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Preservice Teachers Perception of their Preparation Program to Cultivate their Ability to Teach Proof

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Abstract

This study describes twelve preservice teachers perceptions of their preparation program to foster their ability to teach proof. Data were collected via structured interview questions, relative to participants' mathematical background, perceptions about proof, and their readiness to teach proof. The study found that most preservice teachers are not afforded many opportunities to prove outside of geometry or bridge to abstract algebra courses, and perceive that they will be challenged to teach proof effectively. The results suggest that the trajectory of preservice mathematics teachers college experience needs to increase opportunities to prove, and practice teaching proof. Furthermore, the findings unveil fundamental misconceptions preservice teachers have about the role and nature of mathematical proof. An analysis of these misconceptions suggests: explanations of the perceptions, and a need to incorporate explicit reflections on the role and nature of proof in teacher education curricula.

Key words: proof, preservice teachers, content knowledge, preparation program

Overview

Proof is important to the field of mathematics. In the United States, the National Council of Teachers of Mathematics (2000) identified Reasoning and Proof as a Process Standard, and the Common Core State Standards Initiative (2010) Standards for Mathematical Practice suggested that students ought to be able to construct arguments, engage in reasoning and evaluate the reasoning of others. The vitality of proof in school mathematics is dependent on teachers' ability to adequately convey the importance of proof, and promote the usage of proof beyond the roles of validation and verification of known facts. If enacted effectively within the classroom environment, proof can also be used for exploration, systematization, communication, and to develop new ideas (De Villiers, 1999). Hence, it is imperative that teachers are adequately prepared to teach proof.

Research findings suggest that in-service teachers may not be adequately prepared to teach proof in secondary mathematics. Teachers may avoid teaching proof, or limit the depth to which they engage students in proving due to their own limited knowledge or comfort level with proof (Sears, 2012). Knuth (2002) acknowledged that teachers were challenged to differentiate proof from non-proof tasks. Teachers also provided positive feedback for non-proof arguments as though they were generalizable statements (Bieda, 2010). Therefore, teachers' knowledge about proof can impact how they facilitate it. If teachers are not prepared to teach proof, there enacted lessons may limit students' opportunity to engage in proving activities. Whereas, with training, the likelihood that teachers will facilitate the unpacking of proof within the classroom environment may increase.

Similarly, preservice teachers knowledge, or lack thereof can potentially influence how they would eventually enact proof within a classroom environment. Martin and Harel (1989) found that of the 101 preservice teachers studied, 38% accepted incorrect deductive arguments as correct for familiar context, and 52% for non-familiar context. Likewise, Stylianides, Stylianides, and Philippou (2007) found that preservice elementary, secondary and middle school teachers have difficulty with inductive proof arguments, and writing inductive statements into generalizable proof arguments. Since preservice teachers, will eventually be responsible for teaching proof, consideration ought to be given to the extent they are prepared. Therefore, there exists a need to examine preservice teachers' preparations to read and construct mathematical proofs on their own, and to teach proof within the classroom.

This study sought to answer the following research questions: What are preservice teachers perceptions of their preparation program to foster their ability to teach proof? Are there specific needs in the perception of preservice teachers that require new ways or foci of teacher education with respect to proof? In a qualitative, multi-perspective research manner (Creswell, 2012; Gill and Johnson, 1991) our examination documents preservice teachers' viewpoints of their preparation relative to proof. The findings of this study suggests that the core reasons for preservice teachers perceived unreadiness to prove or to teach proof are not a mere lack of meta-knowledge on different logical proof types, or poorly developed abilities to recognize correctness or incorrectness of deductive arguments, but also inadequate beliefs about and conceptions of the role and nature of mathematical proof.

Methods

Twelve preservice teachers, ages ranged from 20 to 57 years, agreed to participate in this study. The participants were enrolled in a middle school methods course, at a university in the southwestern region of the United States. For the methods course, students were required to summarize an article, write a literature review, and analyze videos and lesson sketches relative to reasoning and proof. Hence, at the end of the semester, participants were asked to share their perspectives of their readiness to teach proof. Data were collected via an open-ended questionnaire that comprised of 16 questions, which was distributed electronically and in paper format. Participants were asked to describe their academic background, characteristics needed to teach proof effectively, the extent their preservice preparation program prepared them to teach proof, and their perceived readiness to teach proof. Sample questions posed included: What were your experiences in college with proving? Did your preservice classes help you to teach proof? What could be done to improve your preparation for teaching proof? Do you feel ready or comfortable teaching proof? What challenges would you face if you were asked to teach a proof lesson? What characteristics are needed to teach proof effectively? Do you exhibit such characteristics? For each question students were asked to explain their responses. The questionnaire was generally completed within 60 minutes.

The data were analyzed qualitatively using grounded theory research methods (Creswell, 2012; Strauss, Corbin, and Lynch, 1990). Initially, we sorted the data by items on the questionnaire. Next, we identified codes that were derived from the data. Subsequently, we grouped the codes into themes based on commonalities of ideas. Emergent themes identified embodied courses that provided opportunities to prove, definitive viewpoints as to whether the mathematics education preparation course enhanced preservice teachers proof skills, the need for algorithms, recommendations to increase opportunities to prove, and the desire to increase content knowledge relative to proof.

Results

Generally, the results suggested that preservice teachers have little opportunity to engage with proof during their college experience, outside mathematics content courses (such as geometry or bridge to abstract algebra). A quarter of the participants acknowledged that their preservice teacher preparation program highlighted the importance, shed research findings and exposed them to proof. A preservice teacher stated that in the education course, “I really got to understand why proofs are important and why we should be exposed to them at a younger level. I think that it’s a trait I will get used to, it’ll just take practice to perfect proofs”. However, most of the participants also acknowledged that they are not ready to teach proof because of their limited experience with proof, and that they may be challenged to find creative ways to engage students in proving activities. About three quarter of the participants stated that they were not provided guidance of how to explicitly teach proof. For instance, a preservice teacher stated, “…I believe I am still in need of some practice proving myself, as well as chances to practice teaching in front of a room, something that students might have several different ways of completing a problem”. The statement suggests that the preservice teacher desires a stronger mathematical knowledge and additional pedagogical strategies for teaching proof.

A closer analysis of participants’ comments provided detailed insights about fundamental reasons for preservice teachers perceptions of their own readiness to prove and to teach proof, than just a lack of proving experience and explicit teaching strategies. One of the preservice teachers stated, “All my professors thought that some other professor has taught me how to do proofs so I have never really been formally taught how to do one. Also, each professor has a different idea of what they want proofs to look like”. This response suggest that there exists an uncertainty of who should be responsible for teaching preservice how to prove, and there exists variation in how proof is taught at the tertiary level. It also unveils specific misconceptions about the role and nature of mathematical proof: that there is a formal method of proving, which can be taught in teacher education to enable prospective teachers to teach proof in classroom and that there exists a model or uniform appearance of mathematical proof .

According to another preservice teacher, "… no one has taught me the steps in teaching proofs. Also, when I see a professor do a proof, I wonder how do they know when to stop continuing on with the proof?..." The remark suggests the preservice teacher considers the teaching of proof to be algorithmic in nature, and is unsure about the completeness of a proof. It implies that mathematical proofs always have well determined, context independent, starting and ending points.

To summarize and conclude, preservice teachers suggested that to improve their preparation for teaching proof, they need more opportunities to practice, and extra courses that foster their ability to teach and learn how to prove. According to a preservice teacher, "I have learned the importance of them, so now I just need to actually stand at the board and teach a lesson that revolves around proofs." From a normative point of view, these perceptions of reasons and possible cure for the perceived unreadiness to prove and to teach proof might dismiss the core problem. With respect to a normative concept of mathematical proof as described above, the conceptions of proof are inadequate regarding the role and nature of mathematical proof relevant to actual mathematical practice and proving in the classroom. Thus, although preservice teachers are exposed to proof during their tertiary studies, they do not grasp an adequate concept of proof.

Discussion and Implications

The results suggest that the trajectory of preservice mathematics teachers college experience needs to increase opportunities to prove and to reflect on proof on various dimensions. Proof should not be bounded within the parameters of mathematics content courses. Preservice teachers should not only get the opportunities to prove *more,* or learn more or less *algorithmic step-models* as the only explicit instruction on teaching proof in mathematics courses. Furthermore, exposure to literature about proof and its importance is necessary, but it is not sufficient. Instead, engagement with concrete proofs, both on the content level and on a meta-level of evaluation should be evident in pedagogical and content courses.

Aspects of preservice teachers conceptions of mathematical proof can be seen as misconceived, with respect to certain normative conceptions of proof that underlie the discussion of the importance of proof for school mathematics, in mathematics education (De Villiers, 1999; Harel and Sowder, 2007). According to such conceptions, mathematical proof not only serves as a routine to justify mathematical statements deductively, but also has creative potential and a fundamental exploratory function as a collective activity. Proof is seen as a form of argumentation, and communication that is understood within specific communities (Stylianides, 2007). Although standardized and objective to a seemingly high degree, proof essentially is socially determined in nature. Accordingly, proof in mathematics is not restricted to the adherence of algorithms. Rather proof incorporates creative, argumentative communication with respect to general and objective rules of deductive thinking, and negotiated communicative norms for proving discourses, with the aim to rationally convince the other members of the proving community (Harel and Sowder, 2007; Harel and Rabin, 2010; Schoenfeld, 1988; Stylianides, 2007). From this stance, it cannot be the aim of teacher education programs just to teach desired step-models and algorithms of proving, but first and foremost to change preservice teachers underlying conceptions of the role and nature of mathematical proof. This ought to be seen as an essential part of the didactics courses, rather than just as a side effect of mathematics content courses.

Preservice teachers are challenged to connect their experiences with proof and facilitate discourse about proof during enacted lessons. Hence, teacher education programs ought to increase opportunities for preservice teachers to more actively engage with proof. Preparation programs should seek to facilitate discourse and written arguments for various types of proof in an effort to deepen preservice teachers mathematical content knowledge and ability to communicate proof. Additionally, teacher education programs ought to take into consideration the role and nature of mathematical proof and how to teach it explicitly and effectively to preservice teachers. The role and nature, especially the exploratory, social, communicative, and negotiable aspects of proof, should be reflected upon and discussed to foster acceptable standards of argumentation. The aim should be to develop more adequate and sustainable conceptions of mathematical proof in content and pedagogical courses that can be implemented into preservice teachers' teaching practices. Therefore, the process of proving, and the products of proof ought to be infused in all mathematics and mathematics education courses, to improve preservice teachers abilities to prove, as well as reflective meta-knowledge about proving, as a special form of argumentation.

To provide sustainable, advanced beliefs on mathematical proving and conceptions of the role and nature of mathematical proof, and to develop their skills in proving and teaching proof on this basis, pre-service teachers should pass through various dimensions of experience and reflection with and on proving. It remains an open issue as to how we can initiate such experience and reflection in an effective way. It seems quite clear that this should not be an exclusive part of mathematics content courses. In science education, there exists a vivid debate on how to teach the "nature of science (NOS)" in school as well as in teacher education (Akerson et.al., 2000). Central questions in this regard are in how explicit, instructive versus implicit ways of teaching, meta discussion based on texts or hands-on experience with advanced science content, are more effective, and if historical or socio-scientific issues and themes are more adequate for teaching NOS than "pure" content. This debate could serve as a starting point to answer similar question for the case of mathematics, regarding effective ways of teaching proof.

Based on the results presented, we emphasize the following dimensions for teaching proof in preservice teacher education. We acknowledge that the existence and adequateness of these dimensions are hypothesized as an effective approach to unpack proof in teacher education program; hence empirical data are needed to validate the effectiveness of the identified dimensions. During the first dimension, which may be in direct contrast to the individual's own experiences with more or less algorithmic proving in school mathematics courses, the exploratory, creative and individual element of proving could be highlighted. For the second dimension, the notion of what constitutes a proof and different proving formats and standards, that is, negotiable communicative proving norms *and* deductive standards could be introduced and discussed. The tension between algorithmic and non-algorithmic, but still systematic and structured, rigorous proof could be explicitly reflected. To this end, sufficient time would have to be given to carefully unpack the proof and explicitly emphasize and discuss these aspects. This dimension could also include instructive elements on how to develop proofs conforming to standards established by the field of mathematics, and training elements to gain self-confidence in producing proof on their own. Whereas in the third dimension, the didactical element on how to teach non-algorithmic, creative, but still inter-subjective standard conforming proof in school could be implemented. Therefore, efforts to increase preservice teachers opportunities to prove should be done strategically, such that it fosters a conceptual understanding about proof, as well as improves preservice teachers ability to prove. These dimensions can also be explored in other fields, such as science, which seeks to develop individuals' ability to construct viable arguments.

References

Akerson, V. L., Abd-El-Khalick, F., Lederman, N.G. (2000). Influence of a Reflective Explicit Activity-Based Approach on Elementary Teachers' Conceptions of Nature of Science. *Journal of research in science teaching*, 37 (4), 295-317.

Common Core State Standards Initiative. 2010. *The Standards: Mathematics.* Washington, DC: National Governors Association and the Council of Chief State School Officers. Available online at: http://www.corestandards.org/the-standards/mathematics.

Creswell, J. W. (2012). *Qualitative inquiry and research design: Choosing among five approaches*. Thousand Oak, CA: Sage.

De Villiers, M. D. (1999). *Rethinking proof with the Geometer's Sketchpad*. Emeryville, CA: Key Curriculum Press.

Bieda, K. N. (2010). Enacting proof-related tasks in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education, 41*(4), 351-382.

Gill J & Johnson P (1991) Research Methods for Managers, London: Paul Chapman.

Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. K. J. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 805-842). Charlotte, NC: Information Age Publishing.

Harel, G., & Rabin, J. M. (2010). Teaching practices associated with the authoritative proof scheme. *Journal for Research in Mathematics Education, 41*(1), 14-19.

Knuth, E. J. (2002). Teachers' conceptions of proof in the context of secondary school mathematics. *Journal of Mathematics Teacher Education, 5*(1), 61-88**.**

Martin, W. G., & Harel, G. (1989). Proof frames of preservice elementary teachers. *Journal for Research in Mathematics Education*, 41-51.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Sears, R. (2012). *An examination of how teachers use curriculum materials for the teaching of proof in high school geometry.* Doctoral Dissertation, University of Missouri - Columbia, Columbia, MO.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of 'well-taught' mathematics courses. *Educational psychologist, 23*(2), 145-166.

Strauss, A. L., Corbin, J. M., & Lynch, M. (1990). *Basics of qualitative research: Grounded theory procedures and techniques* (Vol. 270). Newbury Park, CA: Sage.

Stylianides, A. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education, 38*(3), 289-321.

Stylianides, G. J., Stylianides, A. J., & Philippou, G. N. (2007). Preservice teachers' knowledge of proof by mathematical induction. *Journal of Mathematics Teacher Education, 10*(3), 145-166.