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The logic of empirical theories revisited

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Abstract Logic and philosophy of science share a long history, though contacts have gone through ups and downs. This paper is a brief survey of some major themes in logical studies of empirical theories, including links to computer science and current studies of rational agency. The survey has no new results: we just try to make some things into common knowledge.

Keywords Theory structure · Model theory · Formal language · Dynamic logic · Computation · Agency

1 A very brief history of logic and philosophy of science

Looking at famous 19th century authors, it is often hard to separate what we would now call logicians from philosophers of science. Bolzano's *Wissenschaftslehre* (1937) is mainly a classic in logical inference, while Mill's famous work *A System of Logic* (1843) is mainly a classic of scientific methodology. Likewise, Helmholtz' theory of transformations and invariants in the foundations of the empirical sciences (1868) linked to the psychology of perception, reached mathematics, deeply influencing the logical study of definability. But at the end of the 19th century, things changed. Modern logic underwent an agenda contraction toward the foundations of mathematics: just compare the small set of concerns in Frege's *Begriffsschrift* (1879) as a model for the field of logic with the Collected Papers of his contemporary Charles Sanders Peirce

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(1933): a rich mixture of formal and informal themes, ranging from common sense reasoning to science, that is still being mined today.[1]

The foundational turn made mathematics the paradigm for logical method (which it still is) and also the major field of investigation for those methods. Even so, a new brand of philosophers of science soon picked up on the new developments, and in the 20th century, too, many major philosophers contributed to both areas, such as Carnap, Beth, Lewis, Hintikka, or van Fraassen. The main insights and techniques from the foundational phase concern mathematical proof and formal systems. But in the 1930s, members of the Vienna Circle and other groups turned these modern tools to the empirical sciences as well, with Reichenbach and Popper as famous examples. Interests went both ways, and for instance, Carnap also played a role in logical discussions at the time (van Benthem 1978a). Logical methods still dominated 'neo-positivism' in the 1950s.

This marriage came under attack from several sides around 1960. The external criticism of Kuhn's *The Structure of Scientific Revolutions* (1962) seemed to show that logic paints a largely false picture of the reasoning underlying actual practice and progress in science. Added to this, influential internal critics like Suppes observed that the formal language methodology of logic is irrelevant to scientific practice, where one goes for the relevant structures with any symbolism at hand, by-passing system-generated issues like first- versus higher-order languages that logicians delight in.[2]

Contacts did not break off, and Philosophical Logic kept many themes alive, such as conditional reasoning and causality, that meander through logic and the philosophy of science. But through the 1970s, logic became friends with disciplines where languages do play a central role, in particular, computer science and linguistics.[3] Simultaneously, many philosophers of science defected to probabilistic methods. Contacts between the fields atrophied—and sometimes, even a certain animosity could be observed.

Through the 1980s and 1990s, however, many themes have emerged that are again common to the two fields, often with a new impetus from a shared interest in computation. I will discuss a number of these and earlier themes in this paper, and show how a new liaison may be in the air. My emphasis will be, not on shiny new logic tools that philosophers of science should use, but more symmetrically, on shared interests.[4]

2 The logical structure of scientific theories

What logicians call a 'theory' can be as stark as a set of sentences X in some formal language, or much more roughly, abstracting away from details of syntax, the

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[1] While we still celebrate Frege as our founding father (whose limited channel of concerns gave the current of his thoughts torrential force), Peirce's wide range seems closer to the scope of logic today.

[2] Logic did enter 'structuralism' later in the celebrated work of Sneed (1971), and even much more so in the work of Pearce and Rantala (1983) on a variety of abstract logics for analyzing scientific theories.

[3] Probably the bulk of logic research today is at the interface with computer science, broadly conceived.

[4] This paper is not a Handbook-style survey, so no completeness is claimed for our references. Indeed, the interface of logic and philosophy of science is so vast, that no authoritative survey seems to exist.

matching class of models *MOD(X)*.[5] Sometimes one gets a richer 'computational' view with a deductive apparatus: axioms, inference rules, a notion of provability |–, and the resulting theorems.[6]

With a few exceptions, this operational aspect has played little role in contacts between logic and philosophy of science,[7] that have focused on pure syntactic-semantic aspects. But after new contacts with computer science since the 1980s, computational influences have emerged, resulting in sometimes surprising parallels across fields (van Benthem 1989).

2.1 Calculus of theories and inter-theory relations

An early interface was the 'calculus of theories' in the Warsaw School in the 1930s, by Tarski and his students. Mathematics involves a web of formal theories, connected by algebraic operations, but also simple relations like *extension*, or sophisticated ones like *relative interpretation*. And the same is of course true for the empirical sciences, though inter-theory relations are much more varied there. In particular, there is a rich tradition of notions of *reduction* between empirical theories, many of them related to logic (cf. Kuipers 2000). Reductions have to do with the integrated architecture of science, but also help consolidate its growth.

2.2 Empirical and theoretical vocabulary

A masterful historical survey of formal notions of 'scientific theory' is in Suppe (1977), and we only mention a few high-lights. One of the earliest proposals for using a modern semantic conception of theory in the study of empirical sciences like physics is Beth (1948). A richer syntactic-semantic picture of a scientific theory occurs in Hempel and Oppenheim (1948); Quine (1951), with a hierarchical architecture of theoretical laws, empirical regularities, and plain observed facts.[8]

Crucial to this picture is a division not found in mathematical theories, namely, between *observational* and *theoretical* vocabulary. The former refers to directly observable phenomena, the latter superimposes theoretical notions that unify the theory, and provide its computational power. Logical results were used to analyze this, such as the result by Craig and Vaught (1958) that any recursively axiomatizable set of observational sentences can be given a finite axiomatization by adding new theoretical predicates (describing, say, the innards of some enumerating Turing machine). On the semantic side, the pioneering book Przelecki and Wojcicki (cf. Przelecki 1969) turned this into the following simple picture. As opposed to a mathematical theory, an

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[5] An intermediate-level compromise is the association of the set X with a semantic family $\{MOD(\varphi) \mid \varphi \in X.\}$ van Benthem (2005) has a discussion of this notion and its role in current methodological debates.

[6] This is a computational 'operational' aspect, and in modern settings, this even gets broadened. A useful logical system comes with algorithmic procedures for key tasks such as checking for truth of formulas in a model, finding models for given formulas, or comparing models for structural similarities.

[7] Cf. Mittelstaedt (1978) on logical-operational views of measurement in terms of *Lorenzen dialogues*.

[8] In applications, there are also the standard 'auxiliary assumptions' that scientists tend to make.

empirical theory comes with *two disjoint vocabularies*: L_0 and L_t, and two classes of structures are relevant: those whose similarity type matches L_0, and richer structures for the complete language $L_0 + L_t$.[9] Axioms of the theory $T(L_0, L_t)$ can be purely within the component languages, or they are 'bridge principles' between empirical and theoretical vocabulary.

With this two-level picture, an old debate in the philosophy of science enters logic, viz. the ontological status of the theoretical terms. Do the latter denote independent entities and predicates—or are they fictions to oil the wheels of the theory, that might in principle be eliminated by definition? A famous proposal by Ramsey in the 1920s (cf. Ramsey 1960) is that an empirical theory T states an existential second-order claim about L_0-structures $\boldsymbol{M}$ (standing for the empirical situations that satisfy our current data):

> there exist predicates interpreting the theoretical language L_t such that the whole theory T is true on the model $\boldsymbol{M}$ expanded with the new predicates.

Syntactically, with tuples of observational predicates $\boldsymbol{P}$ and theoretical predicates $\boldsymbol{Q}$, the 'Ramsey sentence' of a theory $\varphi(\boldsymbol{P}, \boldsymbol{Q})$ is defined as the second-order formula $\exists \mathbf{Q}.\varphi(\boldsymbol{P}, \boldsymbol{Q})$. This makes sense. By a basic rule of logic, any pure $\boldsymbol{P}$-sentence that follows from the formula $\varphi(\boldsymbol{P}, \boldsymbol{Q})$ also follows from its quantified form $\exists \boldsymbol{Q}.\varphi(\boldsymbol{P}, \boldsymbol{Q})$.

This simple picture suggests many logical questions (cf. the survey van Benthem 1982). We give some examples, to show how philosophy of the empirical sciences and logical model theory can meet in fruitful ways. For a start, the 'empirical content' of a first-order theory T has often been cast as being the models of its Ramsey sentence, or equivalently, the restricted model class $MOD(T)|L_0$. What is the connection between this notion and the set $T|L_0$ of all L_0-consequences of T: the theory's 'empirical part' in a more syntactic sense? (One can think of the latter as the empirical facts and regularities currently known in T.) It is easy to see that the following inclusion holds:

$$MOD\,(T)\,|L_0 \subseteq \mathrm{MOD}\,(T|L_0)\,.$$

When the converse inclusion

$$\mathrm{MOD}\,(T|L_0) \subseteq MOD\,(T)\,|L_0$$

holds, i.e., the situations satisfying our empirical knowledge can be explained by postulating theoretical superstructure, we say that the theoretical terms are *Ramsey eliminable*. But this converse does not always hold (van Benthem 1978b has a counterexample)[10]—and the most general result that pure logic has to offer is this:

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[9] Purely for convenience, we will assume henceforth that these languages have only predicates.

[10] One considers the first-order ordering theory T of the natural numbers $(N, <)$ while adding a unary non-empty predicate that is to be closed under taking immediate successors and predecessors. This theory is consistent, since it holds in all non-standard models of T, taking the predicate to hold of the 'supernatural numbers'. But the natural numbers themselves cannot be expanded to a model of the whole theory. Ketland (2004) rediscovers a similar example in a modern study of Ramseyfication.

> For first-order theories T, each model $\boldsymbol{M}$ of $T|L_0$ has an L_0-*elementary extension* to a model for T in its full language, i.e., a model $\boldsymbol{M}^+$ with possibly new objects where all tuples of $\boldsymbol{M}$-objects still satisfy the same first-order formulas as in $\boldsymbol{M}$.

Thus, sometimes, to make a given empirical situation fall under a theory, we need to postulate not just new predicates (and functions), but also *new objects*.[11]

But there is much more to the logic vs. empirical theory interface. For instance, as we said earlier, one striking theme in the empirical sciences are transformations and *invariance*.[12] Then a natural take on the status of the theoretical terms in a theory T might be that they 'supervene' on the observational vocabulary in the following sense:

> If two models of the theory have an 'empirical' L_0-isomorphism f linking them, then f is automatically an isomorphism w.r.t. the theoretical predicates in L_t.

At least for first-order theories, Beth's Definability Theorem then shows that this is equivalent to the existence, within T, of an *explicit definition* for the theoretical terms in terms of the observational vocabulary.[13] Thus, this invariance criterion is much stronger than Ramsey eliminability, and theoretical terms would become mere abbreviations.[14] Their main function might then be to direct our attention to specific patterns to study, and smoothening matching computations—just as is done by defined concepts in mathematical theories.

The distinction between observational and theoretical vocabulary is relevant to logic in general. For instance, it has been suggested (van Benthem 1984) that 'formal semantics' is largely the art of finding theoretical terms explaining a given linguistic or inferential practice—with the usual representation proofs underpinning logical completeness theorems serving as a sort of Ramsey elimination of these theoretical terms.[15] Likewise, it has been claimed that in logics of agency, the only observable vocabulary concerns agents' actions, while the ubiquitous notions of belief and preference are theoretical terms. Their role is to drive a simple theory of agency based on postulates of rationality as 'going for maximal achievable gain, given one's beliefs'. This is reminiscent of the way that theoretical predicates are added to empirical situations in mechanics, so as to satisfy Newton's Laws,[16] and thereby start a smoother process of computation.[17]

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[11] Demopoulos (2009) reviews the debate about theoretical terms in terms of these model-theoretic results.

[12] For an example, think of the crucial role of the *Lorentz transformations* in relativistic mechanics.

[13] As a special case, this implies implicit definability in Beth's sense. Let a theory $T(\boldsymbol{P}, \boldsymbol{Q})$ have two models (D, $\boldsymbol{P}$, Q), $(D, \boldsymbol{P}, \boldsymbol{Q}')$ on the same domain D with observational predicates $\boldsymbol{P}$ and theoretical $\boldsymbol{Q}$, $\boldsymbol{Q}'$. The identity map f on D is an isomorphism on the observational predicates, and hence it also respects the theoretical predicates. But that means that $\boldsymbol{Q}$ equals $\boldsymbol{Q}'$. In an obvious version with tuples of predicates, Beth's theorem then says that in T, the $\boldsymbol{Q}$-predicates are all definable in terms of the $\boldsymbol{P}$-predicates.

[14] Schurz (2009) has a new perspective on empirical success and the role of theoretical concepts.

[15] Examples are modal accessibility relations and the usual matching Henkin-style completeness proofs.

[16] The analogy becomes particularly nice in versions of mechanics with *shortest-path* and *least-effort*. Minimization over suitable orderings is a ubiquitous logical pattern from common sense to science.

[17] See also (van Benthem 2009a) on some structural strategies for removing observed contradictions from bodies of opinion, or from scientific theories, going back to Weinberger (1965). Such strategies may introduce new 'theoretical' predicates adding argument positions to old predicates, or dividing objects into new

2.3 Inter-theory relations

But the model theory of empirical theories contains many further topics. One important theme in the philosophy of science has been comparison between theories, and given the richer structure discussed just now, this can take place in more ways than in mathematics, with various notion of *reduction* between theories. There has been little research on this topic in logic. An exception is van Benthem and Pearce (1984) that provides a mathematical characterization of the notion of *relative interpretability*.[18] The result extends earlier analyses of Tarski, Sczcerba and Makkai to prove that

> T is syntactically relatively interpretable in T' iff there exists a functor F sending T'-structures $\boldsymbol{M}$ to T-structures $F(\boldsymbol{M})$ that respects isomorphisms between models for the language of T′ and commutes with ultraproducts of such models, where the domain of the model $F(\boldsymbol{M})$ is contained in that of $\boldsymbol{M}$.

The authors relate this notion to structuralist (in Sneed's set-theoretic sense) formats of reduction in the philosophy of science. There is also interesting work of Pearce and Rantala (1983) on using non-standard models with infinite values for the velocity of light to analyze the 'approximation' relation between classical and relativistic mechanics—a common kind of relation in the empirical sciences that has no obvious counterpart in the realm of pure mathematical theories.

More sophisticated calculi of theories that care about different levels of vocabulary occur in computer science. van Benthem (1989) observes how theories of abstract data types with 'visible' and 'hidden' vocabulary reflect the above issues of Ramsey eliminability—while the so-called 'module algebra' of Bergstra et al. (1990) is a sophisticated account of modular theory structure in this two-level setting.[19] Indeed, this is just one instance where computer science meets with core issues in the philosophy of science. We will see a number of further parallels in the next section.[20]

Footnote 17 continued
groupings. The paper also discusses more conversational strategies that defuse contradictions by ascribing different beliefs to agents that are at odds. These, too, seem to involve theoretical terms, since we do not have direct observational access to the minds of people involved in a disagreement.

[18] For theories T, T', this says there is a unary predicate A in the language of T' and a translation τ of T-predicates into possibly complex $T'-$predicates such that T' can prove $\exists xAx$ plus the syntactically relativized version $(\tau\ (T))^A$, i.e., T with its predicates translated and relativized to the 'submodel' of all the $T'-$*objects* satisfying A. A typical case is the relative interpretation of the theory of the natural numbers in set theory, with numbers as finite ordinals, and arithmetical predicates like $<$ translated by set-theoretic ones like $\in$. A more complex notion of is the interpretation of the theory of the rational numbers into that of the integers, taking rationals to be *ordered pairs* of integers, modulo some definable equivalence relation.

[19] An early source here is Maibaum (1986) on logical interpolation theorems in structured programs.

[20] There are many further examples. Doyle (1983) contains a discussion showing how philosophical concerns in *Die Logische Aufbau der Welt* (Carnap 1928) returned naturally in Artificial Intelligence. Glymour (1992) says that Carnap "wrote the first artificial intelligence program".

2.4 Foundations of specific theories

But perhaps the major achievement in the foundations of mathematics has found few repercussions in the philosophy of the empirical sciences, namely, the sustained study of specific important theories by meta-logical means. Landmarks of the latter from the 1930s are Gödel's Theorems on Peano Arithmetic, and Tarski's analysis of 'elementary geometry' (Tarski 1959). These provided spectacular new insights into the expressive power and complexity of these systems, beyond what working mathematicians had realized by themselves. No similar spectacular results are known for theories in physics, such as mechanics. There has only been a small trickle of logical work on the foundations of causal space-time (starting with Robb (1914); cf. also van Benthem (1983) on the formal study of time alone), with topics described in various chapters of the *Handbook of Spatial Logics* (Aiello et al. 2007), and also of 'quantum logic' (Dalla Chiara 1992). Only recently, a new wave of logical studies of the physical sciences seems to be emerging, witness (Andréka et al. 2007) on first-order axiomatizations of the special and general theory of relativity, (Baltag and Smets 2008a) on dynamic logics of information and measurement in quantum mechanics, and (Abramsky and Coecke 2004) on proof-theoretic/computational methods in quantum information theory.

We will now move to the discussion of some more recent logical perspectives, though this by no means implies that the topics that we have discussed here are obsolete. E.g., discussions about the status of theoretical terms are very much alive today, and many of the issues raised so far also make good sense in the broader perspective to which we now turn.

3 The logical structure of scientific activities

But many themes at the interface of logic and philosophy of science are not about static theory structure. They are rather about *scientific activities*, and that even at two levels. First, there is the 'local dynamics' of users engaged in scientific reasoning with some fixed background theory—but also, there is the 'global dynamics' of wholesale theory change as performed by the scientific avant-garde. We list instances of both.

3.1 Working with a theory: varieties of reasoning

One striking feature of the literature in the philosophy of science has been the richer view of inferential activities (in a broad sense) that agents can engage in. For instance, in addition to drawing inferences from a theory, there is the process of *confirming* a given hypothesis from observed data.[21] But maybe the more common type of scientific reasoning is the *explanation* of an observed fact. Explanation is much more than just deriving observed data from a theory. Here is a sketch of what the 'deductive-nomological model' (Hempel and Oppenheim 1948) says about it. I take the liberty of giving

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[21] For a survey of confirmation theory, induction, and related problems, cf. Vickers (2006).

it a Piercean abductive twist (cf. Aliseda 2006 on inference to the best explanation) in terms of a hypothesis backed up by the theory:

> Hypothesis H *explains* evidence E given theory T if (a) T & H imply E, (b) T alone does not imply E, (c) H alone does not imply E.

Several features of this notion go beyond classical logical consequence. First, an inference of this sort is *ternary*, not binary: it involves not just premises and conclusion, but also the third ingredient of a background theory.[22] And then the structural properties of this new notion turn out analogous to, but not identical with those of classical logic. For instance, explanatory reasoning in this style is *non-monotonic* in both the T and H arguments: as opposed to standard classical consequence, strengthening the theory or the hypothesis may clearly violate clauses (b) or (c).[23]

Thus, explanation is a non-classical notion of consequence, whose structural rules show some resemblance to the 'non-monotonic logics' that emerged in AI in the 1980s, and are still a major topic in current research. Such logics allow for valid inferences $\boldsymbol{P} \Rightarrow C$ from a set of premises $\boldsymbol{P}$ to a conclusion C that may fail for stronger premises $\boldsymbol{P}, \boldsymbol{R}$. The reason for this behaviour is often that one does not consider all models for the premises in testing for the conclusion, but just a subclass of most relevant or *minimal* ones in some ordering (McCarthy 1980). In AI, this was motivated by common sense problem solving, where we work with the most plausible scenarios only. But this restriction also fits well with the fact that much empirical reasoning in science has a (usually hidden) proviso of "under normal circumstances".[24] Monotonicity may then fail since minimal models of stronger premises need not be minimal models for the original ones.[25,26]

While these accounts have usually been cast as static relations between propositions, the way logicians have tended to do, confirmation and explanation are of course

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[22] Interestingly, this richer format for what an inference really is bears strong resemblances to the analysis of argument in Toulmin (1958), that rejected modern logic, and then became a major source for modern 'argumentation theory'. In Toulmin's set up, 'data' support a 'claim', with a 'warrant' supplying the bridge, which itself comes with a 'backing'. Moreover, each conclusion has a 'qualifier' indicating its force, a feature reminiscent of the varieties of conclusions in current non-monotonic logics. Historically, transcending the simplistic format of standard logical inference went two ways then: that of HempelOppenheim eventually toward richer logics and more expressive languages, that of Toulmin toward non-logical informal argumentation theory. It would be of interest to see if the two traditions can meet again. For instance, Toulmin's emphasis on the role of 'formalities' (i.e., *procedure*) rather than 'form' in reasoning sounds quite close to the recent dynamic perspectives on logic to be discussed below.

[23] Our discussion does not do justice to a long tradition of accounts of explanation and related notions in the philosophy of science, many with an appeal to logicians (cf. the survey in Wang (2008)).

[24] My colourful high-school teacher in chemistry always told us that the main thing we should understand about chemistry was not its laws, but the meaning of the phrase "under normal circumstances"—since every law that he was going to teach us admitted of lots of exceptions.

[25] The study of confirmation Hempel (1965) even contains an explicit precursor of *circumscription*, the notion of non-monotonic logic that later became famous in AI through the work of McCarthy (1980).

[26] Aliseda (2006) develops many further analogies between scientific reasoning styles and structural rules for non-classical consequence relations, with special attention to Peirce's notion of abduction. van Benthem (2003) points at links with the various notions of consequence distinguished by Bernard Bolzano, and finds some complete sets of structural rules for these styles of reasoning.

also dynamic activities performed by cognitive agents, and we will high-light this perspective as we go on.

3.2 Further examples

There are many more themes in scientific reasoning that have an essential logical slant, and many crucially involve fine-structure of empirical theories. One example is the typical feature of scientific laws that they support *counterfactual* reasoning (cf. Goodman 1955). If you had lit the match, an explosion would have occurred, even though you did not strike it. The laws of elementary physics guarantee the strong counterfactual conclusion—at least 'under normal circumstances': the usual qualification that distinguishes mathematical from ordinary certainty. But this link is just a beginning: philosophers of science have also tried, conversely, to describe the surplus of scientific laws over 'accidentally true generalizations' in terms of their counterfactual or modal nature.[27] van Benthem (2006) discusses this and other themes where logic and philosophy have a shared history, with topics crossing between fields, including linguistics, computer science or economics, before returning.

Note also that, going back to an earlier topic, with a richer view of reasoning activities, a richer picture arises of a scientific theory: not just with layers of observational and theoretical vocabulary, but also in terms of organization of the principles stated in this multi-layered language (cf. Quine 1951). The latter range from deeply entrenched core laws via modal 'dispositional statements' about empirical regularities to brute observed facts, all surrounded by a belt of working assumptions (cf. Rescher 1970). Not surprisingly in the light of the following sections, this is also the sort of structure one finds in current logics of belief revision (cf. Rott 2007), where one's beliefs may be less or more 'entrenched' and hence less or more sensitive to revision. Thus, our discussion returns in a natural way to global issues of theory structure.

3.3 Developing a scientific theory

One can come a long way using a given scientific theory to explain observed facts, especially, since one can invent hypotheses to shield the theory itself from being refuted. But sometimes, the pressure of reality becomes too great, and one wants to change the current theory itself. *Theory change*, too, is a major theme in the philosophy of science, and optimal ways of restructuring theories have been widely studied, both historically and systematically. This process has been emphasized by Popper, who claimed that science learns through refutation of its conjectures, putting a premium on making bold claims, and learning by trial and error. There are various strands of investigation to this. One is the study of *verisimilitude*, where one tries to define when one theory is 'closer to the truth' than another (Zwart 2002), providing an assessment of the rationality and quality of scientific progress. There are also strong connections with *belief revision theory*, as is clear in Gärdenfors (1988). A more computational

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[27] A famous 'modal mechanics' mixing physics and logic was developed in Bressan (1972).

strand is the use of notions and methods from *formal learning theory* in the study of scientific inquiry and theory change, with Kelly (1996) as a pioneering study. Here, 'learners' are cast as computational devices that produce hypotheses over time when exposed to an evidence stream, and one tries to understand long-term convergence behaviour toward identification of correct theories of the world. Again, we see how logic, philosophy of science, and informatics form a natural unity.

3.4 Digression: computational analogies

As we have seen several times now, themes at the interface of logic and the philosophy of science have natural counterparts in *computer science*. The analogies noted with non-monotonic logic and belief revision are a case in point, linking to Artificial Intelligence. But there are also analogies with more standard computer science. For instance, 'structured theories' are important in the study of data bases and knowledge bases (Ryan 1992), and many further examples can be found. Even so, philosophers of science have had less to say on the fundamental function of a theory as a computational device for problem solving. Still, by now, there is a literature connecting progress in science with computational learning mechanisms (cf. Osherson et al. 1986; Glymour 1980; Bod 2006). Finally, the typical *multi-agent systems* view of modern computer science will emerge in the next section.

4 Common sense and scientific agency

For my final topic, I start with a personal perspective. The famous classic *The Structure of Science* (Nagel 1961) explains how science differs from 'common sense', in its standards of rigour, its degree of organization, and many other features. While I assiduously learnt all these criteria by heart as a student, they now seem unconvincing to me—and largely based on ignorance of the delicate workings of common sense, that have only come to light in the work of logicians since the 1970s.[28] I would now think that science is the exercise of certain qualities of our common sense reasoning, but taken further *in isolation*, and also importantly, *simplified* in that many subtle features of actual reasoning and communication are put out of play. Thus, *contra* Nagel, I think that the border line between science and common sense is thin, and this seems a good thing to me: both for the unity of culture, and not to 'let science get out of hand'.[29]

In particular, I think it is worthwhile to compare the agenda of philosophy of science with modern logics of rational agency and intelligent interaction (van Benthem 2010). This will change our perception of interfaces between the two areas. In my view, logic is about processes of information flow, and the intelligent activities supported by these.

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[28] Rereading early philosophers of science I am struck by their uncritical idealized views of the rationality of 'the scientist', often coupled to equally unwarranted disdain for the stupidity of standard philosophers.

[29] To use a not wholly serious linguistic argument: the very *term* "research" sounds dynamic to me, and it is much richer than its 'products' of immutable knowledge and fixed theories. The heart of science seems to lie in its modus operandi: the processes that generate its products, not a museum of 'certified theories'.

4.1 Diversity of information sources: observation on a par with inference

For a start, as has been observed since the earliest days of logic, in the world of common sense, agents with rational skills manipulate at least three major sources of information, namely, *observation*, *inference*, and *communication*. And this moves us with one great step away from the pure mathematics paradigm of inference and proof as the norm for logic toward the reality of the empirical sciences, where observation is equally fundamental.[30] Thus, by its very definition rather than some external motivation of ‘application’, logic moves from the a priori to the a posteriori as its topic, making empirical theories the paradigm to consider, rather than the very special case of mathematical theories.

Indeed, the hard part to understand here may not be the logical rationale of observation, but rather the role of mathematical proof. Even though we all agree that this, too, plays an essential role in science, it is less easy to say in which precise sense valid proof steps generate information. In the philosophy of science, this has been a persistent problem: cf. Fitelson (2006) on the problem of explaining the informativeness of Einstein’s famous deductions from the General Theory of Relativity, or the more general discussion of Bayesian confirmation theory in Earman (1992). The same ‘scandal of deduction’ has been much discussed in contemporary logic: cf. Hintikka (1973) (an early information-based view of logic), Abramsky (2008) (on similar issues in the foundations of computation), van Benthem and Martinez (2008), and van Benthem and Velazquez-Quesada (2009) for some current takes that have immediate relevance to scientific reasoning in general.

4.2 Further attitudes, other informational actions

Very typical for logics of agency is the wider spectrum of attitudes that agents can have toward information, ranging from knowledge and belief to many others, such as neutral ‘entertainment’, and even doubt. My colleague Peter Wesly used to make the same point about scientific activity in the 1970s: science consists not just of what we know, but also of what researchers believe, and even more generally, what we currently ‘entertain’. I would also think that we need a much richer view of what a scientific theory is in terms of surrounding cognitive attitudes, starting with beliefs[31], since, as Popper pointed out so correctly, rational *belief revision* as an engine of learning seems at least as essential to understanding science as peaceful accumulation and regurgitation of knowledge.

And even more than that, there is also the research *agenda* as an object in its own right. What Lakatos and other philosophers of science have said about that fits very

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[30] Newton’s *Principia Mathematica* seems to have mainly mathematical axioms, but read his *Optics*, and you will see that experiments, i.e., the voice of Nature speaking, are treated with equal importance.

[31] van Benthem (2007); Baltag and Smets (2008b) give complete dynamic logics for steps of belief revision. van Benthem (2009a,b) explore belief revision even for purely deductive mathematical theories.

well with current logical interest in *questions*, issues, and agenda dynamics (Hintikka et al. 2002, Girard 2007, van Benthem and Minica 2009).[32]

4.3 Longer term dynamics of information flow

Most of what I have mentioned so far is 'local dynamics' in the sense of single steps of information update, belief revision, or learning, and single questions and steps of agenda change. But science is also a long-term process with features that only emerge in the long run. Thus, dynamic logics interface naturally with *temporal logics* of agency, that can also deal with long-term features of histories, as well as 'protocols' regulating feasible or admissible ways of obtaining evidence. This is also the perspective of formal learning theory as applied to the philosophy of science (cf. Kelly 1996), and attempts are under way to merge the logical and learning-theoretical perspectives (Dégrémont and Gierasimczuk 2009).

4.4 'The others': social aspects of science revisited

One essential source of information mentioned by the earliest logicians was *communication*, given that the discipline of logic arose in a setting of conversation and argumentation with more than one agent. But likewise, science essentially involves different agents. Indeed, various authors have cast empirical inquiry in terms of *games* played by 'Man' and 'Nature' (Giles 1974; Hintikka 1973; Lorenz and Lorenzen 1978; Mittelstaedt 1978). But communication and debate between human agents also seems essential to science: after all, it is one of the most successful social inventions ever. Indeed its internal styles of debate seem one of its main engines of progress. In logic, active contacts with game theory are developing these days (van Benthem 1999; de Bruin 2010; Baltag et al. 2009; Dégrémont and Roy 2009). In the philosophy of science, interfaces have developed more in the area of evolutionary games (Skyrms 1990). Like computer science, game theory might become one point where logic and philosophy of science meet once more.

But there is even more to this 'social aspect' of science than mere interaction of individuals. Scientific theories are usually community creations, and their development tends to be a group activity. But if that is so, we also need to take a further step also visible in logic today, viz. the study of groups as epistemic actors that are *sui generis*, and the way groups can form and evolve, along with their beliefs and actions.

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[32] Indeed, I now think that the earlier emphasis on non-monotonic consequence relations, both in common sense and science, is mistaken. The essential process to understand is the *formation and modification of beliefs*, and non-monotonic features dissolve then into a dynamic logic of belief revision or learning on a classical underlying logic. van Benthem (2008) has details, and shows how this gently 'deconstructs' 'logical pluralism', and McCarthy's circumscription. A similar shift, away from non-classical 'quantum logic' to dynamic logic of measurement actions on a classical base occurs in Baltag and Smets (2008a).

4.5 Science and values

Finally, one crucial aspect about agency is this. Alongside with the dynamics of information flow, there is a second major cognitive system permeating all rational action, viz. the *dynamics of evaluation*. Everything we say and do is coloured, and often driven, by the way we evaluate situations, and set goals accordingly. Indeed, we would not even call a decision or an action 'rational' if it lacks the proper balance between available information and desire. And this evaluational system is dynamic, too: our preferences can change over time, and they interact with available information. Now, for science, it has often been stated that it is 'value-neutral', and one should stick to the mere informational facts. Is that so? Or are we missing a crucial aspect of the scientific activity by ignoring its goals, perhaps even, its changing goals over time?

4.6 A research program

Clearly, this section has only lightly raised a lot of different issues. But there is a serious general program behind the examples that I would advocate here, which seems highly promising to me. I propose taking a systematic second look at the traditional issues in the philosophy of science from the perspective of current dynamic epistemic logics. These explicitly incorporate events of observation and communication producing new information, mainly semantic and in some recent versions also syntactic. It seems to me that much more of what makes science tick will come to light that way.

5 Conclusions

Logic and the philosophy of science share a long contiguous history, and it is often hard to say where one stops and the other starts, both in themes and in persons.[33] Still, the two areas have drifted apart for some decades. Are there irreconcilable differences?

5.1 Structure, or language, or both

We have seen some typical features of the logician's modus operandi that might seem strange to philosophers of science (they certainly are alien to most practicing scientists): the use of formal languages, the search for complete formal systems, and the development of a meta-theory whose results are relative to features of formalization. Dependence on linguistic formulation is usually considered a flaw by philosophers of

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[33] My chapter (van Benthem 2006) in the *Handbook of the Philosophy of Logic* gives many further illustrations of these contacts, including topics like causality and conditionals. In particular, the present survey of contacts has concentrated on qualitative logical methods, and thus, it has left out another story that could be told just as well, the history of interfaces between logic, philosophy of science, and *probability*. The history of such interactions includes inductive logic, and it runs all the way to current work on logic, computation and games. Wheeler et al. (2010) is a recent mix of logic, probability and computational techniques, and a general source is the volume on general methodology in the *Handbook of the Philosophy of Science* (Elsevier, Amsterdam) edited by Theo Kuipers.

science.[34] Personally, I consider such polemics fruitless. 'Language focus' is a natural dual stance to structural semantic approaches to scientific activity, and it forms a natural complement to studies in the philosophy of science. Even more strongly, without proper attention to linguistic code, we cannot even make sense of the crucial *computational* aspect of science, which operates on symbols, not models.

I am a bit ambivalent, however, about the form this language awareness should take. The logician's complete formal systems are whole packages that might serve as eventual replacement of scientific activity. But this misses one of the most intriguing features of science: its ability to create new notions and new notations and then insert these into *existing* reasoning practice. The result is an evolving mixture of natural language and common sense practices with new notations and styles of proof. And it is this mixture, not some projection into pure common sense or mathematics components, that forms the success of the enterprise. Maybe we should all adjust our focus to get a better grasp of this phenomenon that keeps science a part of our culture and life.

5.2 From static structure to science as dynamic social agency

In line with this, I have said that we should look at science from the perspective of dynamic logics of agency. Part of this is still close to the tradition, viz. an emphasis on the *information dynamics* that drives scientists. But there is also the *social* aspect of different agents cooperating, and in the case of science, often also competing, deploying a host of different strategies. Now to many colleagues, science arises precisely by abstracting from this social aspect of the common sense world, and downplaying it. And so, radical critics like Kuhn and diehard logical positivists found themselves on the same side of a divide: logic has nothing to do with social activities. To me, this seems deeply mistaken. Despite the ubiquitous rhetoric of lonely scientists communicating directly with God, science is a major case of a successful social enterprise, with remarkable historical cohesion (longer than that of any current empire or religion) based on rational cooperation and competition. And for logics of agency, precisely the latter are the primary concern.

Indeed Kuhn's own work is a good example. It famously distinguished between 'normal science' and 'scientific revolutions'. But this is not a reason for breaking with logic: it is rather an intriguing shared theme with modern logic. Normal science is about the constant stream of small adjustments that we make to our knowledge and beliefs under information flow, while keeping background theory and conceptual framework fixed. But there is also the more radical form of belief revision, where amongst other things, the very language that we couched our knowledge and beliefs in gets changed. While there is not much logical work on this yet (cf. Rott 2007; van Benthem 2010 for a few thoughts), I would definitely consider language and framework change as logical topics. They are not reasons for leaving logic: taking them seriously rather calls for more logic.

Finally, the Dynamic Turn in logic mirrors similar movements in recent philosophy, from social epistemology (Goldman 1999) to philosophy of science (Friedman 2001).

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[34] Cf. the spectre of 'dependence on translation' in the study of verisimilitude: Miller (1974).

And when all is said and done, I find that even the staunchest logical philosophers of an earlier era admit the social point. Carnap and Nagel saw the main function of science as making things *objective* by making them *intersubjective*. But what is intersubjectivity, if not a virtue that gets created, honed, and maintained through rational social interaction?[35]

In the past, logic has often been held up to philosophers of science, sometimes in a condescending manner, as a field that they should respect, apply, and emulate in its superior rigour and depth. Even philosophers of science like Carnap sometimes gush about the depth of logicians, the way they also wax ecstatic about the rationality of 'the scientist'—disregarding the negative features of actual specimens. Personally, I think that academic fields only cooperate well when there is intellectual symmetry. And then, what I would like to bring to the encounter is not just the usual virtue of precision, but also the *playful* aspect of logic as a way of thinking about rational cognitive activity, the way creative logicians play with their themes and results, rather than preach them.

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[35] A wonderful historical illustration is Staal (2006), who shows how mathematical notations 'democratized' European science since the Middle Ages, allowing for larger groups to get involved in assessing arguments, and finding new ones. On this view, the difference with science in other cultures was not greater intellect, or greater curiosity, but greater participation. While this is not the only cause (cf. Huff 1993 for social dimensions such as legal stability), it is worth seeing that formalization is not 'abstraction away from the world' (cf. Barendregt 2008) but the opposite: an increase in intersubjectivity.

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