relative displacements of each storey, that is, $z(t) = \left[x_{m1}(t)\ x_{m2}(t)\ x_{m3}(t)\right]^T$. Consider the maximum actuator output force limit $u_{\lim_i}$ as 400 N, and suppose that $\underline{\upsilon}_i = 0.2$, $\bar{\upsilon}_i = 1.2$, $\varepsilon_i = 10$ where $i = 1, 2, 3$. Furthermore, we can get the maximum control signal before saturation $u_{bef_{\lim i}} = \varepsilon_i u_{\lim_i} = 10 \times 400 = 4000$ N, that is, when the control signals before saturation u_{bef_i} satisfy $u_{bef_i} \leqslant 4000$ N, the designed controllers should have the desired performances. Firstly, consider the system without uncertainties, that is $\theta_{1i} = 0\,(i = 1, 2, 3)$, $\theta_{2i} = 0\,(i = 1, 2, \ldots, 6)$. By choosing $\tau = 25$ ms, $\beta_1 = 1$, $\beta_2 = 5$, $\beta_3 = 10$, $\gamma = 0.15$, we solve the LMIs Eqs (8) and (9) and obtain the state feedback controller which has the following gain matrix

$$
F = \begin{bmatrix} -2.013 & -1.545 & 4.159 & -1438.8 & -435.10 & -362.48 \\ 3.1274 & -4.7112 & 4.380 & -440.49 & -1816.9 & -809.31 \\ 0.6730 & 3.160 & -2.058 & -369.65 & -820.04 & -2278.78 \end{bmatrix}. \tag{26}
$$

For description in brevity, we denote this designed controller as controller I thereafter.

In order to verify the dynamics of the closed-loop system, a time history of acceleration from EI Centro 1940 earthquake excitation is applied to this system. To simulate the actuator faults conditions, it is assumed that faults

Table 1
The maximum responses of the displacements and accelerations ($\tau = 25$ ms)

	Open-loop	Case A1	Case A2	Case A3	Case A4
$x_{1\max}$ (cm)	4.14	2.01	2.43	2.80	3.06
$x_{2\max}$ (cm)	7.29	3.49	4.24	4.92	5.30
$x_{3\max}$ (cm)	8.95	4.19	5.18	6.05	6.42
$\ddot{x}_{1\max}$ (m/s^2)	8.58	5.20	5.53	5.96	6.99
$\ddot{x}_{2\max}$ (m/s^2)	13.4	7.46	8.00	9.15	10.16
$\ddot{x}_{3\max}$ (m/s^2)	17.7	9.44	9.39	10.99	12.66

Table 2
The maximum control forces before saturation ($\tau = 25$ ms)

	Case A1	Case A2	Case A3	Case A4
u_1	732.6	912.8	1064.9	0
u_2	1344.5	1687.2	2008.1	0
u_3	1741.3	2151.5	2586.0	2633.2

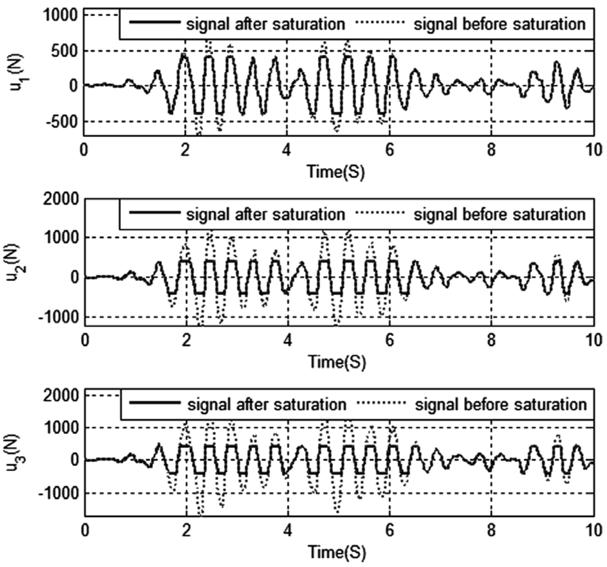

Fig. 6. Control forces in case A1 ($\tau = 25$ ms).

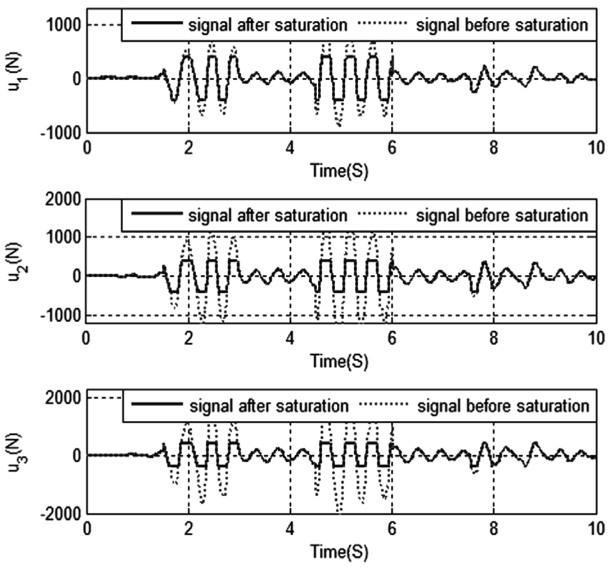

Fig. 7. Control forces in case A2 ($\tau = 25$ ms).

occur periodically, and the percentage scalar $\Psi(t)$ of the signal is defined as

$$\Psi(t) = \begin{cases} \alpha, kT < t \leqslant kT + \Delta(t), & k = 0, 1, 2, 3, \ldots, \\ 1, kT + \Delta(t) < t \leqslant (k+1)\,T, & k = 0, 1, 2, 3, \ldots, \end{cases}$$

where T is a known time period, $\Delta(t)$ is a section of T, which informs how much time there exists faults in a period, and α is the percentage of the signal when faults exist. According to the definition above, the fault condition can be described as: during the first part $kT < t \leqslant kT + \Delta(t)$ of each period, faults exist and the percentage of signal is α; in the second part $kT + \Delta(t) < t \leqslant (k+1)\,T$ of each period, no faults exist and the control system works normally.

For brevity, we consider four cases in this example. Case A1: the control system works normally; case A2: $T = 3$ s, $\Delta(t) = 1.5$ s, and $\alpha = 20$ percent (80 percent loss of the actuator thrusts); case A3: $T = 3$ s, $\Delta(t) = 1.5$ s, and $\alpha = 0$ percent (100 percent loss of the actuator thrusts); case A4: 100 percent loss of the actuator thrusts in storey 1 and 2, and the actuator in storey 3 works normally.

The first storey displacements of open-loop and closed-loop systems which are composed with the controller I in the four cases are compared in Figs 2 to 5. The displacements of the other two storeys have the similar varying trend, which are omitted here for brevity. In addition, the accelerations of the three storeys can give us the same information to explain our results, which are also omitted here. The maximum displacements and accelerations of the open-loop and closed-loop systems in the four cases are compared in Table 1. From Figs 2 to 5 and Table 1, we can obtain that controller I is effective to attenuate the displacements and accelerations in the four cases when control forces input time-delay $\tau = 25$ ms. The control forces in four cases are plotted in Figs 6 to 9, and the maximum control forces are shown in Table 2. From Figs 6 to 9 and Table 2, we can obtain that the maximum control signal before saturation is 2633.2 N, which is less than the permissible limitation 4000 N.

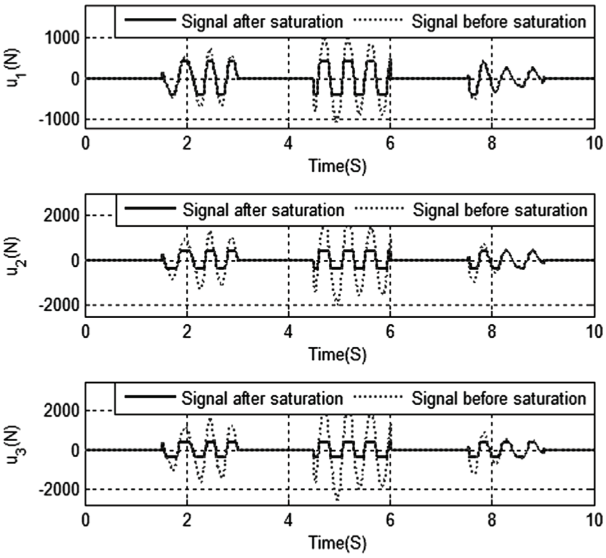

Fig. 8. Control forces in case A3 ($\tau = 25$ ms).

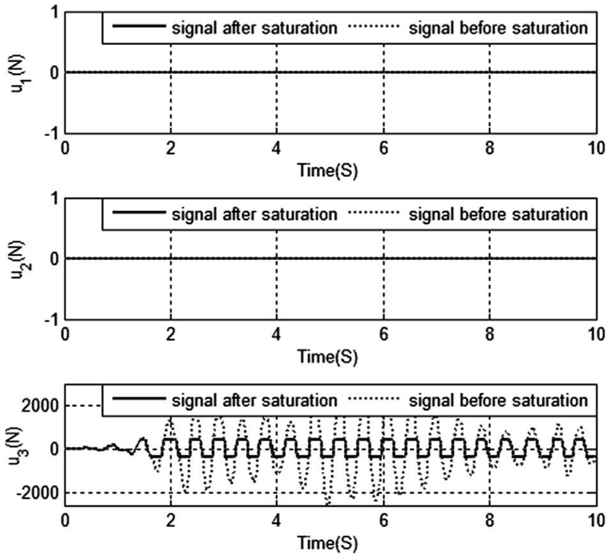

Fig. 9. Control forces in case A4 ($\tau = 25$ ms).

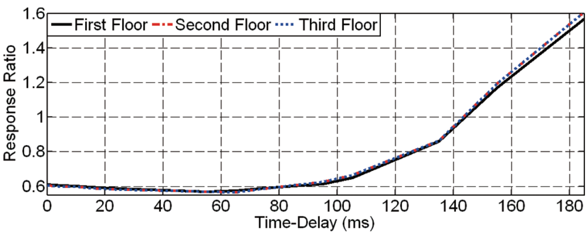

Fig. 10. Maximum response ratio $x_{i\max}(\tau)/x_{i\max}(0)$ vs time delay τ.

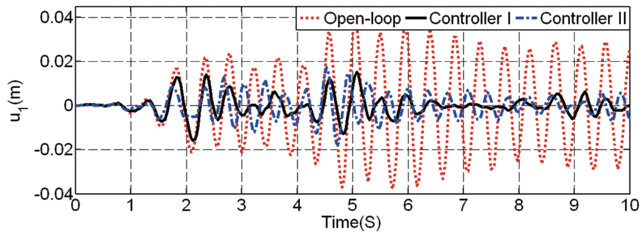

Fig. 11. Displacements of the first floor ($\tau = 0$ ms).

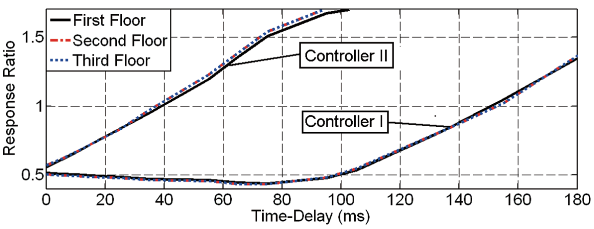

Fig. 12. Comparison of maximum response ratios $x_{i\max}(\tau)/x_{i\max}(0)$ vs time delay τ.

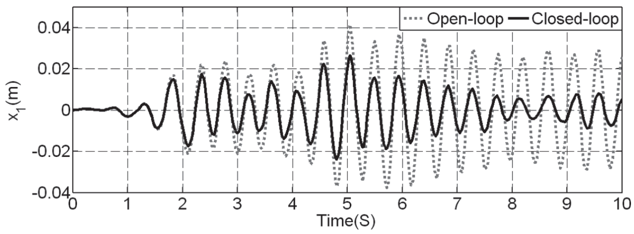

Fig. 13. Displacements of the first storey ($\tau = 25$ ms) in case B1.

To further express the effectiveness of controller I in dealing with the time delay, the effect of time delay on the response of the structural system is studied by calculating the response ratio $x_{i\max}(\tau)/x_{i\max}(0)$ $(i = 1, 2, 3)$ vs the time delay τ, where $x_{i\max}(\tau)$ $(i = 1, 2, 3)$ denotes the maximum displacement of the closed-loop system when the input time delay is τ. $x_{i\max}(0)$ $(i = 1, 2, 3)$ denotes the maximum displacement of the open-loop system. Figure 10 shows the plot of the response ratio $x_{i\max}(\tau)/x_{i\max}(0)$ $(i = 1, 2, 3)$ in case A2 as a function of the time delay τ. The other three cases have the similar results, which are omitted here. It is observed from Fig. 10 that there is no degradation in the displacement attenuation performance of the control system up to the obtained maximal time delay $\tau = 25$ ms. When the time delay exceeds 65 ms, the degradation of the control performance increases.

In order to facilitate the comparison, we obtain another state feedback controller, which does not consider the

Table 3
The maximum responses of the displacements and accelerations ($\tau = 25$ ms)

Cases	Case B1		Case B2		Case B3		Case B4		Case B5	
	Open	Closed	Open	Closed	Open	Closed	Open	Closed	Open	Closed
$x_{1\,\max}$ (cm)	4.14	2.69	7.30	4.31	4.14	3.03	1.47	0.82	3.19	1.76
$x_{2\,\max}$ (cm)	7.29	4.69	13.3	7.66	7.29	5.32	2.61	1.46	5.62	3.04
$x_{3\,\max}$ (cm)	8.95	5.74	16.8	10.0	8.95	6.52	3.25	1.82	6.88	3.69
$\ddot{x}_{1\,\max}$ (m/s^2)	8.58	5.81	7.05	6.53	8.58	6.83	7.15	4.80	6.57	3.89
$\ddot{x}_{2\,\max}$ (m/s^2)	13.4	8.69	11.8	8.33	13.4	9.83	11.8	8.90	10.5	6.19
$\ddot{x}_{3\,\max}$ (m/s^2)	17.7	10.2	15.9	11.5	17.7	11.9	14.6	11.3	11.8	8.54

control input time-delay, by solving Theorem 2 in [29] with $\gamma = 0.15$, and this controller has the following gain

$$F = \begin{bmatrix} -104451 & -980.13 & -0.1193 & -190.5 & -1.4079 & -0.835\times 10^{-3} \\ -980.11 & -104451 & -980.11 & -1.4077 & -190.50 & -1.4078 \\ -0.11738 & -980.12 & -105431 & -0.8216\times 10^{-3} & -1.4079 & -191.91 \end{bmatrix}. \tag{27}$$

For description in brevity, this state feedback controller is denoted as controller II thereafter. We take the EI Centro 1940 earthquake as the disturbance excitation. When there are no control forces input time-delay and saturation in this system, we can obtain the first storey displacements of open-loop and closed-loop systems which are composed with the controller I and II from Fig. 11. The displacements of the other two storeys and the accelerations of three storeys have a similar varying trend, which are omitted here. Form Fig. 11, we can obtain that controller I and II are all effective to attenuate the displacements and accelerations of the system when $\tau = 0$ ms.

We introduce the actuator saturation $u_{\lim_i} = 400$ N $(i = 1, 2, 3)$ into this system. Figure 12 shows the plot of the response ratios $x_{i\,\max}(\tau)/x_{i\,\max}(0)\,(i = 1, 2, 3)$ of the closed-loop systems which are composed with the controller I and II in case A1. It is observed from Fig. 12 that the degradation in control performance of controller II increases rapidly when the time delay is increasing. However, there is no significant degradation in the displacement attenuation performance of the controller I up to time delay $\tau = 75$ ms.

Now, let us consider the uncertain system. Consider the uncertainties are applied to the mass, stiffness and damping coefficients of the first storey, and assume the parameter uncertainties satisfying $|\theta_{11}| \leqslant 0.4, |\theta_{21}| \leqslant 0.4, |\theta_{24}| \leqslant 0.4$. By choosing $\tau = 25$ ms, $\beta_1 = 1, \beta_2 = 10, \beta_3 = 10\ \gamma = 0.15$, we solve the LMIs Eqs (9) and (23), and obtain a robust state feedback controller has the following gain matrix

$$F = \begin{bmatrix} -1.3007 & 0.3681 & -0.0874 & -682.61 & -778.75 & 161.175 \\ -0.3698 & 0.2892 & -0.1256 & 148.262 & -1506.7 & -200.80 \\ -0.3585 & -0.1516 & 0.1813 & 45.425 & -195.48 & -1557.98 \end{bmatrix}. \tag{28}$$

For description in brevity, we denote this designed controller as controller III thereafter.

Set $T = 3$ s, $\Delta(t) = 1.5$ s, and $\alpha = 20$ percent (80 percent loss of the actuator thrusts). For brevity, we consider the nominal case ($\theta_{1i} = 0\,(i = 1, 2, 3)\,, \theta_{2i} = 0\,(i = 1, 2, 3, 4, 5, 6)$, corresponds to case B1) and four-vertex cases where the first storey's mass, stiffness and damping coefficients are given as their vertex values, respectively.

Case B2 corresponds to $\hat{m}_1 = 1.4 \times 1000$ kg, $\hat{k}_1 = 0.6 \times 980$ kN/m, and $\hat{c}_1 = 0.6 \times 1.407$ kNs/m.

Case B3 corresponds to $\hat{m}_1 = 1.4 \times 1000$ kg, $\hat{k}_1 = 1.4 \times 980$ kN/m, and $\hat{c}_1 = 1.4 \times 1.407$ kNs/m.

Case B4 corresponds to $\hat{m}_1 = 0.6 \times 1000$ kg, $\hat{k}_1 = 1.4 \times 980$ kN/m, and $\hat{c}_1 = 0.6 \times 1.407$ kNs/m.

Case B5 corresponds to $\hat{m}_1 = 0.6 \times 1000$ kg, $\hat{k}_1 = 0.6 \times 980$ kN/m, and $\hat{c}_1 = 1.4 \times 1.407$ kNs/m.

Under the same earthquake excitation, the responses of the first storey displacements in case B1 are plotted in Fig. 13. It can be seen from Fig. 13 that the better responses are obtained for the closed-loop system when $\tau = 25$ ms. For detailed comparison, the maximum displacements and accelerations of the open-loop and closed-loop systems are shown in Table 3, where Open means Open-loop system and Closed means Closed-loop system. We can obtain from Table 3 that better responses are reached for all closed-loop cases no matter the parameter uncertainties exist or not. Thus, it is validated that the designed controller III is robust to parameter uncertainties. The control forces in case B1 are plotted in Fig. 14, and the maximum control forces in five cases are shown in Table 4. From Fig. 14 and Table 4, we can obtain that the maximum control signal before saturation is 1492.5 N, which is less than the permissible limitation 4000 N.

Table 4
The maximum control forces before satuaration ($\tau = 25$ ms)

	Case B1	Case B2	Case B3	Case B4	Case B5
u_1	552.49	803.97	619.03	297.71	373.25
u_2	1002.3	1173.0	1137.3	522.76	685.13
u_3	1281.3	1492.5	1478.2	670.14	880.52

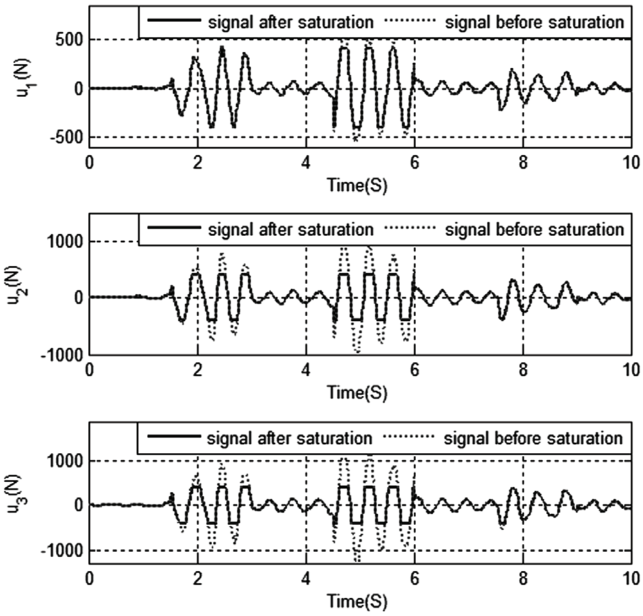

Fig. 14. Control forces in case B1 ($\tau = 25$ ms).

5. Conclusion

In terms of an LMI approach, the problem of fault tolerant vibration control for a class of uncertain structural systems with control input time-delay and saturation is considered in this paper. Based on Newton's second law, the structural system is described as state-space model, which contains actuator fault, input signal saturation and time-delay at the same time. Furthermore, a special Lyapunov functional, which includes some non-positive items, is introduced to research the stability of the structural system, and the LMIs-based conditions for the system to be stabilizable are established. If the feasibility problem of these conditions is solvable, the desired fault tolerant controller can be obtained for the closed-loop system with control input time-delay and saturation to be stable with the performance $\|z\|_2 < \gamma\|\omega\|_2$. The condition is also extended to the uncertain case. Finally, simulation results show that the controllers designed using the presented approach can effectively achieve the attenuation objective when there are actuator failures, control signal input time-delay and saturation.

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