

Research Article

Optimal Replenishment Policy for Weibull-Distributed Deteriorating Items with Trapezoidal Demand Rate and Partial Backlogging

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Received 2 December 2015; Accepted 27 March 2016

Academic Editor: Alireza Amirteimoori

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An inventory model for Weibull-distributed deteriorating items is considered so as to minimize the total cost per unit time in this paper. The model starts with shortage, allowed partial backlogging, and trapezoidal demand rate. By analyzing the model, an efficient solution procedure is proposed to determine the optimal replenishment and the optimal order quantity and the average total costs are also obtained. Finally, numerical examples are provided to illustrate the theoretical results and a sensitivity analysis of the major parameters with respect to the stability of optimal solution is also carried out.

1. Introduction

Deteriorating is a general phenomenon for many products, in that fruits or vegetables are spoiled directly while alcohol undergoes physical depletion over time, and electronic products deteriorate through a gradual loss of potential utility as time passed. Based on the above situation, the effect of deteriorating for items can not be disregarded in many inventory systems. An inventory model with a constant decaying rate for items is proposed by Ghare and Schrader in 1963 [1], while Covert and Philip [2] extended Ghare and Schrader's model to an EOQ model with a two-parameter Weibull-distributed deteriorating rate. Since then, there are many researchers who focused on such topic; Wee et al. [3] considered an inventory model for deteriorating items with quantity discount, pricing, and partial backlogging, and more related articles can be referred to like Yang et al. [4], Shah et al. [5], Yang et al. [6], and so forth.

However, it is impossible for deteriorating items that the demand rate increases continuously during their full life cycle. Based on such reasons, Hill [7] proposed an inventory model with ramp-type demand rate, which extended to allow shortage proposed by Mandal and Pal [8]. Further,

Wu [9] extended Mandal and Pal's model to have Weibull-distributed deterioration and time-dependent backlogging. Skouri et al. [10] considered a model with general ramp-type demand rate, partial backlogging, and Weibull deterioration rate. Tan and Weng [11] considered a discrete inventory model for deteriorating items with partial backlogging. Pal et al. [12] considered a production inventory model for deteriorating items with ramp-type demand rate under the effect of inflation and shortages under fuzziness. There are many other related literatures about such inventory model, such as Ahmed et al. [13], Agrawal et al. [14], Singh and Pattanayak [15], Wu et al. [16], and Rabbani et al. [17].

While, for some short life cycle products, the demand rate may increase up to a certain level in the early stage of marketing, then reach a stabilized period, and finally decrease to zero and reach the end of their life cycle, Cheng and Wang [18] proposed an inventory model for deteriorating items with trapezoidal type demand rate. Further, Cheng et al. [19] extended the model to be of time-dependent deteriorating items with trapezoidal type demand rate and partial backlogging. Uthayakumar and Rameswari [20] studied a model for defective items with trapezoidal type demand rate to determine the optimal product reliability. Lin [21] and

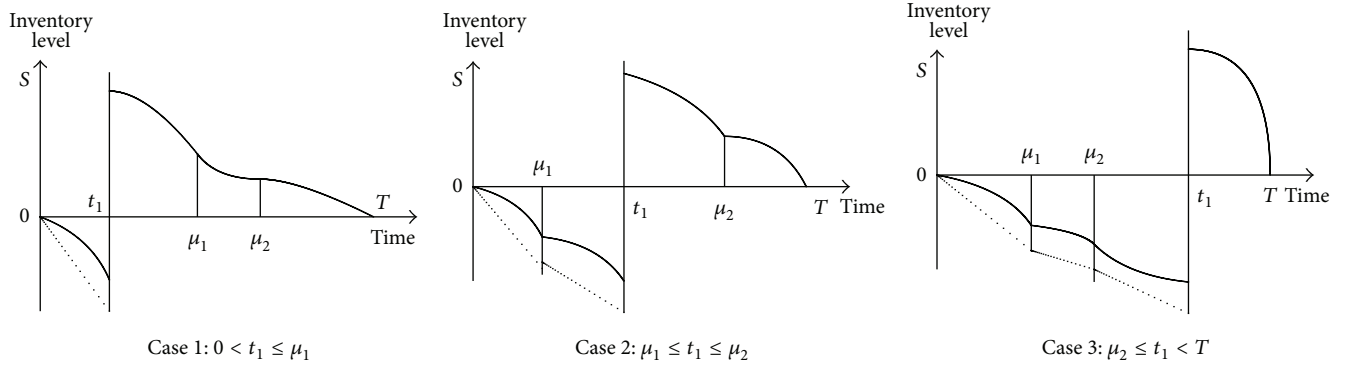


FIGURE 1: Graphical representation of inventory level over the cycle starting with shortage.

Lin et al. [22] considered the inventory model which was proposed by Cheng and Wang [18]. Zhao [23] studied an inventory model for Weibull-distributed deterioration items with trapezoidal type demand rate and partial backlogging in which the model begins with no-shortage. While, in real situation, customers will respond differently in case of shortages, either to wait until replenishment or to look for alternative products, in this paper, we study an inventory model beginning with shortage, Weibull-distributed deterioration items, trapezoidal type demand rate, and time-dependent partial backlogging. By analyzing the studied model, we propose an optimal replenishment strategy for it and also carry out a sensitivity analysis of the main parameters.

The remainder of the paper is organized as follows. Notations and assumptions are described in Section 2. Section 3 constructs and analyzes the inventory model starting with shortage. Some numerical examples to illustrate the solution procedure are provided; then sensitivity analysis of the major parameters is also carried out in Section 4. Finally, a conclusion is presented.

2. Notations and Assumptions

The fundamental notations and assumptions used in this paper are given as follows.

- (i) $I(t)$ is the inventory level at time t , $0 \leq t \leq T$, where T is the length of inventory cycle.
- (ii) t_1 denotes the replenishment time, and t_1^* denotes the optimal t_1 .
- (iii) S is the maximum inventory level for each ordering cycle.
- (iv) Q^* denotes the optimal ordering quantity.
- (v) A_0 denotes the fixed cost per order.
- (vi) c_1 is the cost of each deteriorated item, c_2 is the unit inventory holding cost per unit time, c_3 is the unit shortage cost per unit time, and c_4 is the unit lost sales cost.
- (vii) $C_i(t_1)$, $i = 1, 2, 3$, denotes the average total cost per unit time under different conditions, respectively.

(viii) $TC(t_1)$ denotes the average total cost per unit time.

(ix) The demand rate, $D(t)$, which is positive and consecutive, is assumed to be a trapezoidal type function of time; that is,

$$D(t) = \begin{cases} f(t), & t \leq \mu_1, \\ D_0, & \mu_1 < t < \mu_2, \\ g(t), & \mu_2 \leq t < T, \end{cases} \quad (1)$$

where μ_1 is time point changing from the increasing demand function $f(t)$ to the constant demand D_0 , and μ_2 is time point changing from the constant demand D_0 to the decreasing demand function $g(t)$.

- (x) The replenishment rate is infinite; that is, replenishment is instantaneous.
- (xi) The time to deterioration of the item is distributed as Weibull (α, β) ; that is, the deterioration rate is $\theta(t) = \alpha\beta t^{\beta-1}$, where α is the scale parameter, $0 < \alpha < 1$, and β is the shape parameter, $0 < \beta$.
- (xii) Shortage is allowed and let $e^{-\delta t}$ be fraction of shortages backlogged, where t is the waiting time up to the next replenishment and δ is a tiny positive constant; that is, $\delta \ll 1/T$.

3. Inventory Model Starting with Shortage

The system of model begins with shortage and until t_1 the shortage achieves its maximum; then replenishment occurs at time $t = t_1$. All of the shortage demand during $(0, t_1)$ is partial backlogged (Figure 1). At t_1 , the total number of backlogged items will be satisfied by the replenishment and the inventory level up to S , while the deteriorating occurs during the time of $[t_1, T]$. According to the notations and

assumptions mentioned above, the model is described as shown in Figure 1:

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\delta(t_1-t)}D(t), & 0 < t < t_1, \\ -\theta(t)I(t) - D(t), & t_1 < t < T, \end{cases} \quad (2)$$

with boundary condition $I(0) = 0, I(t_1) = S, I(T) = 0$.

Based on the values of $t_1, \mu_1,$ and $\mu_2,$ three possible cases are presented as follows.

$$I(t) = \begin{cases} -\int_0^t e^{\delta(x-t_1)} f(x) dx, & 0 < t < t_1, \\ \int_t^{\mu_1} e^{\alpha(x^\beta-t^\beta)} f(x) dx + D_0 \int_{\mu_1}^{\mu_2} e^{\alpha(x^\beta-t^\beta)} dx + \int_{\mu_2}^T e^{\alpha(x^\beta-t^\beta)} g(x) dx, & t_1 < t < \mu_1, \\ D_0 \int_t^{\mu_2} e^{\alpha(x^\beta-t^\beta)} dx + \int_{\mu_2}^T e^{\alpha(x^\beta-t^\beta)} g(x) dx, & \mu_1 < t < \mu_2, \\ \int_t^T e^{\alpha(x^\beta-t^\beta)} g(x) dx, & \mu_2 < t < T. \end{cases} \quad (4)$$

The total replenishment quantity can be computed as

$$S = I(t_1) = \int_{t_1}^{\mu_1} e^{\alpha(x^\beta-t_1^\beta)} f(x) dx + D_0 \int_{\mu_1}^{\mu_2} e^{\alpha(x^\beta-t_1^\beta)} dx + \int_{\mu_2}^T e^{\alpha(x^\beta-t_1^\beta)} g(x) dx. \quad (5)$$

The total shortage quantity B_T during the interval $[0, t_1]$ is

$$B_T = \int_0^{t_1} (t_1 - t) e^{\delta(t-t_1)} f(t) dt. \quad (6)$$

The total number of perished items during the interval $[t_1, T],$ say $D_T,$ is

$$D_T = \int_{t_1}^{\mu_1} \left(e^{\alpha(t^\beta-t_1^\beta)} - 1 \right) f(t) dt + D_0 \int_{\mu_1}^{\mu_2} \left(e^{\alpha(t^\beta-t_1^\beta)} - 1 \right) dt + \int_{\mu_2}^T \left(e^{\alpha(t^\beta-t_1^\beta)} - 1 \right) g(t) dt. \quad (7)$$

The total number of inventories carried during the interval $[t_1, T],$ say $H_T,$ is

$$H_T = \int_{t_1}^{\mu_1} \int_t^{\mu_1} e^{\alpha(x^\beta-t^\beta)} f(x) dx dt + D_0 \int_{t_1}^{\mu_1} \int_{\mu_1}^{\mu_2} e^{\alpha(x^\beta-t^\beta)} dx dt$$

Case 1 ($0 < t_1 \leq \mu_1$). If the replenishment time $t_1 \in (0, \mu_1],$ from (2), we have

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\delta(t_1-t)} f(t), & 0 < t < t_1, \\ -\alpha\beta t^{\beta-1} I(t) - f(t), & t_1 < t < \mu_1, \\ -\alpha\beta t^{\beta-1} I(t) - D_0, & \mu_1 < t < \mu_2, \\ -\alpha\beta t^{\beta-1} I(t) - g(t), & \mu_2 < t < T. \end{cases} \quad (3)$$

Solving the differential equations (3) with $I(0) = 0,$ we have

$$\begin{aligned} & + \int_{t_1}^{\mu_2} \int_{\mu_2}^T e^{\alpha(x^\beta-t^\beta)} g(x) dx dt \\ & + D_0 \int_{\mu_1}^{\mu_2} \int_t^{\mu_2} e^{\alpha(x^\beta-t^\beta)} dx dt \\ & + \int_{\mu_2}^T \int_t^T e^{\alpha(x^\beta-t^\beta)} g(x) dx dt. \end{aligned} \quad (8)$$

The total of lost sales during the interval $[0, t_1],$ say $L_T,$ is

$$L_T = \int_0^{t_1} (1 - e^{\delta(t-t_1)}) f(t) dt. \quad (9)$$

Therefore, the average total cost per unit time under the condition $t_1 \leq \mu_1$ can be given by

$$\begin{aligned} C_1(t_1) &= \frac{1}{T} [A_0 + c_1 D_T + c_2 H_T + c_3 B_T + c_4 L_T] \\ &= \frac{1}{T} \left\{ A_0 + c_3 \left[\int_0^{t_1} (t_1 - t) e^{\delta(t-t_1)} f(t) dt \right] \right. \\ &+ c_4 \left[\int_0^{t_1} (1 - e^{\delta(t-t_1)}) f(t) dt \right] \\ &+ c_1 \left[\int_{t_1}^{\mu_1} \left(e^{\alpha(t^\beta-t_1^\beta)} - 1 \right) f(t) dt \right] \\ &+ D_0 \int_{\mu_1}^{\mu_2} \left(e^{\alpha(t^\beta-t_1^\beta)} - 1 \right) dt \end{aligned}$$

$$\begin{aligned}
 & + \int_{\mu_2}^T \left(e^{\alpha(t^\beta - t_1^\beta)} - 1 \right) g(t) dt \Big] \\
 & + c_2 \left[\int_{t_1}^{\mu_1} \int_t^{\mu_1} e^{\alpha(x^\beta - t^\beta)} f(x) dx dt \right. \\
 & + D_0 \int_{t_1}^{\mu_1} \int_{\mu_1}^{\mu_2} e^{\alpha(x^\beta - t^\beta)} dx dt \\
 & + \int_{t_1}^{\mu_2} \int_{\mu_2}^T e^{\alpha(x^\beta - t^\beta)} g(x) dx dt \\
 & + D_0 \int_{\mu_1}^{\mu_2} \int_t^{\mu_2} e^{\alpha(x^\beta - t^\beta)} dx dt \\
 & \left. + \int_{\mu_2}^T \int_t^T e^{\alpha(x^\beta - t^\beta)} g(x) dx dt \right] \Big\}. \tag{10}
 \end{aligned}$$

Taking the first and the second order derivative of $C_1(t_1)$ with respect to t_1 , respectively, we have

$$\begin{aligned}
 \frac{dC_1(t_1)}{dt_1} &= \frac{1}{T} \left\{ c_3 \int_0^{t_1} (1 - \delta t_1 + \delta t) e^{\delta(t-t_1)} f(t) dt \right. \\
 & + c_4 \delta \int_0^{t_1} e^{\delta(t-t_1)} f(t) dt - (c_2 + c_1 \alpha \beta t_1^{\beta-1}) \\
 & \left. \cdot \int_{t_1}^T e^{\alpha(t^\beta - t_1^\beta)} D(t) dt \right\}, \tag{11}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^2C_1(t_1)}{dt_1^2} &= \frac{1}{T} \left\{ c_3 \left[f(t_1) \right. \right. \\
 & + \delta \int_0^{t_1} (\delta t_1 - \delta t - 2) e^{\delta(t-t_1)} f(t) dt \Big] + c_4 \delta \left[f(t_1) \right. \\
 & - \delta \int_0^{t_1} e^{\delta(t-t_1)} f(t) dt \Big] + (c_1 \alpha \beta t_1^{\beta-1} + c_2) f(t_1) \\
 & + \alpha \beta t_1^{\beta-2} \left[c_1 (\alpha \beta t_1^\beta - \beta + 1) + c_2 t_1 \right] \\
 & \left. \cdot \int_{t_1}^T e^{\alpha(t^\beta - t_1^\beta)} D(t) dt \right\}. \tag{12}
 \end{aligned}$$

Then, we have

$$\left. \frac{dC_1(t_1)}{dt_1} \right|_{t_1=0} = -\frac{c_2}{T} \int_0^T e^{\alpha t^\beta} D(t) dt < 0, \tag{13}$$

and let

$$\begin{aligned}
 \Delta_1 &= c_3 \int_0^{\mu_1} (1 - \delta \mu_1 + \delta t) e^{\delta(t-\mu_1)} f(t) dt \\
 & + c_4 \delta \int_0^{\mu_1} e^{\delta(t-\mu_1)} f(t) dt - (c_2 + c_1 \alpha \beta \mu_1^{\beta-1}) \\
 & \cdot \left(D_0 \int_{\mu_1}^{\mu_2} e^{\alpha(t^\beta - \mu_1^\beta)} dt + \int_{\mu_2}^T e^{\alpha(t^\beta - \mu_1^\beta)} g(t) dt \right). \tag{14}
 \end{aligned}$$

Therefore, (i) if $\Delta_1 > 0$ and $d^2C_1(t_1)/dt_1^2 > 0$, there exists a unique solution $t_1^* \in (0, \mu_1)$ satisfying $dC_1(t_1)/dt_1 = 0$, and $C_1(t_1)$ obtains its minimum at $t_1 = t_1^*$; (ii) if $\Delta_1 < 0$ and $d^2C_1(t_1)/dt_1^2 > 0$, it means that $C_1(t_1)$ is a nonincreasing function and obtains its minimum value at point $t_1^* = \mu_1$; (iii) if the sign of $d^2C_1(t_1)/dt_1^2$ is indefinite, then the curve of $C_1(t_1)$ is W-shape, and $C_1(t_1)$ may obtain its minimum at more than one point; without loss of generality, set $t_1 = t_1^*$, where t_1^* is one of the solutions of $dC_1(t_1)/dt_1 = 0$.

Thus, the optimal value of the order level, $S = I(t_1^*)$, is

$$\begin{aligned}
 S^* &= \int_{t_1^*}^{\mu_1} e^{\alpha(x^\beta - t_1^{*\beta})} f(x) dx + D_0 \int_{\mu_1}^{\mu_2} e^{\alpha(x^\beta - t_1^{*\beta})} dx \\
 & + \int_{\mu_2}^T e^{\alpha(x^\beta - t_1^{*\beta})} g(x) dx, \tag{15}
 \end{aligned}$$

and the optimal order quantity Q^* is

$$Q^* = S^* + \int_0^{t_1^*} e^{\delta(t-t_1^*)} f(t) dt. \tag{16}$$

Case 2 ($\mu_1 \leq t_1 \leq \mu_2$). If the replenishment time $t_1 \in [\mu_1, \mu_2]$, then from (2), we have

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\delta(t_1-t)} f(t), & 0 < t < \mu_1, \\ -e^{-\delta(t_1-t)} D_0, & \mu_1 < t < t_1, \\ -\alpha \beta t^{\beta-1} I(t) - D_0, & t_1 < t < \mu_2, \\ -\alpha \beta t^{\beta-1} I(t) - g(t), & \mu_2 < t < T. \end{cases} \tag{17}$$

Solving the differential equation (17) with $I(0) = I(T) = 0$, we have

$$I(t) = \begin{cases} -\int_0^t e^{\delta(x-t_1)} f(x) dx, & 0 < t < \mu_1, \\ -\frac{D_0}{\delta} (e^{\delta(t-t_1)} - e^{\delta(\mu_1-t_1)}) - \int_0^{\mu_1} e^{\delta(x-t_1)} f(x) dx, & \mu_1 < t < t_1, \\ D_0 \int_t^{\mu_2} e^{\alpha(x^\beta - t^\beta)} dx + \int_{\mu_2}^T e^{\alpha(x^\beta - t^\beta)} g(x) dx, & t_1 < t < \mu_2, \\ \int_t^T e^{\alpha(x^\beta - t^\beta)} g(x) dx, & \mu_2 < t < T. \end{cases} \tag{18}$$

From (18), the total replenishment quantity can be computed:

$$S = I(t_1) = D_0 \int_{t_1}^{\mu_2} e^{\alpha(x^\beta - t_1^\beta)} dx + \int_{\mu_2}^T e^{\alpha(x^\beta - t_1^\beta)} g(x) dx. \quad (19)$$

The total shortage quantity during the interval $(0, t_1]$ is

$$B_T = \int_0^{\mu_1} (t_1 - t) e^{\delta(t-t_1)} f(t) dt + \frac{D_0}{\delta} \int_{\mu_1}^{t_1} (e^{\delta(t-t_1)} - e^{\delta(\mu_1-t_1)}) dt. \quad (20)$$

The total number of perished items in the interval $[t_1, T]$ is

$$D_T = D_0 \int_{t_1}^{\mu_2} (e^{\alpha(t^\beta - t_1^\beta)} - 1) dt + \int_{\mu_2}^T (e^{\alpha(t^\beta - t_1^\beta)} - 1) g(t) dt. \quad (21)$$

The total number of inventories carried during the interval $[t_1, T]$ is

$$H_T = D_0 \int_{t_1}^{\mu_2} \int_t^{\mu_2} e^{\alpha(x^\beta - t^\beta)} dx dt + \int_{t_1}^{\mu_2} \int_{\mu_2}^T e^{\alpha(x^\beta - t^\beta)} g(x) dx dt + \int_{\mu_2}^T \int_t^T e^{\alpha(x^\beta - t^\beta)} g(x) dx dt. \quad (22)$$

The total of lost sales during the interval $[0, t_1]$ is

$$L_T = \int_0^{\mu_1} (1 - e^{\delta(t-t_1)}) f(t) dt + D_0 \int_{\mu_1}^{t_1} (1 - e^{\delta(t-t_1)}) dt. \quad (23)$$

Therefore, the average total cost per unit time under the condition $\mu_1 < t_1 \leq \mu_2$ can be given by

$$C_2(t_1) = \frac{1}{T} [A_0 + c_1 D_T + c_2 H_T + c_3 B_T + c_4 L_T] = \frac{1}{T} \left\{ A_0 + c_1 \left[D_0 \int_{t_1}^{\mu_2} (e^{\alpha(t^\beta - t_1^\beta)} - 1) dt + \int_{\mu_2}^T (e^{\alpha(t^\beta - t_1^\beta)} - 1) g(t) dt \right] + c_2 \left[D_0 \int_{t_1}^{\mu_2} \int_t^{\mu_2} e^{\alpha(x^\beta - t^\beta)} dx dt + \int_{t_1}^{\mu_2} \int_{\mu_2}^T e^{\alpha(x^\beta - t^\beta)} g(x) dx dt + \int_{\mu_2}^T \int_t^T e^{\alpha(x^\beta - t^\beta)} g(x) dx dt \right] \right\}$$

$$+ c_4 \left[\int_0^{\mu_1} (1 - e^{\delta(t-t_1)}) f(t) dt + D_0 \int_{\mu_1}^{t_1} (1 - e^{\delta(t-t_1)}) dt \right] + c_3 \left[\int_0^{\mu_1} (t_1 - t) e^{\delta(t-t_1)} f(t) dt + \frac{D_0}{\delta} \int_{\mu_1}^{t_1} (e^{\delta(t-t_1)} - e^{\delta(\mu_1-t_1)}) dt \right] \}. \quad (24)$$

Taking the first and the second order derivative of $C_2(t_1)$ with respect to t_1 , respectively, we have

$$\frac{dC_2(t_1)}{dt_1} = \frac{1}{T} \left\{ c_4 \delta \left[\int_0^{\mu_1} e^{\delta(t-t_1)} f(t) dt + D_0 \int_{\mu_1}^{t_1} e^{\delta(t-t_1)} dt \right] - (c_1 \alpha \beta t_1^{\beta-1} + c_2) \cdot \left[D_0 \int_{t_1}^{\mu_2} e^{\alpha(t^\beta - t_1^\beta)} dt + \int_{\mu_2}^T e^{\alpha(t^\beta - t_1^\beta)} g(t) dt \right] + c_3 \left[\int_0^{\mu_1} (1 - \delta t_1 + \delta t) e^{\delta(t-t_1)} f(t) dt - D_0 (\mu_1 - t_1) e^{\delta(\mu_1-t_1)} \right] \right\}, \quad (25)$$

$$\frac{d^2 C_2(t_1)}{dt_1^2} = \frac{1}{T} \left\{ \alpha \beta t_1^{\beta-2} (c_1 (\alpha \beta t_1^\beta - \beta + 1) + c_2 t_1) \cdot \left[D_0 \int_{t_1}^{\mu_2} e^{\alpha(t^\beta - t_1^\beta)} dt + \int_{\mu_2}^T e^{\alpha(t^\beta - t_1^\beta)} g(t) dt \right] + (c_1 \alpha \beta t_1^{\beta-1} + c_2) D_0 + c_4 \delta \left[D_0 e^{\delta(\mu_1-t_1)} - \delta \int_0^{\mu_1} e^{\delta(t-t_1)} f(t) dt \right] + c_3 \left[\int_0^{\mu_1} (\delta^2 t_1 - \delta^2 t - 2\delta) e^{\delta(t-t_1)} f(t) dt + D_0 (1 + \delta \mu_1 - \delta t_1) e^{\delta(\mu_1-t_1)} \right] \right\}. \quad (26)$$

Let

$$\Delta_2 = c_3 \left[\int_0^{\mu_1} (1 - \delta \mu_2 + \delta t) e^{\delta(t-\mu_2)} f(t) dt + D_0 (\mu_2 - \mu_1) e^{\delta(\mu_1-\mu_2)} \right] + c_4 \delta \left[\int_0^{\mu_1} e^{\delta(t-\mu_2)} f(t) dt + D_0 \int_{\mu_1}^{\mu_2} e^{\delta(t-\mu_2)} dt \right] - (c_2 + c_1 \alpha \beta \mu_2^{\beta-1}) \int_{\mu_2}^T e^{\alpha(t^\beta - \mu_2^\beta)} g(t) dt. \quad (27)$$

Therefore, (i) if $\Delta_1 < 0, \Delta_2 > 0$, and $d^2C_2(t_1)/dt_1^2 > 0$, there exists a unique solution $t_1^* \in (\mu_1, \mu_2)$ satisfying $dC_2(t_1)/dt_1 = 0$, and $C_2(t_1)$ obtains its minimum at $t_1 = t_1^*$; (ii) if $\Delta_1 < 0, \Delta_2 \leq 0$, and $d^2C_2(t_1)/dt_1^2 > 0$, it means that $C_2(t_1)$ is a nonincreasing function and obtains its minimum at point $t_1^* = \mu_2$; (iii) if $\Delta_1 \geq 0, \Delta_2 > 0$, and $d^2C_2(t_1)/dt_1^2 > 0$, it means that $C_2(t_1)$ is a nondecreasing function and obtains its minimum at point $t_1^* = \mu_1$; (iv) if the sign of $d^2C_2(t_1)/dt_1^2$ is indefinite, then the curve of $C_2(t_1)$ is W-shape, and $C_2(t_1)$ may obtain its minimum value at more than one point; without loss of generality, set $t_1 = t_1^*$, where t_1^* is one of the solutions of $dC_2(t_1)/dt_1 = 0$.

Thus, the optimal value of the order level, $S = I(t_1^*)$, is

$$S^* = D_0 \int_{t_1^*}^{\mu_2} e^{\alpha(t_1^* - t)^\beta} dt + \int_{\mu_2}^T e^{\alpha(t_1^* - t)^\beta} g(t) dt, \quad (28)$$

and the optimal order quantity Q^* is

$$Q^* = S^* + \int_0^{\mu_1} e^{\delta(t-t_1^*)} f(t) dt + D_0 \int_{\mu_1}^{t_1^*} e^{\delta(t-t_1^*)} dt. \quad (29)$$

Case 3 ($\mu_2 \leq t_1 < T$). If the replenishment time $t_1 \in [\mu_2, T)$, from (2), we have

$$\frac{dI(t)}{dt} = \begin{cases} -e^{-\delta(t_1-t)} f(t), & 0 < t < \mu_1, \\ -e^{-\delta(t_1-t)} D_0, & \mu_1 < t < \mu_2, \\ -e^{-\delta(t_1-t)} g(t), & \mu_2 < t < t_1, \\ -\alpha\beta t^{\beta-1} I(t) - g(t), & t_1 < t < T. \end{cases} \quad (30)$$

Solving (30) with $I(0) = I(T) = 0$, we have

$$I(t) = \begin{cases} -\int_0^t e^{\delta(x-t_1)} f(x) dx, & 0 < t < \mu_1, \\ \frac{D_0}{\delta} (e^{\delta(\mu_1-t_1)} - e^{\delta(t-t_1)}) - \int_0^{\mu_1} e^{\delta(x-t_1)} f(x) dx, & \mu_1 < t < \mu_2, \\ -\int_0^t e^{\delta(x-t_1)} g(x) dx + \frac{D_0}{\delta} (e^{\delta(\mu_1-t_1)} - e^{\delta(\mu_2-t_1)}) - \int_0^{\mu_1} e^{\delta(x-t_1)} f(x) dx, & \mu_2 < t < t_1, \\ \int_t^T e^{\alpha(x^\beta-t^\beta)} g(x) dx, & t_1 < t < T. \end{cases} \quad (31)$$

From the last equation of (31), the total replenishment quantity can be computed:

$$S = I(t_1) = \int_{t_1}^T e^{\alpha(x^\beta-t_1^\beta)} g(x) dx. \quad (32)$$

The total shortage quantity during the interval $(0, t_1]$ is

$$B_T = \int_0^{\mu_1} (t_1 - t) e^{\delta(t-t_1)} f(t) dt + \int_{\mu_2}^{t_1} (t_1 - t) e^{\delta(t-t_1)} g(t) dt + \frac{D_0}{\delta^2} (e^{\delta(\mu_1-t_1)} - e^{\delta(\mu_2-t_1)}) + \frac{D_0}{\delta} [(t_1 - \mu_1) e^{\delta(\mu_1-t_1)} - (t_1 - \mu_2) e^{\delta(\mu_2-t_1)}]. \quad (33)$$

The total number of perished items in the interval $[t_1, T]$ is

$$D_T = \int_{t_1}^T (e^{\alpha(x^\beta-t_1^\beta)} - 1) g(x) dx. \quad (34)$$

The total number of inventories carried during the interval $[t_1, T]$ is

$$H_T = \int_{t_1}^T \int_t^T e^{\alpha(x^\beta-t^\beta)} g(x) dx dt. \quad (35)$$

The total of lost sales during the interval $[0, t_1]$ is

$$L_T = \int_0^{\mu_1} (1 - e^{\delta(t-t_1)}) f(t) dt + D_0 \int_{\mu_1}^{\mu_2} (1 - e^{\delta(t-t_1)}) dt + \int_{\mu_2}^{t_1} (1 - e^{\delta(t-t_1)}) g(t) dt. \quad (36)$$

Therefore, the average total cost per unit time under the condition $\mu_2 < t_1 \leq T$ can be given by

$$C_3(t_1) = \frac{1}{T} [A_0 + c_1 D_T + c_2 H_T + c_3 B_T + c_4 L_T] = \frac{1}{T} \left\{ A_0 + c_1 \int_{t_1}^T (e^{\alpha(x^\beta-t_1^\beta)} - 1) g(x) dx + c_2 \int_{t_1}^T \int_t^T e^{\alpha(x^\beta-t^\beta)} g(x) dx dt + c_4 \left[\int_0^{\mu_1} (1 - e^{\delta(t-t_1)}) f(t) dt + D_0 \int_{\mu_1}^{\mu_2} (1 - e^{\delta(t-t_1)}) dt \right] \right\}$$

$$\begin{aligned}
 & + \int_{\mu_2}^{t_1} (1 - e^{\delta(t-t_1)}) g(t) dt \Big] \\
 & + c_3 \left[\int_0^{\mu_1} (t_1 - t) e^{\delta(t-t_1)} f(t) dt \right. \\
 & + \int_{\mu_2}^{t_1} (t_1 - t) e^{\delta(t-t_1)} g(t) dt \\
 & + \frac{D_0}{\delta^2} (e^{\delta(\mu_1-t_1)} - e^{\delta(\mu_2-t_1)}) \\
 & \left. + \frac{D_0}{\delta} ((t_1 - \mu_1) e^{\delta(\mu_1-t_1)} - (t_1 - \mu_2) e^{\delta(\mu_2-t_1)}) \right] \Big\}. \tag{37}
 \end{aligned}$$

Taking the first and the second order derivative of $C_3(t_1)$ with respect to t_1 , respectively, we have

$$\begin{aligned}
 \frac{dC_3(t_1)}{dt_1} &= \frac{1}{T} \left\{ - (c_2 + c_1 \alpha \beta t_1^{\beta-1}) \int_{t_1}^T e^{\alpha(t^\beta - t_1^\beta)} g(t) dt \right. \\
 & + c_4 \delta \left[\int_0^{\mu_1} e^{\delta(t-t_1)} f(t) dt + D_0 \int_{\mu_1}^{\mu_2} e^{\delta(t-t_1)} dt \right. \\
 & \left. + \int_{\mu_2}^{t_1} e^{\delta(t-t_1)} g(t) dt \right] \\
 & + c_3 \left[\int_0^{\mu_1} (1 + \delta t - \delta t_1) e^{\delta(t-t_1)} f(t) dt \right. \\
 & + \int_{\mu_2}^{t_1} (1 + \delta t - \delta t_1) e^{\delta(t-t_1)} g(t) dt \\
 & \left. + D_0 ((t_1 - \mu_1) e^{\delta(\mu_1-t_1)} - (t_1 - \mu_2) e^{\delta(\mu_2-t_1)}) \right] \Big\}, \tag{38} \\
 \frac{d^2C_3(t_1)}{dt_1^2} &= \frac{1}{T} \left\{ \alpha \beta t_1^{\beta-2} [c_1 (\alpha \beta t_1^\beta - \beta + 1) + c_2 t_1] \right. \\
 & \cdot \int_{t_1}^T e^{\alpha(t^\beta - t_1^\beta)} g(t) dt + (c_2 + c_1 \alpha \beta t_1^{\beta-1}) g(t_1) \\
 & + c_4 \delta \left[\delta \int_0^{\mu_1} e^{\delta(t-t_1)} f(t) dt + D_0 \delta \int_{\mu_1}^{\mu_2} e^{\delta(t-t_1)} dt \right. \\
 & \left. + \delta \int_{\mu_2}^{t_1} e^{\delta(t-t_1)} g(t) dt + g(t_1) \right] \\
 & + c_2 \left[\alpha \beta t_1^{\beta-1} \int_{t_1}^T e^{\alpha(t^\beta - t_1^\beta)} g(t) dt + g(t_1) \right] \\
 & + c_3 \left[\int_0^{\mu_1} (\delta^2 t_1 - \delta^2 t - 2\delta) e^{\delta(t-t_1)} f(t) dt \right. \\
 & \left. + \int_{\mu_2}^{t_1} (\delta^2 t_1 - \delta^2 t - 2\delta) e^{\delta(t-t_1)} g(t) dt + g(t_1) \right]
 \end{aligned}$$

$$\begin{aligned}
 & + D_0 ((1 + \delta \mu_1 - \delta t_1) e^{\delta(\mu_1-t_1)} \\
 & - (1 + \delta \mu_2 - \delta t_1) e^{\delta(\mu_2-t_1)}) \Big\}. \tag{39}
 \end{aligned}$$

We know that

$$\begin{aligned}
 \frac{dC_3(t_1)}{dt_1} \Big|_{t_1=T} &= \frac{1}{T} \left\{ c_4 \delta \left[\int_0^{\mu_1} e^{\delta(t-T)} f(t) dt \right. \right. \\
 & + D_0 \left[\int_{\mu_1}^{\mu_2} e^{\delta(t-T)} dt + \int_{\mu_2}^T e^{\delta(t-T)} g(t) dt \right] \\
 & + c_3 \left[\int_0^{\mu_1} (1 + \delta t - \delta T) e^{\delta(t-T)} f(t) dt \right. \\
 & + \int_{\mu_2}^T (1 + \delta t - \delta T) e^{\delta(t-T)} g(t) dt \\
 & \left. \left. + D_0 ((T - \mu_1) e^{\delta(\mu_1-T)} - (T - \mu_2) e^{\delta(\mu_2-T)}) \right] \right\} \\
 & > 0.
 \end{aligned} \tag{40}$$

Therefore, (i) if $\Delta_2 < 0$ and $d^2C_3(t_1)/dt_1^2 > 0$, there exists a unique solution $t_1^* \in (\mu_2, T)$ satisfying $dC_3(t_1)/dt_1 = 0$, and $C_3(t_1)$ obtains its minimum at $t_1 = t_1^*$; (ii) if $\Delta_2 \geq 0$ and $d^2C_3(t_1)/dt_1^2 > 0$, it means that $C_3(t_1)$ is a nondecreasing function and obtains its minimum value at point $t_1^* = \mu_2$; (iii) if the sign of $d^2C_3(t_1)/dt_1^2$ is indefinite, then the curve of $C_3(t_1)$ is W-shape, and $C_3(t_1)$ may obtain its minimum value at more than one point; without loss of generality, set $t_1 = t_1^*$, where t_1^* is one of the solutions of $dC_3(t_1)/dt_1 = 0$.

Therefore, the optimal value of the order level, $S = I(t_1^*)$, is

$$S^* = \int_{t_1^*}^T e^{\alpha(x^\beta - t_1^{*\beta})} g(x) dx, \tag{41}$$

and the optimal order quantity Q^* is

$$\begin{aligned}
 Q^* &= S^* + \int_0^{\mu_1} e^{\delta(t-t_1^*)} f(t) dt + D_0 \int_{\mu_1}^{\mu_2} e^{\delta(t-t_1^*)} dt \\
 & + \int_{\mu_2}^{t_1^*} e^{\delta(t-t_1^*)} g(t) dt. \tag{42}
 \end{aligned}$$

From the above analysis, we obtain that the total average cost of the system over the interval $[0, T]$ is

$$TC(t_1) = \begin{cases} C_1(t_1), & 0 < t_1 \leq \mu_1, \\ C_2(t_1), & \mu_1 < t_1 \leq \mu_2, \\ C_3(t_1), & \mu_2 < t_1 < T, \end{cases} \tag{43}$$

where $C_1(t_1)$, $C_2(t_1)$, and $C_3(t_1)$ are obtained from (10), (24), and (37), respectively.

Basing on the analyzed results above, we provide the results which ensure the existence of a unique t_1 , say t_1^* , to minimize the total average cost for the model system starting with shortages. The procedure is as follows:

Step 0. Input the parameters, $\mu_1, \mu_2, T, a_1, a_2, b_1, \alpha, \beta, \delta, D_0$, and $c_i, i = 1, 2, 3, 4$.

Step 1. Calculate Δ_1 and $d^2C_1(t_1)/dt_1^2$.

Step 1.1. If $\Delta_1 > 0$ and $d^2C_1(t_1)/dt_1^2 > 0$, solve a unique t_1^* from $dC_1(t_1)/dt_1 = 0$, and the total average cost and the order quantity can be obtained by (10) and (16), respectively.

Step 1.2. If $\Delta_1 \leq 0$ and $d^2C_1(t_1)/dt_1^2 > 0$, then the optimal average cost and the optimal order quantity can be obtained by (10) and (16) at $t_1^* = \mu_1$, respectively.

Step 1.3. If the sign of $d^2C_1(t_1)/dt_1^2$ is indefinite, then solve equation $dC_1(t_1)/dt_1 = 0$. If the solution is not unique, by comparing their average costs $C_1(t_1)$ to obtain the optimal replenishment point, compute the order quantity by (16).

Step 2. Calculate Δ_2 and $d^2C_2(t_1)/dt_1^2$.

Step 2.1. If $\Delta_1 < 0, \Delta_2 > 0$, and $d^2C_2(t_1)/dt_1^2 > 0$, then seek a unique t_1^* from equation $dC_2(t_1)/dt_1 = 0$, and the optimal total average cost and the optimal order quantity can be obtained by (24) and (29) at $t_1 = t_1^*$, respectively.

Step 2.2. If $\Delta_1 > 0, \Delta_2 > 0$, and $d^2C_2(t_1)/dt_1^2 > 0$, then $C_2(t_1)$ obtains its minimum at $t_1^* = \mu_1$, and compute the order quantity by (29).

Step 2.3. If $\Delta_1 < 0, \Delta_2 \leq 0$, and $d^2C_2(t_1)/dt_1^2 > 0$, then $C_2(t_1)$ obtains its minimum at $t_1^* = \mu_2$, and compute the order quantity by (29).

Step 2.4. If the sign of $d^2C_2(t_1)/dt_1^2$ is indefinite, then solve equation $dC_2(t_1)/dt_1 = 0$. If the solution is not unique, by comparing their average costs $C_2(t_1)$ to obtain the optimal replenishment point, compute the optimal order quantity by (29).

Step 3. Calculate $d^2C_3(t_1)/dt_1^2$.

Step 3.1. If $\Delta_2 < 0$ and $d^2C_3(t_1)/dt_1^2 > 0$, seek the unique t_1^* from equation $dC_3(t_1)/dt_1 = 0$, and the total average cost and the order quantity can be obtained by (37) and (42), respectively.

Step 3.2. If $\Delta_2 \geq 0$ and $d^2C_3(t_1)/dt_1^2 > 0$, then $C_3(t_1)$ obtains its minimum at $t_1^* = \mu_2$, and compute the order quantity by (42).

Step 3.3. If the sign of $d^2C_3(t_1)/dt_1^2$ is indefinite, then solve equation $dC_3(t_1)/dt_1 = 0$. If the solution is not unique, by comparing their average costs $C_3(t_1)$ to obtain the optimal replenishment point, compute the order quantity by (42).

Step 4. Find $TC(t_1^*) = \min\{C_1(t_1^*), C_2(t_1^*), C_3(t_1^*)\}$ and accordingly select the optimum t_1^* , and then obtain corresponding optimal order quantity Q^* .

4. Numerical Examples and Sensitivity Analysis

In order to demonstrate the procedure to obtain the optimal solution of the model, we present four examples for the model. Examples are based on piece-wise demand rate, such as $f(t) = a_1 + b_1t$ and $g(t) = a_2e^{-b_2t}$.

Example 1. The parameter values of an inventory system are given as follows: $T = 12$ weeks, $\mu_1 = 4$ weeks, $\mu_2 = 8$ weeks, $\alpha = 0.005, \beta = 2, \delta = 0.04, a_1 = 30$ units, $b_1 = 5$ units, $a_2 = 100$ units, $A_0 = \$500, c_1 = \$2, c_2 = \$3, c_3 = \$12, c_4 = \$8$.

Since $\Delta_1 = 18.809 > 0, \Delta_2 = 222.869 > 0, d^2C_1(t_1)/dt_1^2 > 0$ for all $t_1 \in (0, \mu_1), d^2C_2(t_1)/dt_1^2 > 0$ for all $t_1 \in [\mu_1, \mu_2]$, and $d^2C_3(t_1)/dt_1^2 > 0$ for all $t_1 \in [\mu_2, T]$, then we get local minimum value $C_2(\mu_1) = 763.891$ in $[\mu_1, \mu_2]$ and $C_3(\mu_2) = 1263.58$ in $[\mu_2, T]$. By solving equation $dC_1(t_1)/dt_1 = 0$, we have $t_1^* = 3.664$. From (10), we obtain $C_1(t_1^*) = 762.759$. Therefore, the optimal ordering quantity is $Q^* = 637.839$ and the total average cost is $TC(t_1^*) = 762.759$.

Example 2. The parameter values of an inventory system are given as follows: $T = 12$ weeks, $\mu_1 = 4$ weeks, $\mu_2 = 8$ weeks, $\alpha = 0.005, \beta = 2, \delta = 0.02, a_1 = 30$ units, $b_1 = 5$ units, $a_2 = 100$ units, $A_0 = \$500, c_1 = \$5, c_2 = \$10, c_3 = \$12, c_4 = \$8$.

Since $\Delta_1 = -257.889 < 0, \Delta_2 = 139.269 > 0, d^2C_1(t_1)/dt_1^2 > 0$ for all $t_1 \in (0, \mu_1), d^2C_2(t_1)/dt_1^2 > 0$ for all $t_1 \in [\mu_1, \mu_2]$, and $d^2C_3(t_1)/dt_1^2 > 0$ for all $t_1 \in (\mu_2, T)$, then we get local minimum value $C_1(\mu_1) = 1805.883$ in $(0, \mu_1]$ and $C_3(\mu_2) = 1582.32$ in $[\mu_2, T]$. By solving equation $dC_2(t_1)/dt_1 = 0$, we have $t_1^* = 6.559$. From (24), we obtain $C_2(t_1^*) = 1480.324$. Therefore, the optimal ordering quantity is $Q^* = 571.827$ and the total average cost is $TC(t_1^*) = 1480.324$.

Example 3. The parameter values of an inventory system are given as follows: $T = 12$ weeks, $\mu_1 = 4$ weeks, $\mu_2 = 6$ weeks, $\alpha = 0.005, \beta = 2, \delta = 0.06, a_1 = 30$ units, $b_1 = 5$ units, $a_2 = 100$ units, $A_0 = \$500, c_1 = \$5, c_2 = \$10, c_3 = \$12, c_4 = \$8$.

Since $\Delta_1 = -249.329 < 0, \Delta_2 = -121.869 < 0, d^2C_1(t_1)/dt_1^2 > 0$ for all $t_1 \in (0, \mu_1], d^2C_2(t_1)/dt_1^2 > 0$ for all $t_1 \in (\mu_1, \mu_2]$, and $d^2C_3(t_1)/dt_1^2 > 0$ for all $t_1 \in [\mu_2, T]$, then we get local minimum value $C_1(\mu_1) = 1417.134$ in $(0, \mu_1]$ and $C_2(\mu_2) = 1052.016$ in $(\mu_1, \mu_2]$. By solving equation $dC_3(t_1)/dt_1 = 0$, we have $t_1^* = 9.120$. From (37), we obtain $C_3(t_1^*) = 889.767$. Therefore, the optimal ordering quantity is $Q^* = 278.960$ and the total average cost is $TC(t_1^*) = 889.767$.

Example 4. The parameter values of an inventory system are given as follows: $T = 12$ weeks, $\mu_1 = 4$ weeks, $\mu_2 = 6$ weeks, $\alpha = 0.005, \beta = 1.6, \delta = 0.02, a_1 = 30$ units, $b_1 = 5$ units, $a_2 = 100$ units, $A_0 = \$500, c_1 = \$5, c_2 = \$10, c_3 = \$12, c_4 = \$8$.

Since $\Delta_1 = -197.263 < 0, \Delta_2 = -86.675 < 0, d^2C_4(t_1)/dt_1^2 > 0$ for all $t_1 \in (0, \mu_1], d^2C_2(t_1)/dt_1^2 > 0$

TABLE 1: The sensitivity of δ for the model in Example 1.

δ	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
t_1^*	3.4366	3.4905	3.5463	3.6042	3.6642	3.7263	3.7907	3.8574	3.9265
S^*	515.0829	515.0829	511.4210	507.5965	503.6066	499.4433	495.0985	490.5641	485.8321
Q^*	648.1034	648.1034	644.8369	641.4184	637.8392	634.0900	630.1614	626.0436	621.7267
$TC_2(t_1^*)$	775.2147	771.8437	768.4699	764.9983	761.4252	757.7473	753.9611	750.063	746.0498

TABLE 2: The sensitivity of α for the models in Example 1.

α	0	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008
t_1^*	3.0141	3.1325	3.2569	3.3870	3.5229	3.6642	3.8105	3.9614	4.1171
S^*	415.8882	432.0629	448.9624	466.5543	484.7907	503.6066	522.9206	542.6345	575.5923
Q^*	522.8982	543.9279	565.9745	589.0106	612.9883	637.8392	663.4729	689.7782	723.0000
$TC_2(t_1^*)$	638.5314	661.4087	685.1585	709.7639	735.1984	761.4252	788.3972	816.0570	845.0147

TABLE 3: The sensitivity of β for the models in Example 1.

β	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2
t_1^*	3.1114	3.1995	3.2513	3.3176	3.40332	3.5148	3.6613	3.8551	4.1128
S^*	427.5053	440.1612	447.1691	456.2029	467.9084	483.1709	503.6066	529.5334	564.1669
Q^*	538.4988	554.7959	563.9494	575.7494	591.0495	611.0248	637.8392	672.0262	717.9964
$TC_2(t_1^*)$	644.4596	670.3559	680.1679	692.9359	709.5918	731.4743	761.4252	797.6508	845.0147

for all $t_1 \in (\mu_1, \mu_2]$, and $d^2C_3(t_1)/dt_1^2 > 0$ for all $t_1 \in [\mu_2, T)$, then we get local minimum value $C_1(\mu_1) = 1252.59$ in $(0, \mu_1]$ and $C_2(u_2) = 1052.016$ in $(\mu_1, \mu_2]$. By solving equation $dC_3(t_1)/dt_1 = 0$, we have $t_1^* = 8.627$. From (37), we obtain $TC(t_1^*) = 842.935$. Therefore, the optimal ordering quantity is $Q^* = 241.524$ obtained from (42), and the minimum cost $TC(t_1^*) = 842.935$.

In order to clearly indicate the effects of parameters such as δ , α , and β on S^* , Q^* , and $TC_i(t_1^*)$ ($i = 1, 2$), respectively, we study the sensitivity of different parameters under the studied inventory model. The sensitivity analysis is performed on the base of Example 1, and the results are shown in Tables 1–3, respectively.

From Table 1, we observe that inventory level S^* , order quantity Q^* , and the total average cost $TC_i(t_1^*)$ ($i = 1, 2$) gradually decrease as the shortage parameter δ increases for the model, and the replenishment time t_1^* increases as δ increases for the model starting with shortage. Also, we observe that the values of t_1^* , S^* , Q^* , and $TC(t_1^*)$ are slightly sensitive to the changes of δ .

From Table 2, we find that when the deterioration parameter α increases, then S^* , Q^* , and $TC(t_1^*)$ increase for both models, and the replenishment time t_1^* increases for the model starting with shortage. We see that the values of t_1^* , S^* , Q^* , and $TC(t_1^*)$ all are temperately sensitive to the changes of α for the model.

From Table 3, we find that S^* , Q^* , and $TC(t_1^*)$ increase as the deterioration parameter β increases for both models. Also, we find that the replenishment time t_1^* increases as β increases for the model starting with shortage. We see that the values of t_1^* , S^* , Q^* , and $TC(t_1^*)$ all are moderately sensitive to the changes of β for the model.

5. Conclusion

This paper studies an inventory model for Weibull-distributed deterioration with trapezoidal type demand rate and partial backlogging. First, an optimal replenishment strategy for deteriorating items with the model starting with shortage is proposed. Then, numerical examples are provided to illustrate the theoretical results. From the analysis of Tables 1–3, it can be found that the replenishment time point t_1^* , order quantity Q^* , and the total average cost $TC(t_1^*)$ are moderately sensitive to the changes of α and β and lowly sensitive to the changes of δ , respectively. However, if the order cycle (T) is a decision variable, such inventory model will be of more practical significance, which provides us an interesting topic in future.

Competing Interests

The author declares that there are no competing interests.

Acknowledgments

The research is supported partly by National Natural Science Foundation of China (Grant no. 71502100).

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