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### Research Article

### A Parameter Estimation Method for Nonlinear Systems Based on Improved Boundary Chicken Swarm Optimization

Shaolong Chen,¹ Renyu Yang,² Renhuan Yang,¹ Liu Yang,¹ Xiuzeng Yang,³ Chuangbiao Xu,¹ Baoguo Xu,⁴ Huatao Zhang,⁵ Yaosheng Lu,¹ and Weiping Liu¹

Correspondence should be addressed to Renhuan Yang; tyangrh@jnu.edu.cn

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Parameter estimation is an important problem in nonlinear system modeling and control. Through constructing an appropriate fitness function, parameter estimation of system could be converted to a multidimensional parameter optimization problem. As a novel swarm intelligence algorithm, chicken swarm optimization (CSO) has attracted much attention owing to its good global convergence and robustness. In this paper, a method based on improved boundary chicken swarm optimization (IBCSO) is proposed for parameter estimation of nonlinear systems, demonstrated and tested by Lorenz system and a coupling motor system. Furthermore, we have analyzed the influence of time series on the estimation accuracy. Computer simulation results show it is feasible and with desirable performance for parameter estimation of nonlinear systems.

### 1. Introduction

In the past decade, control and synchronization of nonlinear system in industry have attracted much attention. Some effective methods for nonlinear system control and synchronization are proposed and applied in engineering [1– 10]. However, most of these methods are based on hypothesis that the parameters of system are known. They are generally inapplicable if system parameters are unknown. However, in practice, parameters of system are difficult to be known or measured due to complexity and unobservability of system. Therefore, parameter estimation is needed in modeling and control of these nonlinear systems.

Dynamic system identification is an inverse problem based on the input data and output data measured by experiment. After a mathematical model is established to reflect the essential characteristics of the system, parameters need to be identified. In general, the dynamics of nonlinear

system in industry can be described by corresponding mathematical model. However, parameters need to be identified according to practice data, which is generally difficult. In the field of parameter estimation for nonlinear system, some effective methods have been proposed during the past few years. For instance, Gao and Hu [11] reported parameter estimation of chaotic system by using discontinuous drive signals. Blanchard et al. [12] proposed a parameter estimation approach that uses polynomial chaos to propagate uncertainties, estimating error covariance in the extended Kalman filter framework. Liu et al. [13] presented a method for estimating one-dimensional discrete chaotic system based on mean value function. In addition, some intelligent optimization algorithms have been proposed for the parameter identification, such as genetic algorithm (GA) [14], particle swarm optimization (PSO) [15-18], differential evolution (DE) [19], ant swarm optimization algorithm (AS) [20], bat algorithm (BA) [21], cuckoo search optimization algorithm

<sup>&</sup>lt;sup>1</sup>College of Information Science and Technology, Jinan University, Guangzhou 510632, China

<sup>&</sup>lt;sup>2</sup>School of Mechanical and Power Engineering, Guangdong Ocean University, Zhanjiang 524088, China

<sup>&</sup>lt;sup>3</sup>Department of Physics and Electronic Engineering, Guangxi Normal University for Nationalities, Chongzuo 532200, China

<sup>&</sup>lt;sup>4</sup>School of Instrument Science and Engineering, Southeast University, Nanjing 210096, China

<sup>&</sup>lt;sup>5</sup>Key Laboratory of Astronomical Optics & Technology, Nanjing Institute of Astronomical Optics & Technology, Chinese Academy of Sciences, Nanjing 210042, China

(CS) [22], and teaching learning based optimization (TLBO) [23]. However, research on the influence of time series on the estimation accuracy of multidimensional nonlinear system is rare.

Recently, a new bioinspired optimization algorithm, namely, chicken swarm optimization (CSO) [24] is proposed, and it mimics the hierarchy and behavior of chickens. The algorithm proved to be very promising and could outperform existing algorithms such as PSO, DE, and BA [24]. Due to the excellent global convergence and robustness, CSO has been widely applied in engineering [25, 26]. Similar to other bioinspired optimization algorithms, the CSO algorithm can be further improved to enhance convergence speed and convergence precision. In this paper, parameter estimation of nonlinear system is transformed into a multidimensional parameter optimization problem by constructing an appropriate fitness function, and then a method based on improved boundary chicken swarm optimization (IBCSO) is proposed for the multidimensional parameters optimization problem. However, to our best knowledge, there is still not research work applying chicken swarm optimization to solve parameter estimation problem of nonlinear system in previous literatures. Furthermore, we have analyzed the influence of time series on the estimation accuracy. We demonstrated and tested the proposed method by Lorenz nonlinear system [16] and coupling motor system [27]. Computer simulation results show the proposed method is feasible with desirable performance for parameter estimation of nonlinear systems.

This paper is organized as follows. The problem formulation is briefly addressed in Section 2. In Section 3, we proposed an improved boundary chicken swarm optimization. In Section 4, we analyze the influence of time series on the estimation accuracy. Computer simulation results are presented in Section 5. Section 6 is the conclusion.

### 2. Problem Description

A general nonlinear system can be described by the following equation:

$$\dot{X} = F\left(X, X_0, \mu_0\right). \tag{1}$$

Here,  $X = (X_1, X_2, \dots, X_n)^T \in \mathbb{R}^n$  represents the state vector of the original system.  $X_0$  is the initial value of the system.  $\mu_0 = (\mu_{10}, \mu_{20}, \dots, \mu_{d0})^T$  are the true value of the parameters of the system.

Assume that the structure of system (1) is known; thus, the estimated system can be written as

$$\dot{Y} = F(Y, Y_0, \mu). \tag{2}$$

Here,  $Y = (Y_1, Y_2, \dots, Y_n)^T \in \mathbb{R}^n$  represents the state vector of the estimated system.  $Y_0$  is the initial value of the system, and  $Y_0 = X_0$ .  $\mu = (\mu_1, \mu_2, \dots, \mu_d)^T$  are the estimated value of the system parameters.

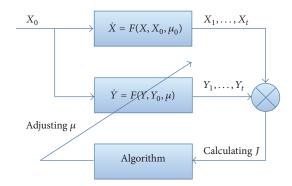


FIGURE 1: The principle of parameter estimation for a nonlinear system.

Based on hereinbefore analysis, the parameter estimation problem can be transformed into the following optimization problem:

$$J = \frac{1}{L} * \sum_{t=1}^{L} \|X_t - Y_t\|.$$
 (3)

Here, L denotes the time series.  $X_t$  and  $Y_t$  coordinates represent the states of the original system and the estimated system at time t, respectively.

The parameter estimation of the nonlinear system can be formulated into multidimensional parameters optimization problem, where the decision vector is  $\mu$  and the optimization goal is to minimize J. The principle of parameter estimation for nonlinear system from an optimizing perspective is shown in Figure 1.

It is difficult to estimate parameters of nonlinear system due to complexity and unobservability of system, so it is challenging to approach satisfactory result by using traditional optimization methods. Therefore, an improved boundary chicken swarm optimization (IBCSO) is proposed to develop an effective parameter estimation method for nonlinear systems in this paper.

### 3. Improved Boundary Chicken Swarm Optimization

3.1. Chicken Swarm Optimization. Chicken swarm optimization (CSO) is a novel swarm intelligence algorithm, which simulates the hierarchy and behavior of chickens. In this algorithm, the chickens were divided into several groups, each of which consists of one rooster and many hens and chicks. Assume  $N_R$ ,  $N_H$ ,  $N_C$ , and  $N_M$  denote the number of the roosters, the hens, the chicks, and the mother hens, respectively. The best  $N_R$  chickens would be assumed to be roosters, while the worst  $N_C$  ones would be regarded as chicks, and the rest are treated as hens. All N virtual chickens, depicted by their positions  $x_{i,j}^t$  ( $i \in [1,N]$ ,  $j \in [1,D]$ ) at time step t, search for food in a D-dimensional space.  $px_{i,j}$  ( $i \in [1,N]$ ,  $j \in [1,D]$ ) represent the optimal position of ith now [24].

Different chickens follow different laws of motions. The roosters with better fitness values have priority for food access

		$ heta_1$	$ heta_2$	$ heta_3$	J
L = 10	Best result	10.000000	28.000000	2.666667	2.595820e – 14
	Worst result	9.999944	27.999901	2.666661	1.832950e - 10
	Average result	9.999991	27.999982	2.666666	2.115565 <i>e</i> – 11
	Standard deviation	1.274592e - 5	2.162007e - 5	1.324675e - 6	3.900723e - 11
	Best result	10.000000	27.999999	2.666666	2.370576e – 11
L = 100	Worst result	9.999854	27.999859	2.666610	7.786259e - 8
L = 100	Average result	9.999976	27.999970	2.666656	7.910590 <i>e</i> – 9
	Standard deviation	2.783841e - 5	3.589249e - 5	1.066062e - 5	1.362798 <i>e</i> – 8
L = 200	Best result	9.999990	27.999997	2.666666	1.264891 <i>e</i> – 9
	Worst result	9.872441	27.934373	2.623538	0.054542
	Average result	9.996019	27.998028	2.665363	0.001121
	Standard deviation	0.018049	0.009255	0.006086	0.007710
L = 500	Best result	9.997352	27.998694	2.665944	1.850371e - 05
	Worst result	9.138323	27.542075	2.353588	2.743463
	Average result	9.712258	27.860182	2.586612	0.378123
	Standard deviation	0.220905	0.104500	0.068190	0.587383

TABLE 1: Estimation results for different time series *L*.

than the ones with worse fitness values, and location update formula is as follows:

$$x_{i,j}^{t} = px_{i,j} * \left(1 + \operatorname{randn}\left(0, \sigma^{2}\right)\right), \tag{4}$$

$$\sigma^{2} = \begin{cases} 1, & \text{if } f_{i} \leq f_{k}, \\ \exp\left(\frac{f_{k} - f_{i}}{|f_{i}| + \epsilon}\right), & \text{otherwise.} \end{cases}$$
 (5)

Here, randn $(0, \sigma^2)$  is a Gaussian distribution with mean 0 and standard deviation  $\sigma^2$ .  $\epsilon$  is the smallest constant in the computer. k is randomly selected from the roosters group, and  $k \neq i$ . f is fitness value of corresponding x.

The hen's location update formula is as follows:

$$x_{i,j}^{t} = px_{i,j} + S_{1} * \text{rand} * (px_{r_{1,j}} - px_{i,j}) + S_{2} * \text{rand}$$
  
  $* (px_{r_{2,j}} - px_{i,j}),$  (6)

$$S_1 = \exp\left(\frac{f_i - f_{r1}}{\operatorname{abs}(f_i) + \epsilon}\right),\tag{7}$$

$$S_2 = \exp\left(f_{r2} - f_i\right). \tag{8}$$

Here, rand is a uniform random distribution of [0, 1]. r1 is the ith hen's group-mate, r2 is randomly chosen from the swarm, and  $r1 \neq r2$ .

The chicks location update formula is as follows:

$$x_{i,j}^{t} = px_{i,j} + FL * (px_{m,j} - px_{i,j}).$$
 (9)

Here, m is the ith chick's mother. FL is a uniform random distribution of [0, 2].

3.2. Improved Boundary Chicken Swarm Optimization. In the standard chicken swarm optimization algorithm, when a

$$\begin{aligned} &\textbf{if } x_{i,j}^t < Lb_j \\ &x_{i,j}^t = Lb_j; \\ &\textbf{else if } x_{i,j}^t > Ub_j \\ &x_{i,j}^t = Ub_j; \\ &\textbf{end if} \end{aligned}$$

ALGORITHM 1: Cross-border processing function.

$$\begin{aligned} & \text{if } x_{i,j}^t < Lb_j \parallel x_{i,j}^t > Ub_j \\ & w = 0.4 * |px_{\text{best},j} - px_{i,j}|; \\ & \text{temp} = px_{\text{best},j} + w * \text{randn}(0,1); \\ & \text{if } Lb_j \leq \text{temp} \leq Ub_j \\ & x_{i,j}^t = \text{temp}; \\ & \text{else} \\ & x_{i,j}^t = px_{i,j}; \\ & \text{end if} \\ & \text{end if} \end{aligned}$$

Algorithm 2: Improved cross-border processing function.

component goes cross the border, it is then replaced with a corresponding value of upper and lower boundary, and the function of cross-border processing is shown in Algorithm 1. In this paper, in order to improve the convergence speed and convergence precision of the CSO, we proposed an improved boundary chicken swarm optimization (IBCSO); when a component goes cross the border, it is then replaced with a random component between the similar component of the individual's best solution and the global best solution so far, and the function of improved cross-border processing is shown in Algorithm 2. Therefore, we get the process of improved boundary chicken swarm optimization shown in Algorithm 3.

```
Initialize a population of N chickens and define the related parameters;
Evaluate the fitness values for each individual, set current each individual's
position and fitness value, and set the current global best individual's position
and fitness value;
for t = 1 to M
  if t \% G == 1
     Rank the chickens' fitness values and establish a hierarchal order in the swarm;
     Divide the swarm into different groups, and determine the relationship
     between the chicks and mother hens in a group;
  end if
  Rank the chickens' fitness values;
  for i = 1 to N
     if i == rooster
                        Update its location using equation (4);
     if i == hen
                     Update its location using equation (6);
     if i == \text{chick}
                      Update its location using equation (9);
                                                                  end if
     Improved cross-border processing function;
     Evaluate the fitness values for i;
     If the new fitness value is better than the current individual's fitness value,
     update the individual's position and fitness value;
     If the new fitness value is better than the current global best individual's
     fitness value, then update the current global best individual's position
     and fitness value;
     If a stopping criterion is met, then output the current global best
     individual's position and fitness value;
  end for
end for
```

ALGORITHM 3: Improved boundary chicken swarm optimization.

TABLE 2: Statistical results from	the IBCSO, CSO,	PSO, GA, and TLBO.

Algorithms		$\theta_1$	$ heta_2$	$\theta_3$	J
IBCSO	Best result	10.000000	28.000000	2.666667	1.311671 <i>e</i> – 14
	Worst result	9.999950	27.999993	2.666662	9.852850e - 11
	Average result	9.999994	27.999998	2.666666	6.801939e - 12
	Standard deviation	7.903017e – 6	1.629054e - 6	7.196902e - 7	1.588317 <i>e</i> – 11
	Best result	9.999997	27.999997	2.666667	9.164669 <i>e</i> – 11
CSO	Worst result	9.999600	27.999761	2.666606	1.395802e - 8
C3O	Average result	9.999875	27.999942	2.666653	2.939848 <i>e</i> – 9
	Standard deviation	9.412609e - 5	5.409299e - 5	1.249644e - 5	3.209969 <i>e</i> – 9
	Best result	9.999725	27.999973	2.666661	1.539265e - 8
PSO	Worst result	9	27.913029	2.660311	0.041631
130	Average result	9.798000	27.984713	2.664953	0.007229
	Standard deviation	0.395116	0.026362	0.001990	0.014707
	Best result	9.987552	27.999037	2.665611	0.000427
GA	Worst result	9.402872	27.743833	2.591086	0.018260
	Average result	9.829154	27.911747	2.642162	0.006100
	Standard deviation	0.145217	0.057552	0.018232	0.004753
TLBO	Best result	9.999999	28.000000	2.666666	6.278703 <i>e</i> – 12
	Worst result	9.999840	27.999899	2.666642	2.610822e - 9
ILDU	Average result	9.999951	27.999976	2.666661	4.760403e - 10
	Standard deviation	4.446187e - 05	1.892609e - 5	5.115130e – 06	5.633741 <i>e</i> – 10

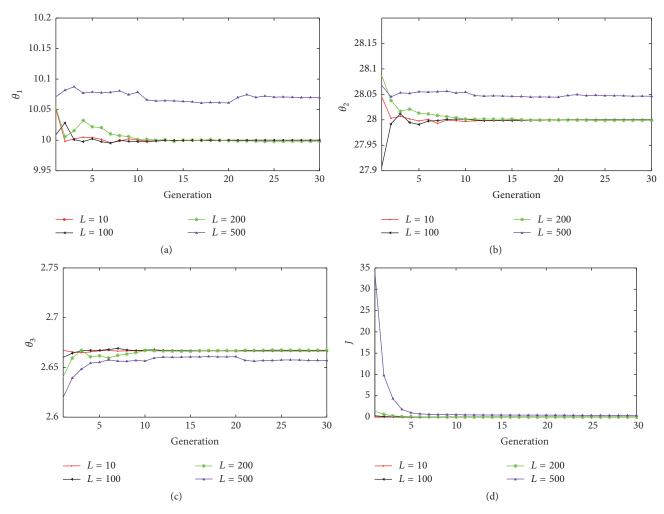


FIGURE 2: The evolving process of the average values for different time series L. (a)  $\theta_1$ ; (b)  $\theta_2$ ; (c)  $\theta_3$ ; (d) J.

## 4. Estimation Accuracy Analysis for a Nonlinear System Example

In this section, in order to discuss the influence of the time series on the estimation accuracy, we consider a Lorenz system.

$$\dot{X} = \theta_1 * (Y - X),$$

$$\dot{Y} = \theta_2 * X - X * Z - Y,$$

$$\dot{Z} = X * Y - \theta_3 * Z.$$
(10)

Here, *X*, *Y*, and *Z* are the state variables;  $\theta_1 = 10$ ,  $\theta_2 = 28$ , and  $\theta_3 = 8/3$  are the original parameters.

We initialize system (10) with a state  $x_0$ , which is randomly selected from the evolution process of the Lorenz system. The searching ranges are set as follows:  $9 < \theta_1 < 11, 20 < \theta_2 < 30,$  and  $2 < \theta_3 < 3$ . The population size and maximum cycle number are set to be N=60, M=30. The parameters of the IBCSO are configured as follows:  $N_R=0.2N, N_H=0.6N, N_C=0.2N, N_M=0.1N, G=10,$  and  $FL\in[0.5,0.9]$  [24]. Let time series L be

different values and run the program of improved boundary chicken swarm optimization algorithm; we use the IBCSO algorithm to estimate the unknown parameters. To make a fair comparison, all cases are run 50 times, and the initial population is set as uniform same value for all the time series L at the same time run. Table 1 lists the estimation results for different time series L. The evolving processes of the average values for different time series L are shown in Figure 2.

Seen from Table 1, the estimation accuracy declines as L increases. In addition, Figure 2 once again shows that estimation accuracy declines as time series L increases. The reason is that the critical sensitivity of the nonlinear system to initial conditions and parameters results in that the objective function becomes very complicated as the increment of L.

# 5. Parameters Estimation Results for Nonlinear Systems and Discussions

5.1. Lorenz System

5.1.1. Offline Estimation. In this simulation, system (10) is used to test the performance of the IBCSO compared with

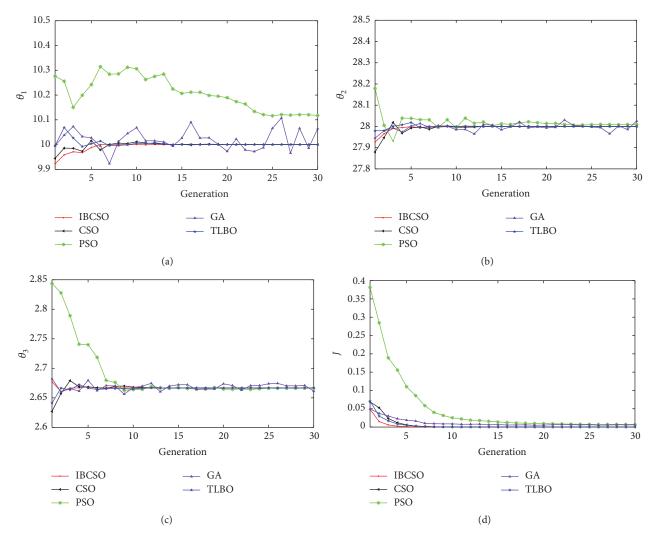


FIGURE 3: The evolving process of the average values obtained by IBCSO, CSO, PSO, GA, and TLBO in offline mode. (a)  $\theta_1$ ; (b)  $\theta_2$ ; (c)  $\theta_3$ ; (d) J.

that of CSO, PSO, GA, and TLBO. We initialize the system with a state  $x_0$ , which is randomly selected from the evolution process of the system. The searching ranges, population size, maximum cycle number, and time series for IBCSO, CSO, PSO, GA, and TLBO are all set as follows: 9 <  $\theta_1$  < 11,  $20 < \theta_2 < 30, 2 < \theta_3 < 3, N = 60, M = 30, and$ L = 10. The parameters of the algorithms are configured as follows. For IBCSO and CSO,  $N_R = 0.2N$ ,  $N_H = 0.6N$ ,  $N_C = 0.2N$ ,  $N_M = 0.1N$ , G = 2, and  $FL \in [0.5, 0.9]$  [24]. For PSO and GA, all the parameters are the same as those used in literature [16]. For TLBO, all the parameters are the same as those used in literature [23]. To make a fair comparison, all algorithms are run 50 times, and the initial population is set as uniform same value for all the optimization algorithms at the same time run. Table 2 lists results obtained by IBCSO, CSO, PSO, GA, and TLBO. The evolving processes of the average values obtained by IBCSO, CSO, PSO, GA, and TLBO are shown in Figure 3. Moreover, to compare the iteration number of the algorithms,  $J \leq 10^{-10}$  is considered as the stopping criteria. The maximum cycle number is set to 1000, and other conditions are the same as above. Table 3 lists the results obtained by IBCSO, CSO, PSO, GA, and TLBO.

5.1.2. Online Estimation. In this simulation, we investigate the capability of the algorithms in chasing the alternations in the parameters of the system. In the first part,  $\theta_1 = 10$ ,  $\theta_2 = 28$ , and  $\theta_3 = 8/3$ . In the second part,  $\theta_1$  moves down to 9.5 from 10,  $\theta_2$  moves down to 27 from 28, and  $\theta_3$  moves down to 2.6 from 8/3 in the 31st iteration. The maximum cycle number is set to 60, and the others conditions in this part are the same as the conditions indicated in the offline mode. The estimation of online parameters of the system can be seen in Figure 4.

5.2. Coupling Motor System. In this section, in order to further prove the performance of the proposed method, we consider a coupling motor system [27].

$$\dot{x} = -\theta_2 x + y (z + \theta_1),$$

$$\dot{y} = -\theta_2 y + x (z - \theta_1),$$

$$\dot{z} = \theta_3 z - x y.$$
(11)

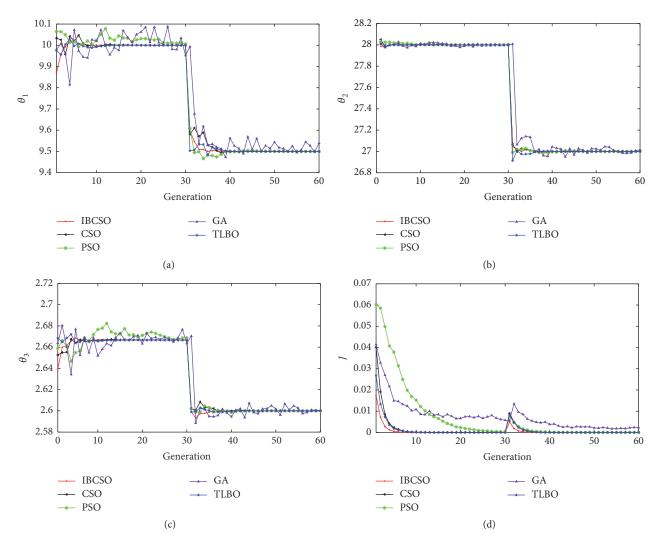


Figure 4: The evolving process of the average values obtained by IBCSO, CSO, PSO, GA, and TLBO in online mode. (a)  $\theta_1$ ; (b)  $\theta_2$ ; (c)  $\theta_3$ ; (d) J.

TABLE 3: Iterations required by IBCSO, CSO, PSO, GA, and TLBO.

Algorithms		Iterations
	Best result	19
IBCSO	Worst result	29
	Average result	24
	Best result	183
PSO	Worst result	1000
	Average result	481
	Best result	24
TLBO	Worst result	32
	Average result	29
	Best result	27
CSO	Worst result	37
	Average result	32
	Best result	1000
GA	Worst result	1000
	Average result	1000

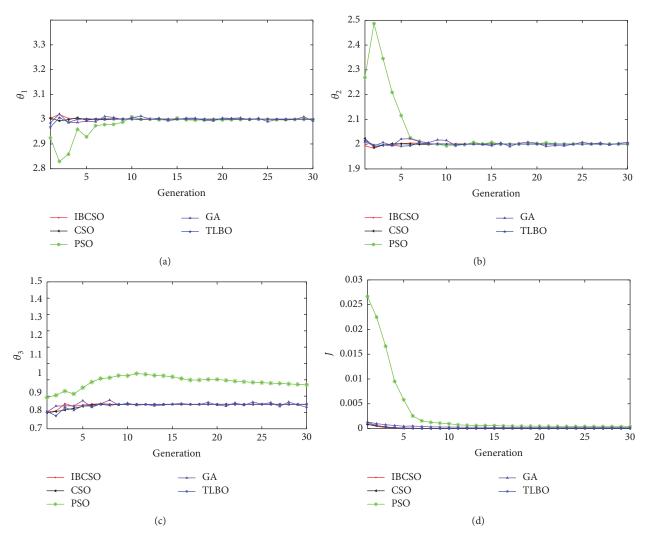


FIGURE 5: The evolving process of the average values obtained by IBCSO, CSO, PSO, GA, and TLBO in offline mode. (a)  $\theta_1$ ; (b)  $\theta_2$ ; (c)  $\theta_3$ ; (d) J.

Here, x, y, and z are the state variables;  $\theta_1 = 3$ ,  $\theta_2 = 2$ , and  $\theta_3 = 0.75$  are the original parameters.

5.2.1. Offline Estimation. In this simulation, system (11) is used to test the performance of the IBCSO compared with that of CSO, PSO, GA, and TLBO. We initialize the system with a state  $x_0$ , which is randomly selected from the evolution process of the system. The searching ranges, population size, maximum cycle number, and time series for IBCSO, CSO, PSO, GA, and TLBO are all set as follows: 2 <  $\theta_1$  < 4, 1 <  $\theta_2 < 3, 0 < \theta_3 < 1, N = 60, M = 30, and L = 10.$ The parameters of the algorithms are configured as follows. For IBCSO and CSO,  $N_R = 0.2N$ ,  $N_H = 0.6N$ ,  $N_C =$  $0.2N, N_M = 0.1N, G = 2, \text{ and } FL \in [0.5, 0.9]$  [24]. For PSO and GA, all the parameters are the same as those used in literature [16]. For TLBO, all the parameters are the same as those used in literature [23]. To make a fair comparison, all algorithms are run 50 times, and the initial population is set as uniform same value for all the optimization algorithms at the same time run. Table 4 lists the results obtained by

IBCSO, CSO, PSO, GA, and TLBO. The evolving processes of the average values obtained by IBCSO, CSO, PSO, GA, and TLBO are shown in Figure 5. Moreover, to compare the iteration number of the algorithms,  $J \leq 10^{-10}$  is considered as the stopping criteria. The maximum cycle number is set to 1000, and other conditions are the same as above. Table 5 lists the results obtained by IBCSO, CSO, PSO, GA, and TLBO.

5.2.2. Online Estimation. In this simulation, we investigate the capability of the algorithms in chasing the alternations in the parameters of the system. In the first part,  $\theta_1 = 3$ ,  $\theta_2 = 2$ , and  $\theta_3 = 0.75$ . In the second part,  $\theta_1$  moves down to 2.5 from 3,  $\theta_2$  moves down to 2.5 from 2, and  $\theta_3$  moves down to 0.5 from 0.75 in the 31st iteration. The maximum cycle number is set to 60, and the other conditions in this part are the same as the conditions indicated in the offline mode. The estimation of online parameters of the system can be seen in Figure 6.

From the above two examples, the results presented demonstrate that a good optimal performance can be achieved by the proposed IBCSO algorithm. As shown in

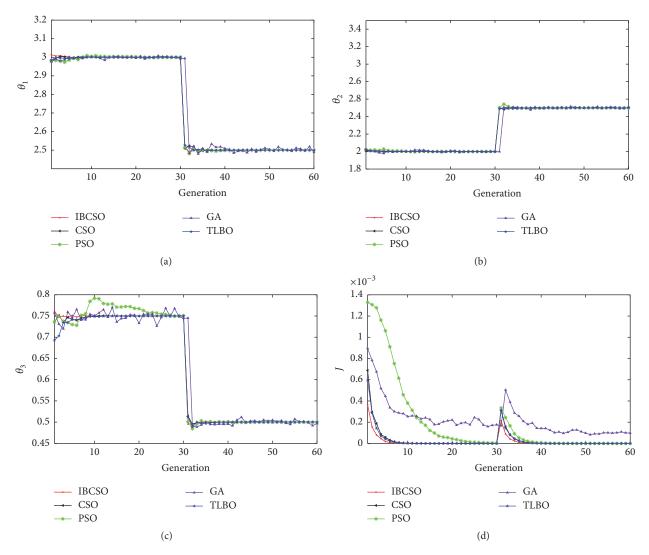


FIGURE 6: The evolving process of the average values obtained by IBCSO, CSO, PSO, GA, and TLBO in online mode. (a)  $\theta_1$ ; (b)  $\theta_2$ ; (c)  $\theta_3$ ; (d) J.

Tables 2 and 4, the best, the average, the worst results and standard deviation obtained by IBCSO are all better than those obtained by CSO, PSO, GA, and TLBO, respectively. In addition, Figures 3 and 5 once again show that IBCSO is of better performance than CSO, PSO, GA, and TLBO in terms of convergence speed and convergence precision. Moreover, from Tables 3 and 5, it is confirmed that the IBCSO spends less iterations to reach a predefined threshold compared with CSO, PSO, GA, and TLBO. Furthermore, as shown in Figures 4 and 6, tracking the changes of the system parameters by the IBCSO is well-performed.

### 6. Conclusion

In this paper, a method based on improved boundary chicken swarm optimization (IBCSO) algorithm is proposed to solve the problem of parameter estimation for nonlinear systems. Computer simulation based on two nonlinear systems examples and comparisons with results obtained by CSO, PSO, GA,

and TLBO demonstrated the effectiveness of the proposed method. Furthermore, we have analyzed the influence of time series on the estimation accuracy. According to theoretical analysis and computer simulation, we achieved the following conclusions: shorter length of time series will benefit the estimation accuracy because that longer time series will make the objective function complicated. Therefore, it is very important to select a suitable time series to reduce the estimation bias of aim nonlinear systems. Although it is demonstrated by two nonlinear systems examples in this paper, the proposed method can also be used as a promising tool for numerical optimization problems in engineering.

### **Competing Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

Algorithms		$ heta_1$	$ heta_2$	$ heta_3$	J
IBCSO	Best result	3.000000	2.000000	0.750000	1.515880e – 14
	Worst result	2.999988	1.999984	0.749954	2.955687 <i>e</i> – 11
	Average result	2.999996	1.999994	0.749992	3.818720e - 12
	Standard deviation	3.528640e - 6	4.394600e - 6	7.615395e - 7	4.837061e - 12
	Best result	2.999998	2.000000	0.750000	1.089261 <i>e</i> – 12
CSO	Worst result	2.999933	1.999945	0.749913	1.445733e - 10
230	Average result	2.999984	1.999982	0.749976	3.833475e - 11
	Standard deviation	1.202585e - 5	1.400733e - 5	2.199347e - 5	3.785071e - 11
	Best result	2.999971	1.999995	0.749776	1.415251e - 8
PSO	Worst result	2.981136	1.984547	0.500000	0.000812
130	Average result	2.997187	1.997420	0.628964	0.000389
	Standard deviation	0.003126	0.002847	0.124597	0.000407
	Best result	2.998662	1.999492	0.746170	1.387276e – 05
GA	Worst result	2.880957	1.883540	0.590241	0.000495
GA	Average result	2.960151	1.962746	0.696085	0.000188
	Standard deviation	0.030768	0.028565	0.038388	0.000136
TLBO	Best result	3.000000	2.000000	0.750000	6.668050e - 14
	Worst result	2.999974	1.999982	0.749947	3.784144e - 11
	Average result	2.999994	1.999995	0.749991	5.022291 <i>e</i> – 12
	Standard deviation	4.982352e - 06	4.686890e - 6	9.583826e - 06	6.587053e - 12

TABLE 4: Statistical results from the IBCSO, CSO, PSO, GA, and TLBO.

Table 5: Iterations required by IBCSO, CSO, PSO, GA, and TLBO.

Algorithms		Iterations
	Best result	22
IBCSO	Worst result	29
	Average result	26
	Best result	175
PSO	Worst result	1000
	Average result	450
	Best result	23
TLBO	Worst result	31
	Average result	27
	Best result	26
CSO	Worst result	34
	Average result	30
	Best result	1000
GA	Worst result	1000
	Average result	1000

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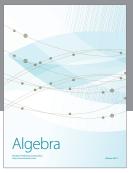
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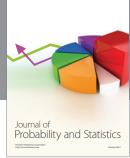
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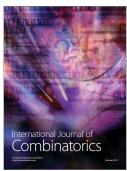








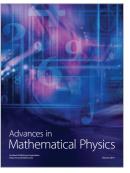


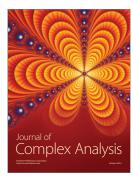




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