

Hindawi Publishing Corporation  
Science and Technology of Nuclear Installations  
Volume 2012, Article ID 915496, 9 pages  
doi:10.1155/2012/915496

## Research Article

# Nuclear Level Density Parameters of $^{203-209}\text{Pb}$ and $^{206-210}\text{Bi}$ Deformed Target Isotopes Used on Accelerator-Driven Systems in Collective Excitation Modes

Şeref Okuducu,<sup>1,2</sup> Nisa N. Aktı,<sup>3</sup> Sabahattin Akbaş,<sup>3</sup> and M. Orhan Kansu<sup>1</sup>

<sup>1</sup> Department of Physics, Faculty of Sciences, Gazi University, 06500 Ankara, Turkey

<sup>2</sup> Department of Physics, Faculty of Science and Arts, Bozok University, 66200 Yozgat, Turkey

<sup>3</sup> Institute of Science, Gazi University, Milas No 15 Street, Teknikokullar, 06500 Ankara, Turkey

Correspondence should be addressed to Şeref Okuducu, [okuducu@gazi.edu.tr](mailto:okuducu@gazi.edu.tr)

Received 27 December 2011; Accepted 17 February 2012

Academic Editor: Alberto Talamo

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The nuclear level density parameters of some deformed isotopes of target nuclei (Pb, Bi) used on the accelerator-driven subcritical systems (ADSs) have been calculated taking into consideration different collective excitation modes of observed nuclear spectra near the neutron binding energy. The method used in the present work assumes equidistant spacing of the collective coupled state bands of the considered isotopes. The present calculated results for different collective excitation bands have been compared with the compiled values from the literature for s-wave neutron resonance data, and good agreement was found.

## 1. Introduction

The knowledge of nuclear level densities is a crucial input in various fields/applications such as the creation of consistent theoretical description of excited nucleus properties and the nuclear reaction cross-section calculations for many branches of nuclear physics, nuclear astrophysics, nuclear medicine, and applied areas (medical physics, etc.) [1–14]. The neutron capture cross-sections, required for both design and nuclear model calculations in nuclear science and technologies, are approximately proportional to the corresponding level densities around the neutron resonance region. In nuclear medicine, the cross-section data obtained from nuclear level density approaches are needed to optimize production of radioactive isotopes for therapeutic purposes, for example, biomedical applications such as production of medical radioisotopes and cancer therapy and accelerator-driven incineration/transmutation of the long-lived radioactive nuclear wastes.

Recently, the accelerator-driven systems (ADSs), which are used for production of neutrons in spallation neutron source and can act as an intense neutron source in accelerator-driven subcritical reactors, and their neutronics have

been studied by many researchers [15–20]. Most of the studies were concerned with specific design concepts and the production of neutrons from spallation reactions. New accelerator-driven technologies make use of spallation neutrons produced in (p,xn) and (n,xn) nuclear reactions on high-Z targets. Through (p,xn) and (n,xn) nuclear reactions, neutrons are produced and moderated by heavy water in the target region and light water in the blanket region. These moderated neutrons are subsequently captured on  $^3\text{He}$ , which flows through the blanket system, to produce tritium via the (n,p) reaction. Therefore, new nuclear cross-section data are needed to improve the theoretical predictions of neutron production, shielding requirements, activation, radiation heating, and material damage [21]. The design of ADS requires precise knowledge of nuclide production cross-sections in order to predict the amount of radioactive isotopes produced inside the spallation target (W, Pb, Bi, and other isotopes), may be liquid or solid [20–24]. However, the precision of models used to estimate production cross-sections is still far from the performance required for technical applications. An important applied field in the ADS systems, which is presently discussed, is the technical application of accelerator-driven subcritical reactors, of

nuclear collision processes for the energy production and the transmutation of nuclear waste in hybrid reactor systems.

Nuclear reactions calculations based on standard nuclear reaction models play an important role in determining the accuracy of various parameters of theoretical models and experimental measurements. Especially, the calculations of nuclear level density parameters for the isotopes can be helpful in the investigation of reaction cross-sections. In this manner, very sophisticated theoretical approaches have been developed to estimate total level densities of atomic nuclei, especially in the region of deformed heavy and light nuclei. However, in practice analytical expressions, which contain several parameters adjusted on scarce experimental data, are generally preferred [25].

The analytical expressions used for the nuclear level density calculations [2, 3, 5] are based on the Fermi gas model. The most widely used description of the nuclear level density is the Bethe formula, based on the thermodynamic relation between entropy and the average energy of a system considered in the framework of noninteracting particles of the Fermi gas. The traditional Bethe theory of the nuclear level density calculation, which uses the assumption that the individual neutrons and protons occupy a set of low energy levels in the ground state and fill up the higher individual states at any excitation energy, has been successfully used so far, with different contributions made to this model in the form of shell, pairing, deformation effects [4, 26–29], finite size effects [30], and thermal and quantal effects [31], as well as improvements in the determination of the spin cut-off factors [32]. However, such contributions do not take into account the collective effects, which may play a basic role in describing the nuclear level density of some deformed nuclides.

Calculations of all parameters of fission and fusion reactors, accelerator-driven systems, and other nuclear technology fields depend strongly on cross-section data. And, it is well known that nuclear level density parameters are of crucial importance for the cross-section. In light of the preceding knowledge in the present study, the nuclear level density parameters of deformed  $^{203-209}\text{Pb}$  and  $^{206-210}\text{Bi}$  target isotopes used on the ADS systems have been calculated by using different collective excitation modes of the observed nuclear spectra near the neutron binding energy and a simple model introduced in our previous works [6–12], in which the collective character of the nuclear excitations is available.

## 2. Theoretical View Point of the Nuclear Level Density

The above-mentioned Bethe theory gives also the dependence of the nuclear level density on the total angular momentum  $J$  of the nucleus. The expression used for the observable nuclear level density at any excitation energy  $U$  and momentum  $J$  can be written as [2, 3]

$$\rho(U, I) = \frac{\sqrt{\pi} \exp(2\sqrt{aU})}{12a^{1/4}U^{5/4}} \frac{(2I+1) \exp\left[-(I+(1/2))^2/2\sigma^2\right]}{2\sqrt{2\pi}\sigma^3}, \quad (1)$$

where  $a$  and  $\sigma$  are the level density parameter and spin distribution parameter, respectively. The parameters  $a$  and  $\sigma$  are defined by

$$a = \frac{\pi^2}{6} g(\varepsilon_F), \quad (2)$$

$$\sigma^2 = g(\varepsilon_F) \langle m^2 \rangle t. \quad (3)$$

Here, the parameter  $g(\varepsilon_F)$  is the sum of the neutron and proton single-particle states density at the Fermi energy  $\varepsilon_F$ ,  $\langle m^2 \rangle$  is the mean square magnetic quantum number for single-particle states, and  $t$  is the nuclear thermodynamic temperature of an excited nucleus in the Fermi gas model. These factors are expressed as follows:

$$g(\varepsilon_F) = \frac{3}{2} \frac{A}{\varepsilon_F}, \quad \langle m^2 \rangle = 0, 146 A^{2/3}, \quad t^2 = \frac{U}{a}, \quad (4)$$

where  $A$  is the mass number of a nucleus.

The experimental observations cannot determine the different orientation of nuclear angular momentum  $J$ . Therefore, it is useful to obtain the observable level density, which has the form [3, 4]

$$\rho(U) = \sum_J \rho(U, J) = \frac{\sqrt{\pi} \exp(2\sqrt{aU})}{12} \frac{1}{a^{1/4} U^{5/4} \sqrt{2\pi}\sigma}. \quad (5)$$

Hence, substitute (2)–(4) into (5) to find the observable level density as

$$\rho(U) = \frac{a}{12\sqrt{2} \times 0.298 A^{1/3} (aU)^{3/2}} \exp(2\sqrt{aU}). \quad (6)$$

The level density parameters of the Bethe theory have been well established in a number of studies [3, 33, 34] on the s-wave neutron resonance for different mass nuclei. However, this theory does not take into account the collective effects of the nuclear particles in the excitation of the nuclei. On the other hand, the measured magnetic and quadrupole moments of the nuclei deviate considerably from the ones calculated using the single-particle shell model in which the closed shells forming the nuclear core play no part. In other words, the excited states and the magnetic and quadrupole moments are the results of collective motion of many nucleons, not just of those nucleons that are outside the closed shell. The collective motion of the nucleons may be described as a vibrational motion about the equilibrium position and a rotational motion that maintains the deformed shape of the nucleus.

## 3. Collective Excitation Modes of Deformed Nuclei

The existence of collective energy level bands of rotational and vibrational types can now easily be identified from nuclear spectra data [35] of many deformed nuclei. In the studies in [34, 36] the contribution of collective motion of nucleons to the energy level density has been considered. However, these studies naturally involve messy equations and

make the model complex for calculation of the nuclear level density parameters of deformed nuclei. A simpler description of collective model was first suggested by Rainwater [37] who made clear the relationship between the motion of individual nuclear particles and the collective nuclear deformation. Later a quantitative development of the nuclear collective model taking into consideration the collective motion of the nuclear particles was given by [30, 38–40]. Recently in considerable studies such as in [6–12], it has been attempted to identify the nuclear level density parameters in the region of some light and large deformed nuclei by the use of a simple model of nuclear collective excitation mechanism. Almost all data on the estimated level density parameters of these deformed nuclei are well identified on a base of collective rotational and collective vibrational bands such as ground state band,  $\beta$  band, octupole band,  $\gamma$ -band, and so forth.

Some deformed isotopes used on ADS systems have also stable deformation in their ground states. Such isotopes studied may rotate due to interactions with an external incident particle or emitting the particle. Rotational energy of an axially symmetric deformed even-even nucleus is given as [30]

$$E_{\text{rot}}(I, K) = \frac{\hbar^2}{2} \left[ \frac{I(I+1)}{J_0} + \left( \frac{1}{J_3} - \frac{1}{J_0} \right) K^2 \right], \quad (7)$$

where  $I$  and  $K$  are the total angular momentum and its projection on the axis of symmetry, respectively, of a nucleus and  $J_3$  and  $J_0$  are moments of inertia about a symmetry axis and an arbitrary axis perpendicular to the symmetry axis, respectively. The authors of [30] have used the hydrodynamic moments of inertia restricting the deformed nuclear surface by a quadrupole term only. In this model one can admit  $J_3 = 0$ , which requires the value of  $K$  in (7) to be identically zero. Then, we come to the following rotational energy equation:

$$E_{\text{rot}} = \frac{\hbar^2}{2J_0} I(I+1), K = 0. \quad (8)$$

The above expression is in good agreement with the observed low-lying energy levels of the even-even large deformed nuclei, which are the values of angular momentum  $I$ ,  $I = 0, 2, 4, 6, \dots$ . As mentioned above the energy level sequence in such a case is called ground state rotational band having positive parity.

In the following collective vibrational modes, we consider two modes, namely, the quadrupole and the octupole vibrational modes. The quadrupole mode, also called  $\beta$ -vibrational band, carries two units of angular momentum and even parity ( $0^+$ ,  $2^+$ ,  $4^+$ ,  $6^+$ , ...) while the octupole vibrational band carries two units of angular momentum and negative parity ( $1^-$ ,  $3^-$ ,  $5^-$ ,  $7^-$ , ...). Here, the  $\beta$  band is associated with vibrations that preserve the axis of symmetry and therefore is  $K = 0$  band with the level sequence given by (8) and the band head  $\hbar\omega_\beta$ . Another excited band is often called the gamma band  $\gamma$  and is associated with the vibrations not preserving the symmetry axis and having the levels given by (7). The spin sequence of  $\gamma$  band with  $K = 2$  is  $I = 2^+, 3^+, 4^+, 5^+, \dots$

In such a case, in deformed odd- $A$  isotopes simple identification of the observed nuclear levels is made on a base

of a strong coupling of a nucleon to an axially symmetric even-even deformed core. The rotational band with a given  $K$  value and spin sequence  $I = K = \Omega, K+1, K+2, \dots$ , where  $K$  and  $\Omega$  are the projections of the total angular momentum and odd nucleon angular momentum, respectively, on the nuclear symmetry axis, has level spacing [41]

$$\Delta E(I, K) = E_{IK} - E_{K,K} = \frac{\hbar^2}{2J_0} [I(I+1) - K(K+1)]. \quad (9)$$

#### 4. Calculation Method of Nuclear Level Density Parameter

The method used in this present study for calculation of nuclear level density parameters of some deformed target isotopes has been given in detail in the studies of [6–12]. Similarly, the mentioned method, in this study, can also be applicable to deformed target isotopes of interest.

In any case, the nuclear energy level density depending on the excitation energy,  $U$ , taking into account different excitation modes can be expressed in the following form:

$$\rho(U) = \sum_i a_i \rho_i(U), \quad (10)$$

where  $\rho_i(U)$  is the partial energy level density at the excitation  $U$  for the  $i$ th excitation mode and  $a_i$  is the weighting coefficient satisfying the condition  $\sum_i a_i = 1$ . In the present work, we use a simple expression for the energy level density, which considered the collective excitation modes. Here, in our determination of the nuclear level density due to excitation bands the “equidistant” condition between energy levels, which is the important property of the observed energy spectrum of isotopes considered, should be satisfied. These properties can approximately be verified for the energies of the coupled state bands in deformed isotopes considered as being the ratios given by

$$R_1 : R_2 : R_3 : R_4 : \dots = 1 : r : 2r : 3r : \dots \quad (11)$$

Here,  $R_1, R_2, R_3, R_4, \dots$  are the ratios of the sequential level energies to the appropriate energy unit of a corresponding band. When the above relation is satisfied, in our study, the nuclear level density formula introduced depending on the excitation energy  $U$  and energy unit  $\varepsilon_o$  for the  $i$ th excitation band can be represented as [6–12]

$$\rho_i(U, \varepsilon_{oi}) \cong \frac{\pi^2 a_{oi}}{24\sqrt{3}(a_{oi}U)^{3/2}} \exp\left(2\sqrt{a_{oi}U}\right), \quad (12)$$

which are fairly simple and contain only one parameter  $a_{oi}$  defined as

$$a_{oi} = \frac{\pi^2}{6\varepsilon_{oi}} \quad (13)$$

and represents a collective level density parameter corresponding to the  $i$ th band with the unit energy  $\varepsilon_{oi}$ . The

unit energies are  $\varepsilon_{0GS} = E(2^+)$ ,  $\varepsilon_{0\beta} = E(2^+) - E(0^+)$ , and  $\varepsilon_{0oct} = E(3^-) - E(1^-)$  for ground state,  $\beta$ , and octupole bands, respectively. Similarly, the other excitation bands can be included. In the even-even and odd- $A$  isotopes it has been shown that the unit energy is either energy of the first excited state (for ground state bands) or the energy separation between the second and first excited states (for excited bands) of the corresponding band with the given projection of the total angular momentum  $K$ . For the applicability of (12) in our identification of the nuclear level density due to different excitation bands, the “equidistant” condition between energy levels should be satisfied. As mentioned before, these band energies clearly should, at least approximately, satisfy (11).

Now, the observable level density expressions of (6) and (12) can be compared which have similar dependence on the energy, although they have been obtained from different approaches. Equation (6) obtained from the Bethe theory has been based on a single-particle nuclear model, whereas (12) has been extracted from the symmetry properties of the nuclear spectra data expressed by (11). In the previous works [7–12], our approach has been successfully used in the classification of the level density parameters for different light and large deformed nuclei.

In the same way, in the present work, this approach takes into consideration the different collective excitation modes in deformed target isotopes that are interesting, and the nuclear level density parameters  $a_{oi}$  defined by (13) can easily be obtained from nuclear spectra data given in [36] regarding nuclear level spectra of collective rotational and collective vibrational bands. The theoretical obtained values for  $^{203-209}\text{Pb}$  and  $^{206-210}\text{Bi}$  isotopes with their different corresponding bands have been listed in Tables 1 and 2.

## 5. Results and Discussion

In the present paper, we have calculated the nuclear level density parameters of deformed target isotopes  $^{203-209}\text{Pb}$  and  $^{206-210}\text{Bi}$  used on ADS by using different collective excitation modes of observed nuclear spectra. It has been seen that the nuclear energy levels of different collective excitation bands (in particular, the bands given in Tables 1 and 2) in the investigated isotopes also approximately satisfy (11). Thus, (13) can be applied for determination of the corresponding level density parameters. The calculated values of the level density parameters due to different excitation bands and the compiled values of those parameters have been represented in Tables 1 and 2 for the deformed target isotopes considered. The demonstrated values of the parameters  $a$  are given in Tables 1 and 2. Figure 1 was compiled by Gilbert and Cameron [3], Baba [33], BSFG model [4], and Mughabghab and Dunford [42] and Figure 2 was compiled by Gilbert and Cameron [3] and Rohr [34] for s-wave neutron resonances near the neutron binding energy.

In Figures 1 and 2 we illustrate the comparison of the single-particle level density parameters  $a$  and the mass number with our calculated values of  $a_0$  corresponding to the different bands for  $^{203-209}\text{Pb}$  and  $^{206-210}\text{Bi}$  deformed isotopes, respectively. From Figures 1 and 2, it is clear that the present values of the level density parameters  $a_0$

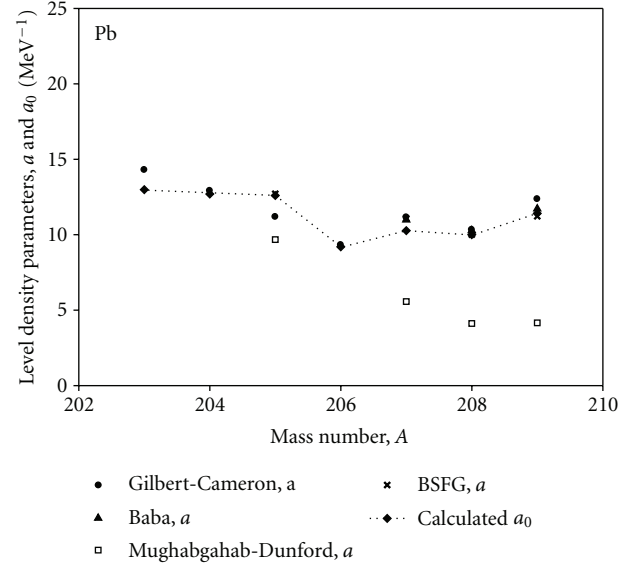


FIGURE 1: Mass dependence of the calculated nuclear level density parameters  $a_0$  and those of the compiled values  $a$  for  $^{203-209}\text{Pb}$  deformed isotopes.

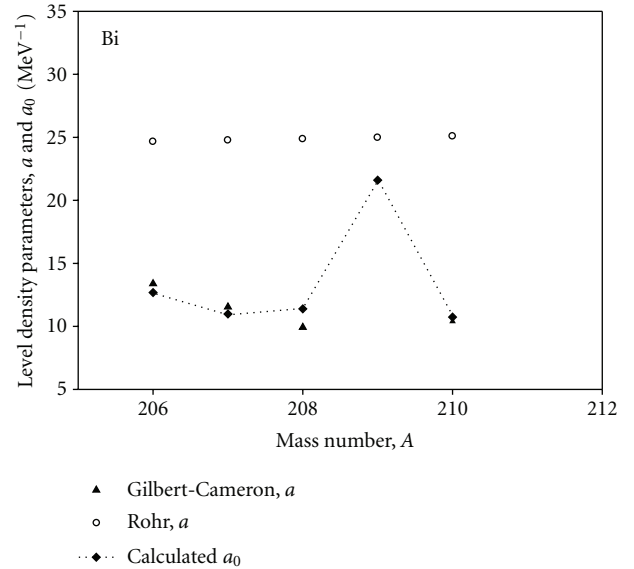


FIGURE 2: Mass dependence of the calculated nuclear level density parameters  $a_0$  and those of the compiled values  $a$  for  $^{206-210}\text{Bi}$  deformed isotopes.

calculated by (13) for these considered isotopes are well consistent with the compiled values of parameters  $a$ . As seen in Figure 1, the dominant bands in the population of  $^{203-205}\text{Pb}$  deformed isotopes generally seem to be the well-known collective bands (especially ground state, octupole, and gamma vibrational bands; see also Table 1), as to  $^{206-209}\text{Pb}$  deformed isotopes it seems to be the mixed bands (negative and positive parity bands). As clear from Figure 2, we can say that the calculated values  $a_0$  for the mixed bands (negative and positive parity bands) of  $^{206-210}\text{Bi}$  are generally good

TABLE 1: The calculated and compiled values of the nuclear level density parameters for  $^{203-209}\text{Pb}$  isotopes.

Radionuclides	Gilbert and Cameron [3], $a$ (MeV $^{-1}$ )	Baba [33], $a$ (MeV $^{-1}$ )	Mughabghab-Dunford [42], $a$ (MeV $^{-1}$ )	BSFG Egidy and Bucurescu [4], $a$ (MeV $^{-1}$ )	Calculated $a_0$ (MeV $^{-1}$ )	Corresponding bands
$^{203}\text{Pb}$ <sub>82</sub>	14.29	—	—	—	12.99	Ground state band
					21.04	Octupole vibration band
					27.39	Octupole vibration band
					2.22	Negative parity band
					1.96	Positive parity band
					17.94	Negative parity band
$^{204}\text{Pb}$ <sub>82</sub>	12.91	—	—	—	7.30	Negative parity band
					1.83	Ground state rotational band
					19.85	Beta vibration band
					12.70	Gamma vibration band
					2.33	Gamma vibration band
					6.48	Gamma vibration band
					7.13	Beta vibration band
					9.33	Negative parity band
					4.67	Octupole vibration band
					20.57	Positive parity band
$^{205}\text{Pb}$ <sub>82</sub>	11.18	—	9.68	12.71	7.04	Positive parity band
					9.54	Positive parity band
					19.85	Beta vibration band
					9.20	Beta vibration band
					6.31	Octupole vibration band
					8.51	Octupole vibration band
					2.40	Positive parity band
					8.87	Negative parity band
					12.60	Octupole vibration band
					15.21	Octupole vibration band
$^{206}\text{Pb}$ <sub>82</sub>	9.32	—	—	—	19.70	Positive parity band
					2.05	Ground state rotational band
					5.44	Beta vibration band
					8.80	Beta vibration band
					3.06	Gamma vibration band
					2.25	Gamma vibration band
					3.98	Gamma vibration band
					12.00	Gamma vibration band
					3.42	Negative parity band
					9.20	Negative parity band
				12.23	Negative parity band	
				20.64	Positive parity band	

TABLE 1: Continued.

Radionuclides	Gilbert and Cameron [3], $a$ (MeV <sup>-1</sup> )	Baba [33], $a$ (MeV <sup>-1</sup> )	Mughabghab-Dunford [42], $a$ (MeV <sup>-1</sup> )	BSFG Egidy and Bucurescu [4], $a$ (MeV <sup>-1</sup> )	Calculated $a_0$ (MeV <sup>-1</sup> )	Corresponding bands
<sup>207</sup> Pb <sub>82</sub>	—	10.98	5.57	11.08	2.88	Neutron hole states and ground state
					2.10	Single-neutron states G9/2, 111/2, J15/2, D5/2, S1/2, D3/2
					20.93	Positive parity band
					6.40	Negative parity band
					10.27	Positive parity band
					20.03	Positive parity band
					13.04	Positive parity band
					13.69	Positive parity band
<sup>208</sup> Pb <sub>82</sub>					1.91	Negative parity band
					6.91	Positive parity band
					9.01	Positive parity band
					10.00	Positive parity band
					2.29	Negative parity band
					14.41	Octupole vibration band
				10.01	17.48	Octupole vibration band
					15.02	Octupole vibration band
					12.45	Octupole vibration band
					12.26	Octupole vibration band
<sup>209</sup> Pb <sub>82</sub>					9.98	Octupole vibration band
					8.47	Octupole vibration band
					7.77	Negative parity band
					2.11	Ground state band
					36.52	Octupole vibration band
				11.22	9.69	Octupole vibration band
					16.43	Negative parity band
				11.41	Negative parity band	
				3.25	Positive parity band	

TABLE 2: The calculated and compiled values of the nuclear level density parameters for  $^{206-210}\text{Bi}$  isotopes.

Radionuclides	Gilbert and Cameron [3], a (MeV <sup>-1</sup> )	Rohr [34], a (MeV <sup>-1</sup> )	Calculated a <sub>0</sub> (MeV <sup>-1</sup> )	Corresponding bands
$^{207}_{83}\text{Bi}$	13.39	24.65	27.43	Ground state band
			14.41	Gamma vibration band
			12.68	Positive parity band
			3.60	Positive parity band
			11.64	Positive parity band
			2.76	Negative parity band
			10.96	Negative parity band
			5.17	Negative parity band
			5.83	Positive parity band
			1.66	Single-proton states: H9/2, F7/2, I13/2
$^{208}_{83}\text{Bi}$	11.54	24.76	22.51	Conf = ((206PB 2+), (P, 1H9/2))
			13.98	Conf = ((206PB 3+), (P, 1H9/2))
			10.39	Conf = ((P, 1H9/2 + 2, 0+), (P, NLJ, 1, 1+))
			1.83	(N, 3P1/2, -2, 0+))J + Conf = ((P, 1H9/2)
			2.45	(N, 1I13/2, -1) (N, NLJ, -1))
			6.27	Ground state band
			10.98	Negative parity band
			26.00	Negative parity band
			3.22	Positive parity band
			14.01	Positive parity band
$^{209}_{83}\text{Bi}$	9.92	24.86	25.96	Ground state band
			15.16	Positive parity band
			25.13	Positive parity band
			2.57	Positive parity band
			11.39	Gamma vibration band
			3.58	Gamma vibration band
			5.52	Negative parity band
			6.77	(208 PB 4-) (P, 1H9/2)
			20.93	(208 PB 5-) (P, 1H9/2)
			1.83	Single-proton states
$^{210}_{83}\text{Bi}$	10.40	24.97	32.89	(210 PO 0+) (P, NLJ, -1)
			14.84	Positive parity band
			32.09	Positive parity band
			21.60	Positive parity band
			11.73	Positive parity band
			2.03	Octupole vibration band
			35.31	Ground state band
			8.98	Negative parity band
			10.74	Positive parity band
			35.82	Negative parity band
17.80	Negative parity band			
7.85	Octupole vibration band			

dominant bands. The values of the calculated parameters of these bands are well consistent with those of the compiled data, in particular with the data of Gilbert and Cameron [3] for  $^{206-210}\text{Bi}$ .

## 6. Conclusions

On the basis of the above presented discussion we can conclude that the nuclear level density parameters of deformed target isotopes  $^{203-209}\text{Pb}$  and  $^{206-210}\text{Bi}$  used on the ADS can be identified by the use of collective vibrational bands taking into consideration the equidistant character of these bands including higher excitations. The nuclear energy level density at any excitation near the neutron binding energy may clearly have generally the same character such as collective rotational, collective vibrational, and intrinsic. Actually, as it has clearly been seen from Tables 1 and 2 and Figures 1 and 2, no dominant band alone is exactly responsible for identification of level density parameters  $a$  for the considered isotopes. Namely, the nuclear level density for such isotopes apparently should involve combination of partial level densities corresponding to the different bands, which is given by (10).

Consequently, we remark that the nuclear collective excitation modes are quite meaningful in order to obtain the level density parameters of different isotopes. The calculation of these parameters based on the properties of the measured nuclear low-lying level spectra should prove a productive area of study that should override the inherent experimental difficulties involved. Hence, at least such parameters can be useful in the design of an ADS system, which requires precise knowledge of isotopes production cross-sections in order to predict the amount of radioactive isotopes produced inside the spallation targets such as Pb and Bi isotopes.

## Acknowledgment

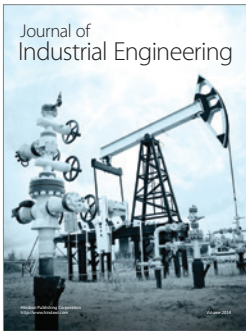
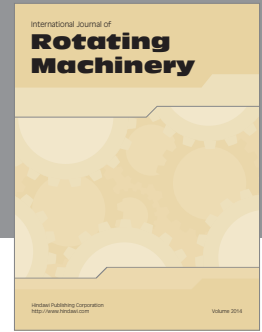
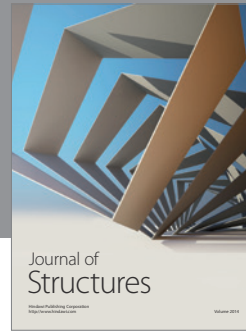
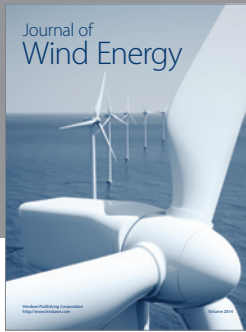
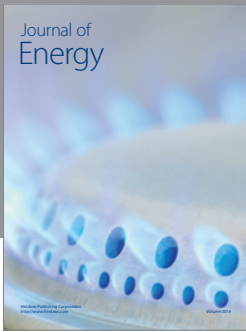
This work has partially been supported by the project of Gazi University (BAP), Code no. 05/2010-28.

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