Park and Bae *Journal of Inequalities and Applications* 2011, **2011**:30
http://www.journalofinequalitiesandapplications.com/content/2011/1/30

Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Generalized ulam-hyers stability of C^*-Ternary algebra n-Homomorphisms for a functional equation

Won-Gil Park[1] and Jae-Hyeong Bae[2*]

* Correspondence: jhbae@khu.ac.kr
[2]Humanitas College, Kyung Hee University, Yongin 446-701, Republic of Korea
Full list of author information is available at the end of the article

Abstract

In this article, we investigate the Ulam-Hyers stability of C^*-ternary algebra n-homomorphisms for the functional equation:

$$f(x_{1,1}+x_{2,1},\cdots,\ x_{1,n}+x_{2,n})=\sum_{1\le i_1,\cdots,i_n\le 2} f(x_{i_1,1},\cdots,x_{i_n,n})$$

in C^*-ternary algebras.

2000 Mathematics Subject Classification: Primary 39B82; 46B03; 47Jxx.

Keywords: n-homomorphisms, C^*-ternary algebra

1. Introduction and preliminaries

Ternary algebraic operations were considered in the nineteenth century by several mathematicians, such as Cayley [1] who introduced the notion of cubic matrix, which, in turn, was generalized by Kapranov et al. [2]. The simplest example of such nontrivial ternary operation is given by the following composition rule:

$$\{a,b,c\}_{ijk}=\sum_{1\le l,m,n\le N} a_{nil}b_{ljm}c_{mkn}\quad (i,j,k=1,2,\cdots,N).$$

Ternary structures and their generalization, the so-called n-array structures, raise certain hopes in view of their applications in physics. Some significant physical applications are as follows (see [3]):

(1) The algebra of *nonions* generated by two matrices

$$\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}\quad \&\quad \begin{pmatrix}0&1&0\\0&0&\omega\\\omega^2&0&0\end{pmatrix}\quad \left(\omega=e^{\frac{2\pi i}{3}}\right)$$

was introduced by Sylvester as a ternary analog of Hamilton's quaternions [4].

(2) The quark model inspired a particular brand of ternary algebraic systems. The so-called *Nambu mechnics* is based on such structures [5].

There are also some applications, although still hypothetical, in the fractional quantum Hall effect, the non-standard statistics, supersymmetric theory, and Yang-Baxter equation [4,6].

A C^*-ternary algebra is a complex Banach space A, equipped with a ternary product $(x, y, z) \alpha [x, y, z]$ of A^3 into A, which is $\mathbb{C}$-linear in the outer variables, conjugate $\mathbb{C}$-linear in the middle variable, and associative in the sense that $[x, y, [z, w, v]] = [x, [w, z, y], v] = [[x, y, z], w, v]$, and satisfies $||[x, y, z]|| \le ||x|| \cdot ||y|| \cdot ||z||$ and $||[x, x, x]|| = ||x||^3$ (see [7,8]). Every left Hilbert C^*-module is a C^*-ternary algebra via the ternary product $[x, y, z] := \langle x, y\rangle z$.

If a C^*-ternary algebra $(A,[\cdot, \cdot, \cdot])$ has an identity, i.e., an element $e \in A$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in A$, then it is customary to verify that A, endowed with $x \circ y := [x, e, y]$ and $x^* := [e, x, e]$, is a unital C^*-algebra. Conversely, if $(A, \circ)$ is a unital C^*-algebra, then $[x, y, z] := x \circ y^* \circ z$ makes A into a C^*-ternary algebra.

Let A and B be C^*-ternary algebras. A $\mathbb{C}$-linear mapping $H : A \to B$ is called a *C^*-ternary algebra homomorphism* if

$$H([x,\ y,\ z]) = [H(x),\ H(y),\ H(z)]$$

for all $x, y, z \in A$.

Definition. Let A and B be C^*-ternary algebras. A multilinear mapping $H : A^n \to B$ over $\mathbb{C}$ is called a *C^*-ternary algebra n-homomorphism* if it satisfies

$$H([x_1, y_1,\ z_1], \cdots,\ [x_n, y_n,\ z_n]) = [H(x_1, \cdots, x_n), \cdots, H(z_1, \cdots, z_n)]$$

for all $x_1, y_1, z_1, \cdots, x_n, y_n, z_n \in A$.

In 1940, Ulam [9] gave a talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of unsolved problems. Among these was the following question concerning the stability of homomorphisms:

We are given a group G and a metric group G' with metric $\rho(\cdot, \cdot)$. *Given* $\varepsilon > 0$, *does there exist a* $\delta > 0$ *such that if* $f : G \to G'$ *satisffies* $\rho(f(xy), f(x) f(y)) < \delta$ *for all* $x, y \in G$, *then a homomorphism* $h : G \to G'$ *exists with* $\rho(f(x), h(x)) < \varepsilon$ *for all* $x \in G$ *?*

In 1941, Hyers [10] gave the first partial solution to Ulam's question for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. Then, Aoki [11] and Bourgin [12] considered the stability problem with unbounded Cauchy differences. In 1978, Rassias [13] generalized the theorem of Hyers [10] by considering the stability problem with unbounded Cauchy differences. In 1991, Gajda [14], following the same approach as that by Rassias [13], gave an affirmative solution to this question for $p > 1$. It was shown by Gajda [14] as well as by Rassias and Šemrl [15], that one cannot prove a Rassias-type theorem when $p = 1$. Găvruta [16] obtained the generalized result of Rassias's theorem which allows the Cauchy difference to be controlled by a general unbounded function. During the last two decades, a number of articles and research monographs have been published on various generalizations and applications of the generalized Hyers-Ulam stability to a number of functional equations and mappings, for example, Cauchy-Jensen mappings, k-additive mappings, invariant means, multiplicative mappings, bounded nth differences, convex functions, generalized orthogonality mappings, Euler-Lagrange functional equations, differential equations, and Navier-Stokes equations. The instability of characteristic flows of solutions of partial differential equations is related to the Ulam's stability of functional equations [17]-[24]. On the other hand, the authors [25], Park [20] and Kim [26] have contributed studies in respect of the stability problem of ternary homomorphisms and ternary derivations.

2. Solution and stability

Let X and Y be real or complex vector spaces and $n \geq 2$ an integer. For a mapping $f: X^n \to Y$, consider the functional equation:

$$\begin{aligned} &f(x_{1,1} + x_{2,1}, \cdots, x_{1,n} + x_{2,n}) \\ &= \sum_{1 \leq i_1, \cdots, i_n \leq 2} f(x_{i_1,1}, \cdots, x_{i_n,n}). \end{aligned} \tag{2.1}$$

The above functional equation is rewritten as

$$f(x_1 + y_1, \cdots, x_n + y_n) = \sum_{(z_1, \cdots, z_n) \in V} f(z_1, \cdots, z_n), \tag{2.2}$$

where

$$\begin{aligned} V &= V(x_1, y_1, \cdots, x_n, y_n) \\ &:= \{ (z_1, \cdots, z_n) | z_1 \in \{x_1, y_1\}, \cdots, z_n \in \{x_n, y_n\}\}. \end{aligned}$$

We solve the general problem in vector spaces for the n-additive mappings satisfying (2.1).

Theorem 2.1. *A mapping $f: X^n \to Y$ satisffies the equation* (2.1) *if and only if the mapping f is n-additive*.

Proof. Assume that f satisfies (2.1). Letting $x_1 = y_1 = \cdots = x_n = y_n = 0$ in (2.2), we get $f(0, \cdots, 0) = 0$. Letting $y_1 = x_2 = y_2 = \cdots = x_n = y_n = 0$ in (2.2), we have

$$f(x_1, 0, \cdots, 0) = 0$$

for all $x_1 \in X$. Similarly, we get

$$f(0, x_2, 0, \cdots, 0) = \cdots = f(0, \cdots, 0, x_n) = 0$$

for all $x_2, \cdots, x_n \in X$. Setting $y_1 = y_2 = 0$ and $x_3 = y_3 = \cdots = x_n = y_n = 0$ in (2.2), we have

$$f(x_1, x_2, 0, \cdots, 0) = 0$$

for all $x_1, x_2 \in X$. Similarly, we get $f(x_1, 0, x_3, 0, \cdots, 0) = \cdots = f(0, \cdots, 0, x_{n-1}, x_n) = 0$ for all $x_1, \cdots, x_n \in X$.

Continuing this process, we obtain that $f(x_1, \cdots, x_n) = 0$ for all $x_1, \cdots, x_n \in X$ with $x_i = 0$ for some $i = 1, \cdots, n$. Letting $y_2 = \cdots = y_n = 0$ in (2.2), we get the additivity in the first variable. Similarly, the additivities in the remaining variables hold.

The converse is obvious. □

We investigate the generalized Ulam's stability in C^*-ternary algebras for the n-additive mappings satisfying (2.1).

Lemma 2.2. *Let X and Y be complex vector spaces and let $f: X^n \to Y$ be a n-additive mapping such that*

$$f(\lambda_1 x_1, \cdots, \lambda_n x_n) = \lambda_1 \cdots \lambda_n f(x_1, \cdots, x_n)$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $x_1, \cdots, x_n \in X$, then f is n-linear over $\mathbb{C}$.

Proof. Since f is n-additive, we get $f(\frac{1}{2}x_1, \cdots, \frac{1}{2}x_n) = \frac{1}{2^n} f(x_1, \cdots, x_n)$ for all $x_1, \cdots, x_n \in X$. Now let $\sigma_1, \cdots, \sigma_n \in \mathbb{C}$ and M be an integer greater than $2(|\sigma_1| + \cdots + |\sigma_n|)$.

Since $|\frac{\sigma_1}{M}|, \cdots, |\frac{\sigma_n}{M}| < \frac{1}{2}$, there is $s_1, \cdots, s_n \in (\frac{\pi}{3}, \frac{\pi}{2}]$ such that $|\frac{\sigma_1}{M}| = \cos s_1 = \frac{e^{is_1}+e^{-is_1}}{2}, \cdots, |\frac{\sigma_n}{M}| = \cos s_n = \frac{e^{is_n}+e^{-is_n}}{2}$. Now

$$\frac{\sigma_1}{M} = \left|\frac{\sigma_1}{M}\right| \lambda_1, \cdots, \frac{\sigma_n}{M} = \left|\frac{\sigma_n}{M}\right| \lambda_n$$

for some $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$. Thus, we have

$$\begin{aligned} &f(\sigma_1 x_1, \cdots, \sigma_n x_n) = f\left(M\frac{\sigma_1}{M}x, \cdots, M\frac{\sigma_n}{M}z\right) \\ &= M^n f\left(\frac{\sigma_1}{M}x, \cdots, \frac{\sigma_n}{M}z\right) = M^n f\left(\left|\frac{\sigma_1}{M}\right| \lambda_1 x_1, \cdots, \left|\frac{\sigma_n}{M}\right| \lambda_n x_n\right) \\ &= M^n f\left(\frac{e^{is_1} + e^{-is_1}}{2}\lambda_1 x_1, \cdots, \frac{e^{is_n} + e^{-is_n}}{2}\lambda_n x_n\right) \\ &= \frac{1}{2^n}M^n f(e^{is_1}\lambda_1 x_1 + e^{-is_1}\lambda_1 x_1, \cdots, e^{is_n}\lambda_n x_n + e^{-is_n}\lambda_n x_n) \\ &= \frac{1}{2^n}M^n \sum_{r_1,\cdots,r_n=1,-1} e^{r_1 is_1} \cdots e^{r_n is_n}\lambda_1 \cdots \lambda_n f(x_1, \cdots, x_n) \\ &= \sigma_1 \cdots \sigma_n f(x_1, \cdots, x_n) \end{aligned}$$

for all $x_1, \cdots, x_n \in X$. Hence, the mapping $f : X^n \to Y$ is n-linear over $\mathbb{C}$. □

Using the above lemma, one can obtain the following result.

Theorem 2.3. *Let X and Y be complex vector spaces and let* $f : X^n \to Y$ *be a mapping such that*

$$\begin{aligned} &f(\lambda_1 x_{1,1} + \lambda_1 x_{2,1}, \cdots, \lambda_n x_{1,n} + \lambda_n x_{2,n}) \\ &= \lambda_1 \cdots \lambda_n \sum_{1\le i_1,\cdots,i_n\le 2} f(x_{i_1,1}, \cdots, x_{i_n,n}) \end{aligned} \tag{2.3}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$ *and all* $x_{1,1}, x_{2,1}, \cdots, x_{1,n}, x_{2,n} \in X$. *Then, f is n-linear over* $\mathbb{C}$.

Proof. Letting $\lambda_1 = \cdots = \lambda_n = 1$, by Theorem 1.1, f is n-additive. Letting $x_{2,1} = \cdots = x_{2,n} = 0$ in (2.3), we get $f(\lambda_1 x_1, \cdots, \lambda_n x_n) = \lambda_1 \cdots \lambda_n f(x_1, \cdots, x_n)$ for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \cdots, x_n \in X$. Hence, by Lemma 2.2, the mapping f is n-linear over $\mathbb{C}$. □

From now on, assume that A is a C^*-ternary algebra *with* norm $||\cdot||_A$ and that B is a C^*-ternary algebra *with* norm $||\cdot||_B$.

For a given mapping $f : A^n \to B$, we define

$$\begin{aligned} &D_{\lambda_1,\cdots,\lambda_n} f(x_{1,1}, x_{2,1}, \cdots, x_{1,n}, x_{2,n}) \\ &:= f(\lambda_1 x_{1,1} + \lambda_1 x_{2,1}, \cdots, \lambda_n x_{1,n} + \lambda_n x_{2,n}) \\ &-\lambda_1 \cdots \lambda_n \sum_{1\le i_1,\cdots,i_n\le 2} f(x_{i_1,1}, \cdots, x_{i_n,n}) \end{aligned}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$ and all $x_{1,1}, x_{2,1}, \cdots, x_{1,n}, x_{2,n} \in A$.

We prove the generalized Ulam-Hyers stability of homomorphisms in C^*-ternary algebras for the functional equation

$$D_{\lambda_1,\cdots,\lambda_n} f(x_{1,1}, x_{2,1}, \cdots, x_{1,n}, x_{2,n}) = 0.$$

Theorem 2.4. *Let* $p_1, \cdots, p_n \in (0, \infty)$ *with* $\sum_{i=1}^{n} p_i < n$*and* $\theta \in (0, \infty)$, *and let* $f : A^n \to B$ *be a mapping such that*

$$\begin{aligned}&||D_{\lambda_1,\cdots,\lambda_n} f(x_1, y_1, \cdots, x_n, y_n)||_B \\ &\le \theta \prod_{i=1}^{n} \max\{\| x_i\|_A, \| y_i\|_A\}^{p_i}\end{aligned} \tag{2.4}$$

and

$$\begin{aligned}&||f([x_1, y_1, z_1], \cdots, [x_n, y_n, z_n]) \\ &- [f(x_1, \cdots, x_n), f(y_1, \cdots, y_n), f(z_1, \cdots, z_n)]||_B \\ &\le \theta(||x_i||_A^{p_1} \cdots ||x_n||_A^{p_n} + ||y_1||_A^{p_1} \cdots ||y_n||_A^{p_n} + ||z_1||_A^{p_1} \cdots ||z_n||_A^{p_n})\end{aligned} \tag{2.5}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$*and all* $x_1, y_1, z_1, \cdots, x_n, y_n, z_n \in A$. *Then, there exists a unique* C^**-ternary algebra n-homomorphism* $H : A^n \to B$ *such that*

$$\begin{aligned}&\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n)\|_B \\ &\le \frac{\theta}{2^n - 2^{\sum_{i=1}^{n} p_i}} \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned} \tag{2.6}$$

for all $x_1, \cdots, x_n \in A$.

Proof. Letting $\lambda_1 = \cdots = \lambda_n = 1$, $y_1 = x_1, \cdots, y_n = x_n$ in (2.4), we gain

$$\begin{aligned}&\| f(2x_1, \cdots, 2x_n) - 2^n f(x_1, \cdots, x_n)\|_B \\ &\le \theta \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned} \tag{2.7}$$

for all $x_1, \cdots, x_n \in A$. Thus, we have

$$\begin{aligned}&\left\| \frac{1}{2^{n(j+1)}} f(2^{j+1}x_1, \cdots, 2^{j+1}x_n) - \frac{1}{2^{nj}} f(2^j x_1, \cdots, 2^j x_n) \right\|_B \\ &\le 2^{(\Sigma_{i=1}^{n} p_i - n)j - n} \theta \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned}$$

for all $x_1, \cdots, x_n \in A$ and all $j \in \mathbb{N}$. For given integer l, $m(0 \le l < m)$, we obtain that

$$\begin{aligned}&\left\| \frac{1}{2^{nm}} f(2^m x_1, \cdots, 2^m x_n) - \frac{1}{2^{nl}} f(2^l x_1, \cdots, 2^l x_n) \right\|_B \\ &\le \sum_{j=l}^{m-1} 2^{(\sum_{i=1}^{n} p_i - n)j - n} \theta \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned} \tag{2.8}$$

for all $x_1, \cdots, x_n \in A$. Since $\sum_{i=1}^{n} p_i < n$, the sequence

$$\left\{ \frac{1}{2^{nj}} f(2^j x_1, \cdots, 2^j x_n) \right\}$$

is a Cauchy sequence for all $x_1, \cdots, x_n \in A$. Since B is complete, the sequence $\left\{ \frac{1}{2^{nj}} f(2^j x_1, \cdots, 2^j x_n) \right\}$ converges for all $x_1, \cdots, x_n \in A$. Define $H : A^n \to B$ by

$$H(x_1, \cdots, x_n) := \lim_{j \to \infty} \frac{1}{2^{nj}} f(2^j x_1, \cdots, 2^j x_n)$$

for all $x_1, \cdots, x_n \in A$. Letting $l = 0$ and taking $m \to \infty$ in (2.8), one can obtain the inequality (2.6). By (2.4), we see that

$$\begin{aligned}&\left\| \frac{1}{2^{ns}} f\left(2^s(x_1+y_1),\ \cdots,\ 2^s(x_n+y_n)\right) \right.\\ &\left. - \sum_{(z_1,\cdots,z_n)\in V} \frac{1}{2^{ns}} f(2^s z_1,\ \cdots,\ 2^s z_n) \right\|_B \\ &\le 2^{(\sum_{i=1}^n p_i - n)s} \theta \prod_{i=1}^n \max\{\| x_i\|_A,\ \| y_i\|_A\}^{p_i}\end{aligned}$$

for all $x_1, y_1, \cdots, x_n, y_n \in A$ and all s. Since $\sum_{i=1}^n p_i < n$, letting $s \to \infty$ in the above inequality, H satisfies (2.1). By Theorem 2.1, H is n-additive.

Letting $y_1 = x_1, \cdots, y_n = x_n$ in (2.4), we gain

$$\begin{aligned}&\| f(2\lambda_1 x_1, \cdots,\ 2\lambda_n x_n) - 2^n \lambda_1 \cdots \lambda_n f(x_1, \cdots,\ x_n)\|_B \\ &\le \theta \ \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned}$$

for all $\lambda_1, \cdots,\ \lambda_n \in \mathbb{T}^1$ and all $x_1, \cdots, x_n \in A$. Thus we have

$$\begin{aligned}&\| f(2^m \lambda_1 x_1, \cdots, 2^m \lambda_n x_n) - 2^n \lambda_1 \cdots \lambda_n f(2^{m-1} x_1, \cdots, 2^{m-1} x_n)\|_B \\ &\le 2^{(m-1)\sum_{i=1}^n p_i} \theta \ \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned}$$

for all $\lambda_1, \cdots,\ \lambda_n \in \mathbb{T}^1$, all $x_1, \cdots, x_n \in A$ and all $m \in \mathbb{N}$. Hence, we get

$$\begin{aligned}&\| f(2^m x_1, \cdots, 2^m x_n) - 2^n f(2^{m-1} x_1, \cdots, 2^{m-1} x_n)\|_B \\ &\le 2^{(m-1)\sum_{i=1}^n p_i} \theta \ \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned}$$

for all $x_1, \cdots, x_n \in A$ and all $m \in \mathbb{N}$, and one can show that

$$\begin{aligned}&\| \lambda_1 \cdots \lambda_n f(2^m x_1, \cdots,\ 2^m x_n) \\ &-2^n \lambda_1 \cdots \lambda_n f(2^{m-1} x_1, \cdots,\ 2^{m-1} x_n)\|_B \\ &= |\lambda_1 \cdots \lambda_n| \cdot \| f(2^m x_1, \cdots,\ 2^m x_n) - 2^n f(2^{m-1} x_1, \cdots,\ 2^{m-1} x_n)\|_B \\ &\le 2^{(m-1)\sum_{i=1}^n p_i} \theta \ \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned}$$

for all $\lambda_1, \cdots,\ \lambda_n \in \mathbb{T}^1$, all $x_1, \cdots, x_n \in A$ and all $m \in \mathbb{N}$. Hence,

$$\begin{aligned}&\| f(2^m \lambda_1 x_1, \cdots,\ 2^m \lambda_n x_n) - \lambda_1 \cdots \lambda_n f(2^m x_1, \cdots,\ 2^m x_n)\|_B \\ &\le \| f(2^m \lambda_1 x_1, \cdots,\ 2^m \lambda_n x_n) - 2^n \lambda_1 \cdots \lambda_n f(2^{m-1} x_1, \cdots,\ 2^{m-1} x_n)\|_B \\ &+ \| 2^n \lambda_1 \cdots \lambda_n f(2^{m-1} x_1, \cdots,\ 2^{m-1} x_n) \\ &-\lambda_1 \cdots \lambda_n f(2^m x_1, \cdots,\ 2^m x_n)\|_B \\ &\le 2^{1+(m-1)\sum_{i=1}^n p_i} \theta \ \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}\end{aligned}$$

for all $\lambda_1, \cdots,\ \lambda_n \in \mathbb{T}^1$, all $x_1, \cdots, x_n \in A$ and all $m \in \mathbb{N}$. Since $\sum_{i=1}^n p_i < n$, we have

$$\frac{1}{2^{nm}} \ \| f(2^m \lambda_1 x_1, \cdots,\ 2^m \lambda_n x_n) - \lambda_1 \cdots \lambda_n f(2^m x_1, \cdots,\ 2^m x_n)\|_B \to 0$$

as $m \to \infty$ for all $\lambda_1, \cdots,\ \lambda_n \in \mathbb{T}^1$ and all $x_1, \cdots, x_n \in A$. Hence

$$\begin{aligned}&H(\lambda_1 x_1, \cdots,\ \lambda_n x_n) = \lim_{m\to\infty} \frac{f(2^m \lambda_1 x_1, \cdots, 2^m \lambda_n x_n)}{2^{nm}} \\ &= \lim_{m\to\infty} \lambda_1 \cdots \lambda_n \frac{f(2^m x_1, \cdots, 2^m x_n)}{2^{nm}} \\ &= \lambda_1 \cdots \lambda_n H(x_1, \cdots,\ x_n)\end{aligned}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \cdots, x_n \in A$. From Lemma 2.2, the mapping $H : A^n \to B$ is n-linear over $\mathbb{C}$. It follows from (2.5) that

$$\begin{aligned}
&\| H([x_1, y_1, z_1], \cdots, [x_n, y_n, z_n]) \\
&-[H(x_1, \cdots, x_n), H(y_1, \cdots, y_n), H(z_1, \cdots, z_n)]\|_B \\
&= \lim_{m\to\infty} \frac{1}{2^{3nm}} \| f(2^{3m}[x_1, y_1, z_1], \cdots, 2^{3m}[x_n, y_n, z_n]) \\
&-[f(2^m x_1, \cdots, 2^m x_n), f(2^m y_1, \cdots, 2^m y_n), f(2^m z_1, \cdots, 2^m z_n)]\|_B \\
&\le \lim_{m\to\infty} \frac{\theta 2^{m\Sigma_{i=1}^n p_i}}{2^{3nm}} (\| x_1 \|_A^{p1} \cdots \| x_n \|_A^{p_n} \\
&+ \| y_1 \|_A^{p1} \cdots \| y_n \|_A^{p_n} + \| z_1 \|_A^{p1} \cdots \| z_n \|_A^{p_n}) \\
&= 0
\end{aligned}$$

for all $x_1, y_1, z_1, \cdots, x_n, y_n, z_n \in A$. So

$$\begin{aligned}
&H([x_1, y_1, z_1], \cdots, [x_n, y_n, z_n]) \\
&= [H(x_1, \cdots, x_n), H(y_1, \cdots, y_n), H(z_1, \cdots, z_n)]
\end{aligned}$$

for all $x_1, y_1, z_1, \cdots, x_n, y_n, z_n \in A$.

Now, let $T : A^n \to B$ be another n-additive mapping satisfying (2.6). Then, we have

$$\begin{aligned}
&\| H(x_1, \cdots, x_n) - T(x_1, \cdots, x_n)\|_B \\
&= \frac{1}{2^{nm}} \| H(2^m x_1, \cdots, 2^m x_n) - T(2^m x_1, \cdots, 2^m x_n)\|_B \\
&\le \frac{1}{2^{nm}} \| H(2^m x_1, \cdots, 2^m x_n) - f(2^m x_1, \cdots, 2^m x_n)\|_B \\
&+ \frac{1}{2^{nm}} \| f(2^m x_1, \cdots, 2^m x_n) - T(2^m x_1, \cdots, 2^m x_n)\|_B \\
&\le \frac{2\left(\sum_{i=1}^n p_i - n\right) m + 1}{2^n - 2^{\sum_{i=1}^n p_i}} \theta \| x_1 \|_A^{p1} \cdots \| x_n \|_A^{p_n},
\end{aligned}$$

which tends to zero as $m \to \infty$ for all $x_1, \cdots, x_n \in A$. Hence, we can conclude that $H(x_1, \cdots, x_n) = T(x_1, \cdots, x_n)$ for all $x_1, \cdots, x_n \in A$. This proves the uniqueness of H.

Thus, the mapping $H : A \to B$ is a unique C^*-ternary algebra n-homomorphism satisfying (2.6). □

Letting $p_1 = \cdots = p_n = 0$ and $\theta = \varepsilon$ in Theorem 2.4, we obtain the Ulam-Hyers stability for the n-additive functional equation (2.1).

Corollary 2.5. *Let* $\varepsilon \in (0, \infty)$ *and let* $f : A^n \to B$ *be a mapping satisfying*

$$\| D_{\lambda_1,\cdots,\lambda_n} f(x_1, y_1 \cdots, x_n, y_n)\|_B \le \varepsilon$$

and

$$\begin{aligned}
&\| f([x_1, y_1, z_1], \cdots, [x_n, y_n, z_n]) \\
&-[f(x_1, \cdots, x_n), f(y_1, \cdots, y_n), f(z_1, \cdots, z_n)]\|_B \le 3\varepsilon
\end{aligned}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$ *and all* $x_1, y_1, z_1 \cdots, x_n, y_n, z_n \in A$. *Then, there exists a unique* C^*-*ternary algebra n-homomorphism* $H : A^n \to B$ *such that*

$$\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n)\|_B \le \frac{\varepsilon}{2^n - 1}$$

for all $x_1, \cdots, x_n \in A$.

Example 2.6. *We present the following counterexample modiffied by the well-known counterexample of Z. Gajda* [14]*for the functional equation* (2.1). *Set* θ > 0 *and let* $\mu := \frac{\theta}{6n2^n}$.

Deffine a function $f : \mathbb{R}^n \to \mathbb{R}$ *by*

$$f(x_1, \cdots, x_n) := \sum_{n=0}^{\infty} \frac{1}{2^n} \phi_\mu(2^n x_1, x_2, \cdots, x_n)$$

for all $x_1, \cdots, x_n \in \mathbb{R}$, *where* $\varphi_\mu : \mathbb{R}^n \to \mathbb{R}$ *is the function given by*

$$\phi_\mu(x_1, \cdots, x_n) := \begin{cases} \mu & \text{if } x_1 \cdots x_n \geq 1 \\ \mu x_1 \cdots x_n & \text{if } -1 < x_1 \cdots x_n < 1 \\ -\mu & \text{if } x_1 \cdots x_n \leq -1 \end{cases}$$

for all $x_1, \cdots, x_n \in \mathbb{R}$. *Deffine another function* $g : \mathbb{R} \to \mathbb{R}$ *by*

$$g(x) := f(x, 1, \cdots, 1) = \sum_{n=0}^{\infty} \frac{1}{2^n} \phi_\mu(2^n x, 1, \cdots, 1)$$

for all $x \in \mathbb{R}$.

For all $m \in \mathbb{N}$ *and all* $x_1, \cdots, x_{2^m} \in \mathbb{R}$, *we assert that*

$$\left| g\left(\sum_{i=1}^{2^m} x_i\right) - \sum_{i=1}^{2^m} g(x_i) \right| \leq \frac{m\theta}{n2^n} \sum_{i=1}^{2^m} |x_i|. \tag{2.9}$$

It was proved in [14]*that*

$$|g(x+y) - g(x) - g(y)| \leq \frac{\theta}{n2^n}(|x| + |y|)$$

for all x, y ∈ ℝ, *that is,* (2.9) *holds for* m = 1. *For a ffixed* $k \in \mathbb{N}$, *assume that* (2.9) *holds for* m = k. *Then, we have*

$$\begin{aligned}
&\left| g\left(\sum_{i=1}^{2^{k+1}} x_i\right) - \sum_{i=1}^{2^{k+1}} g(x_i) \right| \\
&\leq \left| g\left(\sum_{i=1}^{2^{k+1}} x_i\right) - g\left(\sum_{i=1}^{2^k} x_i\right) - g\left(\sum_{i=2^k+1}^{2^{k+1}} x_i\right) \right| \\
&\quad + \left| g\left(\sum_{i=1}^{2^k} x_i\right) - g\left(\sum_{i=2^k+1}^{2^{k+1}} x_i\right) - \sum_{i=1}^{2^{k+1}} g(x_i) \right| \\
&\leq \frac{\theta}{n2^n}\left(\left|\sum_{i=1}^{2^k} x_i\right| + \left|\sum_{i=2^k+1}^{2^{k+1}} x_i\right| \right) + g\left|\left(\sum_{i=1}^{2^k} x_i\right) - \sum_{i=1}^{2^k} g(x_i)\right| \\
&\quad + g\left|\left(\sum_{i=2^k+1}^{2^{k+1}} x_i\right) - \sum_{i=2^k+1}^{2^{k+1}} g(x_i)\right| \\
&\leq \frac{\theta}{n2^n}\left(\left|\sum_{i=1}^{2^k} x_i\right| + \left|\sum_{i=2^k+1}^{2^{k+1}} x_i\right| \right) + \frac{k\theta}{n2^n}\left(\sum_{i=1}^{2^k} |x_i| + \sum_{i=2^k+1}^{2^{k+1}} |x_i| \right) \\
&= \frac{(k+1)\theta}{n2^n} \sum_{i=1}^{2^{k+1}} |x_i|
\end{aligned}$$

for all $x_1, \cdots, x_{2^{k+1}} \in \mathbb{R}$, *that is,* (2.9) *holds for* $m = k + 1$. *Hence,* (2.9) *holds for all* $m \in \mathbb{R}$.

Note that

$$f(x_1, \cdots, x_n) = \begin{cases} \mu & \text{if } 2^n x_1 \cdots x_n \geq 1 \\ \mu 2^n x_1 \cdots x_n & \text{if } -1 < 2^n x_1 \cdots x_n < 1 \\ -\mu & \text{if } 2^n x_1 \cdots x_n \leq -1 \end{cases}$$
$$= f(x_1 \cdots x_n, 1, \cdots, 1) = g(x_1 \cdots x_n) \tag{2.10}$$

for all $x_1, \cdots, x_n \in \mathbb{R}$. *By the inequality* (2.9) *and the above equality, we see that*

$$\left| f(x_1 + y_1, \cdots, x_n + y_n) - \sum_{(z_1,\cdots,z_n)\in V} f(z_1, \cdots, z_n) \right|$$
$$= \left| g\left((x_1 + y_1) \cdots (x_n + y_n)\right) - \sum_{(z_1,\cdots,z_n)\in V} g(z_1 \cdots z_n) \right|$$
$$= \left| g\left(\sum_{(z_1,\cdots,z_n)\in V} z_1 \cdots z_n \right) - \sum_{(z_1,\cdots,z_n)\in V} g(z_1 \cdots z_n) \right|$$
$$\leq \frac{\theta}{2^n} \sum_{(z_1,\cdots,z_n)\in V} |z_1 \cdots z_n| \leq \frac{\theta}{2^n} (|x_1| + |y_1|) \cdots (|x_n| + |y_n|)$$
$$= \theta \prod_{i=1}^{n} \max\{|x_i|, |y_i|\}^1$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathbb{R}$. *However, we observe from* [14] *that*

$$\frac{g(x^n)}{x^n} \to \infty \quad as \quad x \to \infty.$$

and so

$$\frac{f(x, \cdots, x)}{x^n} \to \infty \quad as \quad x \to \infty.$$

Thus,

$$\frac{|f(x, \cdots, x) - h(x, \cdots, x)|}{x^n} (x \neq 0) \text{ is unbounded,}$$

where $h: \mathbb{R}^n \to \mathbb{R}$ *is the function given by*

$$h(x_1, \cdots, x_n) := \lim_{m\to\infty} \frac{1}{2^{mn}} f(2^m x_1, \cdots, 2^m x_n)$$

for all $x_1, \cdots, x_n \in \mathbb{R}$. *Hence, the function* f *is a counterexample for the singular case* $\sum_{i=1}^n p_i = n$*of Theorem* 2.4.

Theorem 2.7. *Let* $p \in (0,n)$ *and* $\theta \in (0, \infty)$, *and let* $f : A^n \to B$ *be a mapping such that*

$$\| D_{\lambda_1,\cdots,\lambda_n} f(x_1, y_1 \cdots, x_n, y_n)\|_B$$
$$\leq \theta \sum_{i=1}^{n} (||x_i||_A^p + \| y_i \|_A^p) \tag{2.11}$$

and

$$\begin{aligned}&\| f([x_1, y_1, z_1], \cdots, [x_n, y_n, z_n]) \\ &-[f(x_1, \cdots, x_n), f(y_1, \cdots, y_n), f(z_1, \cdots, z_n)]\|_B \\ \le\ &\theta \sum_{i=1}^{n}(||x_i||_A^p + ||y_i||_A^p + ||z_i||_A^p)\end{aligned} \tag{2.12}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$*and all* $x_1, y_1, z_1 \cdots, x_n, y_n, z_n \in A$. *Then, there exists a unique* C*-*ternary algebra n-homomorphism* $H : A^n \to B$ *such that*

$$\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n)\|_B \le \frac{2\theta}{2^n - 2^p} \sum_{i=1}^{n} \| x_i \|_A^p$$

for all $x_1, \cdots, x_n \in A$.

Proof. The proof is similar to the proof of Theorem 2.4. □

Example 2.8. *We present the following counterexample modiffied by the well-known counterexample of Z. Gajda* [14]*for the functional equation* (2.1). *Set* $\theta > 0$ *and let* $\mu := \frac{\theta}{6n2^n}$

Let $f : \mathbb{R}^n \to \mathbb{R}$ *and* $g : \mathbb{R} \to \mathbb{R}$ *be the same as in Example* 2.6. *By the same argument as in Example* 2.6, *for all* $m \in \mathbb{N}$ *and all* $x_1, \cdots, x_{2^m} \in \mathbb{R}$, *one can obtain that* g *satisffies the inequality*:

$$\left| g\left(\sum_{i=1}^{2^m} x_i\right) - \sum_{i=1}^{2^m} g(x_i)\right| \le \frac{m\theta}{n2^n} \sum_{i=1}^{2^m} |x_i|.$$

By the equality (2.10) *and the above inequality, we see that*

$$\begin{aligned}&\left| f(x_1 + y_1, \cdots, x_n + y_n) - \sum_{(z_1,\cdots,z_n)\in V} f(z_1, \cdots, z_n)\right| \\ &= \left| g\left((x_1 + y_1)\cdots(x_n + y_n)\right) - \sum_{(z_1,\cdots,z_n)\in V} g(z_1 \cdots z_n)\right| \\ &= \left| g\left(\sum_{(z_1,\cdots,z_n)\in V} z_1 \cdots z_n\right) - \sum_{(z_1,\cdots,z_n)\in V} g(z_1 \cdots z_n)\right| \\ &\le \frac{\theta}{2^n} \sum_{(z_1,\cdots,z_n)\in V} |z_1 \cdots z_n|\end{aligned}$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathbb{R}$. *For each* $x_1, y_1, \cdots, x_n, y_n \in \mathbb{R}$, *let* $\mathrm{M}(x_1, y_1, \cdots, x_n, y_n) := \max\{|x_1|, |y_1|, \cdots, |x_n|, |y_n|\}$. *We have*

$$\frac{\theta}{2^n} \sum_{(z_1,\cdots,z_n)\in V} |z_1 \cdots z_n| \le \frac{\theta}{2^n} \sum_{(z_1,\cdots,z_n)\in V} M^n = \theta M^n \le \theta \sum_{i=1}^{n} \left(|x_i|^n + |y_i|^n\right)$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathbb{R}$. *Thus we have*

$$\left| f(x_1 + y_1, \cdots, x_n + y_n) - \sum_{(z_1,\cdots,z_n)\in V} f(z_1, \cdots, z_n)\right| \le \theta \sum_{i=1}^{n} \left(|x_i|^n + |y_i|^n\right)$$

for all $x_1, y_1, \cdots, x_n, y_n \in \mathbb{R}$. *By the same reason as for Example* 2.6, *the function f is a counterexample for the singular case p = n of Theorem 2.7.*

Theorem 2.9. *Let* $p_1, \cdots, p_n \in (0, \infty)$ *with* $\sum_{i=1}^{n} p_i < n$, $s \in (0, n)$ *and* $\theta, \eta \in (0, \infty)$, *and let* $f: A^n \to B$ *be a mapping such that*

$$\begin{aligned} &\| D_{\lambda_1,\cdots,\lambda_n} f(x_{1,1}, x_{2,1}, \cdots, x_{1,n}, x_{2,n}) \|_B \\ \le\ &\theta \max\{\| x_{1,1}\|_A, \| x_{2,1}\|_A\}^{p_1} \cdots \max\{\| x_{1,n}\|_A, \| x_{2,n}\|_A\}^{p_n} \\ &+\eta \sum_{i=1}^{n} (\| x_{1,i} \|_A^s + \| x_{2,i} \|_A^s) \end{aligned} \tag{2.13}$$

and

$$\begin{aligned} &\| f([x_{1,1}, x_{2,1}, x_{3,1}], \cdots, [x_{1,n}, x_{2,n}, x_{3,n}]) \\ &-[f(x_{1,1}, \cdots, x_{1,n}), f(x_{2,1}, \cdots, x_{2,n}), f(x_{3,1}, \cdots, x_{3,n})] \|_B \\ \le\ &\theta \sum_{i=1}^{3} \| x_{i,1} \|_A^{p1} \cdots \| x_{i,n} \|_A^{p_n} + \eta \sum_{i=1}^{n} (\| x_{1,i} \|_A^s + \| x_{2,i} \|_A^s + \| x_{3,i} \|_A^s) \end{aligned} \tag{2.14}$$

for all $\lambda_1, \cdots, \lambda_n \in \mathbb{T}^1$*and all* $x_{1,1}, x_{2,1}, x_{3,1}, \cdots, x_{1,n}, x_{2,n}, x_{3,n} \in A$. *Then, there exists a unique C*-ternary algebra n-homomorphism* $H: A^n \to B$ *such that*

$$\begin{aligned} &\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n) \|_B \\ \le\ &\frac{\theta}{2^n - 2^{\sum_{i=1}^{n} p^i}} \| x_1 \|_A^{p1} \cdots \| x_n \|_A^{p_n} + \frac{2\eta}{2^n - 2^s} \sum_{i=1}^{n} \| x_i \|_A^s \end{aligned}$$

for all $x_1, \cdots, x_n \in A$.

Proof. The proof is similar to the proof of Theorem 2.4. □

Theorem 2.10. *Let* $p_1, \cdots, p_n \in (0, \infty)$ *with* $\sum_{i=1}^{n} p_i > 3n$*and* $\theta \in (0, \infty)$, *and let* $f: A^n \to B$ *be a mapping satisfying* (2.4) (2.5). *Then, there exists a unique C*-ternary algebra n-homomorphism* $H: A^n \to B$ *such that*

$$\begin{aligned} &\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n) \|_B \\ \le\ &\frac{\theta}{2^{\sum_{i=1}^{n} p^i} - 2^n} \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n} \end{aligned} \tag{2.15}$$

for all $x_1, \cdots, x_n \in A$.

Proof. It follows from (2.7) that

$$\left\| f(x_1, \cdots, x_n) - 2^n f\left(\frac{x_1}{2}, \cdots, \frac{x_n}{2}\right) \right\|_B \le \frac{\theta}{2^{\sum_{i=1}^{n} p^i}} \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}$$

for all $x_1, \cdots, x_n \in A$. Hence,

$$\begin{aligned} &\left\| 2^{nl} f\left(\frac{x_1}{2^l}, \cdots, \frac{x_n}{2^l}\right) - 2^{nm} f\left(\frac{x_1}{2^m}, \cdots, \frac{x_n}{2^m}\right) \right\|_B \\ \le\ &\sum_{j=l}^{m-1} \left\| 2^{nj} f\left(\frac{x_1}{2^j}, \cdots, \frac{x_n}{2^j}\right) - 2^{n(j+1)} f\left(\frac{x_1}{2^{j+1}}, \cdots, \frac{x_n}{2^{j+1}}\right) \right\|_B \\ \le\ &\frac{\theta}{2^{\sum_{i=1}^{n} p^i}} \sum_{j=l}^{m-1} \frac{2^{nj}}{2^{j \sum_{i=1}^{n} p^i}} \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n} \end{aligned} \tag{2.16}$$

for all nonnegative integers m and l *with* $m > l$ and all $x_1, \cdots, x_n \in A$. It follows from (2.16) that the sequence $\left\{2^{nm} f\left(\frac{x_1}{2^m}, \cdots, \frac{x_n}{2^m}\right)\right\}$ is a Cauchy sequence for all $x_1, \cdots, x_n \in A$. Since B is complete, the sequence $\left\{2^{nm} f\left(\frac{x_1}{2^m}, \cdots, \frac{x_n}{2^m}\right)\right\}$ converges. Hence, one can define the mapping $H : A^n \to B$ by $H(x_1, \cdots, x_n) := \lim_{m\to\infty} 2^{nm} f\left(\frac{x_1}{2^m}, \cdots, \frac{x_n}{2^m}\right)$ for all $x_1, \cdots, x_n \in A$. Moreover, letting $l = 0$ and passing the limit $m \to \infty$ in (2.16), we get (2.15).

The remainder of the proof is similar to the proof of Theorem 2.4. □

Theorem 2.11. *Let* $p \in (3n, \infty)$ *and* $\theta \in (0, \infty)$, *and let* f: $A^n \to B$ *be a mapping satisfying* (2.11) (2.7), *and* $f(0, \cdots, 0) = 0$. *Then, there exists a unique C*-ternary algebra n-homomorphism* $H : A^n \to B$ *such that*

$$\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n)\|_B \le \frac{2\theta}{2^p - 2^n} \sum_{i=1}^{n} \| x_i \|_A^p$$

for all $x_1, \cdots, x_n \in A$.

Proof. The proof is similar to that of Theorem 2.10. □

Theorem 2.12. *Let* $p_1, \cdots, p_n \in (0, \infty)$ *with* $\sum_{i=1}^n p_i > 3n$, $s \in (n, \infty)$ *and* $\theta, \eta \in (0, \infty)$, *and let* f: $A^n \to B$ *be a mapping such that* (2.13), (2.14), *and* $f(0, \cdots, 0) = 0$. *Then, there exists a unique C*-ternary algebra n-homomorphism* H: $A^n \to B$ *such that*

$$\begin{aligned} &\| f(x_1, \cdots, x_n) - H(x_1, \cdots, x_n)\|_B \\ &\le \frac{\theta}{2^{\sum_{i=1}^n p_i} - 2^n} \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n} + \frac{2\eta}{2^s - 2^n} \sum_{i=1}^{n} \| x_i \|_A^s \end{aligned}$$

for all $x_1, \cdots, x_n \in A$.

Proof. The proof is similar to that of Theorem 2.10.

Acknowledgements
The authors would like to thank the referee for a number of valuable suggestions regarding a previous version of this paper.

Author details
[1]Department of Mathematics Education, Mokwon University, Daejeon 302-729, Republic of Korea [2]Humanitas College, Kyung Hee University, Yongin 446-701, Republic of Korea

Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 15 December 2010 Accepted: 9 August 2011 Published: 9 August 2011

References

1. Cayley, A: On the 34 concomitants of the ternary cubic. Am J Math. **4**, 1–15 (1881). doi:10.2307/2369145
2. Kapranov, M, Gelfand, IM, Zelevinskii, A: Discriminants, Resultants and Multidimensional Determinants. Birkhäuser, Berlin (1994)
3. Kerner, R: The cubic chessboard: Geometry and physics. Class. Quantum Gravity. **14**, A203–A225 (1997). doi:10.1088/0264-9381/14/1A/017
4. Abramov, V, Kerner, R, Le Roy, B: Hypersymmetry: a is $\mathbb{Z}_3$-graded generalization of supersymmetry. J Math Phys. **38**, 1650–1669 (1997). doi:10.1063/1.531821
5. Daletskii, YL, Takhtajan, L: Leibniz and Lie algebra structures for Nambu algebras. Lett Math Phys. **39**, 127–141 (1997). doi:10.1023/A:1007316732705
6. Vainerman, L, Kerner, R: On special classes of *n*-algebras. J Math Phys. **37**, 2553–2565 (1996). doi:10.1063/1.531526

7. Amyari, M, Moslehian, MS: Approximately ternary semigroup homomor-phisms. Lett Math Phys. **77**, 1–9 (2006). doi:10.1007/s11005-005-0042-6
8. Zettl, H: A characterization of ternary rings of operators. Adv Math. **48**, 117–143 (1983). doi:10.1016/0001-8708(83)90083-X
9. Ulam, SM: A Collection of the Mathematical Problems. Interscience, New York (1960)
10. Hyers, DH: On the stability of the linear functional equation. Proc Natl Acad Sci USA **27**, 222–224 (1941). doi:10.1073/pnas.27.4.222
11. Aoki, T: On the stability of the linear transformation in Banach spaces. J Math Soc Jpn. **2**, 64 (1950). doi:10.2969/jmsj/00210064
12. Bourgin, DG: Classes of transformations and bordering transformations. Bull Am Math Soc. **57**, 223 (1951). doi:10.1090/S0002-9904-1951-09511-7
13. Rassias, ThM: On the stability of the linear mapping in Banach spaces. Proc Am Math Soc. **72**, 297–300 (1978). doi:10.1090/S0002-9939-1978-0507327-1
14. Gajda, Z: On stability of additive mappings. Int J Math Math Sci. **14**, 431–434 (1991). doi:10.1155/S016117129100056X
15. Rassias, ThM, Šemrl, P: On the behaviour of mappings which do not satisfy Hyers-Ulam stability. Proc Am Math Soc. **114**, 989–993 (1992). doi:10.1090/S0002-9939-1992-1059634-1
16. Găvruta, P: A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings. J Math Anal Appl. **184**, 431–436 (1994). doi:10.1006/jmaa.1994.1211
17. Bae, J-H, Park, W-G: On stability of a functional equation with *n* variables. Nonlinear Anal TMA. **64**, 856–868 (2006). doi:10.1016/j.na.2005.06.028
18. Bae, J-H, Park, W-G: On a cubic equation and a Jensen-quadratic equation. Abst Appl Anal. **2007** Article ID 45179 (2007)
19. Bae, J-H, Park, W-G: A functional equation having monomials as solutions. Appl Math Comput. **216**, 87–94 (2010). doi:10.1016/j.amc.2010.01.006
20. Park, C: Isomorphisms between *C**-ternary algebras. J Math Phys. **47**, 103512 (206)
21. Park, W-G, Bae, J-H: On a Cauchy-Jensen functional equation and its stability. J Math Anal Appl. **323**, 634–643 (2006). doi:10.1016/j.jmaa.2005.09.028
22. Park, W-G, Bae, J-H: A functional equation originating from elliptic curves. Abst Appl Anal. **2008** Article ID 135237 (2008)
23. Park, W-G, Bae, J-H: Approximate behavior of bi-quadratic mappings in quasinormed spaces. J Inequal Appl. **2010** Article ID 472721 (2010)
24. Prástaro, A: Geometry of PDEs and Mechanics. World Scientific, River Edge, NJ (1996)
25. Bae, J-H, Park, W-G: Approximate bi-homomorphisms and bi-derivations in *C**-ternary algebras. Bull Korean Math Soc. **47**, 195–209 (2010). doi:10.4134/BKMS.2010.47.1.195
26. Rassias, JM, Kim, H-M: Approximate homomorphisms and derivations between *C**-ternary algebras. J Math Phys. **49**, 063507 (2008). doi:10.1063/1.2942415

doi:10.1186/1029-242X-2011-30
Cite this article as: Park and Bae: Generalized ulam-hyers stability of *C**-Ternary algebra *n*-Homomorphisms for a functional equation. *Journal of Inequalities and Applications* 2011 **2011**:30.