



## RESEARCH ARTICLE

## Sizing optimization of skeletal structures using teaching-learning based optimization

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## ABSTRACT

Teaching Learning Based Optimization (TLBO) is one of the non-traditional techniques to simulate natural phenomena into a numerical algorithm. TLBO mimics teaching learning process occurring between a teacher and students in a classroom. A parameter named as teaching factor,  $T_F$ , seems to be the only tuning parameter in TLBO. Although the value of the teaching factor,  $T_F$ , is determined by an equation, the value of 1 or 2 has been used by the researchers for  $T_F$ . This study intends to explore the effect of the variation of teaching factor  $T_F$  on the performances of TLBO. This effect is demonstrated in solving structural optimization problems including truss and frame structures under the stress and displacement constraints. The results indicate that the variation of  $T_F$  in the TLBO process does not change the results obtained at the end of the optimization procedure when the computational cost of TLBO is ignored.



### 1. Introduction

Optimization tools emerged as obtaining the optimum solution of optimization problems try to maximize or minimize a real function within a domain which contains the acceptable values of variables while some restrictions are to be satisfied. Among the optimization tools developed and used for the solution of optimization problems, the recent novel and innovative meta-heuristic search techniques emerged use nature as a source of inspiration to establish a numerical search algorithm for solving complex engineering problems and they do not suffer the discrepancies of mathematical programming based optimum design methods [1]. Although genetic algorithms (GAs) based on the principle of survival of the fittest as a computational procedure [2-7] seems to be commonly employed to obtain the optimum solution of structural design problems, many meta-heuristic optimization tools occurred in recent years, which were developed inspiring the different process and phenomena from the nature. The optimization algorithms such as ant colony optimization (ACO) working on the behavior of an ant, particle swarm optimization (PSO) implementing the foraging behavior of a bird for searching food, artificial bee

colony (ABC) using the foraging behavior of a honey bee, harmony search (HS) working on the principle of music improvisation in music player, charged system search (CSS) implementing the Coulomb and Gauss's law of electrostatics in physics, and imperialist competitive algorithm (ICA) using a socio-politically motivated strategy might be stated as the new generation meta-heuristic techniques, mine blast algorithm (MBA) simulating the mine bomb explosion, water cycle algorithm (WCA) implementing the main steps of the hydrologic cycle, water wave optimization (WWO) working on the principle of wave motion in recent years, which have been developed mimicking the principles of different natural phenomena and have been effectively employed to attain the optimum solution of structural design problems [1, 8-19]. Moreover, the improved form of these algorithms proposed to enhance performance and ability of those can also be found in the literature [20-22]. On the other hand, the emergence of new computational techniques that are based on the simulation of paradigms found in nature has still continued due to its ability of solving different optimization problems because of their very suitability and effectiveness in finding the solution of

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structural optimization problems.

One of the meta-heuristic techniques offered from inspiring the natural phenomena is the so-called Teaching-Learning Based Optimization (TLBO). TLBO was developed by [23] as a new optimization method, which mimics teaching-learning process in a class between the teacher and the students (learners). [23] tested the TLBO algorithm on constrained benchmark test functions with different characteristics, benchmark mechanical design problems and mechanical design optimization problems taken from the literature. After that, some optimization problems related with the distinct discipline and features were investigated using the standard TLBO algorithm and the enhancement version of its [24-30]. The numerical results presented in the corresponding researches proved exploration and exploitation capacities of TLBO on different kind of optimization problems in comparison to other metaheuristics algorithms used in these optimization cases.

TLBO algorithm contains two main phases known as Teaching phase and Learning phase and it does not need any control parameters values to start its searching process. The teaching factor  $T_F$  placed in the Teaching Phase seems the only tuning parameter although yet  $T_F$  was decided with the help of  $T_F = \text{round}[1 + \text{rand}(0,1) \{2-1\}]$  in [23]. However, the value  $T_F$  was taken as 1 or 2 in the studies conducted using TLBO in contrast to the equation given in [26]. For example, [30], [31], and [24] were adopted it as 2 through the TLBO process while [28] taken as [0, 1]. Therefore this study intends to explore the effect of the variation of teaching factor  $T_F$  on the performances of TLBO. This effect is demonstrated solving structural optimization problems including truss and frame structures under the stress and displacement constraints.

## 2. Optimization problems

A general mathematical statement for the constrained optimization problem is defined in [32] as follows. In  $R^n$  find the design variables  $\mathbf{x} = \{x_1, x_2, \dots, x_n\}^T$  minimizing an objective function and satisfying the constraints:

$$\begin{aligned} \min \quad & f(\mathbf{x}) & \mathbf{x} \in R^n \\ \text{subject to} \quad & g_i(\mathbf{x}) \leq 0 & i =, \dots, m_1 \\ & h_j(\mathbf{x}) = 0 & j =, \dots, m_2 \\ & \mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u \end{aligned} \quad (1)$$

In Eq. (1),  $g_i(\mathbf{x})$  and  $h_j(\mathbf{x})$  represent the inequality and equality constraints,  $\mathbf{x}_l$  and  $\mathbf{x}_u$  are the vectors showing the lower and upper limit for the design variables, respectively. Since the design variables of the optimization problem are discrete  $x_i$  is equal to 1 whereas  $x_u$  is the maximum section number considered for design variables. Therefore, the

optimization problem turns finding a vector of integer values  $\mathbf{x}$  corresponding to the sequence numbers of steel sections in a given list to create a vector of cross-sectional areas  $\mathbf{A} = \{A_1, A_2, \dots, A_M\}^T$  for  $M$  members of the structure. Such that, the objective function  $f$  taken as weight of the structural system is minimized depending on  $\mathbf{A}$ .

$$f = \sum_{i=1}^M \rho L_i A_i \quad (2)$$

In Eq. (2),  $M$  is the number of elements in the structural system.  $L_i$  and  $A_i$  are the length, and the cross-section area of  $i$ -th element respectively,  $\rho$  is the density of the material.

As the meta-heuristic methods are suitable for the unconstrained optimization problems, the constrained optimization problem is converted to the unconstrained one via penalty functions based on the measurement of violation. A penalty functional is added to the objective function to define the fitness value of an infeasible element. The objective function for the design problem incorporating penalty function as well can be expressed as follows;

$$\min W = (1 + f_{penalty}) f \quad (3)$$

In Eq. (3),  $W$  is called the penalized objective function and shows a relative measure of the performance of the solution,  $f_{penalty}$  is the penalty function, and  $f$  is objective function as in Eq. (2). All penalty functions are based on the violation of the constraints, and usually the degree of penalty for a given solution is adjusted through some coefficients placed in the penalty function. The penalty function taken from [8] as given below is used in the current work.

$$f_{penalty} = (1 + C)^\varepsilon \quad (4)$$

where  $C = \sum_{i=1}^{m_1} \max[0, g_i(\mathbf{x})], \varepsilon = 2$

In Eq. (4),  $C$  is the total value of displacement and stress violations,  $\varepsilon$ =penalty function exponent, and  $m_1$  is number of the total constraints considered as the displacement and/or the stress constraints,  $g_i(\mathbf{x})$ .

## 3. Teaching-learning based optimization (TLBO)

TLBO simulates the effect of influence of a teacher on learners (students) which is taken as the source of its inspiration. In accordance with this purpose, the method imitates the set of possible solution alternatives of the problem as teacher-student group in a class which struggles to increase the level of the class by attaining the new information on a subject under the existing conditions. It is intended in this simulation that the students in a class increase and move their knowledge level on a subject taught by the teacher towards his or her own level.

A computational procedure by imitating the above teaching-learning process that occurs between the teacher and the students in a class is developed by [23]

and aforementioned process called TLBO consists of two parts; i) “Teaching Phase”, and ii) “Learning Phase”. In teaching phase, the teacher, who is the most knowledgeable person in a social group and is expected to disseminate information to other learners, is determined whereas in the learning phase, it is provided for the students to acquire new information through the interactions among the learners. As in other meta-heuristic algorithms inspiring from the nature, TLBO is also a population based method.

Each student in a class represents a possible solution, the different subjects offered to learn to students is analogous to different design variables, the students’ result obtained through the exam demonstrates the fitness of solution, the teacher is taken as the best solution achieved so far, and finally whole class is considered as the population in TLBO. After this association, the step-wise procedure for the implementation of TLBO is as follows.

### 3.1. Initialize the optimization problems

The parameters required by the optimization algorithm to be used in solving the structural design problems are defined in this step. These are number of population ( $np$ ), maximum number of cycles ( $C_{max}$ ), number of design variables ( $nd$ ), lower and upper limits of design variables ( $x_l$  and  $x_u$ ), objective function ( $f(x)$ ) and so on, which are selected depending on the type of problem.

### 3.2. Initialize the population and evaluate the solution

The population is randomly generated according to the parameters described in the previous step as follows.

$$pop = \begin{bmatrix} x_{1,1} & \cdots & x_{1,nd-1} & x_{1,nd} \\ \vdots & \cdots & \cdots & \vdots \\ x_{np,1} & \cdots & x_{np-1,nd-1} & x_{np,nd} \end{bmatrix} \rightarrow \begin{matrix} W(x_1) \\ \vdots \\ W(x_{np}) \end{matrix} \quad (5)$$

In Eq. (5), each row shows a possible solution ( $x_i = \{x_{i,1} \dots x_{i,nd-1} \ x_{i,nd}\}$   $i=1, \dots, np$ ),  $W(x_{1,\dots,np})$  represents the value of the penalized objective function for the evaluation of the potential solutions through Eq. (3), and  $pop$  demonstrates the population.

### 3.3. Teaching phase

The solution with a minimum value of the penalized objective function in the population is determined at this stage of TLBO ( $\min(W(x_{1,\dots,np}))$ ). Since this individual is the best of the population it is taken into account as a teacher in the teaching-learning process ( $x_{teacher} = x = \min(W(x))$ ). Then, the other students in the current population are modified in the neighborhood of the teacher by the hope that the level of students will be updated to the level of the teacher. This modification is carried out by using the following equations.

$$x^* = x_i + r(x_{teacher} - T_F x_{mean}) \quad (6a)$$

In Eq. (6a),  $x^*$  shows the renewed form of  $x_i$  by Eq. (6a),  $r$  is a random number varying [0,1],  $T_F$  is a teaching factor being either 1 or 2, which is again a heuristic step and decided randomly with equal probability as  $T_F = \text{round}[1 + \text{rand}(0,1) \{2-1\}]$  (in [23]), and  $x_{mean}$  symbolizes the mean of the population, which is calculated with column-wise manner as follows.

$$x_{mean} = [\bar{x}_{i,1} \ \bar{x}_{i,2} \ \dots \ \bar{x}_{i,nd-1} \ \bar{x}_{i,nd}] \quad (6b)$$

$$x_{mean} = \bar{x}_{i,j} = \frac{\sum_{i=1}^{np} x_{i,j}}{np}$$

In Eq. (6b),  $i=1, \dots, np$ ,  $j=1, \dots, nd$ ,  $np$  and  $nd$  are the number of solutions and the design variables. As a results of these operations,  $x_i$  is taken as  $x^*$  if the obtained  $x^*$  produces a better value of  $W(\cdot)$  than  $x_i$ . Otherwise,  $x_i$  is retained.

### 3.4. Learning phase

After the teacher transfers him or her own knowledge to the students by Eq. (6a), the teaching-learning process continues in the form of interaction among students. At this stage of the TLBO algorithm, a student learns new information by interacting with other students who have more knowledge than him or her. The modification formula representing the learning phase can be expressed as:

$$\begin{aligned} & \text{for } i = 1 : np \\ & \text{randomly select } j, j \neq i \\ & \text{if } f(x_i) < f(x_j) \\ & \text{difference} = x_i - x_j \\ & \text{else} \\ & \text{difference} = x_j - x_i \\ & \text{end if} \\ & x^* = x_i + r \times \text{difference} \\ & \text{end for} \end{aligned} \quad (6c)$$

where,  $x^*$  and  $x_i$  are the new and existing solution of  $i$ ,  $x_j$  is the any solution that is different from  $x_i$ . If the solution gained new information with help of Eq. (6c),  $x^*$ , produces better penalized objective function value than  $x_i$  change  $x_i$  to  $x^*$ , otherwise preserve  $x_i$ .

At the end of the learning phase, a cycle (iteration) is completed for the TLBO and the steps in section 3.3 and 3.4 continues until a termination criterion is satisfied, which is adopted as a pre-determined maximum number of cycles ( $C_{max}$ ) in the current work. The vector  $x^*$  obtained with application of both Eqs. (6a) and (6c) may contain any design variable being less than  $x_l$  or bigger than  $x_u$  due to addition and subtraction in the corresponding expressions. In such a case, a controlling procedure should be performed for  $x^*$  so as not to encounter any abnormal ending in the algorithm. Therefore, it is ensured that any design variable in  $x^*$  must not be bigger than  $x_u$  and less than

$x_l$  and if any design variable of  $x^*$  is less than  $x_l$  or bigger than  $x_u$  it is taken into account as  $x_l$  or  $x_u$ , respectively.

The flowchart of TLBO developed in the light of information given above is demonstrated in Figure 1.

#### 4. Design examples

The design process, that is explained with the implementation steps given above, of a Teaching Learning Based Optimization (TLBO) technique is properly applied to the example designs such as 52 bar truss, 3-bay 15-story frame, and 582 bar space truss in order to exhibit the effecting of varying the value of  $T_F$  on the performance of TLBO algorithm. In the design examples examined, the design variables taken into consideration as the cross-sectional area of the members that make up the structural systems are discrete. In other words, they are represented by the section numbers considered for design variables.

The inequalities shown as follows are kept in mind as constraints in the current work for the examples

$$g_{k_1}(x) = 1.0 - \frac{u_a}{|u|} \leq 0 \quad k_1 = 1, \dots, c_1 \quad (7a)$$

$$g_{k_2}(x) = 1.0 - \frac{\sigma_a}{|\sigma|} \leq 0 \quad k_2 = 1, \dots, c_2 \quad (7b)$$

where, Eq. (7a) and (7b) demonstrate the displacements and stresses constraints, respectively.  $u$  displacement of joint, and  $u_a$  is its upper bound.  $\sigma$  is stress in a member.  $\sigma_a$  is the allowable stresses for the tension and compression members, respectively.  $c_1$  and  $c_2$  are number of restricted displacements and stresses.

The optimizations process performed using TLBO for the structure systems examined in this study is repeated 20 times by the different populations which are generated independently and randomly at every turn. The best (lightest) one of the 20 runs is propounded as the result of the related examples.

The algorithm and finite element analysis program are coded in Matlab software and implemented on PC with Intel Core i5 2.70 GHz processor and 8 GB RAM memory.

#### 4.1. 52 bar truss

A 52 bar plane truss shown in Figure 2 is studied as the first example for demonstrating how to vary the solution process of TLBO depending on the value of  $T_F$ . It is subjected to single-load case given in Table 1. The truss was optimized by [33] using GA, by [34] using GA with adaptive manner penalty function, and by [35] using rank-based ant system that is a variant of the ACO. Moreover the same example was solved by [36] using MBA and [37] using IMBA.

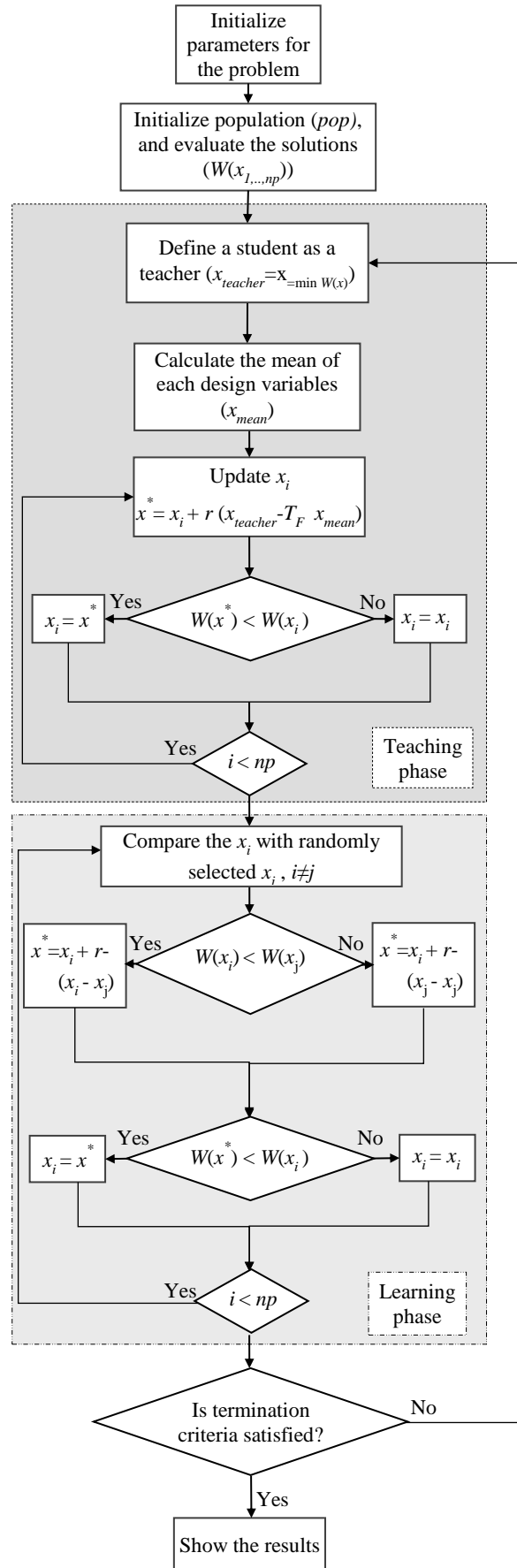


Figure 1. Flowchart diagram for TLBO.

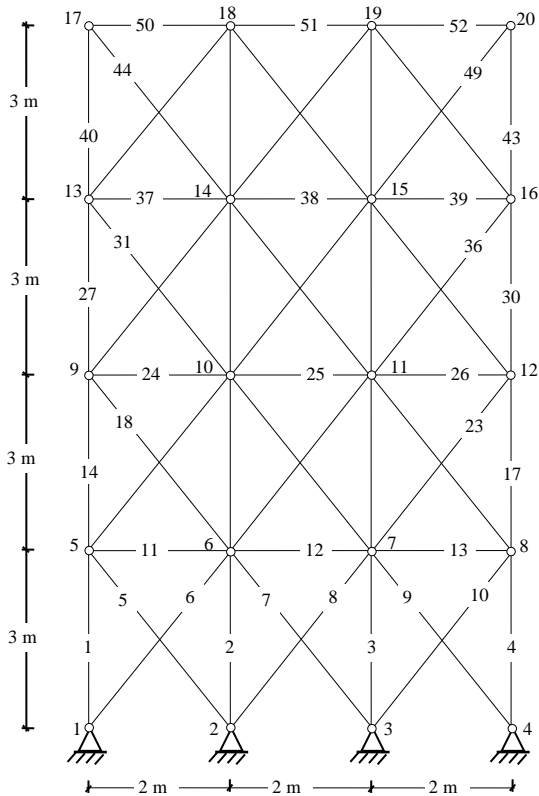


Figure 2. 52 bar planar truss.

The young modulus,  $E$ , is 207 GPa, the density,  $\rho$ , is 7860 kg/m<sup>3</sup>, and the allowable stresses are 180 MPa in tension and compression. Constraints are imposed on member stresses. Members of the truss are divided into 12 groups and the cross-sectional areas are to be selected from a list with 64 sections presented in Table 2.

As mentioned previously, in contrast to [30], [31], [24], and [28] in the current work, to show the dependence of the TLBO on the value of  $T_F$  each design example examined in this study is optimized taking the value of  $T_F$  as 1, 2 and  $\text{round}[1 + \text{rand}(0,1)\{2-1\}]$ , respectively.

The results obtained by the TLBO as well as those from the references cited above are summarized in Table 3. The iteration histories of TLBO process are shown in Figure 3. Figure 3 shows the variations of the penalized objective function during the solution process conducting with TLBO. Figure 3a illustrates this variation for the population size ( $pop$ ) adopted as 50 and a maximum number of cycles ( $C_{max}$ ) taken as 150, 100, and 80 respectively.

Table 1. Load case for the 52 bar truss.

Note	$F_x$ (kN)	$F_y$ (kN)
17	100.0	200.0
18	100.0	200.0
19	100.0	200.0
20	100.0	200.0

variation through the solution process for  $pop=40$  and  $pop=30$ , respectively. Each solution process depicted in Figures 3b and 3c is repeated with different  $C_{max}$  taken as 150, 100, and 80 respectively while the population size remains the same.

It is noticed that for  $T_F = \text{round}[1 + \text{rand}(0,1)\{2-1\}]$ , the results remain the same for  $pop=30, 40,$  and  $50$  when  $C_{max}=150$  and  $100$  as well as for being  $T_F=1$ . In case of  $C_{max}=80$ , the results are also same both  $T_F = \text{round}[1 + \text{rand}(0,1)\{2-1\}]$  and  $T_F=1$  when  $pop=40$  and  $pop=50$ . It is observed that TLBO does not produce the same results for  $T_F=2$  when  $C_{max}=100$  and  $80$ , and  $pop=30, 40,$  and  $50$ .

It might be concluded from the observations that TLBO is capable of finding the same results if the parameters of  $C_{max}$  and  $pop$  are rationally selected for the problem under investigation. In addition, it is worthy said that compared with  $T_F=2$  the results obtained with  $T_F=1$  and  $T_F = \text{round}[1 + \text{rand}(0,1)\{2-1\}]$  are not more sensitive the changes in the population size and the maximum number of cycles.

Table 2. Cross-sectional areas for the 52 bar truss.

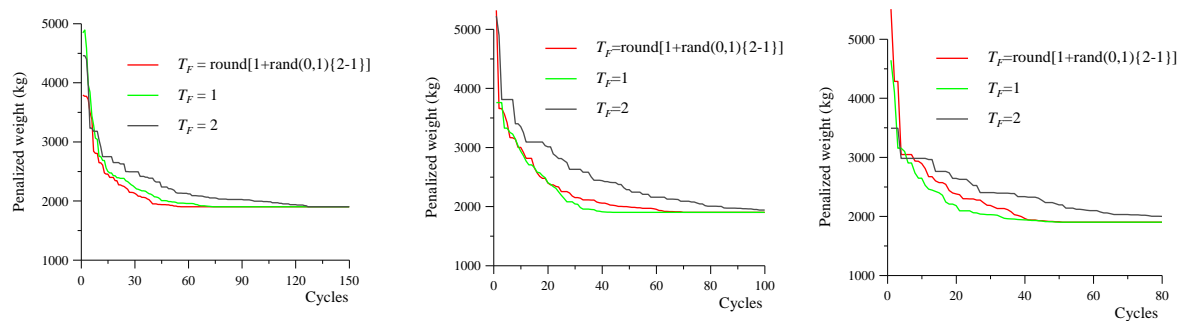
Section no	Area (mm <sup>2</sup> )	Section no	Area (mm <sup>2</sup> )
1	71.613	33	2477.414
2	90.968	34	2496.769
3	126.450	35	2503.221
4	161.290	36	2696.769
5	198.064	37	2722.575
6	252.258	38	2896.768
7	285.161	39	2961.284
8	363.225	40	3096.768
9	388.386	41	3206.445
10	494.193	42	3303.219
11	506.451	43	3703.218
12	641.289	44	4658.055
13	645.160	45	5141.925
14	792.256	46	5503.215
15	816.773	47	5999.988
16	940.000	48	6999.986
17	1008.385	49	7419.340
18	1045.159	50	8709.660
19	1161.288	51	8967.724
20	1283.868	52	9161.272
21	1374.191	53	9999.978
22	1535.481	54	10322.560
23	1690.319	55	10903.204
24	1696.771	56	12129.008
25	1858.061	57	12838.684
26	1890.319	58	14193.520
27	1993.544	59	14774.164
28	2019.351	60	15806.420
29	2180.641	61	17096.740
30	2238.705	62	18064.480
31	2290.318	63	19354.800
32	2341.931	64	21612.860

However, Figures 3b and 3c demonstrate the same

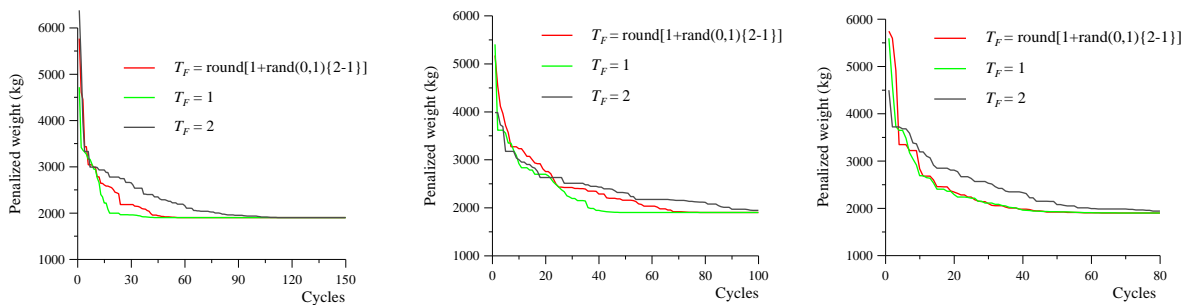
**Table 3.** Design results for the 52 bar truss.

Group no	Members	GA [33]	GA [34]	ACO [35]	MBA [36]	IMBA [37]	TLBO This study
1	1, 2, 3, 4	44	44	44	44	44	44
2	5, 6,...,10	19	19	19	19	19	19
3	11, 12, 13	13	10	11	10	10	11
4	14,...,17	42	42	42	42	42	42
5	18,...,23	18	16	16	16	16	16
6	24, 25, 26	10	12	11	10	10	11
7	27,...,30	33	30	30	30	30	30
8	31,...,36	18	17	17	17	17	17
9	37, 38, 39	7	8	9	10	10	9
10	40,...,43	24	20	20	20	20	20
11	44,...,49	18	19	19	19	19	19
12	50, 51, 52	12	10	11	10	10	11
Best (kg)		1970.142	1903.366	1899.350	1902.605	1902.605	1899.350
Evaluations <sup>+</sup>		60000	17500	17500	5450	4750	6440

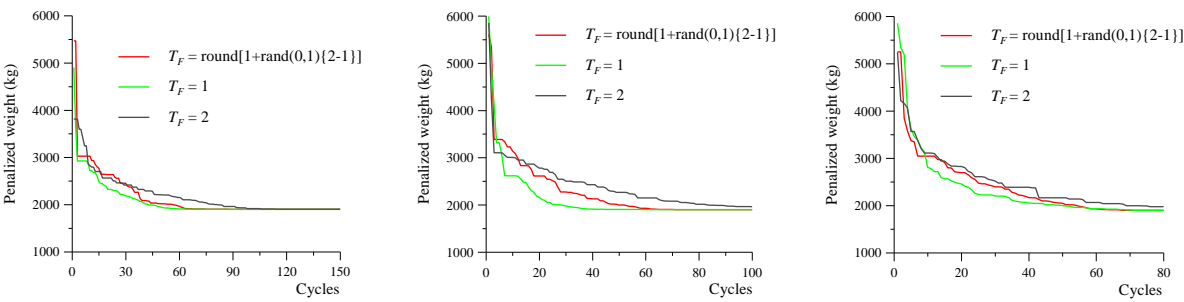
<sup>+</sup> shows the maximum numbers of structural analysis to obtain the optimal design presented in Table



(a) Variation of objective function for  $pop = 50$ .



(b) Variation of objective function for  $pop = 40$ .



(c) Variation of objective function for  $pop = 30$ .

**Figure 3.** Histories of TLBO process of 52-bar truss example.

The TLBO algorithm produces identical design to the design reported by [35]. However, TLBO algorithm uses 80 generations with a population size 40 resulting in 6440 truss analyses to converge to a solution and the required truss analyses to converge to a solution for the TLBO algorithm is more less than 60000, 17500, and 17500 analyses required by GA [33, 34] and ACO [35], respectively. However, [36] and [37] reported the required truss analyses number as 5450 and 4750 to acquire the optimal solutions using MBA and IMBA respectively. Studying the figures given by [36] and [37], it can be stated that maximum number of iteration was set as 500 in their algorithms. Since the results did not change around 100 iterations, it seems that the reported analyses numbers were calculated considering this iteration number in contrast to maximum number of iteration adopted as 500. Keeping this in mind, TLBO find the results presented in Table 3 at 55<sup>th</sup> iteration(see last graphic illustrated in Figure 3b). In this case, TLBO requires 4440 truss analyses to produce the optimal results.

Although both ACO and TLBO reach the same solution the design slightly violates stress constraints (0.012%). In the optimization application taken from the literature, certain results that violate the constraints less than the level of 0.1% might sometimes be encountered. The rationale of this might be meaningful due to the results from the point of view of engineering.

Statistical optimization result of TLBO algorithm is presented in Table 4.

**Table 4.** Load case for the 52 bar truss.

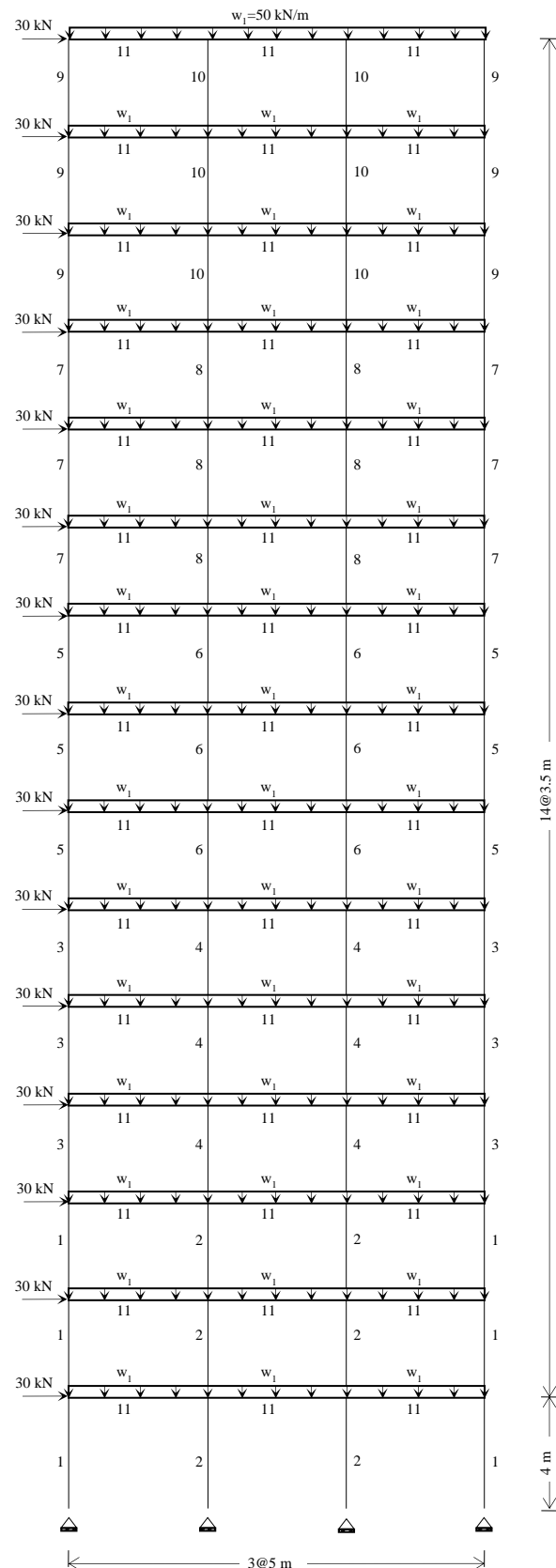
Exp.	Best optimized weight / volume	Average optimized weight / volume	Worst optimized weight / volume	Standard deviation on weight / volume
Exp <sup>1</sup>	1899.350 (kg)	1904.430 (kg)	1920.396 (kg)	6.705 (kg)
Exp <sup>2</sup>	402.94 (kN)	408.44 (kN)	412.13 (kN)	3.99 (kN)
Exp <sup>3</sup>	20.304 (m <sup>3</sup> )	21.073 (m <sup>3</sup> )	24.104 (m <sup>3</sup> )	1.143 (m <sup>3</sup> )

Note: Exp<sup>1</sup>= 52 bar truss; Exp<sup>1</sup>= 3 bay-15 story frame; Exp<sup>3</sup>= 582 bar truss tower

**4.2. Three-bay 15 story frame**

Figure 4 shows configuration of three-bay 15-story frame consisting of 105 members and its node, element numbering patterns and the loading. The material properties are a modulus of elasticity of  $E=200$  GPa and a yield stress of  $f_y=248.2$  MPa. The frame is designed following the AISC-LRFD specification [38] and uses a displacement constraint (the sway of the top story < 23.5 cm). The effective length factors,  $K_x$ , of the members are calculated as  $K_x \geq 0$  for a sway-permitted frame and the out-of-plane effective length factor  $K_y$  is considered as 1.0. All columns are considered as non-braced along their lengths and the non-braced length for each beam

member is specified as one-fifth of the span length.



**Figure 4.** Topology of the 3-bay 15-story frame.

The optimum design of the frame is obtained after 9030 analyses by using the TLBO, having the minimum weight of 402.94 kN. The optimum design for ICA [14] has the weight of 417.466 kN. Table 5 summarizes the optimal designs for ICA and TLBO. The ICA could find the result after 6000 analyses. The results obtained by TLBO is nearly 3.5% lighter than the that of the ICA [14].

As aforementioned, to investigate the effect of  $T_F$  on the results to be obtained, the total number of cycles required for TLBO process is varied by taking the different the population size ( $pop$ ) and by considering distinct  $T_F$  value, i.e.  $T_F=1$ ,  $T_F=2$ , and  $T_F= \text{round}[1 + \text{rand}(0,1)\{2-1\}]$ .

The results reported here correspond to the best having the least weight and they are obtained when the following parameter values are taken into consideration in TLBO process;  $pop=30$ ,  $C_{max} = 150$ ,  $T_F=1$  and  $T_F= \text{round}[1 + \text{rand}(0,1)\{2-1\}]$ . However when  $T_F=2$ , to reach the results presented in the last column of Table 5 TLBO requires more cycles. Figure 5 shows the histories of the best solutions obtained for all cases, which are performed using different values of  $pop$ ,  $T_F$  and  $C_{max}$  in order to shorten the computational cost of TLBO process and to demonstrate the effect of  $T_F$ .

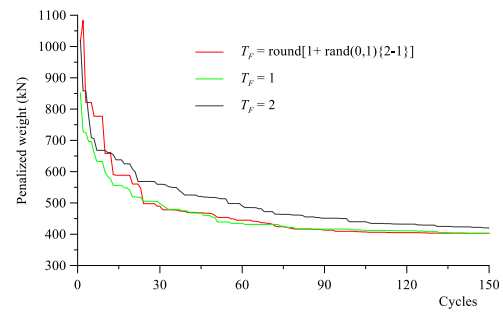
**Table 5.** Design results for the three-bay 15-story frame.

Grp. No	Members	ICA [14]	TLBO This study	TLBO This study
1	column 1-3S, E	W24×117	W24×117	W12×106
2	column 1-3S, I	W21×147	W36×160	W27×161
3	column 4-6S, E	W27×84	W14×82	W24×87
4	column 4-6S, I	W27×114	W30×116	W21×111
5	column 7-9S, E	W14×74	W21×68	W12×65
6	column 7-9S, I	W18×86	W30×90	W16×89
7	column 10-12S, E	W12×96	W12×50	W10×49
8	column 10-12S, I	W24×68	W12×65	W12×65
9	column 13-15S, E	W10×39	W12×30	W8×31
10	column 13-15S, I	W12×40	W12×40	W16×40
11	beams	W21×44	W21×44	W21×44
	Best (kN)	417.466	408.03	402.94
	Evaluations <sup>+</sup>	6000	6030	9030

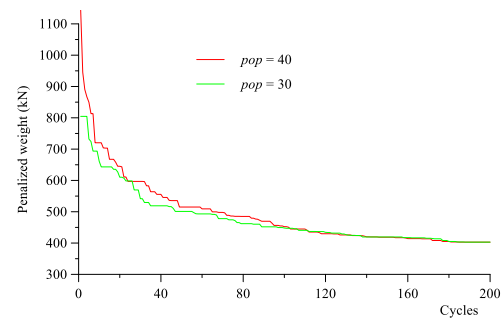
<sup>+</sup> shows the maximum numbers of structural analysis to obtain the optimal design presented in Table  
Note: Grp = Group; S = Story; E = Exterior column; I = Interior column.

It might be realized from Figure 5 that although the design achieved by TLBO for all cases has the same

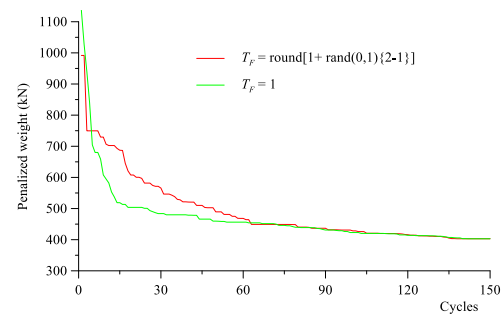
weight of frame, to achieve the results obtained when



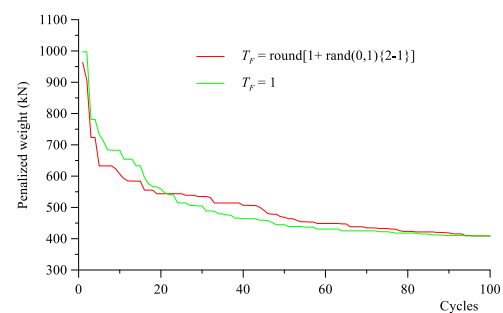
(a) Variation of objective function for  $pop=40$ .



(b) Variation of objective function for  $T_F=2$ .



(c) Variation of objective function for  $pop=30$ .



(d) Variation of objective function for  $pop=30$ .

**Figure 5.** Histories of TLBO process of three-bay 15-story frame example.

$pop=30$ ,  $C_{max}=150$ ,  $T_F=1$ , and  $T_F= \text{round}[1 + \text{rand}(0,1)\{2-1\}]$  the maximum number of cycles should be increased from 150 to 200 when the teaching factor is considered as 2 (see Figure 5b).

Moreover, if the required frame analyses to reach the best design was adopted 6,000 as well as in the ICA



[14] TLBO would produce a frame having a weight of 408.03 kN, which is 2.26% lighter than that of the ICA (see Figure 5d). This indicates that even though varying the  $T_F$  in TLBO process results in different computational cost, the results remain the same or closely the same with small differences.

The global sway at the top story is 13.61 cm, which is less than the maximum sway. The maximum value for the stress ratio is equal to 99.60%. Also, the maximum drift story is equal to 1.11 cm. Statistical optimization result of TLBO algorithm for this example is presented in Table 4.

#### 4.3. 582 bar space truss

The geometry and group numbering of a 582 bar space tower, previously studied by [39] using Particle Swarm Optimization (PSO), is given in Figure 6. The structural members of the space tower are linked together into 32 groups. The modulus of elasticity, the material density of all members and yield stress are 29000 ksi, 0.2836 lb/in.<sup>3</sup> and 36 ksi, respectively. The maximum displacement of all the nodes is not allowed to exceed 8 cm (3.15 in.) for all directions. A single loading condition is considered to be applied such that the lateral loads of 5 kN (1.12 kips) are applied to all nodes in both  $x$  and  $y$ -directions, and vertical loads of -30 kN (-6.74 kips) are applied, respectively, to all nodes in the upper and lower parts of the tower in  $z$  direction. A discrete set of 140 W-shape steel profiles given in Table 6 is used to size the design variables. In association with [39], cross-sectional areas of 140 W-shape steel profiles vary between 6.16 in.<sup>2</sup> (39.74 cm<sup>2</sup>) and 215.0 in.<sup>2</sup> (1387.09 cm<sup>2</sup>).

According to ASD-AISC the maximum slenderness ratio of  $i$ -th member is limited to 300 and 200 for tension and compression, respectively ( $\lambda_i = K_i L_i / r_i \leq \lambda_{allowed}$ , in here  $K_i$  is the effective length factor which was taken to be 1,  $L_i$  is the length and  $r_i$  is minimum radii of gyration). The stress and stability limitations of the members also are imposed according to the provisions of ASD-AISC.

Table 7 lists the designs developed by the PSO [39], the DHPSACO [40] and the IMBA [37]. The TLBO algorithm needs 30050 truss analyses to converge to a solution, while the 50000 analyses are required by PSO [39]. However, studying [39], it can be observed that the results are obtained within 17500 structural analyses although optimization process that ends up 50000 analyses. This case is also the same for the structural analyses number reported by other researchers. For instance, for this example, even though [37] finished the optimization process at the end of the 350 iterations they presented the structural analyses as 15100. This analyses number indicates the obtaining the reported volume firstly. Therefore, the structural analyses number reported as 15550 (155 iteraton) in the current work although TLBO process runs until 300 iterations. Figure 7 shows the convergence histories for the optimum designs

obtained by the TLBO algorithm, which is utilized with  $pop=50$ ,  $T_F=1$ ,  $T_F=2$  and  $T_F = \text{round}[1 + \text{rand}(0,1) \{2-1\}]$  in order to demonstrate the effect of  $T_F$ .

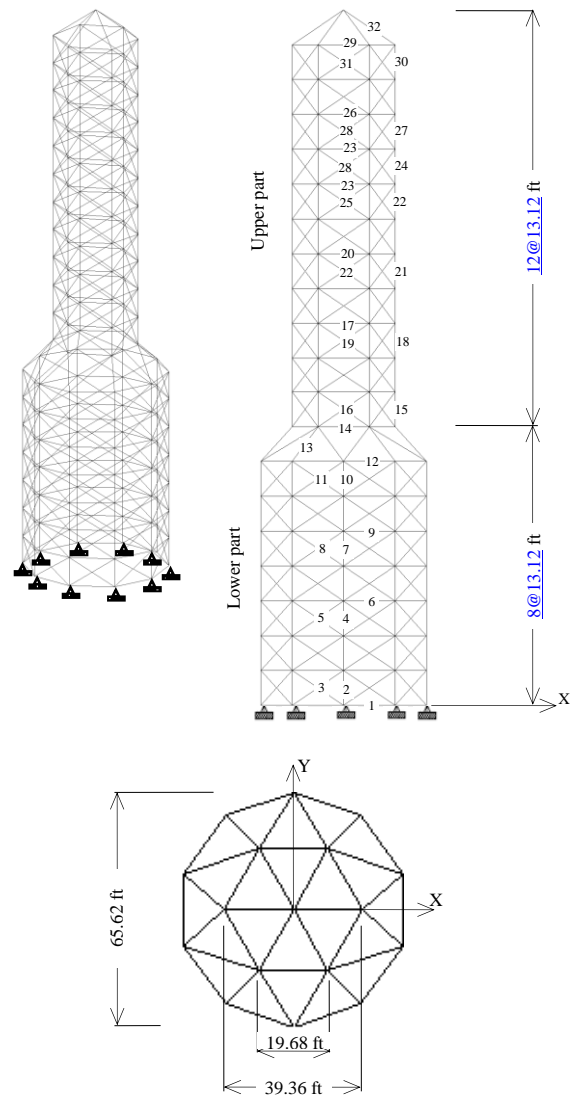


Figure 6. The 582-bar space tower truss.

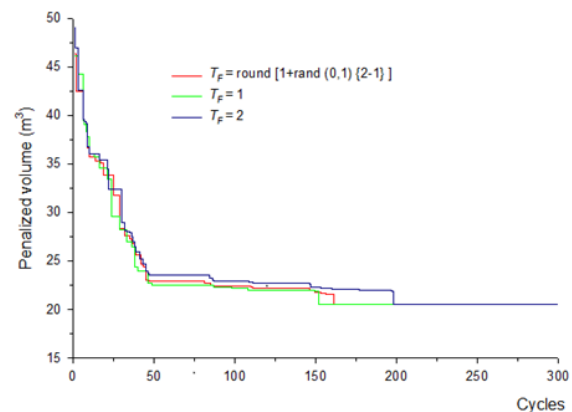


Figure 7. Histories of TLBO process of 582 bar space truss example ( $pop=50$ ).

**Table 6.** Profile list for the 582 bar space tower.

W-shape profile list *			
W27 x 178	W21 x 122	W18 x 50	W14 x 455
W27 x 161	W21 x 111	W18 x 46	W14 x 426
W27 x 146	W21 x 101	W18 x 40	W14 x 398
W27 x 114	W21 x 93	W18 x 35	W14 x 370
W27 x 102	W21 x 83	W16 x 100	W14 x 342
W27 x 94	W21 x 73	W16 x 89	W14 x 311
W27 x 84	W21 x 68	W16 x 77	W14 x 283
W24 x 162	W21 x 62	W16 x 67	W14 x 257
W24 x 146	W21 x 57	W16 x 57	W14 x 233
W24 x 131	W21 x 50	W16 x 50	W14 x 211
W24 x 117	W21 x 44	W16 x 45	W14 x 193
W24 x 104	W18 x 119	W16 x 40	W14 x 176
W24 x 94	W18 x 106	W16 x 36	W14 x 159
W24 x 84	W18 x 97	W16 x 31	W14 x 145
W24 x 76	W18 x 86	W16 x 26	W14 x 132
W24 x 68	W18 x 76	W14 x 730	W14 x 120
W24 x 62	W18 x 71	W14 x 665	W14 x 109
W24 x 55	W18 x 65	W14 x 605	W14 x 99
W21 x 147	W18 x 60	W14 x 550	W14 x 90
W21 x 132	W18 x 55	W14 x 500	W14 x 82
W14 x 74	W12 x 230	W12 x 50	W10 x 45
W14 x 68	W12 x 210	W12 x 45	W10 x 39
W14 x 61	W12 x 190	W12 x 40	W10 x 33
W14 x 53	W12 x 170	W12 x 35	W10 x 30
W14 x 48	W12 x 152	W12 x 30	W10 x 26
W14 x 43	W12 x 136	W12 x 26	W10 x 22
W14 x 38	W12 x 120	W12 x 22	W8 x 67
W14 x 34	W12 x 106	W10 x 112	W8 x 58
W14 x 30	W12 x 96	W10 x 100	W8 x 48
W14 x 26	W12 x 87	W10 x 88	W8 x 40
W14 x 22	W12 x 79	W10 x 77	W8 x 35
W12 x 336	W12 x 72	W10 x 68	W8 x 31
W12 x 305	W12 x 65	W10 x 60	W8 x 28
W12 x 279	W12 x 58	W10 x 54	W8 x 24
W12 x 252	W12 x 53	W10 x 49	W8 x 21

\* the corresponding profile list was taken from Sadollah et al. [37]

Studying on Figure 7 and ignoring the computational cost of TLBO process, it is worthy to state that varying the value of teaching factor, i.e.  $T_F=1$ ,  $T_F=2$ , and  $T_F=\text{round}[1 + \text{rand}(0,1) \{2-1\}]$ , does not affect the results obtained by the TLBO. Statistical optimization result of TLBO algorithm is presented in Table 4.

## 5. Conclusion

Three design examples consisting of two trusses and one frame are considered to illustrate the effect of

teaching factor  $T_F$  on the optimal design for all examples. The comparisons of the numerical results obtained by the TLBO with  $T_F=1$ ,  $T_F=2$ , and  $T_F=\text{round}[1 + \text{rand}(0,1) \{2-1\}]$  and those obtained by other optimization methods based on the meta-heuristic concepts are presented to show the capability of the TLBO algorithm in finding good results. Simulations show that reaching the optimum designs by TLBO is insensitive to the parameter of  $T_F$  and TLBO produces the same results for all case of  $T_F$  when the computational cost of TLBO and the number

**Table 7.** Design results for the 582 bar sapce tower truss.

Elm. grp.	PSO [39]	DHPSACO [40]	IMBA [37]	TLBO This study
1	W8×21	W8×24	W8×21	W8×21
2	W12×79	W12×72	W24×76	W24×84
3	W8×24	W8 × 28	W8 × 21	W8 × 21
4	W10×60	W12×58	W12×65	W24×62
5	W8×24	W8×24	W8×21	W8×21
6	W8×21	W8×24	W8×21	W8×21
7	W8×48	W10×49	W10×54	W16×57
8	W8×24	W8×24	W8×21	W8×21
9	W8×21	W8×24	W8×21	W8×21
10	W10×45	W12×40	W12×50	W12×53
11	W8×24	W12×30	W8×21	W8×21
12	W10×68	W12×72	W10×68	W10×77
13	W14×74	W18×76	W24×76	W21×83
14	W8×48	W10×49	W14×53	W21×57
15	W18×76	W14×82	W12×79	W18×76
16	W8×31	W8×31	W8 × 21	W8 × 21
17	W8×21	W14×61	W12×65	W10×22
18	W16×67	W8×24	W8×21	W18×55
19	W8×24	W8×21	W8×21	W8×21
20	W8×21	W12×40	W12×45	W8×21
21	W8×40	W8×24	W8×21	W14×30
22	W8×24	W14 × 22	W8 × 21	W8 × 21
23	W8×21	W8×31	W16×26	W8 × 21
24	W10×22	W8×28	W8×21	W8×21
25	W8×24	W8×21	W8×21	W8×21
26	W8×21	W8×21	W8×21	W8 × 21
27	W8×21	W8 × 24	W8 × 21	W10×22
28	W8×24	W8 × 28	W8 × 21	W8×21
29	W8×21	W16×36	W8×21	W8×21
30	W8×21	W8×24	W8×21	W8×31
31	W8×24	W8×21	W8×21	W8×21
32	W8×24	W8×24	W8×21	W12×22
Vol.	22.3958	22.0607	20.0688	20.304
Eval <sup>+</sup> .	17500	17500	15300	15550

Note: Vol.= Volume (m<sup>3</sup>); Eval.= Evaluations

<sup>+</sup> shows the maximum numbers of structural analysis to

obtain the optimal design presented in Table

analyses required to obtain the best design are ignored. Comparisons of the numerical results obtained by TLBO with those by other optimization methods are performed to demonstrate the efficiency of the TLBO algorithm in terms of reaching the best designs. Consequently, it is useful to express that  $T_F=1$  and  $T_F=\text{round}[1 + \text{rand}(0,1) \{2-1\}]$  would be more suitable when it is intended to find good results in a less number of iterations.

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