Finite time tracking of a fully actuated biped robot with pre-specified settling time: a second order sliding mode synthesis

Harshal B. Oza, Yury V. Orlov, Sarah K. Spurgeon, Yannick Aoustin and Christine Chevallereau

***Abstract*— A second order sliding mode controller is utilised to track reference trajectories for all the joints of a fully actuated biped robot. The existing tuning rules for the 'twisting' controller are used to guarantee *a priori* attainment of a prescribed settling time between two successive impacts. The joint torque is modeled as the control input. Smoothing of the discontinuous controller is carried out by introducing a high gain linear controller inside a boundary layer defined by an arbitrarily small region around the origin thereby avoiding numerical errors in the simulations. The overall accuracy of motion control is dictated by the size of this layer leading to practical stability of the closed-loop system. The main contribution of the paper is to provide straightforward and realizable engineering guidelines for the reference trajectory generation and for the tuning of a robust finite time controller for achieving stable gait of a biped in the presence of disturbances in both continuous and impact phases. Numerical simulations of a biped robot are shown to support the theoretical results.**

I. Introduction

Second order sliding mode state feedback synthesis and associated tuning for achieving finite time convergence of the joint trajectories of a biped is explored. The biped robot under consideration is a fully actuated robot. Gait stability is to be ensured by designing the reference trajectories and ensuring that they are tracked in finite time. Second order sliding mode (SOSM) controllers [1] are recognized as a good choice for robotics [2] due to their simplicity of use and the underlying robustness properties. There are geometric homogeneity based results [3], [4], [5] for SOSM which highlight finite time convergence in the presence of persisting disturbances.

The main focus of this paper is on finite time tracking of the trajectories of the joints of a fully actuated biped robot to the desired periodic trajectories by using a second order sliding mode controller together with tuning of the controller parameters. There is strong theoretical motivation to study this problem. Firstly, an *a priori* guarantee with appropriate tuning to prescribe finite time convergence for the tracking of periodic trajectories has not been studied for the biped robot when a finite time controller is utilized. Previous work in [6] gives a high gain version of a continuous finite time controller; see [7] but *a priori* tuning which results in finite gains is an open problem. Secondly, the present paper utilizes existing tuning rules [8] for a SOSM controller which is directly applicable to the biped model. The controller is robust to disturbances while requiring only knowledge of an upper bound on the disturbance to cause the tracking errors to converge to zero in finite time between successive impacts. The main motivation to use second order sliding mode synthesis lies in the simplicity of use and in the robustness to persisting disturbances [1]. From a practical viewpoint, the motivation stems from the need to propose tuning rules for a SOSM for the presented class of biped robots. Thus, the goal of the present investigation is to provide engineering guidelines for achieving stable walking gait of a biped in finite time.

The literature on control of biped robots is vast (see [9], [10], [11], or [12] for a comprehensive survey). Previous work utilizing SOSM includes [13], [14]. Previous results on continuous finite time stabilization for biped robots [6] do not give explicit tuning rules. The main contribution of the presented results is that the tuning rules for the controller are given for the tracking problem with an *a priori* guarantee of attaining a pre-specified cyclic walking gait. During this walking gait, which is composed of single support phases and impacts, the biped with feet is always fully actuated. This cyclic gait is the result of an optimization, where the criterion is based on a sthenic criterion and the definition of nonlinear constraints such as the no take-off, no rotation and no sliding of the stance foot on the ground. Then the fully actuated biped, tracking this cyclic gait perfectly, is stable. The tuning rules ensure that the tracking errors always converge to the origin before the subsequent impact event occurs thereby guaranteeing a stable walking biped. To carry out the simulation of the biped, in closed loop, a linear complementary problem (lcp) optimization is solved to determine the exact configuration of the foot of the swing leg when it lands on the ground. The method differs from existing contributions such as [15], [16] that depend on open-loop optimal control. The drawback of the presented method is that it is more conservative. Nevertheless, this work gives theoretical starting values for tuning that guarantee finite time tracking of the states in the presence of disturbances during both impact and continuous phases of the biped dynamics.

Furthermore, a boundary layer is proposed in Section V-A for smoothing of the discontinuous 'twisting' controller. This produces an ultimately bounded closed-loop trajectory. Since the joint torque is modeled as the control input,

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and since it is preferable to avoid a chattering torque in a biped as it produces a discontinuity in the reaction force that may produce take-off or fall down of the robot, the boundary layer is introduced around the origin as there is no sliding anywhere other than the origin [2]. It follows that the closed-loop system trajectory enters the closed region around the origin in finite time due to the proposed tuning. Furthermore, it is shown that there is no sliding mode either on the principal axes within the boundary layer or on the boundary of the layer. These features resemble the boundary layer approach in traditional sliding mode synthesis and hence readily provide a natural extension to SOSM but with the accompanying tuning guidelines intact. The combination of this tuning and boundary layer approach renders these most recent advances in SOSM control suitable for industrial engineering applications encompassing a large class of unilaterally constrained mechanical systems such as biped robots.

II. Problem Statement

Let the system dynamics be given as follows:

$$\dot{\mathbf{e}}_1 = \mathbf{e}_2 \quad \dot{\mathbf{e}}_2 = \mathbf{u} + \boldsymbol{\omega}(t), \tag{1}$$

This system corresponds to the tracking dynamics of a mechanical system (such as a biped) with a computed control law (see section IV.A). Here $\mathbf{e}_1 = \mathbf{x}_1(t) - \mathbf{x}_1^d(t)$ and $\mathbf{e}_2 = \mathbf{x}_2(t) - \mathbf{x}_2^d(t)$ are respectively the error variables in position and in velocity, $\mathbf{u}$ is the control law to be synthesized, and $\boldsymbol{\omega}(t)$ is an external disturbance. Let $(\mathbf{x}_1^d(t), \mathbf{x}_2^d(t) = \dot{\mathbf{x}}_1^d(t))$ represent a desired trajectory as functions of time for the position $\mathbf{x}_1$ and the velocity $\mathbf{x}_2$. The following assumptions are made throughout.

1) $\operatorname*{ess\,sup}_{t\geq 0} |\omega| \leq N$ where N is an *a priori* known positive scalar.
2) The upper bound $\tilde{R}$ on the quantity $\max\{|\mathbf{x}_1(t_0)|, |\mathbf{x}_2(t_0)|, |\mathbf{x}_1^d(t_0)|, |\mathbf{x}_2^d(t_0)|\}$, where t_0 is the initial time, is known *a priori* and finite.

The first assumption appears in the literature [1], [4] and represents a uniform upper-bound on the disturbance. The second assumption dictates that the results presented in the paper are of a local nature[1]. The aim of the paper is to utilize (i) a SOSM state feedback synthesis and (ii) the corresponding tuning rules for the controller gains to give an *a priori* guarantee of finite time convergence of the states of a fully actuated biped robot to the desired trajectory by utilizing the equivalence between the error dynamics of (1) and that of each actuated joint of a biped.

III. SOSM Synthesis

The control law proposed in [8] involves a switching between the initial linear feedback to a second order sliding mode control. It is preferable for an application such as a biped control to have a robust synthesis. For this reason, only the discontinuous robust second order sliding mode control will be employed in the following. The tuning rules proposed in [8] are modified to this effect in this section. Let the control be defined as follows [1]:

$$\mathbf{u}(\mathbf{e}_1, \mathbf{e}_2) = -\mu_1 \mathrm{sign}(\mathbf{e}_2) - \mu_2 \mathrm{sign}(\mathbf{e}_1) \tag{2}$$

the function $\mathrm{sign}(\cdot)$ is defined in the component-wise fashion, i.e, $\mathrm{sign}(\mathbf{e}_i) = \begin{bmatrix} \mathrm{sign}(e_{1i}) & \mathrm{sign}(e_{2i}) & \ldots & \mathrm{sign}(e_{6i}) \end{bmatrix}^{\mathrm{T}}$, $i = 1,2$.

It has been shown [8] that it is possible to *a priori* guarantee the finite time convergence of the error states $(\mathbf{e}_1, \mathbf{e}_2)$ to the origin provided certain tuning rules are employed. It should be noted that the tuning rules in [8] were developed for a switched synthesis where a linear state feedback is utilised to first bring the closed-loop trajectory arbitrarily close to the origin in finite time and then a twisting controller takes over to drive the error states to the origin in finite time. Since in this paper a twisting controller without a linear feedback is being used, a slight adjustment in the tuning rules is needed to make the results of [8] applicable to a purely twisting controller. This can be done, for example, by setting the variable representing the switching boundary $R = \frac{r_0^2}{2}$ (see [8]) where $r_0 = \sqrt{(\mathbf{e}_{1_0})^2 + (\mathbf{e}_{2_0})^2}$ is the upper bound on the Euclidian norm of the system initial conditions where $\mathbf{e}_{1_0} = \mathbf{e}_1(t_0), \mathbf{e}_{2_0} = \mathbf{e}_2(t_0)$. A finite r_0 can always be computed due to assumption 2.

Let the following *a priori* tuning rules be utilized which are modified from [8]:

$$\begin{aligned} &\mu_1 > \max\left\{N, \quad \frac{2\delta}{\mathscr{T}_s\sqrt{1-\eta^2}} + N\right\} \\ &\mu_2 > \max\left\{\sqrt{\frac{R}{2}}, \frac{\mu_1 - N}{\eta}, \mu_1 + N, \rho\sqrt{\frac{R}{2(1-\rho)}}, \rho, \beta\mu_1\right\} \end{aligned} \tag{3}$$

where

$$\delta > \frac{r_0(\beta+1)}{\beta-1}, \quad \mathscr{T}_s = \varepsilon T^d, \tag{4}$$

$r_1 = \sqrt{2R}$, $\beta > 1$ is a tuning variable, $\eta \in (0, \frac{1}{\beta})$, $R = \frac{r_0^2}{2}$, $\varepsilon \in (0,1), \rho \in (0,1)$ are arbitrary scalars and T^d is the desired settling time. Since $\beta > 1$ holds true by choice, the design rule $r_1 = \sqrt{2R}$ always results in $r_0 < r_1 \frac{\beta+1}{\beta-1}$ resulting in the above definition of δ [8, Sec. 6].

IV. Biped Model

The bipedal robot considered in this section is walking on a rigid and horizontal surface. It is modeled as a planar biped, which consists of a torso, hips, two legs with knees and feet (see Fig. 1). The walking gait takes place in the sagittal plane and is composed of single support phases and impacts. The complete model of the biped robot consists of two parts: the differential equations describing the dynamics of the robot during the swing phase, and an impulse model of the contact event (the impact between the swing foot and the ground is modeled as a contact between two rigid bodies [17]). In the single support phase, the dynamic model, considering an implicit contact of the stance foot with the ground (*i.e.* there is no take-off, no rotation and no sliding during the single support phase), can be written as follows:

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \Gamma \tag{5}$$

[1]Such an upper bound is generally known *a priori* for a large class of mechanical systems.

with $\mathbf{q} = (q_1\ q_2\ q_3\ q_4\ q_5\ q_6)^\top \in \mathbb{R}^6$ the vector of the generalized coordinates (see Fig. 1), $\Gamma = (\Gamma_1\ \Gamma_2\ \Gamma_3\ \Gamma_4\ \Gamma_5\ \Gamma_6)^\top \in \mathbb{R}^6$ is the vector of joint torques [2], $\mathbf{D}$ is the symmetric, positive definite 6×6 inertia matrix. As the kinetic energy of the biped is invariant under a rotation of the world frame, q_1 defines the orientation of the biped in the world frame. Terms $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ and $\mathbf{G}(\mathbf{q})$ are the 6×1 matrices of the centrifugal, coriolis and gravity forces respectively. From (5), the state-space form can be written as follows:

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{x}_2 \\ \mathbf{D}^{-1}(-\mathbf{C}-\mathbf{G}+\mathbf{\Gamma}) \end{pmatrix} = f(\mathbf{x}) + g(\mathbf{x}_1)\cdot\mathbf{\Gamma} \tag{6}$$

with $\mathbf{x} = \begin{pmatrix} \mathbf{q}^\top & \dot{\mathbf{q}}^\top \end{pmatrix}^\top = \begin{pmatrix} \mathbf{x}_1^\top & \mathbf{x}_2^\top \end{pmatrix}^\top$. The state space is chosen such that $\mathbf{x} \in \mathscr{X} \subset \mathbb{R}^{12} = \{\mathbf{x} = [\mathbf{x}_1^\top\ \mathbf{x}_2^\top]^\top \mid \mathbf{x}_1 \in \mathscr{N},\ \mathbf{x}_2 \in \mathscr{M}\}$, where $\mathscr{N} = (-\pi,\pi)^6$ and $\mathscr{M} = \{\ \mathbf{x}_2 \in \mathbb{R}^6 \mid |\mathbf{x}_2| < M < \infty\}$ such that $M > 0$ is a positive scalar. One of the difficulties in the biped control is to ensure that the contact with the ground is the expected one in the presence of perturbation. To correctly check the behavior of our controller in simulation a general model able that is able to deal with any condition of contact is used for simulation. The model (6) is used to define the control with the assumption that a flat contact occurs between the stance foot and the ground. This simulation model includes a unilateral contact between the foot and the ground with two contact points, at the heel and the toe, of each foot. Various solutions exist to determine the contact of each corner of the feet with the ground. The contact forces between the feet and the ground reaction are calculated using a constraint-based approach. This approach belongs to the family of time-stepping approaches. Let the vector $\mathbf{R} \in \mathbb{R}^8$ be the reaction force vector, which is obtained by stacking the reaction force vectors of the two corners of each foot. Vector $\mathbf{R}_k$ at $t = t_k$ is expressed at each sampling period as a function of the generalized position vector $\mathbf{q}^k \in \mathbb{R}^9$ composed of the variable orientation of each link and the Cartesian coordinates x_h, y_h of the hips, the associate velocity vector $\mathbf{q}_v^k \in \mathbb{R}^9$ for the biped and Γ^k with an algebraic equation

$$G(\mathbf{R}_k, \mathbf{q}^k, \mathbf{q}_v^k, \Gamma^k) = 0 \tag{7}$$

Let vector $\mathbf{v}^{k+1}$ be the Cartesian velocities of the corners in contact with the ground at $t = t_k$. The normal components must be non negative to avoid interpenetration. The identity $\mathbf{v}_{in}^{k+1} = 0$ means that the contact remains and the inequality $\mathbf{v}_{in}^{k+1} > 0$ means that the contact vanishes. The normal components $\mathbf{r}_{in}^k > 0$ of $\mathbf{R}_k$, when contact occurs, are also subject to non negative constraints. These components can avoid interpenetration but they cannot avoid the stance foot take-off. It is clear that the variables $\mathbf{v}_{in}^{k+1}$ and $\mathbf{r}_{in}^k$ are complementarity quantities:

$$\mathbf{v}_{in}^{k+1} \geq 0 \quad \perp \quad \mathbf{r}_{in}^k \geq 0 \tag{8}$$

Furthermore, the variables $\mathbf{v}_{in}^{k+1}$ and $\mathbf{r}_{in}k$ are subject to constraints imposed by friction which leads to a linear complementarity condition. The valid cases of contact for each corner are determined using constrained optimization [18].

[2]Leg 1 is the stance one, leg 2 the swing one.

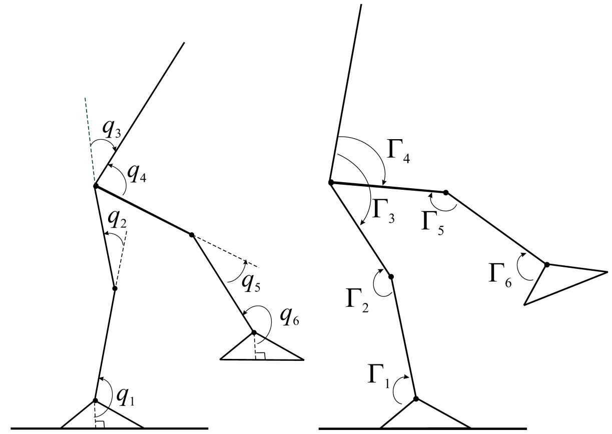

Fig. 1. Seven-link bipedal robot.

A. Pre-feedback and Reference Trajectory

The cyclic walking gait, has been defined by $\mathbf{x}_1^d(t)$, $\mathbf{x}_2^d(t)$ and $\dot{\mathbf{x}}_2^d(t)$ satisfying the conditions of contact using an off-line optimization [19]. Each step is composed of a single support phase on the stance foot which ends with an impact of the foot of the swing leg on the ground, without rebound. In single support the stance foot has a flat contact with the ground. The velocity of the swing foot at its landing is not null. The previous stance foot takes off instantaneously. The reference trajectory is designed such that if the tracking of this reference trajectory is perfect, the stance foot neither rotates nor takes off since the Zero moment point (ZMP) [11] of the biped lies within the interior of the support polygon defined by the foot geometry. Furthermore there is no slipping of the stance foot on ground. The torques and velocities of the actuators are bounded by given values. As a consequence, when the fully actuated biped adopts this cyclic walking gait, no zero dynamics appear. Since the impact is instantaneous and passive, the control law is defined only during the single support phase. The objective of the control is that each joint angle follows its reference trajectory to track the cyclic walking gait. The torque vector $\mathbf{\Gamma}$ is defined based on the dynamic model (5) as follows:

$$\Gamma = \mathbf{D}(\mathbf{x}_1)(\dot{\mathbf{x}}_2^d(t) + \mathbf{u}) + \mathbf{C}(\mathbf{x}_1,\mathbf{x}_2) + \mathbf{G}(\mathbf{x}_1) \tag{9}$$

where u is defined by (2). The pre-feedback (9) enables the system (5) to be transformed into the form (1), thereby rendering the tuning rules (3) applicable to the biped problem.

V. Numerical Simulation

The reference walking minimizes the integral of the norm of the torque vector for a given distance. The walking velocity is selected to be 0.5 m/s. The duration of one step is 0.53s.

A. Boundary Layer Approach For SOSM Controller

A boundary layer approach is briefly introduced in this section before proceeding with the numerical simulation of the biped. Since, in the following, the torque of each joint of the biped will be modeled as a control input, let the following boundary layer be applied to the twisting control to avoid any discontinuity in the reaction force that may produce undesirable take-off or fall down of the biped robot:

$$\begin{array}{ll} u_i(\mathbf{e}_1,\mathbf{e}_2) = -\mu_1 \text{sign}(e_{2_i}) - \mu_2 \text{sign}(e_{1_i}) & \text{if } (e_{1_i}, e_{2_i}) \notin \Omega_{\varepsilon_i} \\ u_i(\mathbf{e}_1,\mathbf{e}_2) = -\mu_1 \frac{e_{2_i}}{|e_{2_i}|+\varepsilon} - \mu_2 \frac{e_{1_i}}{|e_{1_i}|+\varepsilon} & \text{if } (e_{1_i}, e_{2_i}) \in \Omega_{\varepsilon_i} \end{array} \tag{10}$$

where, the subscript $i = 1,2,\ldots,6$ shows the i^{th} component of a vector representing i^{th} joint, $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \ldots & u_6 \end{bmatrix}^{\text{T}} \in \mathbb{R}^6$, the error variables $\mathbf{e}_1 = \mathbf{x}_1 - \mathbf{x}_1^d$, $\mathbf{e}_2 = \mathbf{x}_2 - \mathbf{x}_2^d$ with $\mathbf{e}_1 \in \mathscr{N}, \mathbf{e}_2 \in \mathscr{M}$ with the state $(\mathbf{x}_1, \mathbf{x}_2)$ defined in Section IV and $\Omega_{\varepsilon_i} = \{\frac{1}{2}\mathbf{e}^{\text{T}}\mathbf{e} \le \varepsilon_i\}$ is the boundary layer for the i^{th} joint with $\varepsilon_i > 0$ being the corresponding arbitrarily small parameter to be selected by the user.

B. Robust Walking Cycles

The model (6) is utilized in this section to show numerical simulations of a stable walking gait by achieving a finite settling time via the tuning rules (3). The desired convergence time for tracking the reference trajectories is defined to be 0.5 seconds.

The robustness of the tracking control (9) is verified by introducing a resultant disturbance force $\mathbf{F}_\omega$ on the hip joint of the biped with projections $F_{x\omega} = 50N$ and $F_{y\omega} = 2.5N$ in the horizontal and vertical planes respectively. Such a force is used for the duration of $0.07sec$ to simulate two disturbance effects. On one hand, the effect of $F_{x\omega}$ represents a disturbance in the continuous phase of the dynamics (5) as it starts from $1.08sec$ in the first cycle of the biped which belongs to the continuous phase of the trajectory.

The effect of the aforementioned disturbance force on the hip joint can be studied via the principle of virtual work as follows. Let a disturbance force $\mathbf{F}_\omega$ be applied as mentioned above. Let the effect of the disturbance force $\mathbf{F}_\omega$ on the dynamics of the generalized coordinates $\mathbf{q}$ be denoted by $\mathbf{\Gamma}_\omega = \mathbf{J}^{\text{T}}\mathbf{F}_\omega$ where $\mathbf{J}$ is such that $\mathbf{J}\mathbf{x}_2$ is the velocity of the hip, the point where the force is applied, where

$$\mathbf{J}^{\text{T}} = \begin{pmatrix} l_1 \cos q_1 - l_2 \cos(q_1+q_2) & l_1 \sin q_1 + l_2 \sin(q_1+q_2) \\ -l_2 \cos(q_1+q_2) & l_2 \sin(q_1+q_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{11}$$

Here l_1 and l_2 are the lengths respectively of the shin and the thigh. Hence, the biped model (5) can be revised as follows:

$$\ddot{\mathbf{q}} = \mathbf{D}^{-1}\left(\mathbf{\Gamma} + \mathbf{\Gamma}_\omega - \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{G}(\mathbf{q})\right) \tag{12}$$

It can be seen from the above that the quantity $\mathbf{\Gamma}_\omega$ appears as a disturbance in the $\ddot{\mathbf{q}}$ dynamics. In the following, an *a priori* known upper bound

$$N \triangleq \sup_{t \ge 0} |\mathbf{D}^{-1}\mathbf{\Gamma}_\omega| = 19.2 \tag{13}$$

is utilized by the tuning rules (3) to cover the worst effect produced by the disturbance force F_ω while tuning the gains μ_1, μ_2 for each joint. Thus, the modeling information utilised for the control synthesis (9) lies in the usage of model matrices D, C, G and the *a priori* known upper bound (13).

Next, tuning rules (3) are used to produce a twisting controller with gains $\mu_1 = 20.7$, $\mu_2 = 73.8$. It should be noted here that the tuning rules are conservative because they are based on a Lyapunov function and on a comparison system that encompasses the true trajectories in each quadrant of the planar state-space [8]. Hence, the aforementioned values of gains provide a good starting point from where the gains should be reduced, if needed, to avoid excessive chattering.

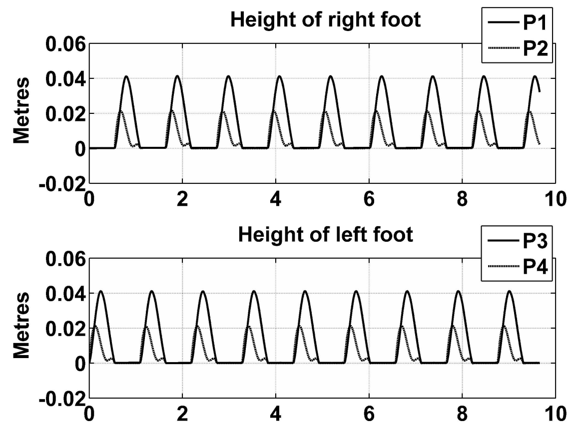

Fig. 2. Feet height in the walking gait with $0.5sec$ settling time

Figure 2 shows the heights of the feet for eight consecutive steps with gain selection $\mu_1 = 20.7, \mu_2 = 65$. The corresponding velocities of the feet in the horizontal and vertical direction can be seen in figures 3 and 4 respectively. Legends 'P1' and 'P3' represent the 'toe' of the right and left foot respectively. Similarly, 'P2' and 'P4' represent the 'heel' of the right and left foot respectively.

Periodic orbits in each of the i^{th} joints are depicted in terms of phase-plane plots of $q_i, \dot{q}_i$ in figures 5, 6 and 7. It can be seen that each joint velocity undergoes a jump at the time of collision of the feet with the walking surface and that the actual trajectory follows the reference trajectory closely due to the robust SOSM control. The swing phase and the impact phases of the biped are diagrammatically shown in Fig. 8 for a step.

C. Robustness Analysis

The effect of the disturbance force can be seen in several plots. For example, the velocity in the vertical direction for the right foot exhibits severe effects of a disturbance as can be seen from the high amplitude impulse like change just at the end of the first step (see Fig. 4). This is the result of the

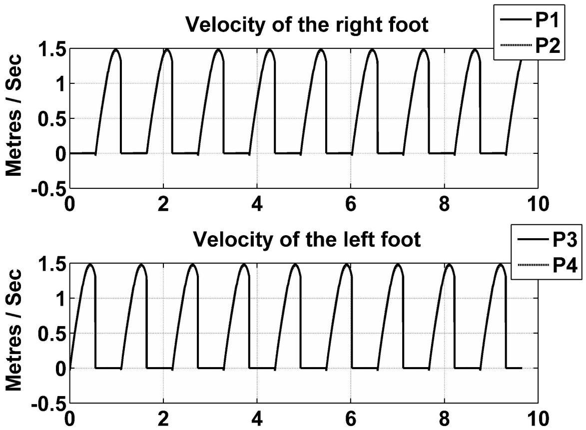

Fig. 3. Feet velocity in horizontal direction in the walking gait with $0.5\,sec$ settling time

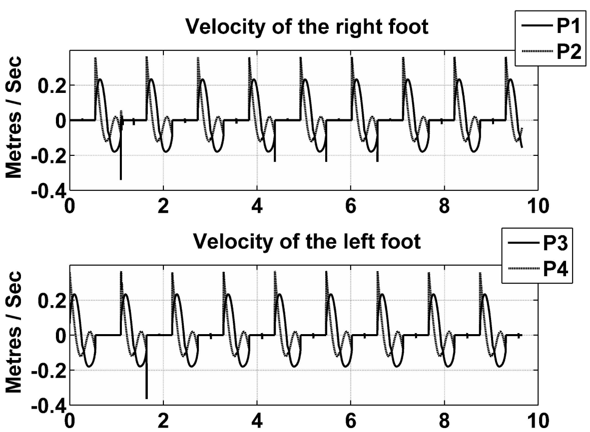

Fig. 4. Feet velocity in vertical direction in the walking gait with $0.5\,sec$ settling time

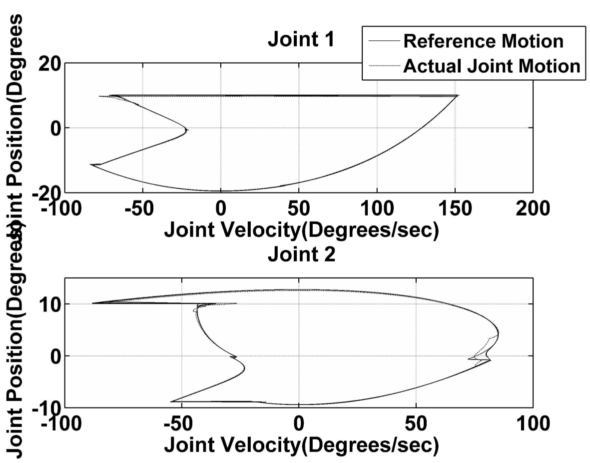

Fig. 5. Periodic orbit in joint 1 and 2 in a walking gait

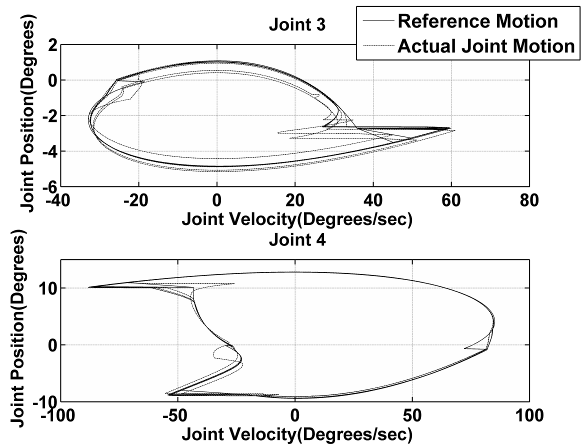

Fig. 6. Periodic orbit in joint 3 and 4 in a walking gait

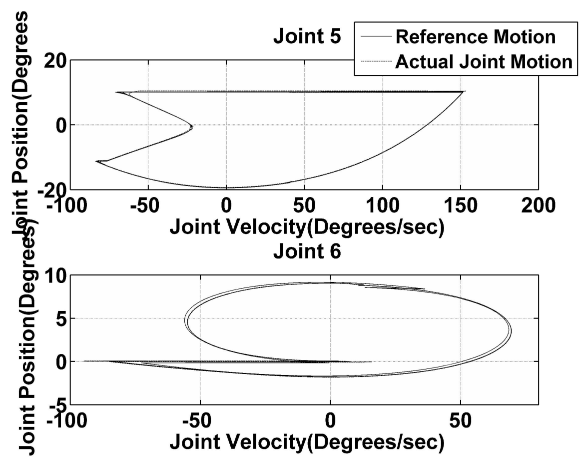

Fig. 7. Periodic orbit in joint 5 and 6 in a walking gait

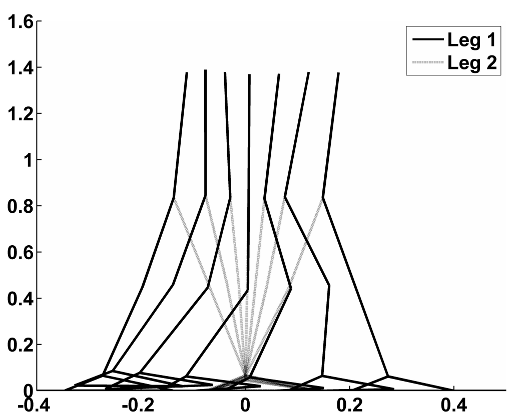

Fig. 8. Stick diagram of the biped

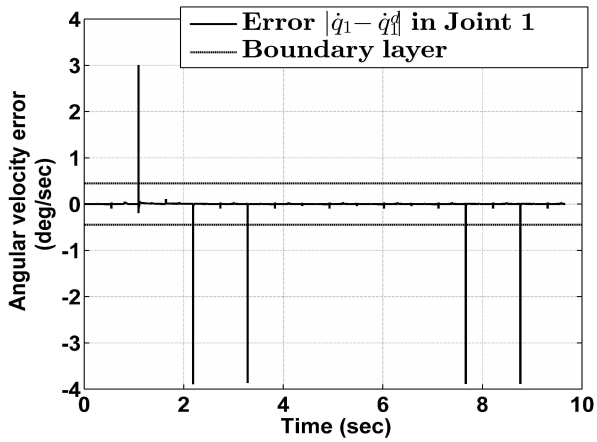

Fig. 9. Convergence time of velocity tracking error $|\dot{q}_1 - \dot{q}_1^d|$ to the boundary layer in joint 1

combination of disturbance forces $F_{x\omega}$ and $F_{y\omega}$ The effect on the biped is a destabilizing one in the continuous phase also. This can be seen in the plot of velocity of the left foot in the vertical direction as it gets affected in its flight in the next step as shown by an abnormal impulse (or stumbling) at the end of the step Fig. 4. This undesired behaviour disappears due to the robustness of the control and the biped returns to its nominal or desired gait as can be seen from figures 2, 4 and subsequent orbits in figures 5, 6 and 7.

D. Convergence Time to the Boundary Layer

A further plot is shown in Fig. 9 which shows the dynamics of the tracking error $\dot{q}_1 - \dot{q}_1^d$ in joint 1. It can be seen that the error always converges to the boundary layer $|\dot{q}_1 - \dot{q}_1^d| < \sqrt{2\varepsilon_1} = 0.48$ with $\varepsilon_1 = 0.1$ in less time than $0.5\,sec$ while also withstanding the disturbance force F_ω in the same duration of time.

VI. Conclusions And Future Work

A robust second order sliding mode controller is utilised along with its *a priori* tuning rules to achieve a finite settling time for the tracking error dynamics of a fully actuated biped robot. The results give straightforward engineering guidelines to achieve stable walking of a biped. Each joint follows its reference trajectory in finite time before the next impact occurs with the ground, thereby producing a stable periodic orbit in this non-linear system. Furthermore, the boundary layer approach makes it possible to produce joint torques without chattering at the origin. Numerical results are presented to show reference trajectory planning, robustness of the non-linear synthesis and ease of tuning. Of course if a premature disturbance such as that caused by severely uneven or slippery surface, which cannot be rejected, occurs before the control reaches this reference trajectories there is no guarantee that the biped does not fall down. Potential future directions include the study of theoretical conditions of stability of the system within the boundary layer in the presence of parasitic dynamics and attainment of similar results for under-actuated bipeds.

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