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Prospects for transient gravitational waves at r-mode frequencies associated with pulsar glitches

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Abstract. t Glitches in pulsars are likely to trigger oscillation modes in the fluid interior of neutron stars. We examined these oscillations specifically at r-mode frequencies. The excited r-modes will emit gravitational waves and can have long damping time scales $\mathcal{O}$(minutes - days). We use simple estimates of how much energy the glitch might put into the r-mode and assess the detectability of the emitted gravitational waves with future interferometers.

1. Introduction

One possible source of burst-like gravitational waves is a *pulsar glitch*. These are anomalies in the observed spin frequency of a neutron star (NS) wherein there is a sudden increase in its rotational frequency followed by an exponential recovery to nearly the pre-glitch rotation rate on timescales of a few days–weeks. In most cases, these glitches are produced by young pulsars. It is believed that these glitches are a consequence of global events within the star's fluid interior and its solid crust [1]. These glitches could excite a variety of quasi-normal modes [2]. A previous search for gravitational waves associated with a glitch in the Vela pulsar in 2005 focussed on f-modes whose frequencies are believed to lie in the range 1–3 kHz [3] and last $\mathcal{O}(100\,\mathrm{ms})$.

A possibility to be considered in gravitational wave searches of this type is the excitation of r-modes oscillations during a pulsar glitch [4]. These r-modes, which are a type of inertial modes (oscillation modes restored by the Coriolis force), have frequencies proportional to the star's angular velocity and they are considered relatively more efficient GW emitters compared to other inertial modes [5]. Also, these modes evolve in time with an exponential decrement that depends on the mode's frequency and dissipation processes [6]. Reference [7] details that when a rotational lag between the fluid component and the crust reach a critical value, an instability that excite r-mode oscillations within the NS interior sets in. Because of this lag, the fluid component (already 'pinned' to the crust by a series of vortices) will be unpinned and will transfer angular momentum to the crust which will consequently spin up. As a result of this process a pulsar glitch will be produced. In that work, the authors considered short-wavelengths modes at which the quadrupole moment $l = m = 2$ would not be excited.

In section 2 we describe the form and duration of the GW signal one might expect from r-mode oscillations. In section 3, we explore the energetics of the r-mode signals associated with pulsar glitches and the plausibility of detecting GWs from r-modes. Finally, in section 4, we summarise our findings.

2. Gravitational waves signals from r-modes in neutron stars

We parameterise the expected GW signal from an r-mode oscillation in terms of harmonic ringdown gravitational waves signals [8],

$$h_+(t) = h_0 \cos(\omega_r t) e^{-t/\tau_r} \qquad h_\times(t) = h_0 \sin(\omega_r t) e^{-t/\tau_r}, \tag{1}$$

where h_+ and $h_\times$ are the gravitational wave signal polarizations 'plus' (+) and 'cross' (×), $\omega_r = 2\pi f_r$ is the frequency of the r-mode oscillation, τ_r is the damping time of the signal and h_0 is the amplitude of the signal at the detector given by equation (23) in [9]

$$h_0 = \sqrt{\frac{8\pi}{5}} \frac{G}{c^5} \frac{1}{r} \alpha \omega_r^3 M R^3 \tilde{J}. \tag{2}$$

This expresses the amplitude of the signal in terms of the NS mass M, radius R, distance from the observer r, a dimensionless parameter $\tilde{J}$ ($1.635\times$ 10^{-2}) that depends on the stellar mass distribution and the amplitude of the mode oscillation α. We consider the amplitude α in more detail in section 3.

According to [10], for sufficiently slowly rotating stars, the r-mode oscillation frequency f_r is proportional to the spin frequency of the star $f_{\rm spin}$. It is expected that the $\ell = m = 2$ mode dominates the GW emission [11]. Under these assumptions the r-mode frequency is given by,

$$\omega_r \approx \frac{4}{3}\Omega, \tag{3}$$

where Ω is the angular spin frequency of the star ($= 2\pi f_{spin}$) [10]. There are several sources of uncertainty in this relation. These include rotational effects [12], which modify the mode frequency by an amount that depends upon the average density, and relativistic effects [11], which introduce fractional corrections of the order of the stellar compactness M/R, so that equation (3) is accurate only to the $\sim 10\%$ level.

The damping time of the r-mode is given by,

$$\frac{1}{\tau_r} = \frac{1}{\tau_{\rm viscosity}} + \frac{1}{\tau_{\rm GRR}}, \tag{4}$$

where $\tau_{viscosity}$ is the dissipation time-scale due to the viscosity in the fluid component of the NS and $\tau_{\rm GRR}$ is the gravitational radiation reaction time-scale. Following [6], the radiation reaction time-scale is a function of the mode frequency,

$$\frac{1}{\tau_{\rm GRR}} = -\frac{32\pi}{15^2} \frac{G}{c^7} \omega_r^6 \tilde{J} M R^4, \tag{5}$$

Note that the damping time of the signal τ_r is a balance between dissipative processes with time-scale $\tau_{\rm viscosity} > 0$ and the radiation reaction time-scale $\tau_{\rm GRR} < 0$; while $\tau_{\rm viscosity}$ acts to dissipate energy and reduce the oscillation amplitude of the mode, $\tau_{\rm GRR}$ forces the mode's amplitude to grow due to the emission of gravitational radiation [13]. In this work we assume that the mode is in the stable regime so that $\tau_{\rm viscosity} << |\tau_{\rm GRR}|$, and so the viscosity time-scale dominates in the duration of the signal.

Levin and Ushomirsky proposed in [14] the existence of a viscous boundary layer in the crust-core interface of the NS which yields the following estimate of the dissipative time-scale,

$$\frac{1}{\tau_{\rm viscosity}} \simeq 0.01 s^{-1} \frac{R_6^2 F^{1/2}}{M_{1.4} T_8} \frac{\rho_b}{\rho} \left(\frac{f_{\rm spin}}{\rm kHz}\right)^{1/2} \left(\frac{\delta u}{u}\right)^2, \tag{6}$$

where R_{10}, $M_{1.4}$, ρ and ρ_b are the radius (10 Km), mass (1.4 $M_\odot$), the density at the crust-core interface, and an estimate of this density ($\rho_{\rm b} = 1.5 \times 10^{14}$ g cm^{-3}), respectively. T_8 is an assumed internal temperature of 10^8 K in the NS. The value of the fitting parameter F depends on the dominant scattering processes in the fluid interior [15]:

$$F \sim \begin{cases} 1/15 & \text{for electron–proton scattering,} \\ (\rho/\rho_b)^{5/4} & \text{for neutron scattering,} \\ 5\rho/\rho_b & \text{for electron scattering.} \end{cases} \tag{7}$$

Here, we assume that neutron-scattering dominates and we have set $\rho = \rho_b$.

Finally, the ratio $\delta u/u$ measures the fractional velocity mismatch between the crust and the core. As demonstrated in [14], for a sufficiently slowly rotating star, the crust does not significantly participate in the r-mode oscillation, so that $\delta u/u \approx 1$. Just how slow the rotation needs to be for this to be the case depends upon the thickness of the crust, but Figure 1 of [14] indicate that all young glitching pulsars are likely to have $\delta u/u \approx 1$. Note, however, that in more rapidly rotating stars $\delta u/u \approx 0.1$, lengthening the viscous decay timescale by a factor of ~ 100.

3. Detectability

The detectability of the GW signal from r-mode oscillations excited by a pulsar glitch is determined by the total amount of energy that the glitch deposits in the r-mode, and by the fraction of that energy which is radiated as GWs by the r-mode oscillations.

Following [9], the instantaneous gravitational wave luminosity from a source at distance r, undergoing damped oscillations with initial amplitude h_0, angular frequency ω and damping time τ is,

$$\dot{E}_{\rm GW} = \frac{c^3}{G}\frac{1}{10} r^2 \omega^2 (h_0 e^{-t/\tau})^2. \tag{8}$$

The total time-integrated energy emitted in GWs is, therefore,

$$\Delta E_{GW} = \frac{c^3}{G}\frac{1}{20} r^2 \omega^2 h_0^2 \tau. \tag{9}$$

The mode energy is

$$\tilde{E} = \alpha^2 \Omega^2 M R^2 \tilde{J}. \tag{10}$$

A rough estimate of the total energy associated with a pulsar glitch is (see e.g. [3]),

$$E_{\rm glitch} \approx I \Omega^2 \left(\frac{\Delta\Omega}{\Omega}\right), \tag{11}$$

where I the moment of inertia of the NS, Ω is the spin frequency and $\Delta\Omega/\Omega$ is the size of the glitch relative to the spin-frequency. An upper limit on the mode amplitude α can be obtained by assuming all of the energy associated with the glitch is channelled into r-mode excitation and we find,

$$\alpha = \left(\frac{\tilde{I}}{\tilde{J}}\right)^{1/2} \left(\frac{\Delta\Omega}{\Omega}\right)^{1/2}, \tag{12}$$

where $\tilde{I}$ (typically $\approx$ 0.261) is a dimensionless parameter dependant on the stellar mass distribution [6]. To evaluate the total power contained in short-duration, narrow-band signals, it is convenient to use the root-sum-squared amplitude:

$$h_{\rm rss} = \left[\int_0^\infty h_+^2(t) + h_\times^2(t)\ {\rm d}t\right]^{\frac{1}{2}} \approx h_0 \tau^{1/2}, \tag{13}$$

where the right hand side is the approximate value for the damped sinusoid we consider for our GW signal. Combining equations (2), (11) and (12), the root-sum-squared GW amplitude from r-mode excitation in terms of the stellar parameters and the size of the glitch is,

$$h_{\rm rss} = \frac{128}{27}\sqrt{\frac{2\pi}{5}\frac{G}{c^5}}\frac{M}{r}(\Omega R)^3\left(\tilde{I}\tilde{J}\frac{\Delta\Omega}{\Omega}\tau\right)^{1/2}. \tag{14}$$

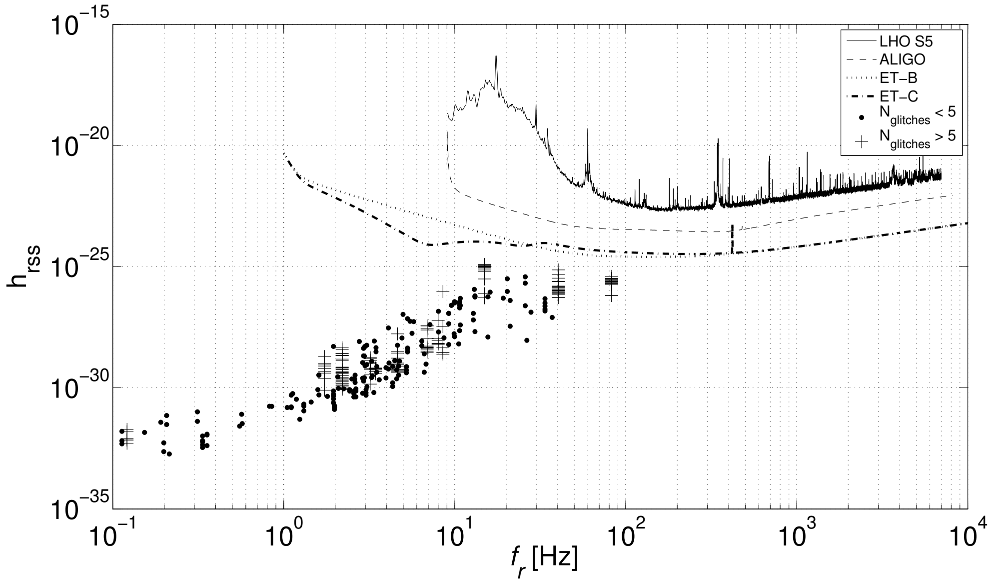

Figure 1. Sensitivity curves of gravitational waves detectors and upper limit estimates $h_{\rm rss}$ of the gravitational wave signal as a function of the r-mode frequency $f_r = \omega_r/(2\pi)$.

In figure 1, we compare the estimated GW upper limits from glitch-induced r-mode excitations in various known pulsars with the noise spectral densities of the initial LIGO instrument during its fifth science run [16], the advanced LIGO detector [17],[18] (currently under construction) and the proposed 3$^{\rm rd}$ generation Einstein Telescope (ET) [19]. We compare the estimates for frequently glitching pulsars (observed to have glitched more than five times, shown in crosses) and all other pulsars, shown in dots. The glitch observations are compiled from [20, 21, 22] and [23]. We find that, in this optimistic case, the estimated GW amplitude is a factor ~ 6 below the noise floor of even ET in its most favorable configuration

4. Discussion

We have presented some estimates of the energy associated with r-modes excited by pulsar glitches and the corresponding estimated upper limit GW amplitudes. We see that, even in the most optimistic scenario of energy transfer from the glitch to the mode, the GW amplitude is well below the sensitivity curves of both existing and planned gravitational wave detectors. Specifically, even for the most rapidly spinning pulsar we consider, where the r-mode frequency lies near the most sensitive part of the detectors, the expected amplitude is $\sim 4 \times 10^{-26}$.

However, the strong frequency dependence of the r-mode oscillation implies that rapidly rotating NS have a significantly greater chance of detection in 3$^{\rm rd}$ generation instruments such as ET. In addition to this, it is possible that these faster rotators possess a significantly smaller slippage parameter $\delta u/u$ (see section 2), leading to longer duration and more detectable GW signals. There is clearly some interest in exploring r-mode excitation in more rapidly rotating stars.

Related to this, we end by noting that, despite the greater frequency of glitches occurring in young pulsars, there is a report of a millisecond pulsar (MSP) glitch in [24]. As well as pushing the expected GW signal closer to the sensitive regions of interferometric GW detectors, higher spin frequencies yield intrinsically stronger signals. Accreting millisecond pulsars present a particularly tantalising opportunity for future analyses and the GW r-mode detectability from such sources is to be considered in future work.

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