Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 343524, 5 pages
http://dx.doi.org/10.1155/2013/343524

Research Article

Fault Diagnosis for Linear Discrete Systems Based on an Adaptive Observer

Jianwei Liu, Bin Jiang, and Ke Zhang

College of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

Correspondence should be addressed to Bin Jiang; ebjiang@yahoo.com

Received 9 July 2013; Revised 9 September 2013; Accepted 23 September 2013

Academic Editor: Zidong Wang

This paper presents a fault diagnosis algorithm to estimate the fault for a class of linear discrete systems based on an adaptive fault estimation observer. And observer gain matrix and adaptive adjusting rule of the fault estimator are designed. Furthermore, the adaptive regulating algorithm can guarantee the first-order difference of a Lyapunov discrete function to be negative, so that the observer is ensured to be stable and fault estimation errors are convergent. Finally, simulation results of an aircraft F-16 illustrate the advantages of the theoretic results that are obtained in this paper.

1. Introduction

As the scale and complexity of modern control systems are increasing, the requirements on system reliability are also increasing. Therefore, the design and analysis of fault detection and diagnosis (FDD) algorithms have received considerable attention during the past three decades. The development of FDD has been addressed by more and more authors and fruitful results have been obtained; see [1–3].

The observer technology is one of the important methods for FDD and fault-tolerant control [4, 5]. Commonly used observer-based fault estimation methods include sliding mode observers [6], unknown input observers [7], H_∞ filtering methods [8], neural networks observers [9], and adaptive observers [10, 11]. But most of them focus on the continuous systems, and only few results have been reported on the fault estimation design in discrete-time systems. The discrete-time is widely used in practical implementations, for example, computer control systems, networked control systems, and so forth [12]. Reference [13] designed an H_∞ FD filter for a class of linear discrete-time systems in a networked environment. Reference [14] used an adaptive fault observer to deal with discrete-time systems, but it did not involve the issue of fault estimation. Reference [15] also dealt with discrete-time nonlinear systems, but its inequality functions were complex and it was difficult to get solutions. On the other hand, aircraft flight control systems are good examples for applications of fault accommodation/active reliable control, and the fault observers have been employed to detect faults [11].

In this paper, a novel discrete-time actuator fault estimation scheme is proposed to deal with abrupt actuator failures. The proposed actuator fault estimation scheme is then applied by using the Lyapunov method. Simulation results of a numerical example are also given.

The paper is organized as follows. Section 2 describes the mathematical preliminaries and problem formulation. In Section 3, concerning the theoretical results of the proposed fault diagnosis scheme, a fault estimation scheme is proposed to deal with the actuator failures of discrete-time system. In Section 4, an example of aircraft flight control system is given to illustrate the performance of the proposed scheme. The concluding remarks are given in Section 5.

2. System Description and Preliminaries

Consider a discrete-time linear system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ef(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state, $u \in R^m$ is the control input, $f(k) \in R^q$ with $q \le m$ is the function to model the actuator faults, and $y(k) \in R^r$ is the measurable output. Matrices A, B,

C, and E are real matrices of appropriate dimensions. Matrix E is of full column rank; that is, rank $(E) = q$.

For estimating the actuator fault $f(k)$, an adaptive observer is constructed as follows:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + E\hat{f}(k) - L[\hat{y}(k) - y(k)],$$
$$\hat{y}(k) = C\hat{x}(k), \tag{2}$$

where $\hat{x}(k) \in R^n$ is the observer state vector, $\hat{y}(k) \in R^r$ is the observer output vector, $\hat{f}(k) \in R^q$ is the estimate of the actuator fault $f(k)$, $L \in R^{n\times r}$ is the observer gain to be designed.

Denoting that

$$e_x = \hat{x}(k) - x(k), \qquad e_y = \hat{y}(k) - y(k),$$
$$e_f = \hat{f}(k) - f(k), \tag{3}$$

then the estimation error dynamics is modeled as follows:

$$e_x(k+1) = (A - LC)e_x(k) + Ee_f(k),$$
$$e_y(k) = Ce_x(k). \tag{4}$$

3. Main Results

3.1. Modified Adaptive Fault Estimation Algorithm

Theorem 1. *For constant actuator fault $f(k)$, if there exist matrices $P > 0$ and $Q > 0$ and positive scalars ε and defined*

$$\Gamma = 2\left(\varepsilon EE^T + Q\right)^{-1} \tag{5}$$

such that the following condition holds:

$$\Xi = \begin{bmatrix} (A-LC)^T P(A-LC) - P & (A-LC)^T PE \\ * & E^T\left(-\Gamma^T Q\Gamma + P\right)E \end{bmatrix} < 0, \tag{6}$$

then the algorithm

$$\Delta\hat{f}(k+1) = -\varepsilon E^T\Gamma[e_x(k+1) - (A-LC)e_x(k)] \tag{7}$$

and the fault estimation

$$\hat{f}(k+1) = \hat{f}(k) + \Delta\hat{f}(k+1) = \hat{f}(0) + \sum_{i=1}^{k+1}\Delta\hat{f}(i) \tag{8}$$

can realize estimation error of both the state and fault uniformly bounded for the entire time period.

Proof. From system (4), one gets $Ee_f(k) = e_x(k+1) - (A - LC)e_x(k)$, so the algorithm (7) becomes

$$\Delta\hat{f}(k+1) = -\varepsilon E^T\Gamma[e_x(k+1) - (A-LC)e_x(k)]$$
$$= -\varepsilon E^T\Gamma Ee_f(k). \tag{9}$$

Then,

$$e_f(k+1) = \hat{f}(k+1) - f(k+1)$$
$$= \hat{f}(k+1) - \hat{f}(k) + \hat{f}(k) - f(k+1). \tag{10}$$

Because $f(k)$ is constant, $f(k+1) = f(k)$ and $\Delta\hat{f}(k+1) = \hat{f}(k+1) - \hat{f}(k)$; then one gets

$$e_f(k+1) = \Delta\hat{f}(k+1) + e_f(k) = \left(I - \varepsilon E^T\Gamma E\right)e_f(k). \tag{11}$$

Based on (4) and (11), we can obtain the following augmented system:

$$\begin{bmatrix} e_x(k+1) \\ e_f(k+1) \end{bmatrix} = \begin{bmatrix} A-LC & E \\ 0 & I-\varepsilon E^T\Gamma E \end{bmatrix}\begin{bmatrix} e_x(k) \\ e_f(k) \end{bmatrix}. \tag{12}$$

Consider the following Lyapunov function:

$$V(k) = e_x^T(k)Pe_x(k) + \varepsilon^{-1}e_f^T(k)e_f(k). \tag{13}$$

Then,

$$\begin{aligned}
\Delta V(k) &= e_x^T(k+1)Pe_x(k+1) - e_x^T(k)Pe_x(k) \\
&\quad + \varepsilon^{-1}e_f^T(k+1)e_f(k+1) - \varepsilon^{-1}e_f^T(k)e_f(k) \\
&= e_x^T(k)\left[(A-LC)^T P(A-LC) - P\right]e_x(k) \\
&\quad + 2e_x^T(k)(A-LC)^T PEe_f(k) + e_f^T(k)E^T PEe_f(k) \\
&\quad + \varepsilon^{-1}e_f^T(k)\left(I - E^T\Gamma E\right)^T\left(I - E^T\Gamma E\right)e_f(k) \\
&\quad - \varepsilon^{-1}e_f^T(k)e_f(k) \\
&= e_x^T(k)\left[(A-LC)^T P(A-LC) - P\right]e_x(k) \\
&\quad + 2e_x^T(k)(A-LC)^T PEe_f(k) \\
&\quad + e_f^T(k)E^T\left(-\Gamma^T Q\Gamma + P\right)Ee_f(k).
\end{aligned} \tag{14}$$

Denoting that $e(k) = [e_x(k)\,e_f(k)]^T$, then $\Delta V(k) = e(k)^T\Xi e(k) < 0$, and Ξ is defined as (6). □

Remark 2. The pair (A, C) is observable, and define the observe matrix $U = [C^T(CA)^T\ldots(CA^{l-1})^T]^T$. So, it gets that the rank of U is n. When C is full column rank,we have

$$\Delta\hat{f}(k+1) = -\varepsilon E^T\Gamma[e_x(k+1) - (A-LC)e_x(k)]$$
$$= -\varepsilon E^T\Gamma\left[C^{-1}e_y(k+1) - \left(AC^{-1} - L\right)e_y(k)\right]. \tag{15}$$

At the first step, the online fault estimation is as follows:

$$\hat{f}(1) = \hat{f}(0) + \Delta\hat{f}(1)$$
$$= \hat{f}(0) - \varepsilon E^T\Gamma\left[C^{-1}e_y(1) - \left(AC^{-1} - L\right)e_y(0)\right]. \tag{16}$$

And at the kth step, fault estimation is as (8).

If C is not full column rank, then more system outputs and observer estimations should be used to get the state estimation.

When the output $y(k)$ satisfies Lipschitz condition, define $l_{\min} = \operatorname{argmax}_l[\operatorname{rank}(U)]$, and define $F = (A-LC)^{-1}$ and $U_F = [C^T (CF)^T \cdots (CF^{l\min -1})^T]^T$; then one can get

$$\begin{aligned}\Delta \hat{f}(k+1) &= -\varepsilon E^T \Gamma \Big\{ U_F^{-1}\left[e_y(k+1)^T \ \cdots \ e_y(k+2-l\min)^T\right]^T \\ &\quad - AU_F^{-1}\left[e_y(k)^T \ \cdots \ e_y(k+1-l\min)^T\right] \\ &\quad + Le_y(k) \Big\}.\end{aligned} \quad (17)$$

Then, fault estimation is $\hat{f}(k+1) = \hat{f}(k) + \Delta\hat{f}(k+1) = \hat{f}(0) + \sum_{i=1}^{k+1}\Delta\hat{f}(i)$.

Remark 3. Equation (6) is a nonlinear matrix inequality about matrices Γ and Q, and it is not easy to be calculated, so let $Q = \varepsilon EE^T + \delta \operatorname{tr}(EE^T)$. And after we choose $0 < \delta < \varepsilon$, the solution can be found easily.

Remark 4. If the fault $f(k)$ is not constant but is a linear function, such as $f(k+1) = Hf(k)$, then

$$e_f(k+1) = \left(H - E^T\Gamma E\right) e_f(k). \quad (18)$$

Consider the same Lyapunov function $V(k) = e_x^T(k)Pe_x(k) + \varepsilon^{-1}e_f^T(k)e_f(k)$. We can also get

$$\Xi = \begin{bmatrix} (A-LC)^T P (A-LC) - P & (A-LC)^T PE \\ * & E^T\left(-\varepsilon\Gamma^T Q\Gamma + P\right)E + \varepsilon\left(H^T H - I\right)\end{bmatrix} < 0. \quad (19)$$

The proof is smilar to that of Theorem 1 and it is omitted here for brevity.

4. Simulation Results

In this section, the fault estimation algorithm is applied to a model of the vertical dynamics of an F-16 aircraft. The model is taken from [16]. The signals and their generation in the simulations are summarized in Table 1, where size means the variance for the inputs and constant magnitude for the faults, respectively.

Table 1: Signals in the F-16 simulation study.

Signal	Not.	Meaning	Size
Inputs	u_1	Spoiler angle (0.1 deg)	1
	u_2	Forward accelerations (m/s^2)	1
	u_3	Elevator angle (deg)	1
Outputs	y_1	Relative altitude (m)	10^{-4}
	y_2	Forward speed (m/s)	10^{-6}
	y_3	Pitch angle (deg)	10^{-6}
States	x_1	Altitude (m)	10^{-4}
	x_2	Forward speed (m/s)	10^{-4}
	x_3	Pitch angle (deg)	10^{-4}
	x_4	Pitch rate (deg/s)	10^{-4}
	x_5	Vertical speed (deg/s)	10^{-4}
Faults	f_1	Spoiler angle actuator	0.5
	f_2	Forward acceleration actuator	0.1
	f_3	Elevator angle actuator	1

We have the following numerical values in (1):

$$\begin{gathered} A = \begin{bmatrix} 1 & 0.0014 & 0.1133 & 0.0004 & -0.0997 \\ 0 & 0.9945 & -0.0171 & -0.0005 & 0.0070 \\ 0 & 0.0003 & 1 & 0.0957 & -0.0049 \\ 0 & 0.0061 & 0 & 0.9130 & -0.0966 \\ 0 & -0.0286 & 0.0002 & 0.1004 & 0.9879 \end{bmatrix}, \\ B = \begin{bmatrix} -0.0078 & 0 & 0.0003 \\ -0.0115 & 0.0997 & 0 \\ 0.0212 & 0 & -0.0081 \\ 0.4150 & 0.0003 & -0.1589 \\ 0.1794 & -0.0014 & -0.0158 \end{bmatrix}, \\ E = \begin{bmatrix} -0.0078 & 0 & 0.0003 \\ -0.0115 & 0.0997 & 0 \\ 0.0212 & 0 & -0.081 \\ 0.4150 & 0.0003 & -0.1589 \\ 0.1794 & -0.0014 & -0.0158 \end{bmatrix}, \\ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \\ l_{\min} = 2, \qquad U_F = \left[C^T \ \ (CF)^T\right]^T. \end{gathered} \quad (20)$$

By solving conditions in Theorem 1, one can obtain the following solutions after iterations: $L = [0.0001\ 0\ 0;\ 0\ 0.0001\ 0;\ 0\ 0\ 0.0002;\ 0.0001\ 0.0001\ 0;\ 0\ 0.0001\ 0.0001]$; $\varepsilon = 1$, $\delta = 0.1$. Then, one can take the learning rate $\Gamma = [189923\ \ -147.6\ \ 646.2\ \ 2443\ \ 1727.8;\ \ -147.6\ \ 100.6\ \ 1.31 -0.58\ \ 1.24;\ \ 646.2\ \ 1.31\ \ 9365\ \ -465.8\ \ -1.27;\ \ 2443\ \ -0.589\ \ -465.8\ \ 71.3 -5.26;\ \ 1727.8\ \ 1.24 -1.27\ \ -5.26\ \ 87.5]$.

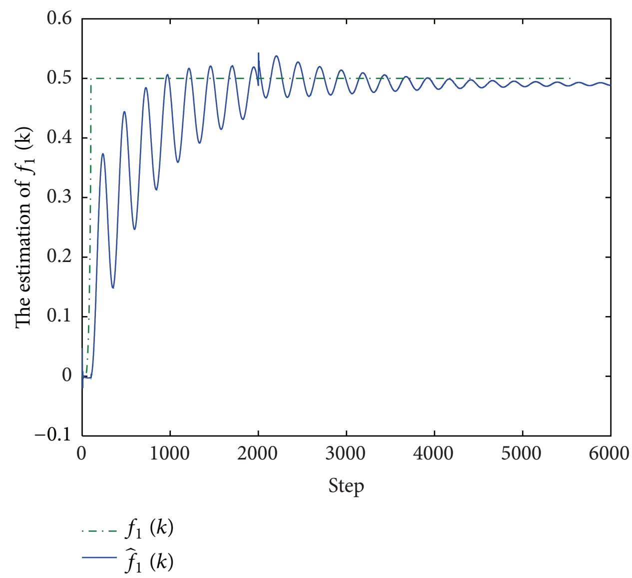

Figure 1: Estimation of the fault $f_1(k)$.

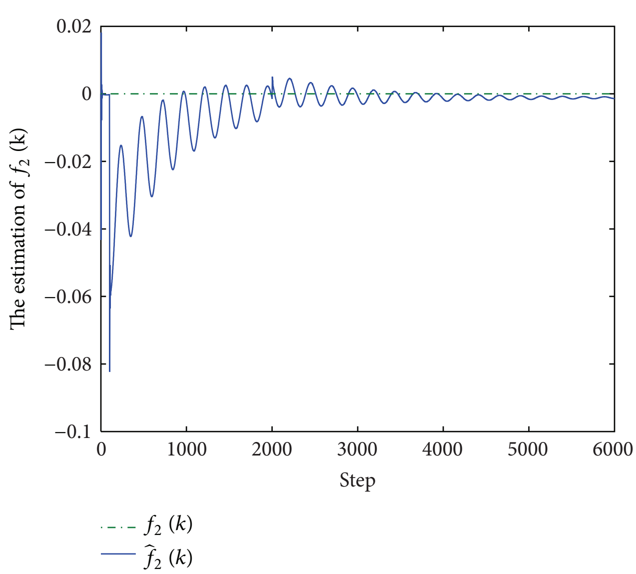

Figure 2: Estimation of the fault $f_2(k)$.

In this simulation, it is assumed that three kinds of actuator faults $f_i(k)$ are, respectively, created as

$$f_1(k) = \begin{cases} 0, & k < 100, \\ 0.5, & k > 100, \end{cases} \quad f_2(k) \text{ is free,}$$
$$f_3(k) = \begin{cases} 0, & k < 2000 \\ 1, & k > 2000 \end{cases}. \tag{21}$$

The fault $f_1(k)$ estimation result is shown in Figure 1, while Figure 2 illustrates the estimation of $f_2(k)$, and Figure 3 illustrates the estimation of $f_3(k)$.

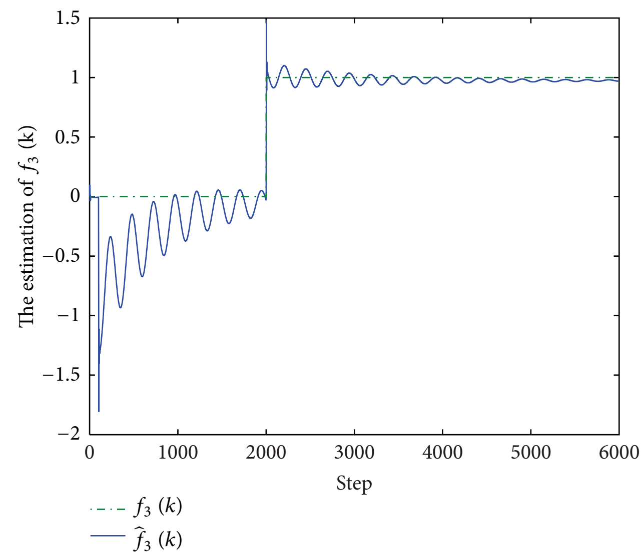

Figure 3: Estimation of the fault $f_3(k)$.

5. Conclusions

In this paper, a fault estimation algorithm is established for linear discrete systems with actuator faults. The algorithm can enhance the performance of fault estimation. And simulation results show that using the algorithm, the accuracy of fault estimation can be improved evidently. Extension of the proposed fault estimation method to more general nonlinear systems is an interesting issue, which will be investigated in our future research work.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (nos. 61273171, 61304112, and 61104020), the Natural Science Foundation of Jiangsu Province (BK20131364), and the Doctoral Fund of Ministry of Education of China (no. 20113218110011).

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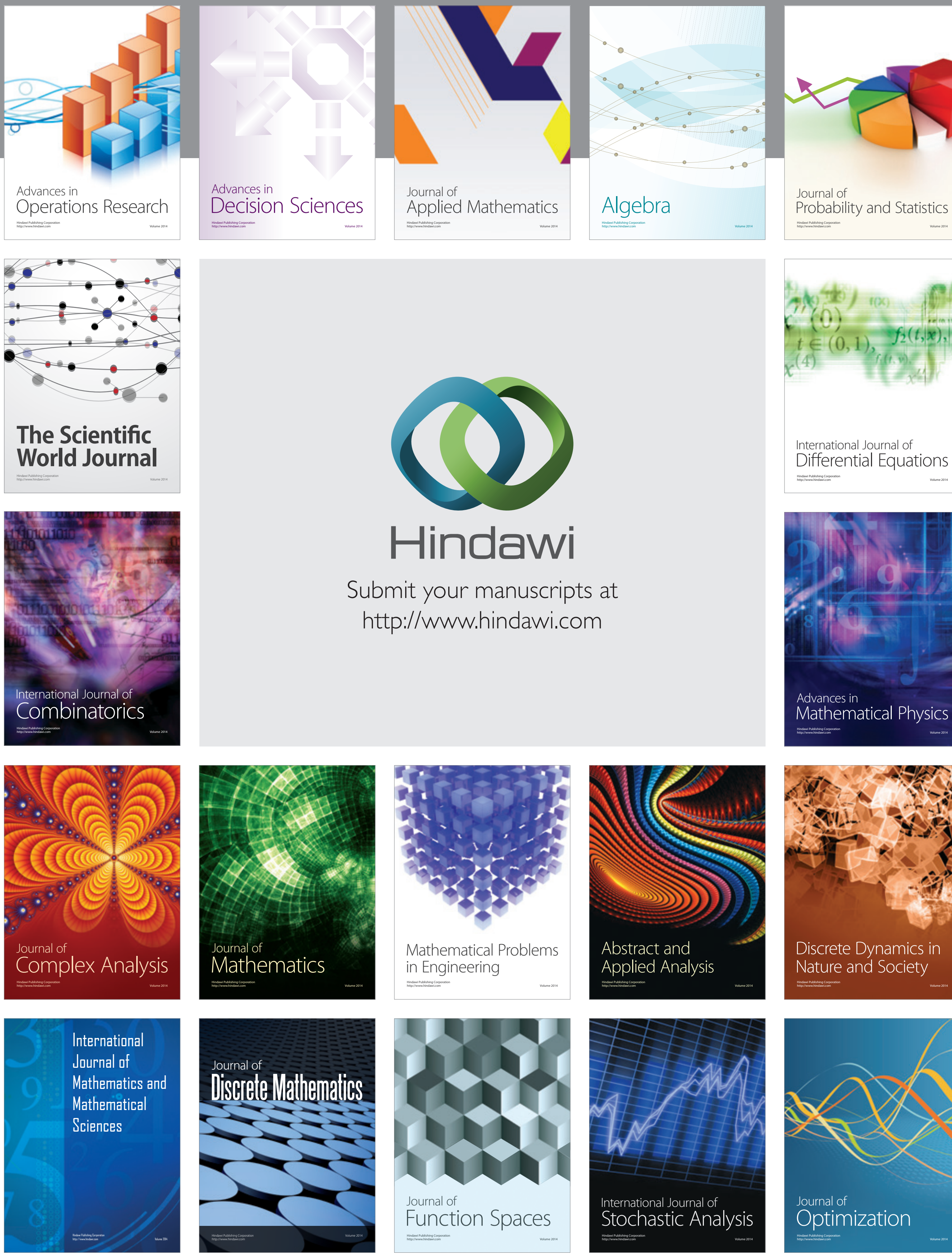
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