Research Article

A Lagrange Relaxation Method for Solving Weapon-Target Assignment Problem

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We study the weapon-target assignment (WTA) problem which has wide applications in the area of defense-related operations research. This problem calls for finding a proper assignment of weapons to targets such that the total expected damaged value of the targets to be maximized. The WTA problem can be formulated as a nonlinear integer programming problem which is known to be NP-complete. There does not exist any exact method for the WTA problem even small size problems, although several heuristic methods have been proposed. In this paper, Lagrange relaxation method is proposed for the WTA problem. The method is an iterative approach which is to decompose the Lagrange relaxation into two subproblems, and each subproblem can be easy to solve to optimality based on its specific features. Then, we use the optimal solutions of the two subproblems to update Lagrange multipliers and solve the Lagrange relaxation problem iteratively. Our computational efforts signify that the proposed method is very effective and can find high quality solutions for the WTA problem in reasonable amount of time.

1. Introduction

Weapon-target assignment (WTA) problem is a fundamental problem arising in defense-related applications, which involve calling for finding a proper assignment of weapons to targets such that the total expected damaged value of the targets is to be maximized. It is a specific case of the more general resource allocation problem.

WTA problem has been of interest to many researchers for several decades [1–4]. This problem can be formulated as a nonlinear integer programming problem which is known to be NP-complete. Ravindra et al. provided a brief and comprehensive survey of the relevant literature in this area and proposed a network flow-based construction heuristic algorithm [2]. There does not exist any exact methods for the WTA problem even relatively small size problems, and much research has focused on developing heuristic algorithms based on metaheuristic techniques, such as neural networks [5], genetic algorithms [6–9], tabu

search algorithm [10], simulated annealing algorithm [11], and other expert systems [12]. These heuristic methods may not be able to produce an acceptable or even feasible solution, which is not allowed in a warfare scenario. Moreover, since no exact algorithm is available to solve WTA problem, it is unavailable to estimate the quality of solutions produced by such heuristics. A feasible solution corresponds to a WTA schedule. The purpose of this paper is to find feasible solutions in a reasonably fast time to help decision makers to make proper scheme on the battlefield.

In this paper, we present a Lagrange relaxation method to solve WTA problem. First, a discrete variable is introduced in the formulation to transform the nonlinear integer programming problem into a linear optimization problem with the objective function is linear and the constraints respect to the original variables are linear and the introduced variable is nonlinear. Then we construct a Lagrange relaxation problem where the constraints are relaxed with Lagrange multipliers. The relaxation problem can be decomposed into two: one concerned with the introduced discrete variable is nonlinear optimization problem and the other concerned with the original variables is linear. We get the advantages of working with these two natural subproblems: the former subproblem can be decomposed into several one-dimensional discrete optimization problems which can be parallel processed easily. The latter subproblem's constraint matrix is totally unimodular and thus can be solved by applying the simplex method. Therefore, the Lagrange relaxation problem is quite easy to solve, and any optimal integer solutions to the linear programming with respect to the original variables is feasible to the WTA problem. Then we use the optimal solutions of the two subproblems to update Lagrange multipliers and solve the Lagrange relaxation problem iteratively. The algorithm terminates due to a lack of improvement in the best solution over a number of generations. Computational results show that this method can be very successful for WTA problem.

This paper is organized as follows: in Section 2, we describe WTA problem and its formulation. In Section 3, we present a Lagrange relaxation method for solving WTA problem. Then, the results of employing the proposed algorithm to solve WTA problem are presented in Section 4. Finally, in Section 5 we give some concluding remarks and possible future work in this area.

2. The WTA Problem

To formulate the weapon-target assignment problem, we use the following notation and variables.

W: The number of weapon types.

T: The number of targets that must be engaged.

u_j: The value of target j. This is determined during the threat evaluation phase and used to priorities target engagement.

w_i: The number of weapons of type i available to be assigned to targets.

t_j: The minimum number of weapons required for target j.

p_{ij}: The probability of destroying target j by a single weapon of type i, also referred to as the kill probability for weapon i on target j. It's known for all i and j.

x_{ij}: an integer decision variable indicating the number of weapons of type i assigned to target j.

Then the WTA problem may now be modeled as the following nonlinear integer programming formulation in terms of the above variables [1, 4],

$$\max \quad \sum_{j=1}^{T} u_j \left(1 - \prod_{i=1}^{W} (1 - P_{ij})^{x_{ij}} \right), \tag{2.1}$$

$$\text{s.t.} \quad \sum_{j=1}^{T} x_{ij} \leq w_i \quad i = 1, 2, \ldots, W, \tag{2.2}$$

$$\sum_{i=1}^{W} x_{ij} \geq t_j \quad j = 1, 2, \ldots, T, \tag{2.3}$$

$$x_{ij} \geq 0, \quad \text{integral}, \quad i = 1, 2, \ldots, W, \ j = 1, 2, \ldots, T. \tag{2.4}$$

Since p_{ij} is the destroying probability for weapon i on target j, the $1 - p_{ij}$ term in (2.1) therefore denotes the probability of survival for target j if weapon i is assigned to it. Objective function (2.1) maximizes the probability of the total expected damaged value of the targets. Equation (2.2) provides a constraint to the problem that ensures that the total number of weapons used does not exceed what is available, (2.3) provides a constraint to the problem that ensures that the total number of weapons used should exceed the minimum number of weapons required for target j. Equation (2.4) provides a constraint that ensures that the number of weapons assigned to target j is nonnegative and discrete.

3. Lagrange Relaxation Method

WTA is computationally intractable because of the nonlinear of objective function and the integrality of variable. For problems involving small numbers of weapons and targets, these problems can be solved using any general purpose nonlinear integer programming package. For larger problems, faster algorithms are desired. So we are interested in solving these problems using Lagrange relaxation method as follow. For $j = 1, 2, \ldots, T$, let

$$y_j = 1 - \prod_{i=1}^{W} (1 - P_{ij})^{x_{ij}}, \tag{3.1}$$

$$D_j = \left\{ y_j \mid y_j = 1 - \prod_{i=1}^{W} (1 - P_{ij})^{x_{ij}}, \ x_{ij} = 0, 1, 2, \ldots, w_i \right\}. \tag{3.2}$$

Then D_j is a discrete set. Let

$$(x, y) = (x_{11}, x_{12}, \ldots, x_{1T}, x_{21}, x_{22}, \ldots, x_{2T}, \ldots, x_{W1}, x_{W2}, \ldots, x_{WT}, y_1, \ldots, y_T). \tag{3.3}$$

Then for (x, y), we have

$$\sum_{i=1}^{W} (\ln(1 - P_{ij}) x_{ij}) = \ln(1 - y_j), \quad j = 1, 2, \ldots, T. \tag{3.4}$$

WTA problem can be transformed as follows:

$$\max \quad \sum_{j=1}^{T} u_j y_j, \tag{P}$$

$$\text{s.t.} \quad (x,y) \in \{(x,y) \mid (x,y) \text{ satisfy (2.2), (2.3), (2.4), and (3.4), } y_j \in D_j\},$$

We observed that problem $((P))$ remove the nonlinear of objective function by increasing T discrete variable y_j and will lead to some difficulties in the calculation. To avoid the problem we guide searching with y_j by mapping between y_j and x_{ij} in (3.1). We first derive the Lagrangian function as follows:

$$(L(\lambda)) \quad \max \quad L(x,y,\lambda),$$
$$\text{s.t.} \quad (x,y) \in \{(x,y) \mid (x,y) \text{ satisfy (2.2), (2.3), (2.4), and (3.4), } y_j \in D_j\}, \tag{3.5}$$

where

$$L(x,y,\lambda) = \sum_{j=1}^{T} u_j y_j + \sum_{j=1}^{T} \lambda_j \left(\sum_{i=1}^{W} (\ln(1-P_{ij})) x_{ij} - \ln(1-y_j) \right)$$
$$= \sum_{j=1}^{T} (u_j y_j - \lambda_j \ln(1-y_j)) + \sum_{j=1}^{T} \sum_{i=1}^{W} \lambda_j (\ln(1-P_{ij})) x_{ij}, \tag{3.6}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_T)$ is Lagrange multiplier, $\lambda_i \in R$, $i = 1, 2, \ldots, T$, $L(x,y,\lambda)$ is Lagrange function. It is obvious that $((P))$ can be decomposed into two subproblems as follows:

$$(L_1(x)) \quad \max \quad \sum_{j=1}^{T} \sum_{i=1}^{W} \lambda_j (\ln(1-P_{ij})) x_{ij}, \tag{3.7}$$
$$\text{s.t.} \quad x \in \{x \mid \text{for fixed } y, \, x \text{ satisfy (2.2), (2.3), and (2.4)}\},$$

$$(L_2(y)) \quad \max \quad \sum_{j=1}^{T} (u_j y_j - \lambda_j \ln(1-y_j)), \tag{3.8}$$
$$\text{s.t.} \quad y = (y_1, y_2, \ldots, y_T) \in \{y \mid y_j \in D_j, \, j = 1, 2, \ldots, T\}.$$

Theorem 3.1. *Let x^*, y^* be optimal solutions of the two subproblems $L_1(x)$ and $L_2(y)$, respectively. If x^*, y^* satisfy (3.4), then x^* is optimal solution of WTA.*

Proof. Since x^*, y^* are optimal solutions of the two subproblems $L_1(x)$ and $L_2(y)$, respectively so x^*, y^* are also feasible solutions of the two subproblems $L_1(x)$ and $L_2(y)$, respectively. If

x^*, y^* satisfy (3.4), then x^* is optimal solution of WTA. Let x is any feasible solution of WTA, for $j = 1, 2, \ldots, T$, let

$$y_j = 1 - \prod_{i=1}^{W} (1 - P_{ij}) x_{ij}. \tag{3.9}$$

Then (x, y) also satisfy (3.4) and we have

$$
\begin{aligned}
\sum_{j=1}^{T} u_j &\left(1 - \prod_{i=1}^{W} (1 - P_{ij}) x_{ij}^* \right) \\
&= \sum_{j=1}^{T} u_j y_j^* \\
&= \sum_{j=1}^{T} u_j y_j^* + \sum_{j=1}^{T} \lambda_j \left(\sum_{i=1}^{W} (\ln(1 - P_{ij})) x_{ij}^* - \ln\left(1 - y_j^*\right) \right) \\
&= \sum_{j=1}^{T} \left(u_j y_j^* - \lambda_j \ln\left(1 - y_j^*\right) \right) + \sum_{j=1}^{T} \lambda_j \left(\sum_{i=1}^{W} (\ln(1 - P_{ij})) x_{ij}^* \right) \\
&\geq \sum_{j=1}^{T} \left(u_j y_j - \lambda_j \ln(1 - y_j) \right) + \sum_{j=1}^{T} \lambda_j \left(\sum_{i=1}^{W} (\ln(1 - P_{ij})) x_{ij} \right) \\
&= \sum_{j=1}^{T} u_j y_j + \sum_{j=1}^{T} \lambda_j \left(\sum_{i=1}^{W} (\ln(1 - P_{ij})) x_{ij} - \ln(1 - y_j) \right) \\
&= \sum_{j=1}^{T} u_j y_j \\
&= \sum_{j=1}^{T} u_j \left(1 - \prod_{i=1}^{W} (1 - P_{ij}) x_{ij} \right).
\end{aligned}
\tag{3.10}
$$

Hence, x^* is optimal solution of WTA.   □

Obviously, subproblem $L_2(y)$ can be decomposed into T 1-dimensional discrete problem as follows ($j = 1, 2, \ldots, T$):

$$
\begin{aligned}
(L_2(y)) \quad &\max \quad (u_j y_j - \lambda_j \ln(1 - y_j)), \\
&\text{s.t.} \quad y_j \in \{ y_j \mid y_j \in D_j \}.
\end{aligned}
\tag{3.11}
$$

Theorem 3.2. *For $j = 1, 2, \ldots, T$, let*

$$\alpha_j = 1 - \left(1 - \left(\min_{1 \leq i \leq W} P_{ij}\right)\right)^{T_j},$$

$$\beta_j = 1 - \left(1 - \left(\max_{1 \leq i \leq W} P_{ij}\right)\right)^{\sum_{i=1}^{W} w_i}, \tag{3.12}$$

one has $\alpha_j \leq y_j \leq \beta_j$.

Proof. For $j = 1, 2, \ldots, T$, we have

$$
\begin{aligned}
y_j &= 1 - \prod_{i=1}^{W} (1 - P_{ij})^{x_{ij}} \\
&\leq 1 - \prod_{i=1}^{W} \left(1 - \left(\max_{1 \leq i \leq W} P_{ij}\right)\right)^{x_{ij}} \\
&= 1 - \left(1 - \left(\max_{1 \leq i \leq W} P_{ij}\right)\right)^{\sum_{i=1}^{W} x_{ij}} \\
&\leq 1 - \left(1 - \left(\max_{1 \leq i \leq W} P_{ij}\right)\right)^{\sum_{i=1}^{W} w_i} \\
&= \beta_j.
\end{aligned}
\tag{3.13}
$$

Note that

$$
\begin{aligned}
y_j &= 1 - \prod_{i=1}^{W} (1 - P_{ij})^{x_{ij}} \\
&\geq 1 - \prod_{i=1}^{W} \left(1 - \left(\min_{1 \leq i \leq W} P_{ij}\right)\right)^{x_{ij}} \\
&= 1 - \left(1 - \left(\min_{1 \leq i \leq W} P_{ij}\right)\right)^{\sum_{i=1}^{W} x_{ij}} \\
&\geq 1 - \left(1 - \left(\min_{1 \leq i \leq W} P_{ij}\right)\right)^{T_j} \\
&= \alpha_j.
\end{aligned}
\tag{3.14}
$$

Thus, $\alpha_j \leq y_j \leq \beta_j$.

Consider the relaxation form of 1-dimensional discrete problem $L_{2j}(y_j)$ as follows:

$$
\begin{aligned}
(SL_{2j}(y_j)) \ \max, \quad & (u_j y_j - \lambda_j \ln(1 - y_j)), \\
\text{s.t.} \quad & y_j \in \{y_j \mid \alpha_j \leq y_j \leq \beta_j\}.
\end{aligned}
\tag{3.15}
$$

For given u_j and λ_j $SL_{2j}(y_j)$ is a convexity programming on a closed interval that can be solved easily, when $\lambda_j \neq 0$, the optimal solution is

$$
y_j^s = \begin{cases}
\alpha_j, & 1 + \dfrac{\lambda_j}{u_j} \leq \alpha_j, \\[2ex]
1 + \dfrac{\lambda_j}{u_j}, & 1 + \dfrac{\lambda_j}{u_j} \in (\alpha_j, \beta_j), \\[2ex]
\beta_j, & 1 + \dfrac{\lambda_j}{u_j} \geq \beta_j,
\end{cases}
\tag{3.16}
$$

when $\lambda_j = 0$, the optimal solution is

$$
y_j^s = \beta_j.
\tag{3.17}
$$

□

Theorem 3.3. *Let* y_j^1, $y_j^2 \in D_j$, $y_j^1 = \max\{y_j \mid y_j \leq y_j^s, y_j \in D_j\}$, $y_j^2 = \min\{y_j \mid y_j \geq y_j^s, y_j \in D_j\}$, *then one has*

$$
y_j^* = \mathrm{argmax}\left\{ u_j y_j^1 - \lambda_j \ln\left(1 - y_j^1\right), \ u_j y_j^2 - \lambda_j \ln\left(1 - y_j^2\right) \right\},
\tag{3.18}
$$

which is the optimal solution of 1-dimensional discrete problem $L_{2j}(y_j)$.

Since $SL_{2j}(y_j)$ *is a convexity programming on a closed interval, Theorem 3.3 is obvious.*

Next, we prove that the subproblem $L_1(x, y, \lambda)$ can be solved as a linear programming.

Definition 1 (see [13]). An $m \times n$ integral matrix A is totally unimodular (TU) if the determinant of each square submatrix of A is equal to 0, 1, or −1.

Corollary 3.4 (see [13]). *If A is TU, then* $P(b) = \{x \in R_+^n : Ax \leq b\}$ *is integral for all* $b \in Z^m$ *for which it is not empty.*

Corollary 3.5 (see [13]). *If the (0,1,−1) matrix A has no more than two nonzero entries in each column and if* $\sum_i a_{ij} = 0$ *if column j contains two nonzero coefficients, then* A *is TU.*

Theorem 3.6. *The optimal solution of integer linear programming* $L_1(x)$ *can be obtained by solving* $L_1(x)$ *as a linear programming using simplex method.*

Proof. Integer linear programming $L_1(x)$ can be described as follows:

$$
\begin{aligned}
\max \quad & \sum_{j=1}^{T}\sum_{i=1}^{W}\lambda_j\left(\ln(1 - P_{ij})\right)x_{ij}, \\
\text{s.t.} \quad & \sum_{j=1}^{T}x_{ij} \leq w_i \quad i = 1,2,\ldots,W, \\
& \sum_{i=1}^{W}(-x_{ij}) \leq -t_j, \quad j = 1,2,\ldots,T, \\
& x_{ij} \geq 0, \quad \text{integral}, \quad i = 1,2,\ldots,W, \ j = 1,2,\ldots,T.
\end{aligned}
\tag{3.19}
$$

□

Note that the constraint matrix is a (0,1,–1) matrix A. Without loss of generality, assume that the column of A corresponds with the variable x_{ij} and contains two nonzero elements, for example, the i element is 1 and the $w + j$ element is –1. Thus, the sum of two nonzero elements is zero. By Corollary 3.5, constraint matrix A is TU. Hence, the optimal solution of integer linear programming $L_1(x)$ can be obtained by solving $L_1(x)$ as a linear programming using simplex method.

4. Proposed Algorithm and Numerical Result

By Theorem 3.1, for given Lagrange multiplier $\lambda^k = (\lambda_1^k, \lambda_2^k, \dots, \lambda_T^k)$, assume that $x^k y^k$ are the optimal solutions of two subproblems $L_1(x)$ and $L_2(y)$, if x^k and y^k satisfy (3.4), then x^k is the optimal solution of WTA. Otherwise, update Lagrange multiplier. Let

$$g_j\left(x^k, y^k\right) = \sum_{i=1}^{W}(\ln(1 - P_{ij}))x_{ij}^k - \ln\left(1 - y_j^k\right). \tag{4.1}$$

Update λ with

$$\lambda_j^{k+1} = \begin{cases} -1, & g_j(x^k) > 0, \\ \lambda_j^k, & g_j(x^k) = 0, \\ 1, & g_j(x^k) < 0. \end{cases} \tag{4.2}$$

The proposed Lagrange relaxation algorithm solving WTA problem can be summarized as follows.

Step 1. Set the maximum iterate number K, Lagrange multiplier $\lambda^0 = (\lambda_1^0, \lambda_2^0, \dots, \lambda_T^0)$, $\lambda_i^0 = 0$ ($i = 1, 2, \dots, T$) and $k = 0$, $z = 0$, $x^* = 0$.

Step 2. For fixed λ^k, solve $L_1(x)$ as linear programming using simplex method, obtain integral optimal solution x^k and optimal value z^k if $z^k \geq z$, then $z = z^k$, $x^* = x^k$.

Step 3. If $k \geq K$, stop.

Step 4. Solve $L_2(y)$ and get the optimal solution y^k. Compute $g_j(x^k, y^k)$ by (4.1). If

$$g_j\left(x^k, y^k\right) = 0, \quad j = 1, 2, \dots, T \tag{4.3}$$

Stop. Otherwise, update Lagrange multiplier λ^k with (4.2) and get λ^{k+1}, set $k := k + 1$, return Step 2.

The proposed algorithm can make the computation more efficient. Because solving the relaxation linear programming is much easier than solving binary integer linear programming in Step 2. In Step 4, we first calculate the optimal solutions of T 1-dimensional discrete problem $SL_{2j}(y_j)$, then solve $L_2(y)$ with dichotomy. In order to narrow the search

Table 1: Numerical results of Lagrange relaxation method.

Dataset	Metrics (W,T)	Number of integral variables	Initial objective value	Best objective value	Improve rate	Computing time (sec)	
						Initial solution	Best solution
1	$W = 4$, $T = 7$	28	3.37	4.1995	24.61%	0.002	0.005
2	$W = 20$, $T = 30$	600	11.7959	13.7753	16.78%	0.003	0.012
3	$W = 30$, $T = 40$	1200	14.6271	17.3206	18.41%	0.003	0.015
4	$W = 40$, $T = 50$	2000	18.7093	22.1186	18.22%	0.003	0.015
5	$W = 40$, $T = 70$	2800	30.0852	34.0953	13.33%	0.004	0.016
6	$W = 40$, $T = 70$	2800	25.6031	30.625	19.61%	0.004	0.018
7	$W = 30$, $T = 100$	3000	40.0767	45.1007	12.54%	0.007	0.015
8	$W = 30$, $T = 100$	3000	40.321	45.255	12.24%	0.006	0.016
9	$W = 50$, $T = 100$	5000	42.5188	47.0363	10.62%	0.008	0.038
10	$W = 50$, $T = 100$	5000	41.511	45.2753	9.07%	0.009	0.024
11	$W = 100$, $T = 100$	10000	40.6264	44.7785	10.22%	0.031	0.061
12	$W = 100$, $T = 100$	10000	45.0092	49.8157	10.68%	0.016	0.041
13	$W = 100$, $T = 150$	15000	62.9031	67.2282	6.88%	0.047	0.703
14	$W = 100$, $T = 150$	15000	67.0165	72.3343	7.94%	0.063	0.835
15	$W = 150$, $T = 150$	22500	69.5433	74.1215	6.58%	0.078	1.927
16	$W = 150$, $T = 150$	22500	68.9607	74.9836	8.73%	0.062	1.006
17	$W = 100$, $T = 300$	30000	122.0562	129.9045	6.43%	0.094	2.313
18	$W = 100$, $T = 300$	30000	133.6946	145.4427	8.79%	0.093	3.142
19	$W = 200$, $T = 200$	40000	92.3307	98.6923	6.89%	0.125	9.315
20	$W = 200$, $T = 200$	40000	96.3919	106.7961	10.79%	0.112	9.637

space, the algorithm guide searching by updating Lagrange multiplier using mapping between y_j and x_{ij} in $\{-1, 0, 1\}^T$.

Our algorithm has been coded in Matlab 2009 and implemented on an Intel Core 2, CPU 2.53 Ghz, RAM 2 GB, Windows XP-System. We generate twenty random datasets in Matlab 2009. The settings are as follows: $u_j \in [0.3, 0.9]$, $P_{ij} \in [0.2, 0.9]$, $w_i \in [10, 15]$, $t_j \in [1, 4]$, $K = 100$. T, and W are given in Table 1. The number of integral variables in datasets from 28 to 40000. The numerical results are given in Table 1.

From the numerical results, we observe that our algorithm can find a good initial feasible solution quickly and make some improvements based on the obtained initial feasible solutions after some iteration. With the increasing of problem scales, the computing time required is acceptable. Hence, for large-scale problems, our algorithm can find a good feasible solution in reasonable computing time and is an efficient method to deal with WTA problem on the battlefield.

5. Conclusion

In this paper, Lagrange relaxation method is proposed for WTA. The method first decompose the Lagrange relaxation problem into two subproblems, and each subproblem can be easy to solve to optimality based on its specific features. Then use the optimal solutions of the two subproblems to update Lagrange multipliers and solve the Lagrange relaxation problem
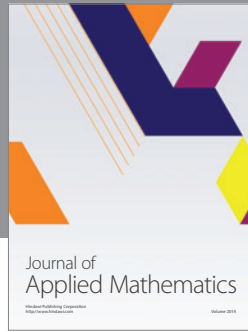
iteratively. The proposed method can find a good initial feasible solution quickly and make some improvements. The computational results obtained show that the proposed method is efficient. One direction of further research is to solve $L_2(y)$ in Theorem 3.3 accurately. The solution obtained by dichotomy usually is near optimal solution. This can lead to some difficulties in solving WTA and increases the number of iteration in calculating Lagrange relaxation problem.

Acknowledgment

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