Hindawi Publishing Corporation
International Journal of Microwave Science and Technology
Volume 2015, Article ID 646029, 6 pages
http://dx.doi.org/10.1155/2015/646029

Research Article

Negative Group Delay Circuit Based on Microwave Recursive Filters

Mohammad Ashraf Ali and Chung-Tse Michael Wu

Department of Electrical and Computer Engineering, Wayne State University, 5050 Anthony Wayne Drive, Detroit, MI 48202, USA

Correspondence should be addressed to Chung-Tse Michael Wu; ctmwu@wayne.edu

Received 13 June 2015; Revised 27 July 2015; Accepted 5 August 2015

Academic Editor: Tanmay Basak

This work presents a novel approach to design a maximally flat negative group delay (NGD) circuit based on microwave recursive filters. The proposed NGD circuit is realized by cascading N stages of quarter-wavelength stepped-impedance transformer. It is shown that the given circuit can be designed to have any prescribed group delay by changing the characteristic impedance of the quarter-wave transformers (QWTs) cascaded with each other. The proposed approach provides a systematic method to synthesize NGD of arbitrary amount without including any discrete lumped component. For various prescribed NGD, the characteristic impedance of QWT has been tabulated for two and three stages of the circuit. The widths and lengths of microstrip transmission lines can be obtained from characteristic impedance and the frequency of operation of the transmission line. The results are verified in both simulation and measurement, showing a good agreement.

1. Introduction

Following the classical paper published by Brillouin and Sommerfeld in 1960 [1], Garrett and McCumber were the first to analytically prove that group velocity of wave can be greater than the speed of light [2]. However, its first practical demonstration was performed after a decade by Chu and Wong [3]. At first, the concept of a wave having velocity greater than the speed of light (superluminal velocity [1]) seems to defy the causality. Nevertheless, there exist enough practical experiments showing that the concept of superluminal velocity follows the causal system definition [3–5]. Since the development of this concept, there has been a lot of work done by scientists to utilize this property and apply it to practical applications. This concept has been used to enhance the efficiency of the feed-forward amplifier, broadband and constant phase shifter, and shortening of delay lines [6–8]. By using NGD circuits, positive group delays introduced by the circuit components and electrical connections in electronic systems can be compensated. In NGD circuits, electromagnetic waves can have superluminal velocity in the region of anomalous dispersion such that the phase of the higher frequency component of the wave will move in advance with respect to the lower frequency components.

Although there have been a lot of efforts put towards generating NGD using both active and passive techniques, little work has been done towards generating the prescribed NGD at the desired frequency. Recently, a maximally flat NGD circuit based on transversal filter concept has been proposed to synthesize the desired group delay [9]. In this paper, we will demonstrate that a microwave recursive filter can also be used to generate NGD of predetermined values in the desired frequency band by using only distributed components [10]. It is noted that very recently distributed components-based NGD circuits have also been proposed; however, few systematic methods were provided to synthesize the desired NGD [11]. The contribution in this work expands the theory derived in [9] and provides a full analysis of a microwave recursive-filter based NGD circuitry.

To illustrate, multistage QWTs are used to generate NGD of predetermined values as shown in Figure 1. Essentially, it behaves as a recursive filter since the reflected waves will bounce back and forth among the impedance interfaces and form an infinite impulse response (IIR). Following this concept, we will show a comprehensive methodology to synthesize the desired NGD by treating the multistage QWT as an IIR filter. In addition, it is worth mentioning that there are no lumped components present, which makes this design easy to fabricate and to be scaled up to higher frequency. We will also demonstrate the synthesis procedure of a two-stage QWT with −0.5 ns, −1 ns, and −2 ns group delays. In addition, a branch-line coupler is used to transfer NGD from a one-port circuit to a two-port one, making the NGD circuit more applicable in practices.

2. Theory and Formulae

A microstrip-line based transmission line structure is used to fabricate QWTs. As depicted in Figure 1, the QWTs have the characteristic impedance of $Z_1, Z_2, Z_3 \cdots Z_N$ with an identical electrical length of θ. The impedance of all QWTs has been normalized to the termination impedance of 50 ohms. The design flow is as follows. First, we need to obtain the characteristic impedance of each of the microstrip transmission line QWTs that can generate the desired group delay. Once we obtain the characteristic impedance, we can calculate the width and length of the transmission line QWT according to the synthesis procedure shown in [12]. In order to solve for the desired characteristic impedance, we first relate it to the transmission matrix or ABCD matrix parameters. The relation between the characteristic impedance of the transmission line and the transmission matrix parameters for the ith stage of QWT is as follows:

$$\begin{bmatrix} \cos\theta & jZ_i \sin\theta \\ \left(\frac{j}{Z_i}\right)\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} A_i & jB_i \\ jC_i & D_i \end{bmatrix}. \quad (1)$$

Furthermore, when cascading N stages of QWT we will obtain

$$\prod_{i=1}^{N} \begin{bmatrix} \cos\theta & jZ_i \sin\theta \\ \left(\frac{j}{Z_i}\right)\sin\theta & \cos\theta \end{bmatrix} = \prod_{i=1}^{N} \begin{bmatrix} A_i & jB_i \\ jC_i & D_i \end{bmatrix}, \quad (2)$$

where N is the number of stages in the circuit, Z_i corresponds to the impedance of the ith QWT, and θ is the electrical length of each stage of the transmission line which is equal to $\pi/2$ at the center frequency ($\theta = \beta l$, where $l = \lambda/4$ and $\beta = 2\pi/\lambda$). We can then represent the product of the above transmission matrix parameter for N stage as follows:

$$\prod_{i=1}^{N} \begin{bmatrix} A_i & jB_i \\ jC_i & D_i \end{bmatrix} = \begin{bmatrix} A_n & jB_n \\ jC_n & D_n \end{bmatrix}, \quad (3)$$

in which A_n, B_n, C_n, and D_n represent the transmission matrix parameters for the entire N-stage quarter-wave transformer circuit. The reflection coefficient (Γ) for the N-stage circuit can be written as

$$\Gamma = \frac{V_{\text{in}}^-}{V_{\text{in}}^+} = \frac{(A_n - D_n) + j(B_n - C_n)}{(A_n + D_n) + j(B_n + C_n)}. \quad (4)$$

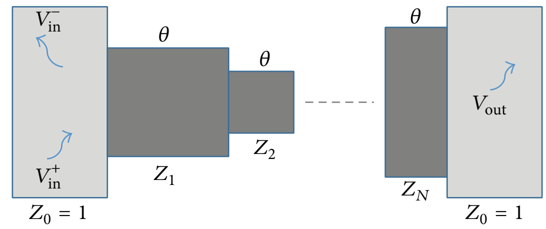

Figure 1: An N-stage quarter-wave transformer.

The phase of the reflection coefficient can be expressed as

$$\angle\Gamma = \tan^{-1}\frac{(B_n - C_n)}{(A_n - D_n)} - \tan^{-1}\frac{(B_n + C_n)}{(A_n + D_n)}. \quad (5)$$

In order to obtain the prescribed maximally flat group delay, we need to obtain the characteristic impedance of each of the transmission lines $[Z_1, Z_2, Z_3 \cdots Z_N]$ by solving the following equations:

$$\left|\Gamma\left(\theta = \frac{\pi}{2}\right)\right| = \Gamma_p, \quad (6)$$

$$-\left.\frac{\partial \angle\Gamma(\theta)}{\partial\theta}\right|_{\theta=\pi/2} = \tau_{gp}, \quad (7)$$

$$-\left.\frac{\partial^{2n-3} \angle\Gamma(\theta)}{\partial\theta}\right|_{\theta=\pi/2} = 0, \quad (8)$$

$$\text{where } n = 3, 4, \ldots, (N-1).$$

Here Γ_p is the prescribed magnitude, and τ_{gp} is the prescribed NGD with a unit of the sampling period T. In order to have the maximally flat response, we set all the higher order derivatives shown in (8) to zero. To give a quantitative example, let us take the two-stage QWTs case ($N = 2$) and assume τ_{gp} to be −4 and Γ_p to be 0.04 at the center frequency 1 GHz. Since the sampling period T of a 90-degree ($\pi/2$) delay line at 1 GHz is 0.25 ns, we can generate group delay of $-4 \times 0.25 = -1$ ns. Similarly, by changing τ_{gp} to −8 we can obtain group delay of −2 ns and so on. It is noted that we terminate our device with standard 50 ohms load, and the characteristic impedance of the transmission lines obtained here is normalized to the load impedance. Table 1 shows the different values of normalized characteristic impedance for various group delays by setting Γ_p equal to 0.04 derived from (6)–(8). After obtaining the characteristic impedance, we can easily calculate the dimension of QWT [12]. The width and length of the QWT corresponding to the characteristic impedance which we calculate from (6)–(8) for negative group delay of −0.5 ns, −1 ns, and −2 ns are tabulated in Table 2. The pictorial representation of width and length of the transmission line is shown in Figure 2.

3. Simulation and Measurement

To validate our concept of generating the prescribed NGD using transversal filter methodology, we designed a NGD circuit consisting of two-stage quarter-wave transformer

Table 1: Normalized characteristic impedance for two and three stages ($\Gamma_p = 0.04$).

$\tau_{gp}(T)$	$N = 2$		$N = 3$		
	Z_1	Z_2	Z_1	Z_2	Z_3
−1	1.0833	1.0408	1.10527	1.0834	1.02024
−2	1.1052	1.0618	1.15044	1.15051	1.0409
−3	1.1275	1.0833	1.20535	1.23801	1.06904
−4	1.1503	1.1052	1.27094	1.34942	1.1051
−5	1.1735	1.1274	1.34819	1.489	1.14954
−6	1.1971	1.1501	1.43799	1.66171	1.20276
−7	1.2211	1.1732	1.54106	1.87306	1.26507
−8	1.2455	1.1966	1.65783	2.12887	1.33657
−9	1.2703	1.2205	1.78835	2.43493	1.41715
−10	1.2956	1.2448	1.9323	2.79674	1.50646

Table 2: Dimension of transmission line for different group delay (substrates: Rogers RO3010 with dielectric constant of 10.2 and thickness of 25 mils).

Group delay (ns)	Dimensions	
	Width (mm)	Length (mm)
−0.5 ns		
$TL_1(Z_1)$	0.459072	29.3245
$TL_2(Z_2)$	0.503547	29.2085
−1.0 ns		
$TL_1(Z_1)$	0.417143	29.4401
$TL_2(Z_2)$	0.459072	29.3245
−2.0 ns		
$TL_1(Z_1)$	0.340949	29.6696
$TL_2(Z_2)$	0.376	29.5539

using microstrip-line structures on a printed circuit board (PCB). The substrate that we used is RO3010 from Rogers Corporation, with a dielectric constant of 10.2 and thickness of 25 mils. Normalized characteristic impedance of two transmission lines for −1 ns group delay is obtained from Table 1: that is, $Z_1 = 1.1503$, $Z_2 = 1.1052$. After that we fabricated the quarter-wave transformer according to this normalized characteristic impedance. Figure 3 shows the simulated results of three different NGD at 1 GHz for a two-stage quarter-wave transformer using the tabulated coefficients. In addition, it is worth mentioning that one can also design a three-stage transformer to generate prescribed NGD as shown in Figure 4 with the coefficients obtained from Table 1. For a given NGD, the bandwidth can be enhanced when more stages are cascaded.

In practical application, it is often desired to introduce the negative group delay between two ports. In fact, we can transfer the group delay from one port to another port by simply using a branch-line coupler. The schematic for the complete circuit is shown in Figure 5. The S parameters of the two-port network can be obtained by solving the branch-line coupler with odd and even mode analysis [12]. The port reduction technique [13] can be applied to get the reflection coefficient of two-port negative group delay circuit. Figure 5

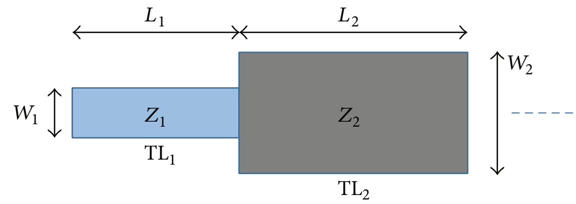

Figure 2: Dimensions of the quarter-wave transformers.

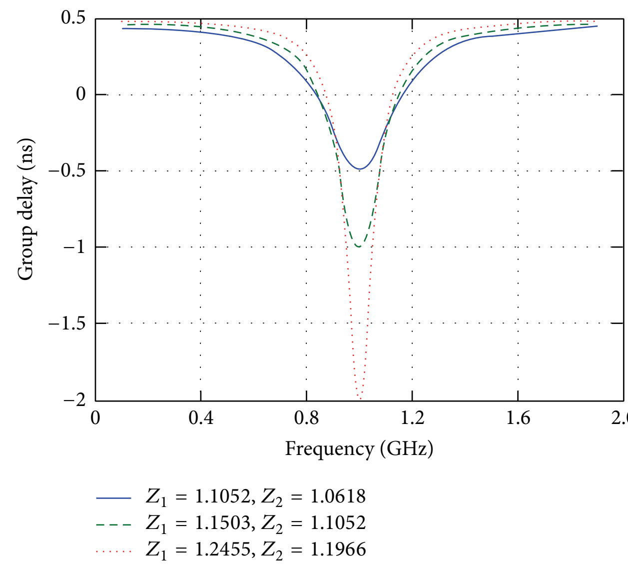

Figure 3: Simulated results for prescribed group delay of −0.5 ns, −1 ns, and −2 ns for a two-stage quarter ($N = 2$) wave transformer.

shows all the voltages that we are required to find out the S matrix for the circuit. V_1^+ and V_1^- are the incident and reflected voltage at the input port 1. V_2^- is the reflected voltage at the output port 2. V_{A1}^+ represents incident and V_{A1}^- represents reflected voltage at junction A; similarly V_{B1}^+ represents incident and V_{B1}^{+-} represents the reflected voltage at junction B. We can calculate S_{21} by the following procedure:

$$S_{21} = \frac{V_2^-}{V_1^+}. \quad (9)$$

The reflected voltage from the output port 2, V_2^-, can be written in terms of reflected voltages V_{A1}^- and V_{B1}^- as

$$\begin{aligned} V_2^- &= \frac{e^{-j2\theta}}{\sqrt{2}} V_{A1}^- + \frac{e^{-j(2\theta-\pi/2)}}{\sqrt{2}} V_{B1}^- \\ &= \frac{e^{-j2\theta}}{\sqrt{2}} \Gamma_A V_{A1}^+ + \frac{e^{-j(2\theta-\pi/2)}}{\sqrt{2}} \Gamma_B V_{B1}^+ \\ &= \frac{e^{-j(2\theta+\pi/2)}}{2} \Gamma_A V_1^+ + \frac{e^{-j(2\theta+\pi/2)}}{2} \Gamma_B V_1^+ \\ &= \frac{e^{-j(2\theta+\pi/2)}}{2} V_1^+ \left(\Gamma_A + \Gamma_B\right). \end{aligned} \quad (10)$$

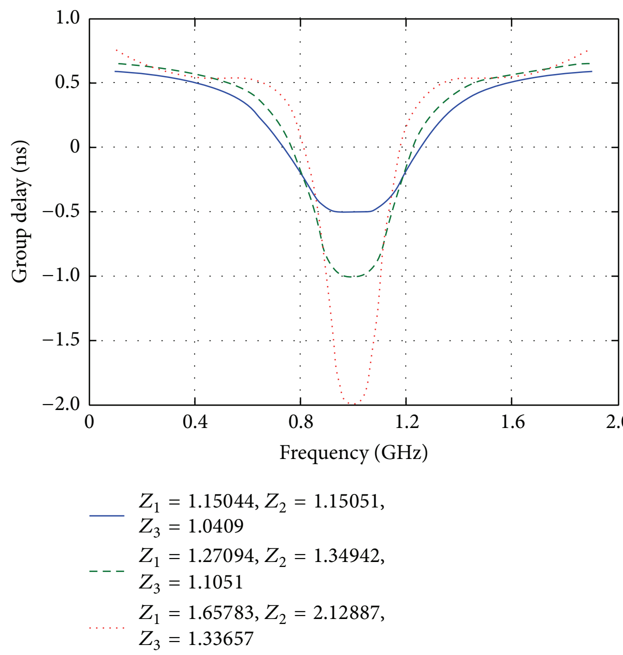

Figure 4: Simulated results for prescribed group delay of −0.5 ns, −1 ns, and −2 ns for a three-stage ($N = 3$) QWT.

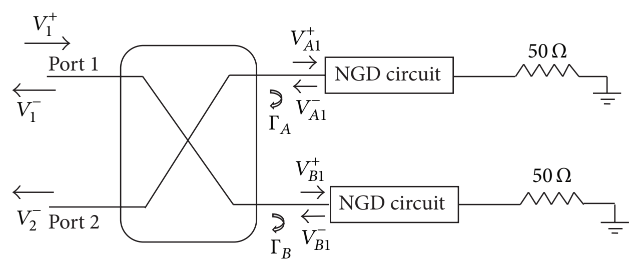

Figure 5: Schematic of two-stage quarter-wave transformers integrated with a branch-line coupler.

Therefore, $S_{21} = (e^{-j(2\theta+\pi/2)}/2)(\Gamma_A + \Gamma_B)$, and we know $\Gamma_A = \Gamma_B$. Let us assume $\Gamma_A = \Gamma_B = \Gamma$; this will give us

$$S_{21} = e^{-j(2\theta+\pi/2)}\Gamma. \quad (11)$$

Similarly, S_{11} will be obtained as

$$V_1^- = \frac{1}{\sqrt{2}}V_{A1}^- + \frac{e^{-j\theta}}{\sqrt{2}}V_{B1}^- = \frac{1}{\sqrt{2}}\Gamma_A V_{A1}^+ + \frac{e^{-j\theta}}{\sqrt{2}}\Gamma_B V_{B1}^+ = \frac{1}{2}V_1^+\left(\Gamma_A + e^{-2j\theta}\Gamma_B\right). \quad (12)$$

Thus, we can write

$$S_{11} = \frac{V_1^-}{V_1^+} = \frac{1}{2}\left(\Gamma_A + e^{-2j\theta}\Gamma_B\right). \quad (13)$$

Since $\Gamma_A = \Gamma_B$, we can put $\Gamma_A = \Gamma_B = \Gamma$, which will give us $S_{11} = (\Gamma/2)(1+e^{-2j\theta})$. The reflection coefficient matrix for the

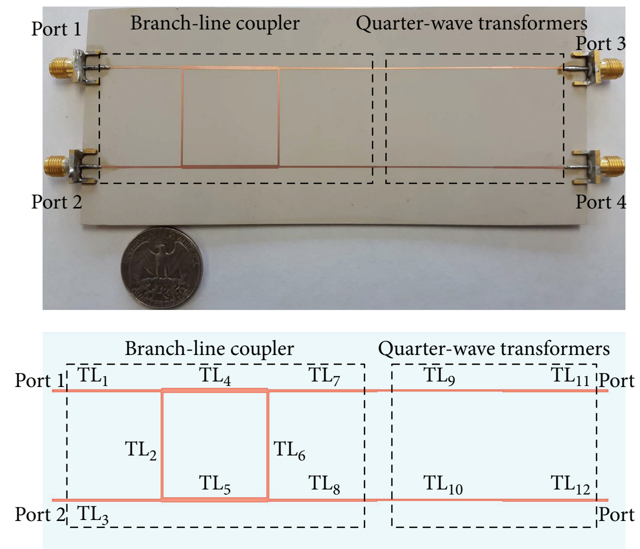

Figure 6: Fabricated prototype of NGD circuit using two-stage quarter-wave transformers integrated with a branch-line coupler.

two-port negative group delay circuit can thus be represented as

$$S = \begin{bmatrix} \frac{\Gamma}{2}\left(1+e^{-2j\theta}\right) & e^{-j(2\theta+\pi/2)}\Gamma \\ e^{-j(2\theta+\pi/2)}\Gamma & \frac{\Gamma}{2}\left(1+e^{-2j\theta}\right) \end{bmatrix}. \quad (14)$$

From (7) and (14), the group delay at desired frequency becomes

$$-\left.\frac{\partial \angle S_{21}}{\partial \theta}\right|_{\theta=\pi/2} = \tau_{gp} + 2. \quad (15)$$

The constant term in (15) represents the extra group delay caused by the coupler, which also agrees with the result in [13]. The prototype of the design including a branch-line coupler along with two-stage quarter-wave transformers is shown in Figure 6. Transmission lines TL_1, TL_3, TL_7, and TL_8 are included in the prototype for interconnections, which will result in addition of group delay in the circuit. The effects can be nullified by fine-tuning the dimensions of the NGD circuit based on quarter-wave transformers. Table 3 indicates the length and width of the branch-line coupler and quarter-wave transformer as depicted in Figure 6.

Figure 7 depicts the comparison of group delays between the simulation and measurement in which we use the branch-line coupler. The simulated and measured results agree with each other very well. The resulting NGD of the entire circuit is around −1 ns at 1.1 GHz. Furthermore, the magnitude of corresponding S_{21} is plotted in Figure 8, which indicates around 23 dB signal attenuation during the NGD region.

4. Conclusion

In this paper, we present a systematic technique of generating prescribed NGD by simply using multistage quarter-wave

Table 3: Dimensions of branch-line coupler and quarter-wave transformers for group delay of −1 ns (substrates: Rogers RO3010 with dielectric constant of 10.2 and thickness of 25 mils).

	Branch-line coupler		Quarter-wave transformer	
			First stage	Second stage
	TL_1, TL_2, TL_3, TL_6, TL_7, and TL_8	TL_4 and TL_5	TL_9 and TL_{10}	TL_{11} and TL_{12}
Width (mm)	0.575	1.111	0.417	0.560
Length (mm)	29.034	28.054	33.440	28.000

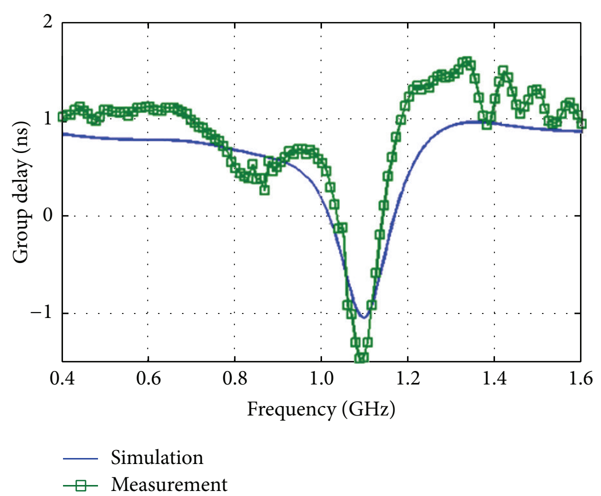

Figure 7: Simulation and measurement of group delay for two-stage quarter-wave transmission line with branch-line coupler NGD circuit.

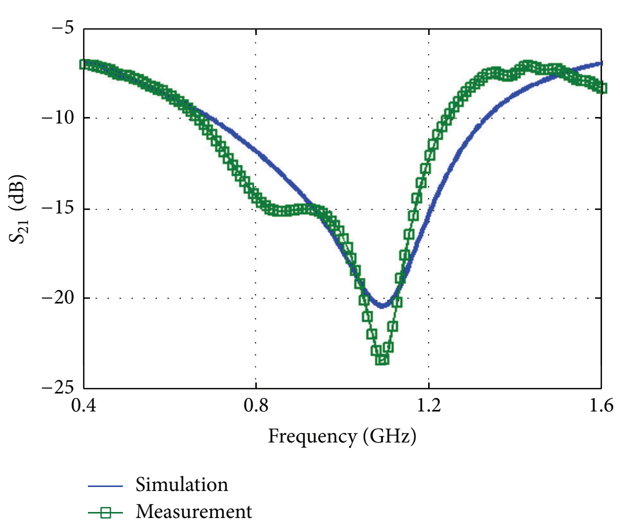

Figure 8: Comparison between magnitude of S_{21} in simulation and measurement for two-stage quarter-wave transmission line with branch-line coupler NGD circuit.

transformers to form microwave recursive filters. By properly choosing the impedance of transformer, we can realize the desired NGD. A table providing the associated values for desired NGD is given and utilized to synthesize the desired NGD. In addition, a branch-line coupler is used to transfer the group delay from one-port circuit to a two-port NGD circuit. The proposed method is promising to be further used in high frequency circuitries to synthesize any desired amount of NGD in order to compensate the undesired excessive group delay such as in feed-forward amplifiers and envelop tracking power amplifier, which can improve the system overall efficiency.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] L. Brillouin and A. Sommerfeld, *Wave Propagation and Group Velocity*, Academic Press, New York, NY, USA, 1960.

[2] C. G. B. Garrett and D. E. McCumber, "Propagation of a Gaussian light pulse through an anomalous dispersion medium," *Physical Review A*, vol. 1, no. 2, pp. 305–313, 1970.

[3] S. Chu and S. Wong, "Linear pulse propagation in an absorbing medium," *Physical Review Letters*, vol. 48, no. 11, pp. 738–741, 1982.

[4] L. J. Wang, A. Kuzmich, and A. Dogariu, "Gain-assisted superluminal light propagation," *Nature*, vol. 406, no. 6793, pp. 277–279, 2000.

[5] M. S. Bigelow, N. N. Lepeshkin, and R. W. Boyd, "Superluminal and slow light propagation in a room-temperature solid," *Science*, vol. 301, no. 5630, pp. 200–202, 2003.

[6] H. Choi, Y. Jeong, C. D. Kim, and J. S. Kenney, "Efficiency enhancement of feedforward amplifiers by employing a negative group-delay circuit," *IEEE Transactions on Microwave Theory and Techniques*, vol. 58, no. 5, pp. 1116–1125, 2010.

[7] S.-S. Oh and L. Shafai, "Compensated circuit with characteristics of lossless double negative materials and its application to array antennas," *IET Microwaves, Antennas and Propagation*, vol. 1, no. 1, pp. 29–38, 2007.

[8] H. Noto, K. Yamauchi, M. Nakayama, and Y. Isota, "Negative group delay circuit for feed-forward amplifier," in *Proceedings of the IEEE MTT-S International Microwave Symposium (IMS '07)*, pp. 1103–1106, June 2007.

[9] C.-T. M. Wu and T. Itoh, "Maximally flat negative group-delay circuit: a microwave transversal filter approach," *IEEE Transactions on Microwave Theory and Techniques*, vol. 62, no. 6, pp. 1330–1342, 2014.

[10] C. Rauscher, "Microwave active filters based on transversal and recursive principles," *IEEE Transactions on Microwave Theory and Techniques*, vol. 33, no. 12, pp. 1350–1360, 1985.

[11] G. Chaudhary and Y. Jeong, "Distributed transmission line negative group delay circuit with improved signal attenuation,"

IEEE Microwave and Wireless Components Letters, vol. 24, no. 1, pp. 20–22, 2014.

[12] D. M. Pozar, *Microwave Engineering*, John Wiley & Sons, 2009.

[13] S. Lucyszyn and I. D. Robertson, "Analog reflection topology building blocks for adaptive microwave signal processing applications," *IEEE Transactions on Microwave Theory and Techniques*, vol. 43, no. 3, pp. 601–611, 1995.