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Compressed Sensing Electron tomography using adaptive dictionaries: a simulation study

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Abstract. Electron tomography (ET) is an increasingly important technique for examining the three-dimensional morphologies of nanostructures. ET involves the acquisition of a set of 2D projection images to be reconstructed into a volumetric image by solving an inverse problem. However, due to limitations in the acquisition process this inverse problem is considered ill-posed (i.e., no unique solution exists). Furthermore reconstruction usually suffers from missing wedge artifacts (e.g., star, fan, blurring, and elongation artifacts). Compressed sensing (CS) has recently been applied to ET and showed promising results for reducing missing wedge artifacts caused by limited angle sampling. CS uses a nonlinear reconstruction algorithm that employs image sparsity as a priori knowledge to improve the accuracy of density reconstruction from a relatively small number of projections compared to other reconstruction techniques. However, The performance of CS recovery depends heavily on the degree of sparsity of the reconstructed image in the selected transform domain. Prespecified transformations such as spatial gradients provide sparse image representation, while synthesising the sparsifying transform based on the properties of the particular specimen may give even sparser results and can extend the application of CS to specimens that can not be sparsely represented with other transforms such as Total variation (TV). In this work, we show that CS reconstruction in ET can be significantly improved by tailoring the sparsity representation using a sparse dictionary learning principle.

1. Introduction

In materials science, Electron tomography (ET) is considered an effective technique that provides indispensable information for the study of particles and structures in the Nanoworld. The 3D morphology of nanostructures can be provided by reconstruction of an aligned set of 2D TEM images acquired around a single or double tilting axis. The reconstruction step in ET is performed using an image reconstruction algorithm such as weighted back projection (WBP) or simultaneous iterative reconstruction technique (SIRT). SIRT algorithm provides reconstruction of a relatively higher Signal-to-noise ratio than those obtained using WBP. However, the reconstruction still suffers from elongation, blurring and artifacts due to the missing wedge limitation in ET. The number of projections that can be recorded is limited by the maximum tilt angle (typically $\pm$ 72) above which projections can not be acquired. A segmentation step is usually needed to overcome such artifacts and to distinguish between artifacts and main components in the reconstruction. This segmentation process is often performed manually leading to potentially subjective and time-consuming interpretation of the data.

The reconstruction process of ET can be explained by considering a 3D object as a set of 2D sections, the reconstruction of that object can then be created by reconstructing each 2D section from the corresponding 1D projection. An object may be reconstructed from its projections by solving a system of linear equations which can be modelled as:

$$Px = b \qquad (1)$$

Where P is the discrete Radon transform that converts the measurements (1D projections) b into object domain (2D imaged object) x. To create a reconstruction (tomogram), the unknown vector x needs to be calculated from measurements b which is not straightforward because of the underdetermined nature of the reconstruction inverse problem due to the limited number of projections. Also the reconstruction will be further degraded with the presence of noise and alignment errors.

It is well-known that the quality of a tomographic reconstruction can be enhanced by including additional prior knowledge about the specimen throughout the reconstruction process as in SIRT and recently CS-based approaches. CS has been shown to be powerful technique to reducing the artifacts that arise from ET reconstruction. The key prior knowledge employed in CS is that the signal is sparse in a transform domain, meaning that it can be approximated in a more compact form. This sparsity assumption can then be formulated as a problem of simultaneously minimising a cost function consisting of a data consistency term and one or more sparsity constraints in an appropriate transform domain (e.g., wavelets and TV). It should be noted that, in order for the CS algorithm to be successful and effective, an object to be reconstructed need to be compressible in a certain domain (for example, the TV can be used for reconstruction, if the object under study can be described as a piecewise constant). However, many samples do not satisfy such constraints, and therefore CS reconstruction can not be effective. Recently, an algorithm (referred to as K-SVD) for training a dictionary for sparse signal representation was proposed [1]. K-SVD is a signal representation approach which, from a set of signals, can derive a dictionary able to approximate each signal with a sparse combination of the atoms. The goal in this work is to make use of the K-SVD algorithm for designing sparsity dictionaries and utilizing these dictionaries for CS recovery in ET.

2. CS Recovery with Adaptive Transform using K-SVD

In [2], Candes *et al* , discovered important results that formed the theoretical basis of compressed sensing as currently studied, where an exact recovery of the SheppLogan phantom was obtained from 22 radial samples of its discrete Fourier transform. This work kicked off the field of compressed sensing theory and lead to successful application to other Inverse Problems such as Magnetic Resonance Imaging (MRI) [3] and recently ET as in [4, 5, 6] and [7]. The reconstruction in ET based on CS can be expressed in terms of minimizing the cost function in Eq.(2)

$$\arg \min_x \left\{ \|Px - b\|_2^2 + \lambda \|\Psi x\|_1 \right\} \qquad (2)$$

where Ψ is a predefined sparse transform (such as wavelets and total variation). While these transforms, especially total variation that have been proven to work well for the general class of ET samples [5, 7], they are not optimized for each ET application. In EM, significant prior information exists about the object being imaged and this prior information can be used to design more efficient sparsity dictionaries for CS ET. The K-SVD [1] algorithm aims for learning adaptive transforms (dictionary) Ψ and a sparse matrix Ω, such that for any signal x, there exists a sparse linear combination from Ψ that minimizes the representation error by solving:

$$\arg \min_{\Psi, \Omega} \|x - \Psi\Omega\|_2^2 \ \ s.t \ \ \|\omega_i\|_0 \le v, \forall i \qquad (3)$$

Where ω_i are columns of Ω, and v is the desired sparsity. In this work, the K-SVD algorithm will be used for designing adaptive sparsity dictionaries for a restricted class of ET images and utilizing these dictionaries for CS recovery. The sparsity in the proposed method is enforced on overlapping patches of the reconstructed image x and the dictionary is directly adapted to the image leading to higher reconstruction fidelity. For reconstruction in ET, the proposed algorithm can be formulated as in Eq.(4)

$$\arg \min_{\alpha, \Psi, x} \sum_{ij} \left\{ \|B_{ij}x - \Psi\omega_{ij}\|_2^2 + \lambda \|Px - y\|_2^2 \right\} \ s.t \ \|\omega_{ij}\|_0 \leq v, \ \forall i,j \quad (4)$$

Here, B_{ij} is a mapping operator that extracts a square patches of size $\sqrt{n} \times \sqrt{n}$ from image x; $\Psi\omega_{ij}$ is the sparse approximation of the patch, with the maximum number of non-zero values $\leq v$; Ψ is the trained dictionary; Ω is the set $\{\omega_{ij}\}_{ij}$; and v is a positive constant.

The first term in Eq.(4) measures the quality of the sparse approximations of the image patches with respect to the dictionary Ψ. The second term in the cost forces data fidelity in image domain. In one step (dictionary learning), x is assumed fixed, and the dictionary is learned with the sparse representations of the image patches. In the other step (reconstruction update), Ψ and Ω are fixed, and x is updated to satisfy data consistency. The reconstruction update step involves a least squares problem that can be solved using the corresponding normal equation and employing the conjugate gradient method that is initialized with a zero-filled Fourier reconstruction for x. v is the sparsity threshold in the K-SVD algorithm: higher v produces fewer atoms to approximate a patch in the sparse coding step.

3. Results

To evaluate the quality and accuracy of proposed reconstruction algorithm, the CS test phantom (CS-phantom) proposed in [8] is used (Figure 1-a). CS-phantom is tailored for testing the accuracy and properties of CS solvers in the noise-free domain. For comparisons, we performed the reconstruction using a WBP method (Figure 1-b), a TV-based method (Figure 1-c), and proposed approach (Figure 1-d) using simulated data set consisted of 27 projections. Peak Signal to Noise Ratio (PSNR) in dB is showed for the reconstruction in (Table 1). The reconstruction scheme outlined in this work is shown to produce substantially better reconstruction both visually and in terms of PSNR, as compared to the WBP and TV-based method.
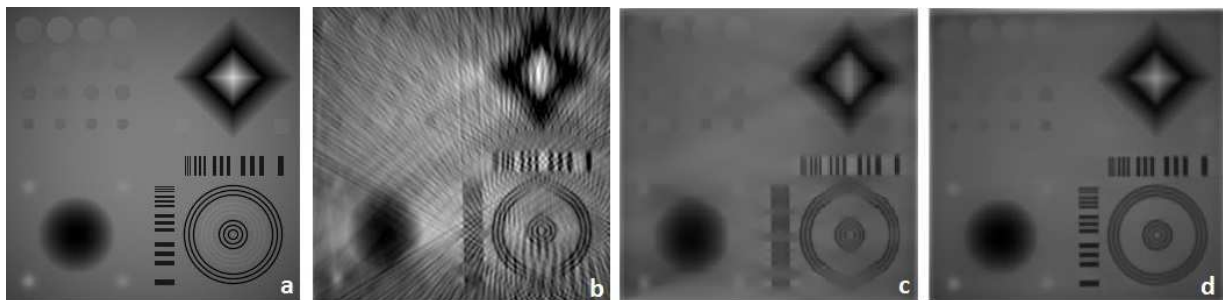

Figure 1. Reconstructed phantom images, (a) CS-phantom used for the simulation of a tilt series from $-65°$ to $65°$ with an increment of $5°$. (b) WBP (c) CS TV based algorithm (d) proposed method.

Table 1. PSNR of reconstructions.

Technique	CS-phantom
WBP	8.20 dB
CS-TV	21.17 dB
CS-Dictionary	30.5 dB

4. Conclusions and Discussion

A novel sparse reconstruction technique that incorporates prior information through dictionary training is introduced for CS in ET. Reconstruction results illustrate that the proposed technique can yield significantly improved image quality compared to commonly used sparsity transforms in CS ET. Future work will explore other interesting properties of the dictionary that may provide even more advanced reconstruction for highly under sampled ET.

4.1. Acknowledgments

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