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Research Article

Optimization of Aeroengine Shop Visit Decisions Based on Remaining Useful Life and Stochastic Repair Time

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Considering the wide application of condition-based maintenance in aeroengine maintenance practice, it becomes possible for aeroengines to carry out their preventive maintenance in just-in-time (JIT) manner by reasonably planning their shop visits (SVs). In this study, an approach is proposed to make aeroengine SV decisions following the concept of JIT. Firstly, a state space model (SSM) for aeroengine based on exhaust gas temperature margin is developed to predict the remaining useful life (RUL) of aeroengine. Secondly, the effect of SV decisions on risk and service level (SL) is analyzed, and an optimization of the aeroengine SV decisions based on RUL and stochastic repair time is performed to carry out JIT manner with the requirement of safety and SL. Finally, a case study considering two CFM-56 aeroengines is presented to demonstrate the proposed approach. The results show that predictive accuracy of RUL with SSM is higher than with linear regression, and the process of SV decisions is simple and feasible for airlines to improve the inventory management level of their aeroengines.

1. Introduction

Both purchase cost and maintenance cost of civil aeroengines are very expensive. For example, CFM56-5 is about 4.5 million dollars, and its shop visit (SV) cost is up to one million dollars per visit [1, 2]. According to the definition given by the world airlines technical operations glossary (WATOG), an aeroengine removal is classified as an SV whenever the subsequent engine maintenance performed prior to reinstallation entails separation of pairs of major mating flanges or removal of a disk, hub, or spool. SV is due to two main reasons: (a) ultralimit of exhaust gas temperature margin (EGTM) or (b) repair/replacement of life limited part (LLP). EGTM is the most important measure of aeroengine health, and once EGTM has dropped to the threshold value (usually 0), which means that there is a potential for failure, an SV is required to restore the aeroengine. A spare aeroengine is needed when an SV occurs, so the airlines try to use as few spare aeroengines as possible by decreasing utilization of installed aeroengines in order to avoid that several SVs occur simultaneously [3]. In fact, one spare aeroengine is sufficient if there is no overlap of SVs, which is similar to the concept of just-in-time (JIT) if a repaired aeroengine back from shop is regarded as a spare aeroengine. With the development of condition-based maintenance (CBM) in aeroengine maintenance practice, it is now possible to implement the concept of JIT through reasonably planning the aeroengine SVs by considering an essential condition that the previous aeroengine will return from the shop before the next aeroengine needs a spare aeroengine. Therefore, the repair time (i.e., SV duration) and remaining useful life (RUL) are key factors for an SV decision. Because repair time is always stochastic due to differences in repair range, repair level, transport conditions, and so on, the aeroengine SV decision will be optimized based on both RUL and stochastic repair time in this study.

There has been a lot of research on RUL prediction [4–11], but very few studies considering RUL prediction of aeroengines have appeared in the literatures. A hybrid PSO-SVM-based model for the RUL prediction of aeroengines was described [12]. An approach is presented based on the so-called shapelet extraction to estimate the RUL of turbofan engines [13]. EGTM is taken as a measure of aeroengines health to predict the RUL of aeroengines [9, 14–16]. There are many papers dealing with JIT inventory management, but JIT for an aeroengine SV has not been considered. In order to reasonably plan aeroengine SVs and smooth the SV rate, the performance ranking of aeroengines has been researched [9, 17–20]. On the basis of the performance ranking, an aeroengine maintenance cost was developed for a scheme of aeroengine fleet with different quantity of spare aeroengines in a finite period of time [21]. Further, SV prediction of aeroengines was focused on and the take-off exhaust gas temperature was considered for SVs decisions [22]. An approach was proposed for aeroengine preventive repair based on an LLP life time distribution [23]. Recently, a graphical technique was presented [24], which considers a rule for decision-making based on both condition-based reliability function and a stochastic/fixed lead time. However, no papers have been published considering the aeroengine SV decisions based on the concept of JIT. Therefore, in this study, according to the concept of JIT, the RUL of aeroengine would not only be predicted but also be applied to make SV decisions to avoid the overlap of SVs and reduce the number of spare aeroengines; furthermore, stochastic repair time of aeroengine is also considered as a major factor during making SV decisions.

Obviously, SV ahead of schedule leads to life utilization loss of an aeroengine, but, on the other hand, postponing SV may result in failure and expensive downtime. Therefore, the effect of SV decision on risk and service level (SL) should be analyzed, and then aeroengine SV decisions should be optimized based on RUL and stochastic repair time, in order to minimize the number of spare aeroengines and to maintain required safety and SL. The rest of this paper is organized as follows: a state space model (SSM) for RUL prediction is developed firstly; then the optimization of the aeroengine SV decisions is presented. Finally, a case study using real data is developed, and the conclusions from the work presented in this study and suggestions for future research are given.

2. RUL Prediction of Aeroengine

EGTM is a kind of time series data which measures performance deterioration of aeroengine, and the SSM approach offers a very general and powerful framework to provide best estimates and forecast performance trends by modeling such data. An SSM based on EGTM data will be developed in this study and used for the RUL prediction.

2.1. State Space Model. EGTM is defined as

$$\mathrm{EGTM} = \mathrm{EGT}_{\mathrm{RED}} - \mathrm{EGT}_{E}, \tag{1}$$

where $\mathrm{EGT}_{\mathrm{RED}}$ is the red line temperature of aeroengine provided by manufacturers and EGT_E is the exhaust gas temperature of aeroengine with full power take-off in standard conditions (sea-level pressure and turnover temperature).

Given the observations up to time T flight cycles (FC), the observed and actual EGTM value at time t ($t = 1, 2, \ldots, T$) are, respectively, represented by y_t and x_t. Because there is the observation noise, y_t can be expressed by the following equation:

$$y_t = x_t + \varepsilon_t. \tag{2}$$

It is supposed that observation noise has Gaussian distribution, that is, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$, where variance σ_ε^2 needs to be estimated. Throughout the whole life cycle of an aeroengine, EGTM has characteristic of piecewise linear degradation, that is, at the beginning of putting into operation, the degradation of EGTM is fast because of initial wear, and, after a period of time, the degradation of EGTM gradually slows down. So the degradation rate of EGTM changes over time, and a piecewise linear degradation model is suitable for describing the degradation of EGTM [25]. Therefore, a linear growth model is adopted as state equations to describe the aeroengine degradation path [26]:

$$\begin{aligned} x_t &= x_{t-1} + \beta_{t-1} + \upsilon_x \quad \upsilon_x \sim N\left(0, \sigma_x^2\right), \\ \beta_t &= \beta_{t-1} + \upsilon_\beta \quad \upsilon_\beta \sim N\left(0, \sigma_\beta^2\right), \end{aligned} \tag{3}$$

where β_t is the rate of change for EGTM degradation; υ_x and υ_β are the process noises with Gaussian distributions, where variances σ_x^2 and σ_β^2 need to be estimated. Using (2)-(3), a Gaussian linear SSM for describing aeroengine performance degradation can be obtained as follows:

$$\begin{aligned} y_t &= \mathbf{F}_t \mathbf{H}_t + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right) \\ \mathbf{H}_t &= \mathbf{G}_t \mathbf{H}_{t-1} + \mathbf{w}_t \quad \mathbf{w}_t \sim N\left(\mathbf{0}, \mathbf{\Sigma}\right), \end{aligned} \tag{4}$$

where $\mathbf{H}_t = \begin{bmatrix} x_t \\ \beta_t \end{bmatrix}$, $\mathbf{\Sigma} = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_\beta^2 \end{bmatrix}$, $\mathbf{G}_t = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $\mathbf{F}_t = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

2.2. Bayesian State Estimation and Prediction. It is assumed that initial information $(\mathbf{H}_0 \mid D_0)$ is subject to the distribution $N(\mathbf{m}_0, \mathbf{C}_0)$, where D_0 is the initial prior information, $\mathbf{m}_0 \in \mathbb{R}^2$ and $\mathbf{C}_0 \in \mathbb{R}^{2\times 2}$. At time t, the available information set is $D_t = \{y_t, D_{t-1}\}$. Given that the posterior probability distribution of $(\mathbf{H}_{t-1} \mid D_{t-1})$ is $N(\mathbf{m}_{t-1}, \mathbf{C}_{t-1})$ at time $t-1$, so that the prior probability distribution of $(\mathbf{H}_t \mid D_{t-1})$ can be expressed by [26]

$$\left(\mathbf{H}_t \mid D_{t-1}\right) \sim N\left(\mathbf{a}_t, \mathbf{R}_t\right), \tag{5}$$

where $\mathbf{a}_t = \mathbf{G}_t \mathbf{m}_{t-1}$, $\mathbf{R}_t = \mathbf{G}_t \mathbf{C}_{t-1} \mathbf{G}_t' + \mathbf{\Sigma}$.

A one-step predictive observed EGTM Y_t is given by

$$\left(Y_t \mid D_{t-1}\right) \sim N\left(f_t, Q_t\right), \tag{6}$$

where $f_t = \mathbf{F}_t' \mathbf{a}_t$, $Q_t = \mathbf{F}_t \mathbf{R}_{t-1} \mathbf{F}_t' + \sigma_\varepsilon^2$.

Hence, the posterior probability distribution of $(\mathbf{H}_t \mid D_t)$ at time t is given by

$$(\mathbf{H}_t \mid D_t) \sim N(\mathbf{m}_t, \mathbf{C}_t), \tag{7}$$

where

$$\begin{aligned} \mathbf{m}_t &= \mathbf{a}_t + \mathbf{A}_t e_t, \\ \mathbf{C}_t &= \mathbf{R}_t - \mathbf{A}_t Q_t \mathbf{A}_t', \\ \mathbf{A}_t &= \mathbf{R}_t \mathbf{F}_t Q_t^{-1}, \\ e_t &= y_t - f_t. \end{aligned} \tag{8}$$

Based on the above method, up to operation time T, $(\mathbf{H}_T \mid D_T) \sim N(\mathbf{m}_T, \mathbf{C}_T)$ can be obtained. It is assumed that $\mathbf{m}_T = \begin{bmatrix} \mu_x \\ \mu_\beta \end{bmatrix}$, $\mathbf{C}_T = \begin{bmatrix} \sum_{xx} & \sum_{\beta x} \\ \sum_{\beta x} & \sum_{\beta\beta} \end{bmatrix}$, and the k-step predictive $\mathbf{H}_{T+k}$ is given using the following equations:

$$(\mathbf{H}_{T+k} \mid D_T) = \left(\begin{array}{c} x_{T+k} \\ \beta_{T+k} \end{array} \middle| D_T \right) \sim N(\mathbf{a}_T(k), \mathbf{R}_T(k)). \tag{9}$$

$\mathbf{a}_T(k)$ and $\mathbf{R}_T(k)$ can be recursively calculated by

$$\begin{aligned} \mathbf{a}_T(k) &= \mathbf{G}_{T+k} \mathbf{a}_T(k-1) \\ \mathbf{R}_T(k) &= \mathbf{G}_{T+k} \mathbf{R}_T(k-1) \mathbf{G}_{T+k}' + \mathbf{\Sigma}. \end{aligned} \tag{10}$$

Based on (5) and (9), k-step predictive EGTM value $(x_{T+k} \mid D_T) \sim N(\mu_T(k), \sigma_T^2(k))$ can be obtained, where $\mu_T(k) = \mu_x + k \cdot \mu_\beta$, $\sigma_T^2(k) = \sum_{xx} + k^2 \sum_{\beta\beta} + 2k \sum_{\beta x} + k\sigma_x + ((k-1)k(2k-1)/6)\sigma_\beta$.

Hence the probability density function (pdf) of x_{T+k} can be expressed as

$$\pi(x_{T+k}) = \frac{1}{\sqrt{2\pi}\sigma_T(k)} \exp\left(-\frac{(x_{T+k} - \mu_T(k))^2}{2\sigma_T^2(k)} \right). \tag{11}$$

2.3. RUL Distribution Function. When EGTM drops to the threshold value x_F, there is a potential for failure, and preventive maintenance needs to be performed for the aeroengine. So at time T, the cumulative distribution function (CDF) and pdf of the k-step predictive RUL can be expressed, respectively, by

$$F_T(k) = \int_{-\infty}^{x_F} \pi(x_{T+k}) \, \mathrm{d}x_{T+k} = \phi\left(\frac{x_F - \mu_T(k)}{\sigma_T(k)} \right) = \phi\left(\frac{x_F - \mu_x - k \cdot \mu_\beta}{\sqrt{\sum_{xx} + k^2 \sum_{\beta\beta} + 2k \sum_{\beta x} + k\sigma_x + ((k-1)k(2k-1)/6)\sigma_\beta}} \right) \tag{12}$$

$$f_T(k) = \frac{\partial(F_T(k))}{\partial(k)} = \frac{1}{\sqrt{8\pi}} \exp\left(-\frac{(x_F - \mu_x - k\mu_\beta)^2}{2\sigma_T^2(k)} \right) \frac{(\sigma_T^2(k))' (\mu_x + k\mu_\beta - x_F) - 2\mu_\beta \sigma_T^2(k)}{\sigma_T^3(k)}, \tag{13}$$

where $(\sigma_T^2(k))' = 2k \sum_{\beta\beta} + 2 \sum_{\beta x} + \sigma_x + ((6k^2 - 6k + 1)/6)\sigma_\beta$.

3. Optimization of the Aeroengine SV Decisions

3.1. SV Decision Policy. As mentioned above, SV is due to two main reasons. According to (12), the time of ultralimit of EGTM obeys a certain distribution; however, the time of repair/replacement of LLP is a fixed value, but it can be described by a normal distribution with small variance. Thus, RUL is a probability distribution function no matter whether the aeroengine removal is due to ultralimit of EGTM or repair/replacement of LLP.

JIT inventory management requires that the aeroengines should undergo SV in turn, so the different aeroengines have the different times of putting into operation. It is assumed that the #1 and #2 aeroengines have operated for T_1 and T_2 FC, respectively, and the corresponding predictive RUL of #1 and #2 aeroengine is P and Z, respectively. Based on the historical EGTM data, using (5)–(13), the pdf of P and Z, respectively, represented by $f_{1,T_1}(p)$ and $f_{2,T_2}(z)$, can be obtained and shown in the Figure 1(a), where subscripts "1" and "2" indicate #1 and #2 aeroengines, respectively (similarly later in the text).

In order to reduce the number of spare aeroengines with the requirement of safety and SL, an SV decision policy is proposed and illustrated with the curves of the two obtained predictive pdfs in Figure 1(b). It is assumed that the SV time of #1 aeroengine will be after k FC, for example, $k = 215$ FC, so the predictive probability of the risk of #1 aeroengine is equal to the blue shaded area in Figure 1(b). Let U be the repair time random variable and $s(u)$ be its pdf. If $u = 400$ FC, the return time of #1 aeroengine from shop will be after 615 FC, so the corresponding predictive probability of the shortage of #2 aeroengine is equal to the green-shaded area in Figure 1(b). If the shortage occurs, the airline must spend much money on renting an aeroengine urgently.

Based on the analysis above, it can be found that variable k has effect on the life utilization and risk of #1 aeroengine as well as the SL of #2 aeroengine. For example, taking k as a smaller value would lead to a higher life utilization loss of #1 aeroengine; on the other hand, a bigger value of k would result in the higher risk of #1 aeroengine and lower SL of #2 aeroengine.

3.2. Optimization Formulation. Based on (9), T_1 FC after putting into operation, k-step predictive EGTM value of #1 aeroengine can be estimated as $x_{1,T_1+k} \sim N(\mu_{1,T_1}(k), \sigma_{1,T_1}^2(k))$.

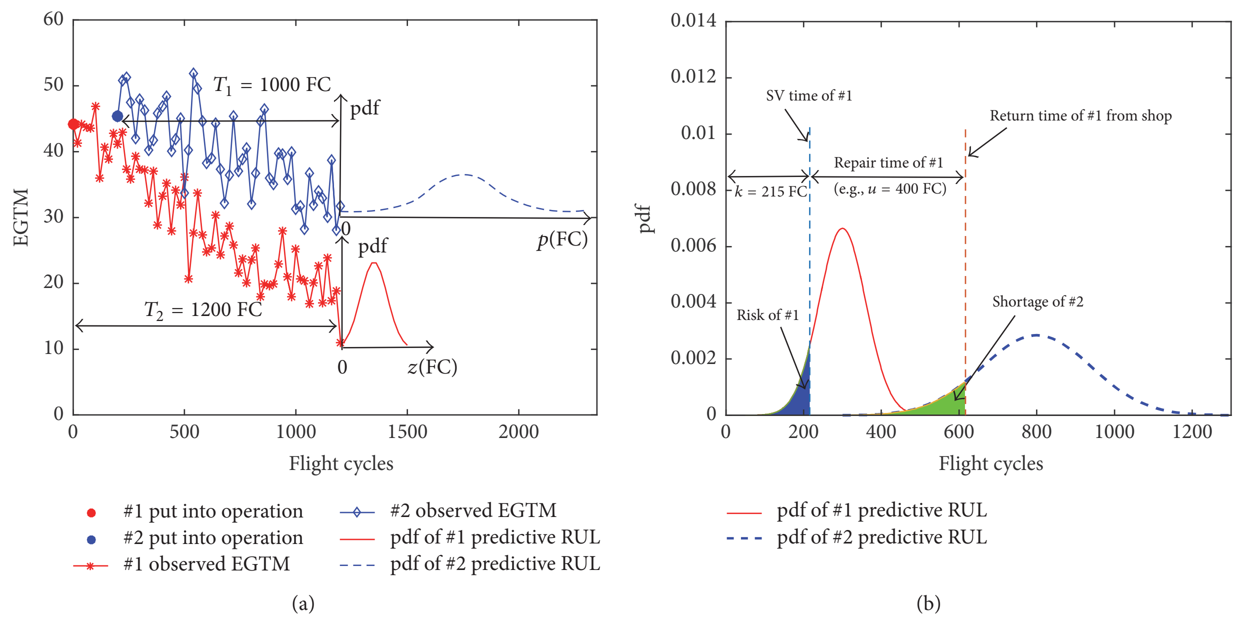

Figure 1: (a) The pdf of the predictive RUL. (b) SV decision policy.

It is assumed that F_{REQ} is the accepted probability of risk of aeroengine; therefore, based on (12), the following inequality needs to be satisfied:

$$\phi\left(\frac{x_F-\mu_{1,T_1}(k)}{\sigma_{1,T_1}(k)}\right)\leq F_{\mathrm{REQ}}. \tag{14}$$

Similarly, T_2 FC after putting into operation, z-step predictive EGTM value of #2 aeroengine can be estimated as $x_{2,T_2+z}\sim N(\mu_{2,T_2}(z),\sigma^2_{2,T_2}(z))$, and, according to (13), the corresponding pdf of #2 aeroengine, represented by $f_{2,T_2}(z)$, can be obtained.

Therefore, if #2 aeroengine has operated for T_2 FC and #1 aeroengine will be taken to the shop after k FC, the predictive probability of the SL of #2 aeroengine, represented by $\mathrm{SL}_{T_2}(k)$, is [24]

$$\begin{aligned}\mathrm{SL}_{T_2}(k)&=P(k+U\leq Z)\\&=1-\int_{-\infty}^{\infty}\left[\int_{-\infty}^{z-k}s(u)\,\mathrm{d}u\right]f_{2,T_2}(z)\,\mathrm{d}z.\end{aligned} \tag{15}$$

The following inequality needs to be satisfied:

$$\mathrm{SL}_{T_2}(k)\geq P_{\mathrm{SUP}}, \tag{16}$$

where P_{SUP} is the accepted probability of SL.

Therefore, the SV time k can be optimized by the following inequalities:

$$\begin{gathered}\phi\left(\frac{x_F-\mu_{1,T_1}(k)}{\sigma_{1,T_1}(k)}\right)\leq F_{\mathrm{REQ}}\\1-\int_{-\infty}^{\infty}\left[\int_{-\infty}^{z-k}s(u)\,\mathrm{d}u\right]f_{2,T_2}(z)\,\mathrm{d}z\geq P_{\mathrm{SUP}}.\end{gathered} \tag{17}$$

4. Case Study

The initial EGTM data of two CFM56-5B aeroengines in one of the Chinese airlines are shown in Figures 2 and 3. The threshold value x_F is 0 and the two aeroengines were removed, respectively, after 2080 and 2253 FC. #1 aeroengine was put into operation 200 FC earlier than #2 aeroengine; that is, $T_1-T_2=200$ FC.

4.1. RUL Prediction. Set observation time data as $\mathbf{T}$ and observed EGMT data as $\mathbf{Y}$, and the correlation coefficient of $\mathbf{T}$ and $\mathbf{Y}$ is given by [27]

$$\rho_{\mathbf{TY}'}=\frac{E(\mathbf{TY}')-\mu_{\mathbf{T}}\mu_{\mathbf{Y}}}{\sqrt{E(\mathbf{T}^2)-\mu_{\mathbf{T}}^2}\sqrt{E(\mathbf{Y}^2)-\mu_{\mathbf{Y}}^2}}, \tag{18}$$

where $\mu_{\mathbf{T}}$ and $\mu_{\mathbf{Y}}$ are mean of $\mathbf{T}$ and $\mathbf{Y}$, respectively, and $E(\cdot)$ is the expectation function.

Substituting the initial EGTM data into (18), the correlation coefficients can be obtained as

$$\begin{aligned}\rho_{1,\mathbf{TY}'}&=-0.9365\\\rho_{2,\mathbf{TY}'}&=-0.9476.\end{aligned} \tag{19}$$

Since the correlation coefficients are very close to −1, the performance degradation process can approximately be regarded as linear degradation, as shown in Figures 2 and 3. Therefore, a linear growth model can be adopted as state equations to describe the aeroengine degradation path as (3).

The software package dlm is an R package for Bayesian analysis of dynamic linear models [26, 28], which is applied in this study to estimate the aeroengine degradation state and

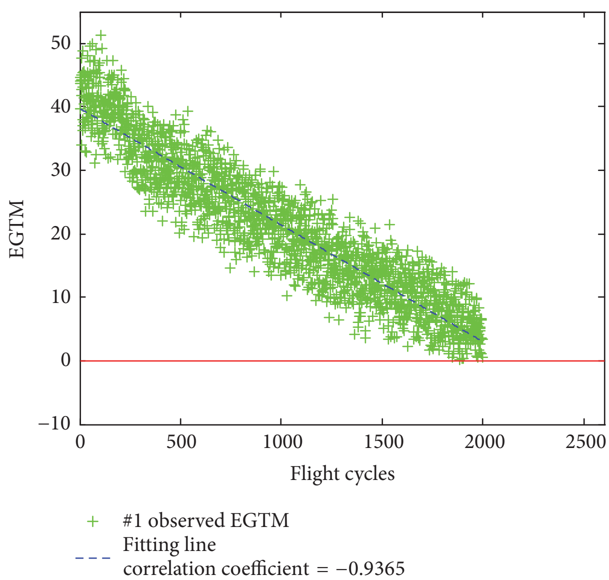

Figure 2: EGTM data of #1 CFM56-5B aeroengine.

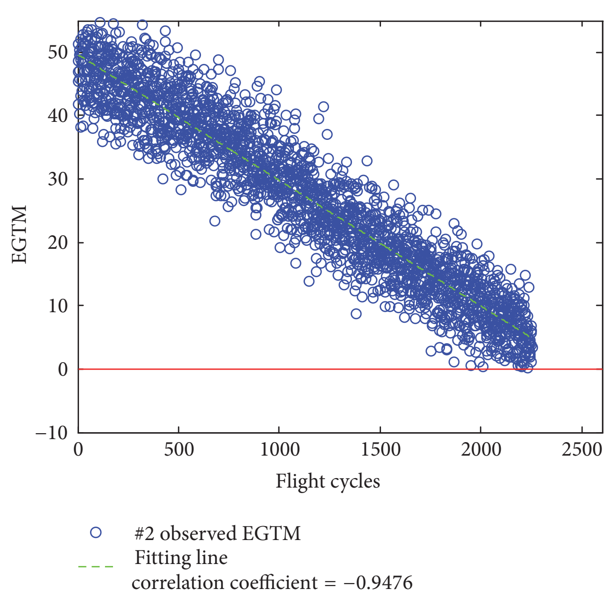

Figure 3: EGTM data of #2 CFM56-5B aeroengine.

the unknown model parameters. For example, if T_1 is equal to 1200 FC, that is, $T_2 = 1000$ FC, with function dlmMLE, variances σ_ε^2, σ_x^2, and σ_β^2 of #1 aeroengine can be estimated as follows:

$$\sigma_{1,\varepsilon}^2 = 15.19,$$
$$\sigma_{1,x}^2 = 0.0119, \quad (20)$$
$$\sigma_{1,\beta}^2 = 1.019 \times 10^{-15}.$$

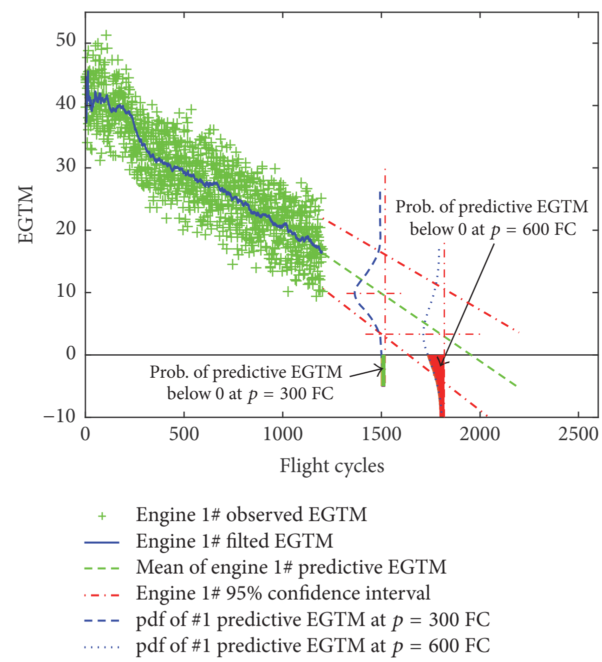

Figure 4: EGTM prediction ($T_1 = 1200$ FC) with SSM for #1.

And variances σ_ε^2, σ_x^2, and σ_β^2 of #2 aeroengine can be estimated as follows:

$$\sigma_{2,\varepsilon}^2 = 20.88,$$
$$\sigma_{2,x}^2 = 0.0002, \quad (21)$$
$$\sigma_{2,\beta}^2 = 2.22 \times 10^{-9}.$$

Then, with function dlmFilter, the following can be obtained:

$$\left(x_{1,1200} \mid D_{1200}\right) \sim N\left(16.37, 0.811^2\right),$$
$$\left(\beta_{1,1200} \mid D_{1200}\right) \sim N\left(-0.022, 0.003^2\right)$$
$$\left(x_{2,1000} \mid D_{1000}\right) \sim N\left(31.46, 0.566^2\right), \quad (22)$$
$$\left(\beta_{2,1000} \mid D_{1000}\right) \sim N\left(-0.0173, 0.002^2\right).$$

Finally, with the function dlmForecast, the predictive distribution of EGTM value can be obtained. For example, the pdfs of #1 aeroengine predictive EGTM at $p = 300$ and 600 FC are described as blue-dashed and blue-dotted curve, respectively, in Figure 4, and their corresponding probabilities of predictive EGTM below 0 at $p = 300$ and 600 FC are equal to the green-shaded area and the red-shaded area, respectively, in Figure 4. The corresponding predictive parameters and expected RUL are shown in Tables 1 and 2, respectively. Meanwhile, in order to verify the correctness of expected RUL, the linear regression as a reference method is also applied and the corresponding results are shown in Figure 5 and Table 2.

Table 1: Predictive parameters of #1 aeroengine.

Predicted parameters	k-step prediction when $T_1 = 1200$ (FC)		k-step prediction when $T_1 = 1500$ (FC)	
	$p = 300$	$p = 600$	$p = 200$	$p = 500$
$\mu_{1,T_1}(p)$	9.87	3.36	6.30	2.43
$\sigma_{1,T_1}(p)$	2.93	4.62	2.07	2.85
$F_{1,T_1}(p)$	3.78×10^{-4}	0.2335	0.0012	0.1969

Table 2: Expected RUL of #1 and #2 aeroengine.

Aeroengine	T_1/T_2 (FC)	Actual RUL (FC)	Excepted RUL with SSM (FC)	SSM prediction error	Excepted RUL with linear regression (FC)	Linear regression prediction error
#1	1200	880	754	14.31%	699	20.57%
#1	1500	580	625	7.76%	528	8.97%
#2	1000	1253	1734	38.4%	1868	49.1%
#2	1300	953	1045	9.65%	728	23.61%

Table 3: Predictive parameters of #2 aeroengine.

Predicted parameters	z-step prediction when $T_2 = 1000$ (FC)		z-step prediction when $T_2 = 1300$ (FC)	
	$z = 800$	$z = 1500$	$z = 500$	$z = 900$
$\mu_{2,T_2}(z)$	16.85	4.22	12.27	3.27
$\sigma_{2,T_2}(z)$	3.16	6.32	3.51	8.45
$F_{2,T_2}(z)$	4.85×10^{-8}	0.2522	2.36×10^{-4}	0.35

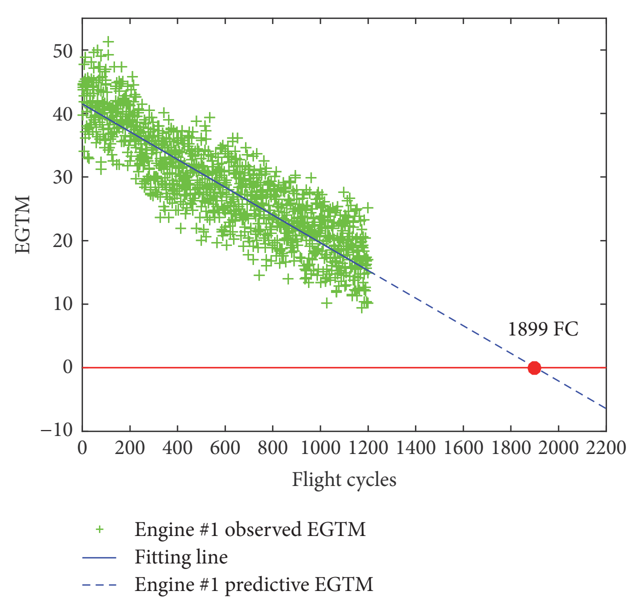

Figure 5: EGTM prediction ($T_1 = 1200$ FC) with linear regression for #1.

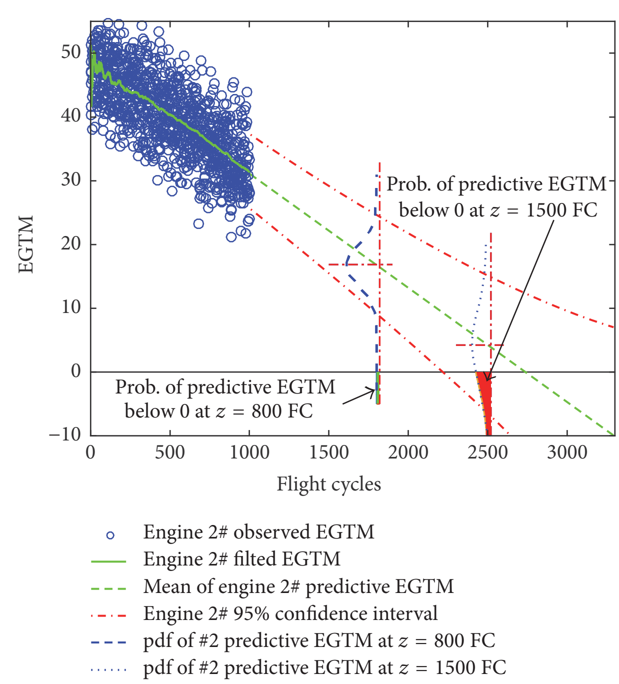

Figure 6: EGTM prediction ($T_2 = 1000$ FC) with SSM for #2.

The pdfs of #2 aeroengine predictive EGTM at $z = 800$ and 1500 FC are described as blue-dashed and blue-dotted curve, respectively, in Figures 6 and 7, and the corresponding predicted parameters and expected RUL are shown in Tables 3 and 2, respectively.

Figures 8 and 9 show the pdf of #1 aeroengine predictive EGTM at time $p = 200$ and 500 FC, and Figures 10 and 11 show the pdf of #2 aeroengine predictive EGTM at time $z = 500$ and 900 FC. The corresponding predicted parameters and expected RUL are shown in Tables 1–3.

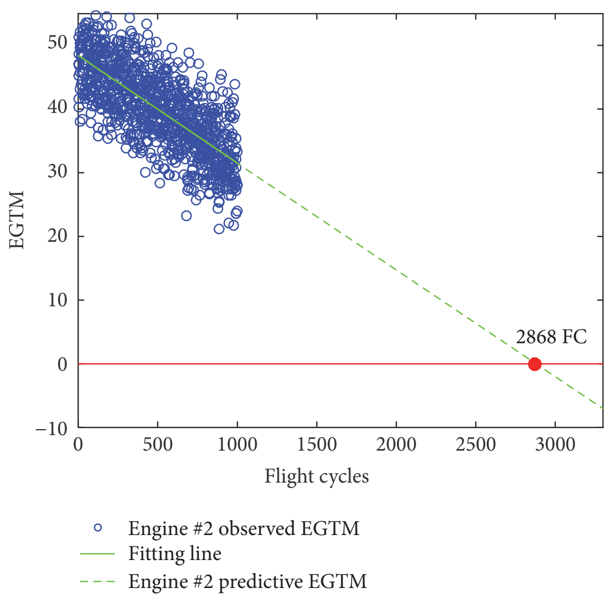

Figure 7: EGTM prediction (T_2 = 1000 FC) with linear regression for #2.

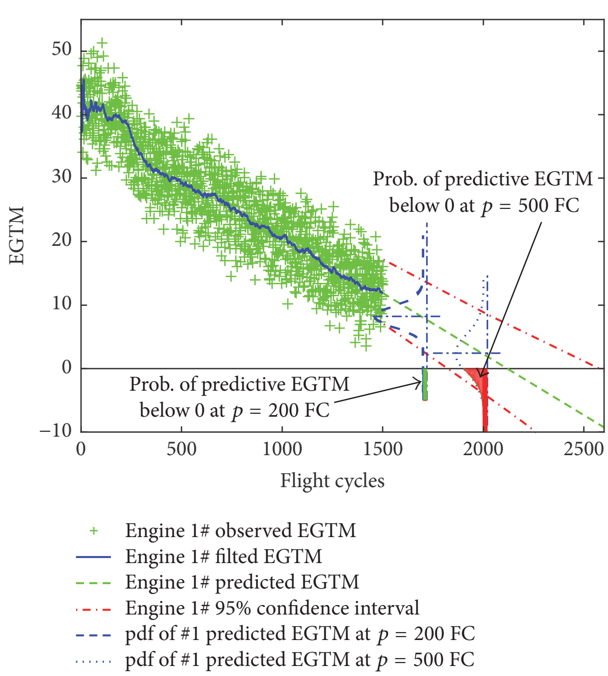

Figure 8: EGTM prediction (T_1 = 1500 FC) with SSM for #1.

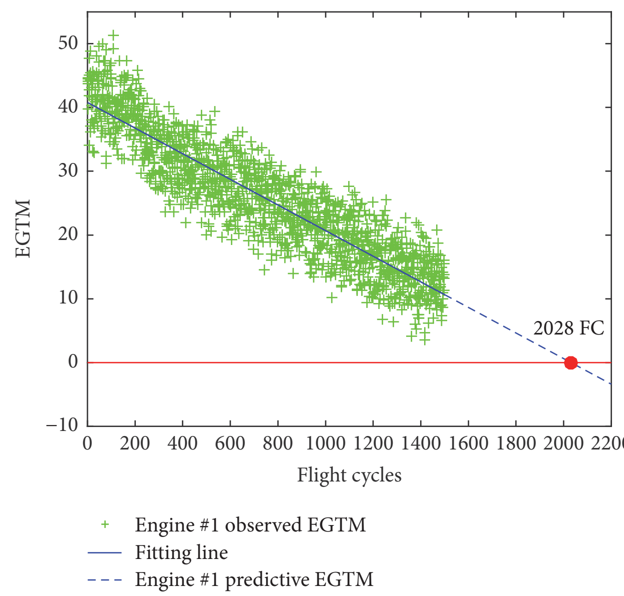

Figure 9: EGTM prediction (T_1 = 1500 FC) with linear regression for #1.

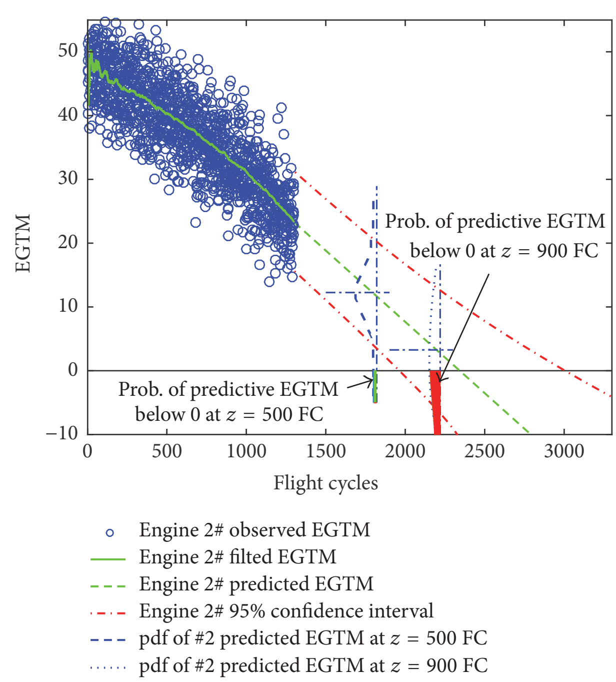

Figure 10: EGTM prediction (T_2 = 1300 FC) with SSM for #2.

From Table 2, it can be found that SSM can predict RUL more accurately than linear regression. Moreover, it is clear that the predictive accuracy of #1 aeroengine RUL at time T_1 = 1500 FC is 6.55% higher than at time T_1 = 1200 FC, and the predictive accuracy of #2 aeroengine RUL at time T_2 = 1300 FC is 28.75% higher than at time T_2 = 1000 FC. So with increasing historical EGTM data, the predictive accuracy of the RUL is getting higher, which is illustrated with #2 aeroengine in Figure 12.

4.2. Stochastic Repair Time Determination. As mentioned Section 1, the repair time scatters since it is affected by many factors. Based on the historical SV data, with the general distribution fitting method and Kolmogorov-Smirnov test, the results show that the normal distribution has a good fit with P value of 0.91198 for the repair time, as shown in Figure 13. And the pdf of the repair time is

$$s(u) = \frac{1}{\sqrt{2\pi}\sigma_u} e^{-(1/2)((u-\mu_u)/\sigma_u)^2}, \quad \mu_u = 329, \ \sigma_u = 69.14. \tag{23}$$

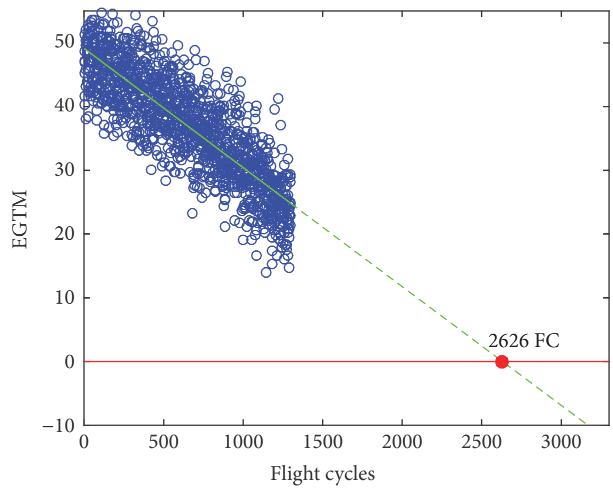

Figure 11: EGTM prediction (T_2 = 1300 FC) with linear regression for #2.

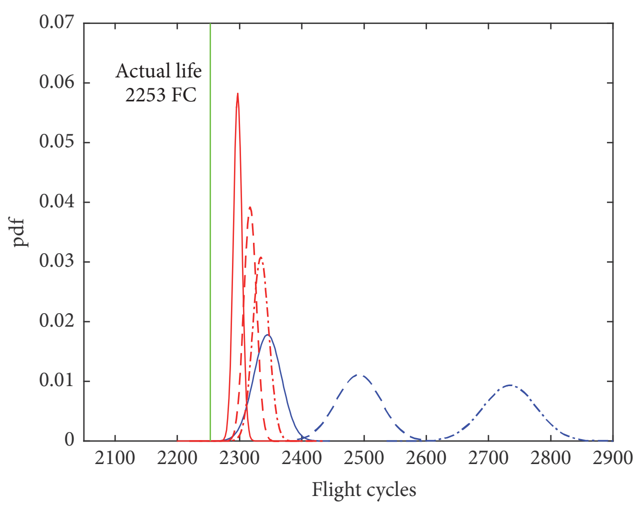

Figure 12: Predictive life of #2 aeroengine with different time T_2.

4.3. SV Decision. Carrying out SV is a complex work and preparations should be made in advance; in order not to affect the airlines operation, an aeroengine SV decision is always made approximately 2 weeks (50 FC) ahead of its real SV time. So the shortest interval between two consecutive optimizations of SV time should be less than 50 FC. However, because the performance deterioration of an aeroengine is a gradual process, overmuch frequent optimizations of SV time is unnecessary, especially at the early stage of operation. So the following rules would be adopted in this study:

(1) If the optimal SV time k is more than 150 FC, #1 aeroengine will continue to be used and the next time of optimizing SV time k is after 100 FC.

(2) If the optimal SV time k is between 100 FC and 150 FC, #1 aeroengine will continue to be used and the next time of optimizing SV time k is after 50 FC.

(3) If the optimal SV time k is between 50 FC and 100 FC, the SV decision should be made in (k-50) FC and #1 aeroengine needs to be taken to the shop in k FC.

(4) If the optimal SV time k is less than 50 FC, the SV decision should be made immediately and #1 aeroengine needs to be taken to the shop in k FC.

According to inequalities (17), the effect of SV time k on the risk of #1 aeroengine and the SL of #2 aeroengine needs to be analyzed simultaneously in order to obtain the optimal SV time. It is assumed that $F_{\mathrm{REQ}} = 1\times10^{-6}$ and $P_{\mathrm{SUP}} = 99.9\%$. Based on (12), the failure probability distribution of #1 aeroengine at different operation time can be obtained, as shown in Figure 14(a). For example, the red-solid curve in Figure 14(a) represents failure probability distribution of #1 aeroengine at operation time T_1 = 1200 FC. And the failure probability distribution changes over time because the predictive parameters as shown in Table 1 are constantly being updated. Because $F_{T_1}(k) \le F_{\mathrm{REQ}}$ should be satisfied, the optimal SV time k on the view of risk can be obtained at different operation time T_1, as shown in Figure 14(b). From Figure 14(b), it can be known that the more F_{REQ}, the less optimal SV time.

In the same way, based on inequality (15), the service level of #2 aeroengine at different operation time can be obtained, as shown in Figure 15(a). Because $\mathrm{SL}_{T_2}(k) \ge P_{\mathrm{SUP}}$ should be satisfied, the optimal SV time k on the view of service level can be obtained at different operation time T_2, as shown in Figure 15(b). From Figure 15(b), it can be known that the more P_{SUP}, the less optimal SV time.

Based on Figure 14, it can be found that when T_1 = 1200 FC, the optimal SV time is 310 FC because $F_{1200}(310)$ is equal to 1×10^{-6}. However, at the same time, that is, when $T_2 = T_1 - 200$ FC = 1000 FC, it can be found from Figure 15 that the optimal SV time is 923 FC because $\mathrm{SL}_{1000}(923)$ is equal to 99.9%. Therefore, according to inequalities (17), the optimal k is 310 FC. Because the optimal k is more than 150 FC, the next time of optimizing SV time is after 100 FC.

However, over time, at operation time T_1 = 1800 FC (T_2 = 1600 FC), $F_{1800}(111)$ is equal to 1×10^{-6} and $\mathrm{SL}_{1600}(159)$ is equal to 99.9%, so the optimal k is 111 FC (between 100 FC and 150 FC) and the next time of optimizing SV time is after 50 FC. And then when T_1 = 1850 FC (T_2 = 1650 FC), $F_{1850}(57)$ is equal to 1×10^{-6} and $\mathrm{SL}_{1650}(108)$ is equal to 99.9%, so the optimal k is 57 FC (between 50 FC and 100 FC) and the SV decision should be made in 7 FC. The SV decision process can be shown in Table 4.

The study case indicates that (1) the predictive accuracy of the RUL is getting higher with increasing historical EGTM data, (2) the SV decision process is dynamic and the optimal SV time changes over time, and (3) both P_{SUP} and F_{REQ} have an effect on SV decisions as well as RUL and repair time, and the more F_{REQ} or P_{SUP}, the less optimal SV time; that is, the

Table 4: SV decision process.

T_1 (FC)	1200	1300	1400	1500	1600	1700	1800	1850
T_2 (FC)	1000	1100	1200	1300	1400	1500	1600	1650
k when $F_{T_1}(k) = 1 \times 10^{-6}$	310	289	277	242	210	157	111	57
k when $\mathrm{SL}_{T_2}(k) = 99.9\%$	923	639	514	342	303	257	159	108
Decision	Continue, optimize SV time every 100 FC						Continue, optimize SV time after 50 FC	SV decision should be made in 7 FC

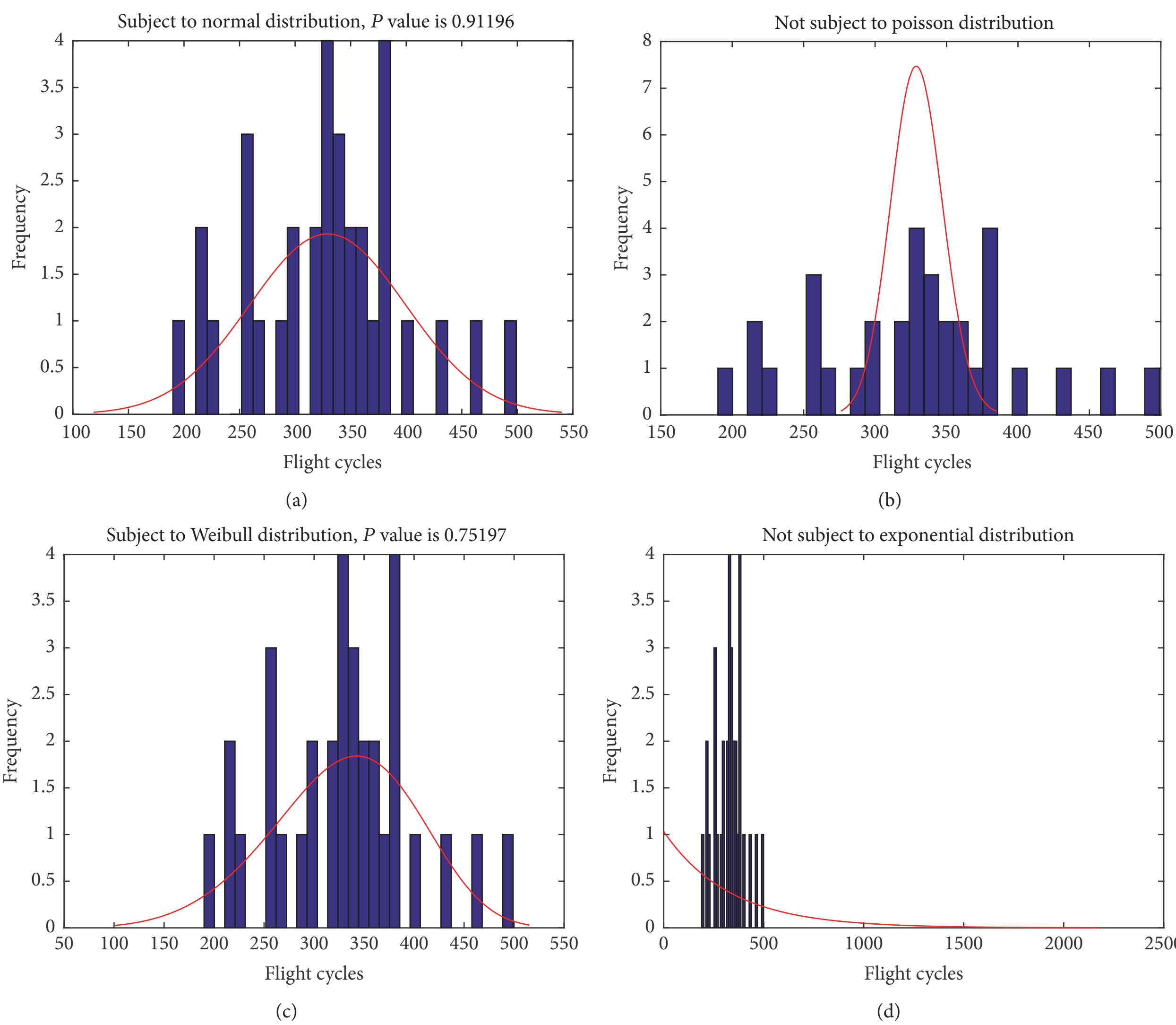

Figure 13: Goodness-of-fit test for aeroengine repair time.

requirements of lower risk or higher service level means that aeroengine visits shop earlier.

5. Conclusions

Reasonably planning aeroengine SVs can avoid the overlap of SVs and reduce the number of spare aeroengines; so according to the concept of JIT, an optimization of the aeroengine SV decision based on RUL and stochastic repair time is developed in this study, by analyzing the effect of SV decision on the risk and the SL. A case study of two CFM-56 aeroengines was given to demonstrate the proposed optimization, and the results show that (1) SSM can predict RUL more accurately than linear regression, and the expected

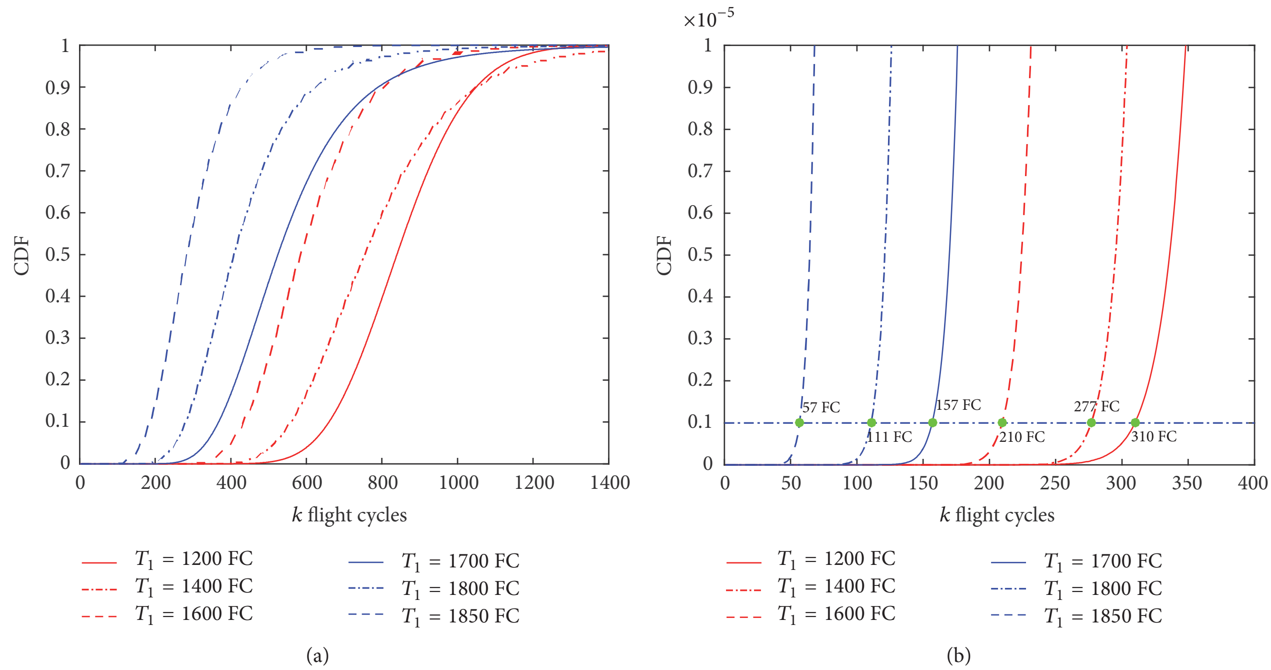

Figure 14: $F_{T_1}(k)$ at different operation time T_1.

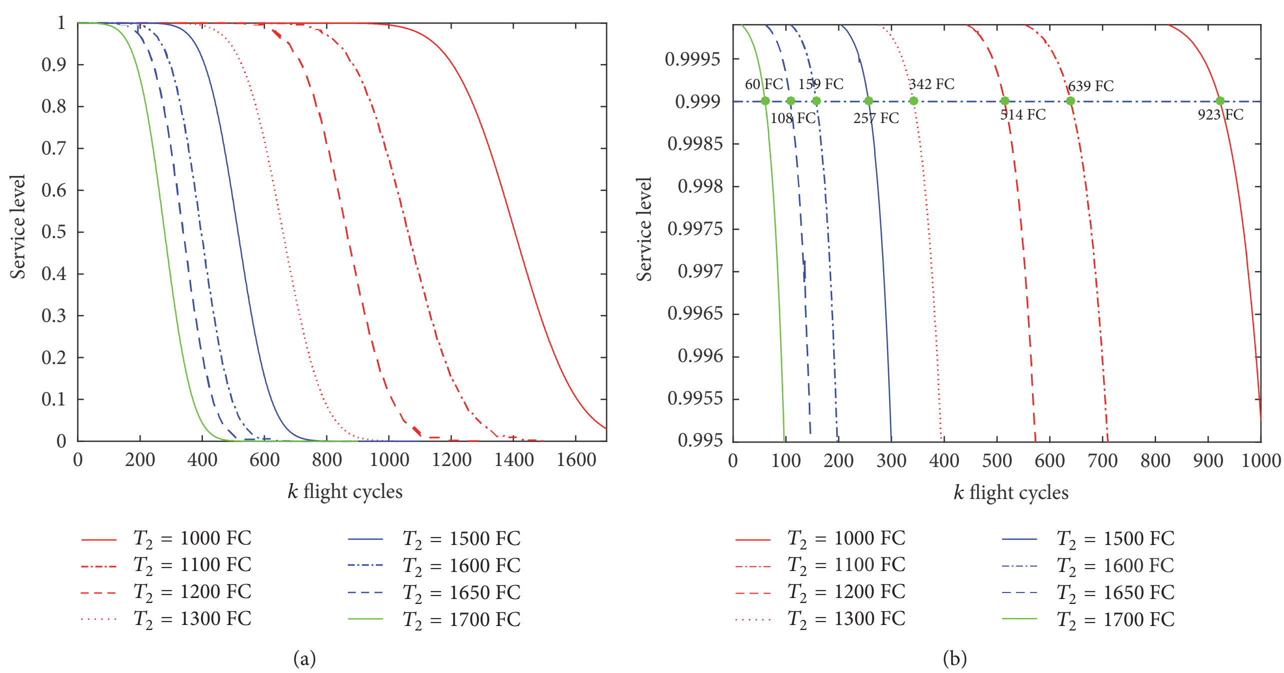

Figure 15: $\mathrm{SL}_{T_2}(k)$ at different operation time T_2.

RUL is closer to the actual RUL with increasing historical EGTM data; (2) the process of SV decisions is simple and feasible for airlines to implement; (3) the optimization of the aeroengine SV decisions not only takes full advantage of the CBM but also decreases the spare aeroengines with the requirement of safety and SL. In further study, an optimization of the aeroengine SV decisions for more than two aeroengines will be developed, and, in order to obtain more accurate optimization results, the economic analysis on aeroengine cost rate will be incorporated into the proposed optimization.

Competing Interests

The authors declare that they have no competing interests regarding the publication of this paper.

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