Research Article

Event-Based Control for Average Consensus of Wireless Sensor Networks with Stochastic Communication Noises

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This paper focuses on the average consensus problem for the wireless sensor networks (WSNs) with fixed and Markovian switching, undirected and connected network topologies in the noise environment. Event-based protocol is applied to each sensor node to reach the consensus. An event triggering strategy is designed based on a Lyapunov function. Under the event trigger condition, some sufficient conditions for average consensus in mean square are obtained. Finally, some numerical simulations are given to illustrate the effectiveness of the results derived in this paper.

1. Introduction

Wireless sensor network (WSN) has attracted significant attention as an emerging communication architecture. It has many practical applications in such areas as robotics, surveillance and environment monitoring, and information collection.

A WSN can be viewed as a multiagent system (MAS) from a network-theoretic perspective. Each node represents a sensor and each edge performs information exchange between sensors. In some cases, the agreement is a common value which may be the average of the initial states of the system, is often called average consensus, and has wide application background in the areas such as formation control [1], distributed filtering [2], and distributed computation [3]. It means to achieve the accordance of the states of MAS. In [4], Olfati-Saber and Murray consider the average consensus control for the directed and undirected networks with fixed and switching topologies. In [5], Kingston and Beard extend the results of [4] to the discrete-time models and weakened the condition of instantaneous strong connectivity. In [6], Xiao and Boyd consider the distributed averaging consensus of the networks with fixed and undirected topologies. In [7], Q. Zhang and J. Zhang design a distributed consensus protocol to analyze the multiagent systems in uncertain

communication environments including the communication noises and Markov topology switches. In [8], Wang et al. investigate the H∞ consensus control problem for a class of discrete time-varying multiagent systems with both missing measurements and parameter uncertainties. Also, the distributed estimation problems over sensor networks have been widely discussed in [9–11].

For the node of the WSNs with limited energy, control over networks with limited resources is a challenging task. Consequently, the most important problem in WSN is the energy consumption, which is directly proportional to the transmit power of the information exchange between the sensors. The event-based control can facilitate the efficient usage of the resources. Based on the event-based control mechanism, it can reduce the useless communication between neighboring agents along with the energy consumption. In [12], Wang and Lemmon discuss the event-triggered data transmission in distributed networked control systems with packet loss and transmission delays. In [13], Mazo and Tabuada focus on reducing the number of messages from sensors to controllers and from controllers to actuators by the use of the decentralized event-triggered mechanism. In [14], Seyboth et al. propose a novel control strategy for multiagent coordination with event-based broadcasting. The proposed control strategy guarantees either asymptotic convergence to

average consensus or convergence to a ball centered at the average consensus. In [15], Meng and Chen study an average consensus problem for multiagent systems by event-based control, which is used on each agent to drive the state to their initial average eventually.

The main contribution of this paper is studying average consensus problem for the WSN with fixed topology and Markovian switching topology in the noise environment; the event-based average consensus control protocol is designed for the WSN based on a Lyapunov function. Under the event trigger condition, sufficient conditions for average consensus in mean square are obtained.

The remainder of the paper is organized as follows. In Section 2, some concepts in graph theory are described, and the problem to be investigated is formulated. In Section 3, the main results are presented. In Section 4, some numerical examples show the reliability of the main results. In Section 5, some conclusions are given.

2. Problem Formulation and Preliminaries

2.1. Concepts in Graph Theory. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be an undirected graph, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the set of nodes, node i represents the ith sensor node, $\mathcal{E}$ is the set of edges, and an edge in $\mathcal{G}$ is denoted by an ordered pair (j, i). $(j, i) \in \mathcal{E}$ if and only if the jth sensor node can send information to the ith sensor node directly. The neighborhood of the ith sensor node is denoted by $N_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$.

$\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ is called the adjacency matrix of $\mathcal{G}$. For any $i, j \in \mathcal{V}$, $a_{ij} \geq 0$, and $a_{ij} > 0 \Leftrightarrow j \in N_i$. $\deg_{\text{in}}(i) = \sum_{j=1}^{n} a_{ij}$ is called the in-degree of i; $\deg_{\text{out}}(i) = \sum_{j=1}^{n} a_{ij}$ is called the out-degree of i; $L = \mathcal{D} - \mathcal{A}$ is called the Laplacian matrix of $\mathcal{G}$, where $\mathcal{D} = \text{diag}(\deg_{\text{in}}(1), \ldots, \deg_{\text{in}}(n))$. Its eigenvalues are real and can be ordered as

$$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n \tag{1}$$

with $\lambda_1 = 0$ and λ_2 being the smallest nonzero eigenvalue for connected graphs.

2.2. Average Consensus for WSNs. In this paper, we study the average consensus control for a WSN with dynamics

$$\dot{x}_i(t) = u_i(t), \tag{2}$$

where $x_i(t) \in \mathbb{R}$ is the state of the ith sensor and $u_i(t) \in \mathbb{R}$ is the control input. The initial state $x_i(0)$ is deterministic.

Each sensor includes a digital microprocessor and dynamics. The microprocessor of sensor i monitors its own measurement value $x_i(t)$ continuously and decides when to communicate with the neighboring sensors by broadcasting the actual measurement value. Therefore the latest broadcasted value of sensor i can be described by the piecewise constant function

$$\widehat{x}_i(t) = x_i(t_k^i), \quad t_k^i \leq t < t_{k+1}^i, \tag{3}$$

where $t_0^i, t_1^i, \ldots$ is sequence of event-times of sensor i. We can see that the discrete-time signal $x_i(t_k^i)$ is converted into the continuous-time signal $\widehat{x}_i(t)$.

The ith sensor can receive information from its neighbors

$$\widehat{y}_{ji}(t) = \widehat{x}_j(t) + \sigma_{ji}\omega_{ji}(t), \tag{4}$$

where $y_{ji}(t)$ denotes the measurement of the jth sensor's state $x_j(t)$ by the ith sensors, $\{\omega_{ji}(t) \mid i, j = 1, 2, \ldots, n\}$ are the communication noises, and σ_{ji} is the noise intensity function.

For the dynamic network $(\mathcal{G}, X)$, we give the event-based consensus protocl.

$$u_i(t) = \sum_{j \in N_i} a_{ij} \left(\widehat{y}_{ji}(t) - \widehat{x}_i(t) \right). \tag{5}$$

The dynamics of agent i for $t \in [t_k^i + lh, t_k^i + lh + h)$ is given by

$$
\begin{aligned}
\dot{x}_i(t) &= \sum_{j \in N_i(t)} a_{ij} \left[x_j(t_{k'}^j) - x_i(t_k^i) + \sigma_{ji}\omega_{ji}(t) \right] \\
&= \sum_{j \in N_i(t)} a_{ij} \left[x_j(t_k^i + lh) - x_i(t_k^i + lh) \right] \\
&\quad + \sum_{j \in N_i(t)} a_{ij} \left[x_i(t_k^i + lh) - x_i(t_k^i) \right] \\
&\quad + \sum_{j \in N_i(t)} a_{ij} \left[x_j(t_{k'}^j) - x_j(t_k^i + lh) \right] \\
&\quad + \sum_{j \in N_i(t)} a_{ij}\sigma_{ji}\omega_{ji}(t) \\
&= \sum_{j \in N_i(t)} a_{ij} \left[x_j(t_k^i + lh) - x_i(t_k^i + lh) \right] \\
&\quad + \sum_{j \in N_i(t)} a_{ij} \left[e_j(t_k^i + lh) - e_i(t_k^i + lh) \right] \\
&\quad + \sum_{j \in N_i(t)} a_{ij}\sigma_{ji}\omega_{ji}(t),
\end{aligned}
\tag{6}
$$

where $t_{k'}^j$ is defined as

$$t_{k'}^j = \max \left\{ t \mid t \in \left\{ t_k^j, k = 0, 1, \ldots \right\}, t \leq t_k^i + lh \right\}. \tag{7}$$

Substituting the protocol (5) into the system (2) leads to

$$dx(t) = -[L(x(kh) + e(kh))] dt + G dW(t), \tag{8}$$

where L is the Laplacian matrix of $\mathcal{G}$, G is the noise intensity matrix, and $W(t) = (W_1(t), \ldots, W_n(t))^T$ is an n-dimensional Brownian motion.

The event condition for agent i has the following form:

$$\lambda_n \left\| x_i(t_k^i + lh) \right\|^2 \leq \lambda_2 \left\| e_i(t_k^i + lh) \right\|^2 + 2\|G\|^2, \tag{9}$$

where

$$e_i(t_k^i + lh) = x_i(t_k^i) - x_i(t_k^i + lh). \tag{10}$$

Remark 1. Each sensor broadcasts its state information to the neighbors and also receives state information from its neighbors for event detection at each sampling instant. The event detector in (9) guarantees that it reduces the sensor energy consumption and network bandwidth usage because it only checks the event condition at discrete sampling instants.

Next, we consider the average consensus control protocol for the system (8) as follows:

$$d\delta(t) = -L\left[\delta(kh) + (I - J) e(kh)\right] dt$$
$$+ (I - J) G dW(t), \tag{11}$$

where $\delta(t) = x(t) - Jx(t)$ and $J = (1/n)1_n 1_n^T$.

If we define

$$f(\delta, t) = -L\left[\delta(kh) + (I - J) e(kh)\right],$$
$$g(\delta, t) = (I - J) G, \tag{12}$$

then (11) becomes

$$d\delta(t) = f(\delta, t) dt + g(\delta, t) dW(t). \tag{13}$$

In WSNs, each sensor node communicates with other sensor nodes through the unreliable networks. If the communication channel between sensors i and j is $(j, i) \in \mathscr{E}_f$, and $\mathscr{E}_f$ is the set of the communication channels which probably lost the signal, then the time-varying topologies under link failure or creation can be described by the Markov switching topology.

Let $\{r(t)_{t \geq 0}\}$ be a right-continuous Markov chain on the probability space taking values in a finite state set $S = \{1, 2, \ldots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N \times N}$ given by

$$P\left\{r(t + \Delta) = j \mid r(t) = i\right\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases} \tag{14}$$

where $\Delta > 0$ and $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, while

$$\gamma_{ii} = -\sum_{j \neq i} \gamma_{ij} \tag{15}$$

and $r(0) = r_0$.

We denote the undirected communication graph by $\mathscr{G} = \{\mathscr{G}(1), \mathscr{G}(2), \ldots, \mathscr{G}(N)\}$, where $\mathscr{G}(k) = \{\mathscr{V}, \mathscr{E}_{\mathscr{G}(k)}, \mathscr{A}_{\mathscr{G}(k)}\}$ is the undirected graph. Denote the topology graph by $\mathscr{G}_t$ at moment t $(t \geq 0)$, so $r(t) = k$ if and only if $\mathscr{G}_t = \mathscr{G}(k)$.

Under Markovian switching topology, we have

$$dx(t) = -\left[L(r(t))(x(kh) + e(kh))\right] dt$$
$$+ G(r(t)) dW(t), \tag{16}$$

$$d\delta(t) = -L(r(t))\left[\delta(kh) + (I - J) e(kh)\right] dt$$
$$+ (I - J) G(r(t)) dW(t). \tag{17}$$

So, under this condition, the event condition for agent i has the following form:

$$\lambda_n\left(L(r(t))\right) \left\|x_i(t_k^i + lh)\right\|^2$$
$$\leq \lambda_2\left(L(r(t))\right) \left\|e_i(t_k^i + lh)\right\|^2 + 2\|G\|^2, \tag{18}$$

where

$$e_i\left(t_k^i + lh\right) = x_i\left(t_k^i\right) - x_i\left(t_k^i + lh\right). \tag{19}$$

3. Main Results

In this section, we consider the average consensus for the WSNs with fixed topology and Markovian switching topology by event-based control.

3.1. WSNs with the Fixed Topology

Theorem 2. *Consider the system* (8) *over a connected communication graph driven by event condition in* (9). *Then the system* (11) *is asymptotically stable in mean square; it means that the system* (8) *reaches average consensus in mean square if* $0 \leq h \leq 1/2\lambda_n$.

Proof. Define the function $V : R^n \times R_+ \to R^+$ by

$$V(\delta, t) = \frac{1}{2}\delta^T(t) \delta(t). \tag{20}$$

Computing $dV(\delta, t)$ along the trajectory generated by the system (11) for any $t \in [kh, (k+1)h)$, we have

$$dV(\delta, t) = \left[\delta^T(t) f(\delta, t) + \text{trace}\left[g^T(\delta, t) g(\delta, t)\right]\right] dt$$
$$+ \delta^T(t) (I - J) G dW(t)$$
$$= \left[-\delta^T(t) L\left[\delta(kh) + (I - J) e(kh)\right]\right.$$
$$\left. + \|(I - J)G\|^2\right] dt + \delta^T(t) (I - J) G dW(t)$$
$$= \left[(t - kh)\left[\delta(kh) + (I - J) e(kh)\right]^T\right.$$
$$\times L^T L\left[\delta(kh) + (I - J) e(kh)\right]$$
$$- \delta^T(kh) L\left[\delta(kh) + (I - J) e(kh)\right]$$
$$\left. + \|(I - J)G\|^2\right] dt$$
$$- \left(W^T(t) - W^T(kh)\right) G (I - J)$$
$$\times L\left[\delta(kh) + (I - J) e(kh)\right]$$
$$+ \delta^T(t) (I - J) G dW(t)$$
$$\leq \left(h\lambda_n[\delta(kh) + (I - J) e(kh)]^T\right.$$
$$\times L\left[\delta(kh) + (I - J) e(kh)\right]$$
$$- \delta^T(kh) L\left[\delta(kh) + (I - J) e(kh)\right]$$
$$\left. + \|(I - J)G\|^2\right) dt$$
$$- \left(W^T(t) - W^T(kh)\right) G (I - J)$$
$$\times L\left[\delta(kh) + (I - J) e(kh)\right]$$
$$+ \delta^T(t) (I - J) G dW(t)$$

$$
\begin{aligned}
= & \left((h\lambda_n - 1)\,\delta^T\,(kh)\,L\delta\,(kh) + (2h\lambda_n - 1)\right.\\
& \times \delta^T\,(kh)\,L\,(I - J)\,e\,(kh)\\
& + h\lambda_n\|I - J\|^2 e^T\,(kh)\,Le\,(kh)\\
& \left. + \|(I - J)\,G\|^2\right)dt\\
& - \left(W^T\,(t) - W^T\,(kh)\right)G\,(I - J)\\
& \times L\,[\delta\,(kh) + (I - J)\,e\,(kh)]\\
& + \delta^T\,(t)\,(I - J)\,GdW\,(t)\,.
\end{aligned}
\tag{21}
$$

Applying the inequality

$$
\begin{aligned}
\delta^T\,(kh)\,L\,(I - J)\,e\,(kh) \le &\; \frac{1}{2}\|L\|^2\delta^T\,(kh)\,\delta\,(kh)\\
& + \frac{1}{2}\|I - J\|^2 e^T\,(kh)\,e\,(kh)\,,
\end{aligned}
\tag{22}
$$

it can be derived that

$$
\begin{aligned}
dV\,(\delta\,(t)\,,t) \le &\left(-\frac{1}{2}\delta^T\,(kh)\,L\delta\,(kh)\right.\\
& + \frac{1}{2}\|I - J\|^2 e^T\,(kh)\,Le\,(kh)\\
& \left. + \|(I - J)G\|^2\right)dt\\
& - \left(W^T\,(t) - W^T\,(kh)\right)G\,(I - J)\\
& \times L\,[\delta\,(kh) + (I - J)\,e\,(kh)]\\
& + \delta^T\,(t)\,(I - J)\,GdW\,(t)
\end{aligned}
\tag{23}
$$

with $2h\lambda_n \le 1$.

We can see that

$$
\begin{aligned}
& d\left[e^{-t}V\,(\delta\,(t)\,,t)\right]\\
& = e^{-t}\left[-V\,(\delta\,(t)\,,t)\,dt + dV\,(\delta\,(t)\,,t)\right]\\
& \le e^{-t}\left[-V\,(\delta\,(t)\,,t) - \frac{1}{2}\delta^T\,(kh)\,L\delta\,(kh)\right.\\
& \qquad + \frac{1}{2}\|I - J\|^2 e^T\,(kh)\,Le\,(kh)\\
& \qquad \left. + \|(I - J)G\|^2\right]dt\\
& \quad - e^{-t}\left(W^T\,(t) - W^T\,(kh)\right)G\,(I - J)\\
& \qquad \times L\,[\delta\,(kh) + (I - J)\,e\,(kh)]\\
& \quad + e^{-t}\delta^T\,(t)\,(I - J)\,GdW\,(t)\,.
\end{aligned}
\tag{24}
$$

Integrating both sides of the previous inequality from kh to t ($t \in [kh, (k + 1)h)$), making use of (9), and taking the expectation, one can obtain that

$$
\begin{aligned}
e^{-t}\mathbb{E}V\,(\delta\,(t)\,,t) \le &\int_{kh}^{t} -e^{-t}\mathbb{E}V\,(\delta\,(t)\,,t)\,dt\\
& + \frac{1}{2}e^{-kh}\delta^T\,(kh)\,\delta\,(kh)\\
& - \left(\frac{1}{2}\delta^T\,(kh)\,L\delta\,(kh)\right.\\
& \quad - \frac{1}{2}\|I - J\|^2 e^T\,(kh)\,Le\,(kh)\\
& \quad \left. - \|(I - J)\,G\|^2\right)\left(e^{-t} - e^{-kh}\right)\\
\le &\int_{kh}^{t} -e^{-t}\mathbb{E}V\,(\delta\,(t)\,,t)\,dt\\
& + \frac{1}{2}e^{-kh}\delta^T\,(kh)\,\delta\,(kh)\\
& + \left(\frac{1}{2}\delta^T\,(kh)\,L\delta\,(kh)\right.\\
& \quad - \frac{1}{2}\|I - J\|^2 e^T\,(kh)\,Le\,(kh)\\
& \quad \left. - \|(I - J)\,G\|^2\right)\left(e^{-kh} - e^{-(k+1)h}\right)\\
= &\int_{kh}^{t} -e^{-t}\mathbb{E}V\,(\delta\,(t)\,,t)\,dt\\
& + \frac{1}{2}e^{-kh}\delta^T\,(kh)\,\delta\,(kh)\\
& + \left(\frac{1}{2}\|I - J\|^2 x^T\,(kh)\,Lx\,(kh)\right.\\
& \quad - \frac{1}{2}\|I - J\|^2 e^T\,(kh)\,Le\,(kh)\\
& \quad \left. - \|(I - J)G\|^2\right)\left(e^{-kh} - e^{-(k+1)h}\right)\\
\le &\int_{kh}^{t} -e^{-t}\mathbb{E}V\,(\delta\,(t)\,,t)\,dt\\
& + \frac{1}{2}e^{-kh}\delta^T\,(kh)\,\delta\,(kh)\,.
\end{aligned}
\tag{25}
$$

By using Gronwall inequality, we have

$$
\mathbb{E}V\,(\delta\,(t)\,,t) \le \frac{1}{2}e^{-kh}\|\delta(kh)\|^2 e^{-(t-kh)} = \frac{1}{2}\|\delta(kh)\|^2 e^{-t}\,.
\tag{26}
$$

Letting $t \to \infty$ ($k \to \infty$), we obtain

$$
\lim_{t \to \infty}\mathbb{E}\|\delta(t)\|^2 = 0\,.
\tag{27}
$$

So the system (8) reaches average consensus in mean square. $\square$

3.2. WSNs with Markovian Switching Topology

Theorem 3. *Consider the system* (16) *over a number of connected switching communication graphs driven by event condition in* (18). *Then the system* (17) *is asymptotically stable in mean square; it means that the system* (16) *reaches average consensus in mean square if* $0 \leq h \leq 1/2\lambda_n(r(t))$.

Proof. Define the function $V : R^n \times R_+ \times S \rightarrow R^+$ by

$$V(\delta, t, r(t)) = \frac{1}{2}\delta^T(t)\delta(t). \tag{28}$$

Computing $dV(\delta, t, r(t))$ along the trajectory generated by the system (11) for any $t \in [kh, (k+1)h)$, we have

$$
\begin{aligned}
dV(\delta, t, r(t)) &= \Big[\delta^T(t)f(\delta, t, r(t)) \\
&\quad + \text{trace}\Big[g^T(\delta, t, r(t))g(\delta, t, r(t))\Big]\Big]dt \\
&\quad + \delta^T(t)(I-J)G(r(t))dW(t) \\
&= \Big[-\delta^T(t)L(r(t))[\delta(kh) + (I-J)e(kh)] \\
&\quad + \|(I-J)G\|^2 + \sum_{j=1}^{N}\gamma_{ij}V(\delta, t, r(t))\Big]dt \\
&\quad + \delta^T(t)(I-J)G(r(t))dW(t) \\
&\leq \Big((h\lambda_n - 1)\delta^T(kh)L(r(t))\delta(kh) \\
&\quad + (2h\lambda_n - 1)\delta^T(kh)L(r(t))(I-J)e(kh) \\
&\quad + h\lambda_n\|I-J\|^2 e^T(kh)L(r(t))e(kh) \\
&\quad + \|(I-J)G(r(t))\|^2 + \overline{\gamma}V(\delta, t, r(t))\Big)dt \\
&\quad - \Big(W^T(t) - W^T(kh)\Big)G(r(t))(I-J) \\
&\quad \times L(r(t))[\delta(kh) + (I-J)e(kh)] \\
&\quad + \delta^T(t)(I-J)G(r(t))dW(t) \\
&\leq \Big(-\frac{1}{2}\delta^T(kh)L(r(t))\delta(kh) \\
&\quad + \frac{1}{2}\|I-J\|^2 e^T(kh)L(r(t))e(kh) \\
&\quad + \|(I-J)G(r(t))\|^2 + \overline{\gamma}V(\delta, t, r(t))\Big)dt \\
&\quad - \Big(W^T(t) - W^T(kh)\Big)G(r(t))(I-J) \\
&\quad \times L(r(t))[\delta(kh) + (I-J)e(kh)] \\
&\quad + \delta^T(t)(I-J)G(r(t))dW(t)
\end{aligned}
\tag{29}
$$

with $2h\lambda_n \leq 1$.

For $\overline{\gamma} > 0$, $\overline{\gamma} = \max\{|\gamma_{ii}| : i \in S|\}$, compute

$$
\begin{aligned}
&d\Big[e^{-2\overline{\gamma}t}V(\delta(t), t, r(t))\Big] \\
&= e^{-2\overline{\gamma}t}\Big[-2\overline{\gamma}V(\delta(t), t, r(t))dt + dV(\delta(t), t, r(t))\Big] \\
&\leq e^{-2\overline{\gamma}t}\Big[-2\overline{\gamma}V(\delta(t), t, r(t)) \\
&\quad - \frac{1}{2}\delta^T(kh)L(r(t))\delta(kh) \\
&\quad + \frac{1}{2}\|I-J\|^2 e^T(kh)L(r(t))e(kh) \\
&\quad + \|(I-J)G(r(t))\|^2 + \overline{\gamma}V(\delta, t, r(t))\Big]dt \\
&\quad - e^{-2\overline{\gamma}t}\Big(W^T(t) - W^T(kh)\Big)G(I-J) \\
&\quad \times L(r(t))[\delta(kh) + (I-J)e(kh)] \\
&\quad + e^{-2\overline{\gamma}t}\delta^T(t)(I-J)G(r(t))dW(t).
\end{aligned}
\tag{30}
$$

Integrating both sides of the above inequality from kh to t ($t \in [kh, (k+1)h)$), making use of (18), and taking the expectation, one can obtain that

$$
\begin{aligned}
&e^{-2\overline{\gamma}t}\mathbb{E}V(\delta(t), t, r(t)) \\
&\leq \int_{kh}^{t} -\overline{\gamma}e^{-2\overline{\gamma}t}\mathbb{E}V(\delta(t), t, r(t))dt \\
&\quad + \frac{1}{2}e^{-2\overline{\gamma}kh}\delta^T(kh)\delta(kh) \\
&\quad - \Big(\frac{1}{2}\delta^T(kh)L(r(t))\delta(kh) \\
&\qquad - \frac{1}{2}\|I-J\|^2 e^T(kh)L(r(t))e(kh) \\
&\qquad - \|(I-J)G(r(t))\|^2\Big)\Big(e^{-2\overline{\gamma}t} - e^{-2\overline{\gamma}kh}\Big) \\
&\leq \int_{kh}^{t} -\overline{\gamma}e^{-2\overline{\gamma}t}\mathbb{E}V(\delta(t), t, r(t))dt \\
&\quad + \frac{1}{2}e^{-2\overline{\gamma}kh}\delta^T(kh)\delta(kh) \\
&\quad + \Big(\frac{1}{2}\delta^T(kh)L(r(t))\delta(kh) \\
&\qquad - \frac{1}{2}\|I-J\|^2 e^T(kh)L(r(t))e(kh) \\
&\qquad - \|(I-J)G(r(t))\|^2\Big) \\
&\quad \times \Big(e^{-2\overline{\gamma}kh} - e^{-2\overline{\gamma}(k+1)h}\Big)
\end{aligned}
$$

$$= \int_{kh}^{t} -\overline{\gamma}e^{-2\overline{\gamma}t}\mathbb{E}V\left(\delta\left(t\right),t,r\left(t\right)\right)dt$$

$$+ \frac{1}{2}e^{-2\overline{\gamma}kh}\delta^{T}\left(kh\right)\delta\left(kh\right)$$

$$+ \left(\frac{1}{2}\|I - J\|^{2}x^{T}\left(kh\right)L\left(r\left(t\right)\right)x\left(kh\right)\right.$$

$$\left. - \frac{1}{2}\|I - J\|^{2}e^{T}\left(kh\right)L\left(r\left(t\right)\right)\right.$$

$$\left. \times e\left(kh\right) - \|(I - J)G\left(r\left(t\right)\right)\|^{2}\right)$$

$$\times \left(e^{-2\overline{\gamma}kh} - e^{-2\overline{\gamma}(k+1)h}\right)$$

$$\leq \int_{kh}^{t} -\overline{\gamma}e^{-2\overline{\gamma}t}\mathbb{E}V\left(\delta\left(t\right),t,r\left(t\right)\right)dt$$

$$+ \frac{1}{2}e^{-2\overline{\gamma}kh}\delta^{T}\left(kh\right)\delta\left(kh\right).$$

$$(31)$$

By using Gronwall inequality, we have

$$\mathbb{E}V\left(\delta\left(t\right),t,r\left(t\right)\right) \leq \frac{1}{2}e^{-2\overline{\gamma}kh}\|\delta(kh)\|^{2}e^{-\overline{\gamma}(t-kh)}$$

$$= \frac{1}{2}e^{-\overline{\gamma}(t+kh)}\|\delta(kh)\|^{2}.$$

$$(32)$$

Letting $t \rightarrow \infty$ $(k \rightarrow \infty)$, we obtain that

$$\lim_{t \rightarrow \infty}\mathbb{E}\|\delta(t)\|^{2} = 0.$$

$$(33)$$

So the system (16) reaches average consensus in mean square.   □

4. Numerical Examples

In this section, we give two examples to examine the average consensus of the systems (8) and (16).

Example 1. Consider a WSN composed by four sensors in which each dynamic state of the sensor is $\dot{x}_{i} = u_{i}$, where $i = 1, 2, 3, 4$ (see the topology in Figure 1, which is used in Dimarogonas [16]) and the initial state is $X(0) = (4 - 1, 5, 2)^{T}$.

The adjacent matrix is

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$(34)$$

The related degree matrix is

$$D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$(35)$$

The related Laplacian matrix is

$$L = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}.$$

$$(36)$$

The nonzero largest and smallest eigenvalues are $\lambda_{n} = 4$ and $\lambda_{2} = 1$, respectively.

Let the noise intensity matrix G be

$$G = \begin{pmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 \\ 0 & 0 & 0.003 & 0 \\ 0 & 0 & 0 & 0.001 \end{pmatrix}.$$

$$(37)$$

The sampling period for all sensor nodes is chosen as $h = 0.003$ which satisfies $0 \leq h \leq 1/2\lambda_{n}$. Using the event condition in (9), we can draw the dynamic curve of the states of the sensors by Matlab as Figure 2. It shows us that the four sensor nodes reach the average consensus in mean square with fixed topology. The control signal when the events occur for each sensor is shown in Figure 3. We can see that the number of sensor control updates that only occur at the sampling instant under the event trigger condition is rapidly reduced to reach average consensus.

Example 2. The switching topology of the four sensors is determined by the Markov chain $r(t)$ whose state space is $S = \{1, 2, 3\}$. The related topology graph is $\mathscr{G}(i) = \{\mathscr{V}(i), \mathscr{E}(i), \mathscr{A}(i)\}$ (see the topologies in Figure 4).

The adjacent matrices are

$$A\left(1\right) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad A\left(2\right) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix},$$

$$A\left(3\right) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

$$(38)$$

The related degree matrices are

$$D\left(1\right) = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad D\left(2\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix},$$

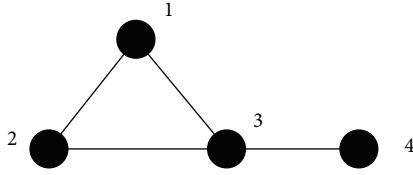$$D\left(3\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

$$(39)$$

FIGURE 1: The topology of the four sensors.

The related Laplacian matrices are

$$
L(1) = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix},
$$

$$
L(2) = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix},
$$

$$
L(3) = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}.
$$

(40)

The nonzero largest and smallest eigenvalue are $\lambda_n = 3.4142$ and $\lambda_2 = 0.5858$, respectively.

Let the noise intensity matrices G be

$$
G(1) = \begin{pmatrix} 0.001 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 \\ 0 & 0 & 0.003 & 0 \\ 0 & 0 & 0 & 0.001 \end{pmatrix},
$$

$$
G(2) = \begin{pmatrix} 0.002 & 0 & 0 & 0 \\ 0 & 0.003 & 0 & 0 \\ 0 & 0 & 0.003 & 0 \\ 0 & 0 & 0 & 0.001 \end{pmatrix},
$$

(41)

$$
G(3) = \begin{pmatrix} 0.003 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{pmatrix}.
$$

The sampling period for all sensor nodes is chosen as $h = 0.002$ which satisfies $0 \leq h \leq 1/2\lambda_n(i)$ $(i = 1, 2, 3)$. Using the event condition in (18), we can draw the dynamic curve of the states of the sensors by Matlab as Figure 5. It shows us that the four sensor nodes reach the average consensus in mean square with Markovian switching topologies.

5. Conclusions

In this paper, we have dealt with the problem of average consensus in mean square of the WSNs. By using the event-based control mechanism, we have obtained several sufficient conditions to ensure the average consensus in mean square for WSNs with fixed and Markovian switching topologies. There are many other topics worth investigating, such as the
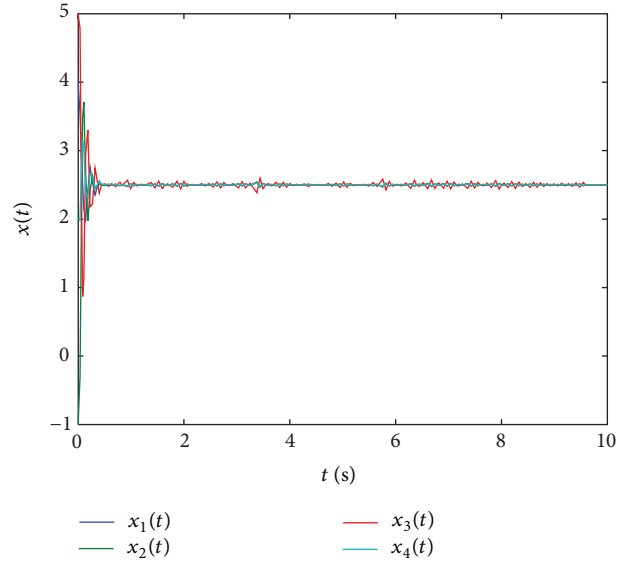

FIGURE 2: The trajectories of the state vectors $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$.
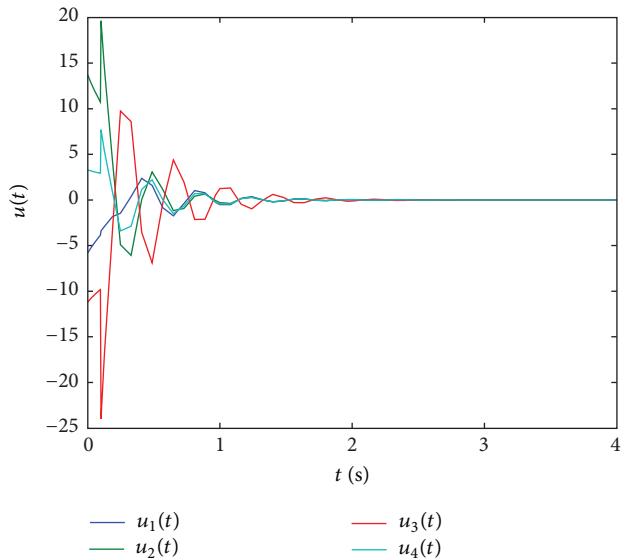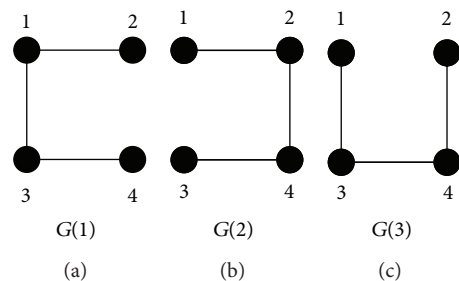

FIGURE 3: Control inputs for the sensors.

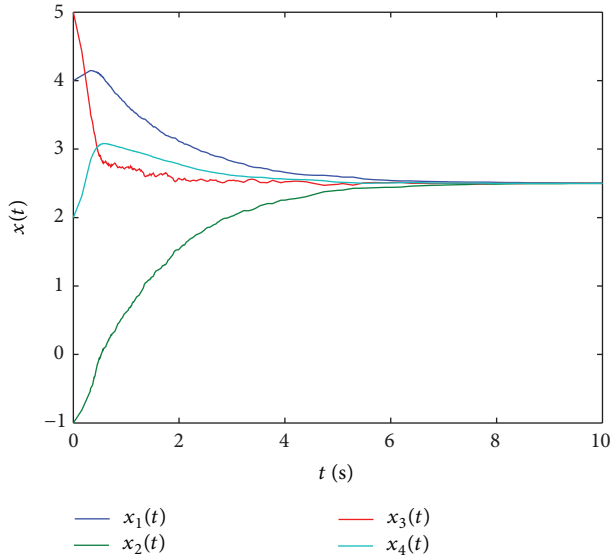FIGURE 4: The topologies of the four sensors in states 1, 2, and 3.

FIGURE 5: The trajectories of the state vectors $x_1(t)$, $x_2(t)$, $x_3(t)$, and $x_4(t)$.

packet loss and time-delay cases of the WSN's consensus problem under event condition.

Acknowledgments

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