Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 291702, 11 pages
http://dx.doi.org/10.1155/2015/291702

*Research Article*

# T-S Fuzzy Model Based Control Strategy for the Networked Suspension Control System of Maglev Train

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Received 16 June 2014; Accepted 15 August 2014

Academic Editor: Chaudry M. Khalique



The control problem for the networked suspension control system of maglev train with random induced time delay and packet dropouts is investigated. First, Takagi-Sugeno (T-S) fuzzy models are utilized to represent the discrete-time nonlinear networked suspension control system, and the parameters uncertainties of the nonlinear model have also been taken into account. The controllers take the form of parallel distributed compensation. Then, a sufficient condition for the stability of the networked suspension control system is derived. Based on the criteria, the state feedback fuzzy controllers are obtained, and the controller gains can be computed by using MATLAB LMI Toolbox directly. Finally, both the numerical simulations and physical experiments on the full-scale single bogie of CMS-04 maglev train have been accomplished to demonstrate the effectiveness of this proposed method.

## 1. Introduction

Maglev train has been considered to be a popular type of track transportation vehicle for its merits of low noise, no danger of derailment, small turning radius, and easy maintenance, which has extensively been studied in many countries [1, 2]. The suspension control system is the most pivotal part of the maglev train. In traditional way, point-to-point cables are used to connect the system components including sensors, controllers, and actuators, which make the transmission circuits very complex. Besides, the complicated electromagnetic interference which is derived from the electromagnets and linear motor will affect the reliability of the data transmission of the sensors and thus will deteriorate the stability of the suspension control system. As the network technology is developed rapidly, it is clear that the traditional control system will be replaced by the networked control system [3]. With regard to the suspension control system, a real-time network is adopted to construct the networked suspension control system to avoid the electromagnetic interference with wires and to improve the reliability of data transmission [4]. By doing that, it brings the advantages of reducing system complexity, realizing data sharing and communication among the suspension control units of the maglev train. However, network-induced delay and data packet dropouts bring new challenges for the networked suspension control system. Hence, it is necessary to pay attention to the control problem on the networked suspension control system.

For the present, lots of the researches have been focused on the modeling, stability, and controller design of the networked control system and many important results have been reported. Peng and Yang [5] study an event-triggered communication scheme and an $H_\infty$ control codesign method for networked control systems with communication delay and packet loss, which can both maintain the desired system $H_\infty$ performance and make better use of network resources. Shi et al. [6] investigate robust step tracking control methods for networked control systems, and the random time delay is modeled by Markov chains. Pang et al. [7] study the stability of output tracking for the networked control systems with bounded packet loss. Besides, the design method of a two-stage controller which can guarantee the stability and good tracking performance has also been given. Surveys of the main methodologies to cope with typical network-induced constraints have been presented in [8]. However, most of the mentioned researches are based on linear models, which make applying those methods to the nonlinear networked suspension control system difficult. Since the Takagi-Sugeno

(T-S) fuzzy modeling method can approximate the nonlinear model by many local linear models in different state space regions [9], it has been wildly adopted in the modeling, analysis, and control synthesis of the nonlinear networked control systems. Up until now, lots of valuable researches on T-S fuzzy model based continuous nonlinear networked control system have been reported. The T-S fuzzy modeling and stability analysis for nonlinear networked control system are investigated in [10–12]. The guaranteed cost networked control method for T-S fuzzy systems with time delay was presented in [13, 14]. To cope with the approximation errors between the T-S fuzzy model and the nonlinear model, the T-S fuzzy model based robust control design for nonlinear networked control system is discussed in [15–17]. In practice, the adopted digital controller in the CMS04 maglev train is based on a discrete-time model. In recent five years, fuzzy control of nonlinear discrete-time networked control system with induced time delay and packet dropouts has also been reported, but not frequently, such as [18, 19]. Moreover, most of those researches are focused on how to reduce the conservation of the conclusions theoretically, which have been demonstrated only by numerical simulations. And little research pays attention to the engineering application of the developed methods. In addition, the networked suspension control system is expected for the engineering applications in CMS04 low speed maglev train. In view of that, stability analysis and control synthesis for the networked suspension control system from the viewpoint of engineering applications motivate this work. Firstly, the nonlinear networked suspension control model is represented by discrete T-S fuzzy models. Then, the sufficient condition for testing the stability of the networked suspension control system is presented, based on which the sufficient condition for controllers design is also obtained. Finally, simulations and experiments are finished to demonstrate the effectiveness of this method.

*Notation.* The superscript "$T$" stands for matrix transposition; $\mathbf{R}^n$ denotes the $n$-dimensional Euclidean space, and the notation $X > 0$ ($<0$) means that $X$ is real symmetric and positive definite (negative definite). In symmetric block matrices or complex matrix expressions, we use an asterisk $*$ to represent a term that is induced by symmetry, and diag$\{\cdots\}$ stands for a block diagonal matrix. If not explicitly stated, matrices are assumed to be compatible for algebraic operations.

## 2. Problem Formulation and Modeling

The low speed EMS (electromagnetic suspension) train consists of car body, levitation bogies, air springs, and levitation, and guidance magnets. Figure 1 shows a lateral view of the CMS04 low speed maglev train, from which it can be founded that there are ten suspension units distributed on each side of the vehicle. The suspension unit is the basic element of the maglev train. Therefore, the research of this paper focuses on the single suspension unit. Figure 2 shows the scheme of networked suspension control system (single node), from which it can be seen that the system consists of a suspension controller, a sensors group, the CAN bus network, the wave chopper, and the electromagnetic suspension system. The sensors group contains gap sensors, current sensors, and acceleration sensors. In the networked suspension control scheme, CAN bus network frame is adopted to realize the transmission of sensors message including the gap sensors, the current sensors, and the acceleration. The ten suspension units of one side are linked to a CAN bus frame, which means that each suspension unit becomes a network node. Each network node samples the information of sensors with specified frequency and transmits them to corresponding controller node through CAN bus network. Once the sensors signals are transmitted to the controller, the controlling quantity can be computed immediately. Then, a PWM wave is also generated to drive the wave chopper, which generated the desired current to adjust the movement of the electromagnets. By doing those, the closed control loop is formed.

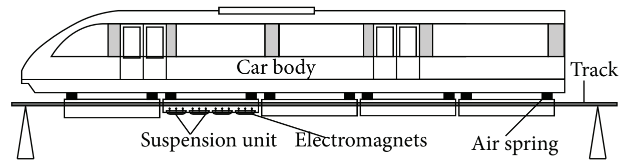


Figure 1: Lateral view of the CMS04 low speed maglev vehicle.

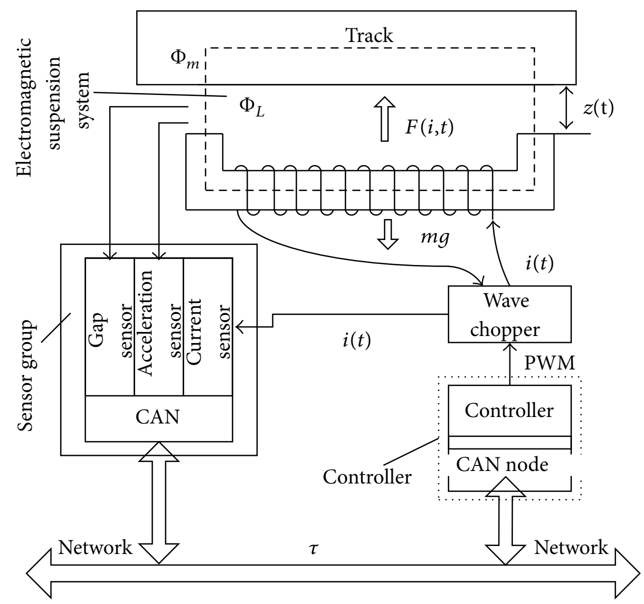


Figure 2: The scheme of the networked suspension control system (single node).

The total electromagnetic force generated by the electromagnet can be given by

$$F_m(t) = \frac{\mu_0 A_s N^2}{4}\left(\frac{i(t)}{z(t)}\right)^2, \quad (1)$$

where $N$ is the number of turns of the electromagnet, $A_s$ is the pole area, $i(t)$ is the current through the electromagnet, $z(t)$ is the suspension gap of the system, and $\mu_0$ is the space permeability.

Suppose that $u$ is the control voltage and that $R$ is the DC resistance of the electromagnet. The relationship between the current and the voltage of the electromagnet can be derived as

$$u(t) = i(t) R + \frac{\mu_0 A_s N^2}{2z(t)} \dot{i}(t) - \frac{\mu_0 A_s N^2 i(t)}{2z^2(t)} \dot{z}(t). \quad (2)$$

According to Newton's law, the motion equation of the electromagnet can be described as follows:

$$m\frac{\ddot{z}(t)}{dt^2} = -F_m(t) + mg, \quad (3)$$

where $m$ is the total mass which a single suspension unit supports and $g$ is the acceleration of gravity.

Define that the state vector of the system is $x(t) = (x_1(t), x_2(t), x_3(t))'$, where $x_1(t) = z(t)$, $x_3(t) = i(t)$, and $x_2(t)$ is the velocity in the vertical direction that can be obtained by integration of the acceleration. Here, the state equations of the suspension control system can be obtained directly as follows:

$$\dot{x}(t) = f(x(t)) + g(x(t)) u(t),$$

$$f(x(t)) = \begin{bmatrix} x_2(t) & g - \dfrac{k_e x_3^2(t)}{m x_1^2(t)} & \dfrac{x_2(t) x_3(t)}{x_1(t)} - \dfrac{R x_1(t) x_3(t)}{2k_e} \end{bmatrix}^T,$$

$$g(x) = \begin{bmatrix} 0 & 0 & \dfrac{x_1(t)}{2k_e} \end{bmatrix}^T, \qquad k_e = \frac{\mu_0 A_s N^2}{4}. \quad (4)$$

It is obvious that the magnetic suspension system is a nonlinear system. Here, due to the terrific approximation quality of the T-S fuzzy model between the linear system and the nonlinear system, we introduce it into the modeling, analysis, and control synthesis of the networked suspension control system. From [9], the nonlinear system can be represented by a T-S fuzzy plant model with some simple local linear dynamic systems. In this paper, the local linear model at the static equilibrium point $x_e = (x_{10}, x_{20}, x_{30})$ is obtained using Taylor's series. By neglecting the higher order terms, the local linear model is given as follows:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (5)$$

where

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_e \\ u=u_0}} = \begin{bmatrix} 0 & 1 & 0 \\ \dfrac{2k_e x_{30}^2}{m x_{10}^3} & 0 & -\dfrac{2k_e x_{30}}{m x_{10}^2} \\ 0 & \dfrac{x_{30}}{x_{10}} & -\dfrac{R x_{10}}{2k_e} \end{bmatrix}, \quad (6)$$

$$B = \left.\frac{\partial \widehat{F}}{\partial u}\right|_{\substack{x=x_e \\ u=u_0}} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{x_{10}}{2k_e} \end{bmatrix},$$

where $x_{20} = 0$, $x_{30} = x_{10}\sqrt{mg/k_e}$, $x_{10}$ is the given suspension gap.

In this paper, the presented approach is focused on the discrete-time case, so the discrete model of (5) with the sampling time $T$ is obtained as follows:

$$x(k+1) = \widehat{A}x(k) + \widehat{B}u(k), \quad (7)$$

where $\widehat{A} = e^{AT}$, $\widehat{B} = \int_0^T e^{At} B dt$.

For maglev train, the desired suspension gap between the electromagnets and the track is 9 mm when it is levitated steadily, and the initial gap is set to be 20 mm. The levitating procedure is to adopt the suspension controller to produce a desired electromagnetic force which can change the suspension gap from 20 mm to 9 mm. Besides, a 3 mm thick copper billet is embedded on the electromagnets pole to prevent that the electromagnets trash into the track. So, the smallest suspension gap is set to be 3 mm. To build the T-S fuzzy model of the networked suspension control system, this paper denotes three rules which represent the dynamics around the static equilibrium point $x_1 = 3$ mm, $x_1 = 9$ mm, and $x_1 = 20$ mm, respectively. The three rules are with the following formats:

Plant Rule 1: IF $x_1(k)$ is about 3 mm, then $x(k+1) = \widehat{A}_1 x(k) + \widehat{B}_1 u(k)$;

Plant Rule 2: IF $x_1(k)$ is about 9 mm, then $x(k+1) = \widehat{A}_2 x(k) + \widehat{B}_2 u(k)$;

Plant Rule 3: IF $x_1(k)$ is about 20 mm, then $x(k+1) = \widehat{A}_3 x(k) + \widehat{B}_3 u(k)$,

where $\widehat{A}_i$ and $\widehat{B}_i$ $(i = 1, 2, 3)$ are the known parameter matrices from the system (7), when the equilibrium position of $x_{10}$ is supposed at 3 mm, 9 mm, and 20 mm accordingly.

Due to the fact that the state variable $x_1(k)$ is measurable, the fuzzy membership function can be chosen as

$$\begin{aligned} h_1(x_1(k)) &= 0, \\ h_2(x_1(k)) &= 1 - \frac{(x_1(k) - 9)^2}{121}, \\ h_3(x_1(k)) &= \frac{(x_1(k) - 9)^2}{121}, \\ &\quad \text{if } 20\text{ mm} \ge x_1(k) \ge 9\text{ mm}, \\ h_1(x_1(k)) &= \frac{(x_1(k) - 9)^2}{36}, \\ h_2(x_1(k)) &= 1 - \frac{(x_1(k) - 9)^2}{36}, \\ h_3(x_1(k)) &= 0, \\ &\quad \text{if } 3\text{ mm} \le x_1(k) < 9\text{ mm}, \end{aligned} \quad (8)$$

where $h_i(x_1(k)) \geq 0$, $\sum_{i=1}^{3} h_i(x_1(k)) = 1$. The whole T-S fuzzy model of the magnetic suspension system can be written as follows:

$$x(k+1) = \sum_{i=1}^{3} h_i(x_1(k)) \left(\widehat{A}_i x(k) + \widehat{B}_i u(k)\right). \tag{9}$$

Because model (9) is obtained by linearization, nonlinearities and unmodeled dynamics may cause parametric uncertainties in the practical control system. Assume that $\Delta\widehat{A}_i$ and $\Delta\widehat{B}_i$ are the bounded matrixes which can represent the time varying parametric uncertainties of the system model. Inspired by [20], we make the following supposition:

$$\left[\Delta\widehat{A}_i, \Delta\widehat{B}_i\right] = D_i H_i(k) \left[E_{ai}, E_{bi}\right], \tag{10}$$

where $D_i, E_{ai}, E_{bi}$ are known real constant matrices with appropriate dimensions and $H_i(k)$ is the unknown time varying matrix function with Lebesgue measurable elements and it satisfies $H_i(k)^T H_i(k) \leq I$. Then, the T-S fuzzy model of suspension control system with parametric uncertainties can be rewritten as

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{3} h_i(x_1(k)) \left(\widehat{A}_i + \Delta\widehat{A}_i\right) x(k) + \left(\widehat{B}_i + \Delta\widehat{B}_i\right) u(k) \\ &= \sum_{i=1}^{3} h_i(x_1(k)) \left(\widehat{A}_i + D_i H_i(k) E_{ai}\right) x(k) \\ &\quad + \left(\widehat{B}_i + D_i H_i(k) E_{bi}\right) u(k). \end{aligned} \tag{11}$$

In this paper, the parallel distributed compensation (PDC) is utilized to construct a networked T-S fuzzy model based state feedback controller [21]. For the networked suspension control system, the network framework is placed between the sensors and the controller. According to the data stream path in the CAN bus network, the network-induced delay from sensors to controllers contains transform processing delay, CAN bus access waiting delay, and receiving processing delay. Besides, packet dropouts also happen when the band of the network is congested. The problems such as induced delay and packet dropouts in the sensors information transmission will degrade the performance of the suspension control system and even cause instability under some extreme circumstances. Hence, the mathematical model of the suspension control system must take those issues into consideration. Throughout this paper, some assumptions are given below.

*Assumption 1.* Both the sensors and the controller are time-driven and synchronized. Considering that the computational delay is very small, it is omitted in this paper.

*Assumption 2.* When packet dropouts occur, the latest packet will be used again, which is equal to the increment of the time delay [22]. Once the new packet reaches the controller before the old one, the old one will be discarded.

*Assumption 3.* The network induced time delays and the number of packet dropouts are commonly bounded [23].

Based on the assumptions on the induced time delay and packet dropouts mentioned above, one can merge the networked induced time delay and packet dropouts into a time-varying random input delay $\tau_k$. From [18, 24], it can also be concluded that the time-varying input delay $\tau_k$ will have a limit of $[\tau_0, \tau_m]$. Hence, the designed T-S fuzzy controllers are given as follows.

Rules:

IF $x_1$ is about 3 mm, then $u(k) = K_1 x(k-\tau_k)$, $k \in [k_m + \tau_k, k_{m+1} + \tau_{k+1})$;

IF $x_1$ is about 9 mm, then $u(k) = K_2 x(k-\tau_k)$, $k \in [k_m + \tau_k, k_{m+1} + \tau_{k+1})$;

IF $x_1$ is about 20 mm, then $u(k) = K_3 x(k-\tau_k)$, $k \in [k_m + \tau_k, k_{m+1} + \tau_{k+1})$;

where $K_i$ $(i = 1, 2, 3)$ are the controller gains to be determined and $k_m$ $(m = 0, 1, 2, \ldots)$ is the transmitting instant from sensors to the controller. Hence, the overall control laws with time delay is given as follows:

$$\begin{aligned} u(k) &= \sum_{i=1}^{3} h_i(x_1(k-\tau_k)) K_i x(k-\tau_k), \\ &\quad k \in [k_m + \tau_k, k_{m+1} + \tau_{k+1}). \end{aligned} \tag{12}$$

It is assumed that output of the controller is 0 before the first control signal reaches the system. For convenience, $h_i(x_1(k))$ and $h_i(x_1(k-\tau_k))$ are denoted by $h_i$ and $h_{\tau i}$, respectively. Substituting (12) into (11) yields to the closed loop model of the networked suspension control system with time-varying input time delays:

$$\begin{aligned} x(k+1) = \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_{\tau j} &\left[\left(\widehat{A}_i + \Delta\widehat{A}_i\right) x(k) \right. \\ &\left. + \left(\widehat{B}_i + \Delta\widehat{B}_i\right) K_j x(k-\tau_k)\right], \\ & k \in [k_m + \tau_k, k_{m+1} + \tau_{k+1}), \end{aligned} \tag{13}$$

$$x(k) = x_0, \qquad k \in [-\tau_m, 0], \tag{14}$$

where $x_0$ is the given initial condition of the networked suspension control system.

## 3. Main Results

The main aim of this section is to develop the stability analysis and control synthesis approach for the system model (13). Firstly, we introduce some lemmas which are useful in following derivation.

**Lemma 4** (see [13]). *For any real matrices $X_iY_i$, $1 \le i \le r$, and $S > 0$ with appropriate dimensions, we have*

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{p=1}^{r}\sum_{l=1}^{r}h_ih_jh_ph_lX_{ij}^TSY_{pl} \le \sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\left(X_{ij}^TSX_{ij}+Y_{ij}^TSY_{ij}\right). \tag{15}$$

**Lemma 5** (see [25]). *For any matrix $T_1 \in \mathbf{R}^{n\times n}$, $T_2 \in \mathbf{R}^{n\times n}$, and $S = S^T > 0$, a positive scalar $\tau < \tau_m$ and vector function $x(k) \in \mathbf{R}^n$, $e(k) = x(k+1) - x(k)$ such that the following integration is well defined; then, the following inequality holds:*

$$-\sum_{l=k-\tau_m}^{k-1}e^T(l)Se(l) \le \begin{bmatrix}x(k)\\ x(k-\tau)\end{bmatrix}^T\left(\tau_m\begin{bmatrix}T_1\\ T_2\end{bmatrix}S^{-1}\begin{bmatrix}T_1^T & T_2^T\end{bmatrix} + \begin{bmatrix}T_1+T_1^T & -T_1+T_2^T\\ -T_1^T+T_2 & -T_2-T_2^T\end{bmatrix}\right)\begin{bmatrix}x(k)\\ x(k-\tau)\end{bmatrix}. \tag{16}$$

*3.1. Stability Analysis of the Networked Suspension Control System.* In the stability analysis of networked suspension control system, it is assumed that the state feedback gain matrices $K_j$ have been well designed. Rewrite the networked control system described by (13) as

$$x(k+1) = \sum_{i=1}^{3}\sum_{j=1}^{3}h_ih_{\tau j}\left[\widehat{A}_ix(k)+\widehat{B}_iK_jx(k-\tau_k)+D_iv_{ij}(k)\right], \quad k\in[k_m+\tau_k, k_{m+1}+\tau_{k+1}) \tag{17}$$

subject to uncertain feedback

$$w_{ij}(k) = E_{ai}x(k)+E_{bi}K_jx(k-\tau_k), \quad v_{ij}(k) = H_i(k)w_{ij}(k). \tag{18}$$

In view of (10) and (18), we have

$$v_{ij}^T(k)v_{ij}(k) \le \left[E_{ai}x(k)+E_{bi}K_jx(k-\tau_k)\right]^T \times\left[E_{ai}x(k)+E_{bi}K_jx(k-\tau_k)\right]. \tag{19}$$

The following theorem gives the sufficient condition to guarantee the stability of the networked suspension control system (13).

**Theorem 6.** *For a given controller gain matrix $K_j \in \mathbf{R}^{m\times n}$, $(j = 1,2,3)$, system (13) is asymptotically stable, if there exist real symmetric positive definite matrixes $P \in \mathbf{R}^{n\times n}$, $Q \in \mathbf{R}^{n\times n}$, and $R \in \mathbf{R}^{n\times n}$ and real matrixes $T_1 \in \mathbf{R}^{n\times n}$ and $T_2 \in \mathbf{R}^{n\times n}$ satisfying the following matrix inequality:*

$$\Psi_{ij} = \begin{bmatrix}\psi_{11} & \psi_{12} & \psi_{13}\\ \psi_{12}^T & \psi_{22} & \psi_{23}\\ \psi_{13}^T & \psi_{23}^T & \psi_{33}\end{bmatrix} < 0, \tag{20}$$

*where*

$$\begin{aligned}\psi_{11} &= \widehat{A}_i^TP\widehat{A}_i - P + (\tau_m-\tau_0+1)Q + \tau_m^2\left(\widehat{A}_i-I\right)^TR\left(\widehat{A}_i-I\right)+\tau_m\left(T_1+T_1^T\right) + \tau_m^2T_1R^{-1}T_1^T;\\ \psi_{12} &= \widehat{A}_i^TP\widehat{B}_iK_j+\tau_m^2\left(\widehat{A}_i-I\right)^TR\widehat{B}_iK_j-\tau_m\left(T_1-T_2^T\right) + \tau_m^2T_1R^{-1}T_2^T;\\ \psi_{13} &= \widehat{A}_i^TPD_i+\tau_m^2\left(\widehat{A}_i^T-I\right)RD_i;\\ \psi_{22} &= K_j^T\widehat{B}_i^TP\widehat{B}_iK_j+\tau_m^2K_j^T\widehat{B}_i^TR\widehat{B}_iK_j-\tau_m\left(T_2+T_2^T\right) - Q+\tau_m^2T_2R^{-1}T_2^T;\\ \psi_{23} &= K_j^T\widehat{B}_i^TPD_i+\tau_m^2K_j^T\widehat{B}_i^TRD_i;\\ \psi_{33} &= D_i^TPD_i+\tau_m^2D_i^TRD_i.\end{aligned} \tag{21}$$

*Proof.* For convenience, the following symbols are defined at first:

$$e(k) = x(k+1)-x(k), \quad \zeta(k) = \left[x^T(k), x^T(k-\tau_k), v_{ij}^T(k)\right]^T, \quad \xi(k) = \left[x^T(k), x^T(k-\tau_k)\right]^T. \tag{22}$$

Then, we have

$$e(k) = \sum_{i=1}^{3}\sum_{j=1}^{3}h_ih_{\tau j}\left[\left(\widehat{A}_i-I\right)x(k)+\widehat{B}_iK_jx(k-\tau_k)+D_iv_{ij}(k)\right], \quad k\in[k_m+\tau_k, k_{m+1}+\tau_{k+1}). \tag{23}$$

Define a Lyapunov-Krasovskii functional as follows:

$$V(x(k)) = V_1(x(k))+V_2(x(k))+V_3(x(k))+V_4(x(k)), \tag{24}$$

where

$$V_1(x(k)) = x^T(k)Px(k),$$
$$V_2(x(k)) = \sum_{j=k-\tau_k}^{k-1} x^T(j)Qx(j),$$
$$V_3(x(k)) = \tau_m \sum_{i=-\tau_m+1}^{0} \sum_{j=k-1+i}^{k-1} e^T(j)\operatorname{Re}(j) \tag{25}$$
$$V_4(x(k)) = \tau_m \sum_{i=-\tau_{m+1}}^{-\tau_0} \sum_{j=k+i}^{k-1} x^T(j)Qx(j).$$

Define $\Delta V_i = V_i(k+1) - V_i(k)$, firstly; one obtains

$$\begin{aligned}\Delta V_1 &= V_1(k+1) - V_1(k)\\ &= x^T(k+1)Px(k+1) - x^T(k)Px(k).\end{aligned} \tag{26}$$

According to Lemma 4, we have

$$\begin{aligned}x^T(k+1)Px(k+1) &\le \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_{\tau j} S_1^T(k)PS_1(k),\\ S_1(k) &= \widehat{A}_i x(k) + \widehat{B}_i K_j x(k-\tau_k) + D_i v_{ij}(k).\end{aligned} \tag{27}$$

So,

$$\Delta V_1 \le \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_{\tau j}\left\{S_1^T(k)PS_1(k) - x^T(k)Px(k)\right\}. \tag{28}$$

Secondly,

$$\begin{aligned}\Delta V_2 &= V_2(k+1) - V_2(k)\\ &= \sum_{j=k-\tau_k}^{k-1} x^T(j)Qx(j)\\ &= x^T(k)Qx(k) - x^T(k-\tau)Qx(k-\tau)\\ &\quad + \sum_{j=k+1-\tau_{k+1}}^{k-1} x^T(j)Qx(j) - \sum_{j=k+1-\tau_k}^{k-1} x^T(j)Qx(j)\\ &\le x^T(k)Qx(k) - x^T(k-\tau)Qx(k-\tau)\\ &\quad + \sum_{j=k+1-\tau_m}^{k-\tau_0} x^T(j)Qx(j).\end{aligned} \tag{29}$$

Thirdly,

$$\begin{aligned}\Delta V_3 &= \tau_m \sum_{i=-\tau_m+1}^{0}\left[\sum_{j=k+i}^{k} e^T(j)\operatorname{Re}(j) - \sum_{j=k-1+i}^{k-1} e^T(j)\operatorname{Re}(j)\right]\\ &= \tau_m^2 e^T(k)\operatorname{Re}(k) - \tau_m \sum_{i=k-\tau_m}^{k-1} e^T(i)\operatorname{Re}(i).\end{aligned} \tag{30}$$

Applying Lemma 4 again, the following inequality is obtained:

$$\begin{aligned}e^T(k)\operatorname{Re}(k) &\le \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_{\tau j} S_2^T(k)RS_2(k),\\ S_2(k) &= \left(\widehat{A}_i - I\right)x(k) + \widehat{B}_i K_j x(k-\tau_k) + D_i v_{ij}(k).\end{aligned} \tag{31}$$

Besides, applying Lemma 5, the following can be obtained:

$$\begin{aligned}&-\sum_{i=k-\tau_m}^{k-1} e^T(i)\operatorname{Re}(i)\\ &\le \xi^T(k)^T\left(\tau_m \begin{bmatrix}T_1\\ T_2\end{bmatrix} R^{-1}\begin{bmatrix}T_1^T & T_2^T\end{bmatrix}\right.\\ &\qquad \left. + \begin{bmatrix}T_1 + T_1^T & -T_1 + T_2^T\\ -T_1^T + T_2 & -T_2 - T_2^T\end{bmatrix}\right)\xi^T(k).\end{aligned} \tag{32}$$

Hence,

$$\begin{aligned}&\Delta V_3\\ &\le \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_{\tau j}\\ &\quad\times\left\{S_2^T(k)\tau_m^2 RS_2(k)\right.\\ &\quad + \xi^T(k)\left(\tau_m^2 \begin{bmatrix}T_1\\ T_2\end{bmatrix} R^{-1}\begin{bmatrix}T_1^T & T_2^T\end{bmatrix}\right.\\ &\quad \left.\left. + \tau_m \begin{bmatrix}T_1 + T_1^T & -T_1 + T_2^T\\ -T_1^T + T_2 & -T_2 - T_2^T\end{bmatrix}\right)\xi(k)\right\}.\end{aligned} \tag{33}$$

In the end, we get

$$\begin{aligned}\Delta V_4 &= V_4(k+1) - V_4(k)\\ &= \left(\tau_m - \tau_0\right)x^T(k)Qx(k) - \sum_{j=k+1-\tau_m}^{k-\tau_0} x^T(j)Qx(j).\end{aligned} \tag{34}$$

Based on the derivations, one obtains

$$\begin{aligned}\Delta V &= \Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4\\ &\le \sum_{i=1}^{3}\sum_{j=1}^{3} h_i h_{\tau j}\left\{S_1^T(k)PS_1(k) + x^T(k)\right.\\ &\qquad \times\left[\left(\tau_m - \tau_0 + 1\right)Q - P\right]x(k)\\ &\qquad - x^T(k-\tau)Qx(k-\tau)\\ &\qquad + S_2^T(k)\left(\tau_m^2 R\right)S_2(k) + \xi^T(k)\end{aligned}$$

$$\times \left( \tau_m^2 \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} R^{-1} \begin{bmatrix} T_1^T & T_2^T \end{bmatrix} + \tau_m \begin{bmatrix} T_1 + T_1^T & -T_1 + T_2^T \\ -T_1^T + T_2 & -T_2 + T_2^T \end{bmatrix} \right) \xi(k) \Bigg\}$$

$$= \sum_{i=1}^{3} \sum_{j=1}^{3} h_i h_{\tau j} \zeta^T(k) \Psi_{ij} \zeta(k). \tag{35}$$

According to the Lyapunov stability theory, it can be concluded from (35) that system (13) is asymptotically stable if the matrix inequality (20) holds. The proof is completed. □

Applying $S$-procedure [26], the matrix inequality (20) is satisfied if the following matrix inequalities hold:

$$\Psi_{ij} + \begin{bmatrix} \varepsilon^{-1} E_{ai}^T E_{ai} & \varepsilon^{-1} E_{ai}^T E_{bi} K_i & 0 \\ * & \varepsilon^{-1} K_j^T E_{bi}^T E_{bi} K_i & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0. \tag{36}$$

Using Schur complement lemma, the matrix inequality (36) holds if and only if the following matrix inequality holds:

$$\Gamma = \begin{bmatrix} \Gamma_1 & -\tau_m (T_1 - T_2^T) & 0 & \tau_m T_1 & \widehat{A}^T & \tau_m (\widehat{A}_i - I)^T & E_{ai}^T \\ * & -Q - \tau_m (T_2 + T_2^T) & 0 & \tau_m T_2 & K_j^T \widehat{B}_i^T & \tau_m K_j^T \widehat{B}_i^T & K_j^T E_{bi}^T \\ * & * & -\varepsilon I & 0 & D_i^T & \tau_m D_i^T & 0 \\ * & * & * & -R & 0 & 0 & 0 \\ * & * & * & * & -P^{-1} & 0 & 0 \\ * & * & * & * & * & -R^{-1} & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{37}$$

where $\Gamma_1 = -P + (\tau_m - \tau_0 + 1)Q + \tau_m (T_1 + T_1^T)$. Hence, the matrix inequality (20) can be guaranteed if the matrix inequality (37) holds. Based on the Theorem 6 and the matrix inequality (37), the procedure of the controller design can be given in Section 3.2.

*3.2. Controller Design.* In the following, we will give the design procedure of state feedback controllers based upon Theorem 6.

**Theorem 7.** *Considering system* (13), *it is asymptotically stable with* $K = YX^{-1}$, *if there exist a scalar* $\varepsilon > 0$, *symmetric positive definite matrices* $X \in \mathbf{R}^{n\times n}$, $\widehat{Q} \in \mathbf{R}^{n\times n}$, $\widehat{R} \in \mathbf{R}^{n\times n}$, *real matrixes* $T_1 \in \mathbf{R}^{n\times n}$, $T_2 \in R^{n\times n}$, $Y_j \in \mathbf{R}^{m\times n}$ $(j = 1, 2, 3)$, *such that*

$$\Gamma = \begin{bmatrix} \Gamma_2 & -\tau_m (\widehat{T}_1 - \widehat{T}_2^T) & 0 & \tau_m \widehat{T}_1 & X\widehat{A}^T & \tau_m X(\widehat{A}_i - I)^T & XE_{ai}^T \\ * & -\widehat{Q} - \tau_m (\widehat{T}_2 + \widehat{T}_2^T) & 0 & \tau_m \widehat{T}_2 & Y_j^T \widehat{B}_i^T & \tau_m Y_j^T \widehat{B}_i^T & Y_j^T E_{bi}^T \\ * & * & -\varepsilon I & 0 & D_i^T & \tau_m D_i^T & 0 \\ * & * & * & -\widehat{R} & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & \alpha^2 \widehat{R} - 2\alpha X & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{38}$$

*where* $\Gamma_2 = -X + (\tau_m - \tau_0 + 1)\widehat{Q} + \tau_m (\widehat{T}_1 + \widehat{T}_1^T)$.

*Proof.* Defining $X = P^{-1}$, $\widehat{Q} = XQX$, $\widehat{R} = XRX$, $\widehat{T}_1 = XT_1X$, $\widehat{T}_2 = XT_2X$, and pre- and postmultiplying the matrix inequality (37) by $\mathrm{diag}\{X, X, I, X, I, I, I\}$, the matrix inequality (37) is further equivalent to the following expression:

$$\Gamma = \begin{bmatrix} \Gamma_2 & -\tau_m (\widehat{T}_1 - \widehat{T}_2^T) & 0 & \tau_m \widehat{T}_1 & X\widehat{A}^T & \tau_m X(\widehat{A}_i - I)^T & XE_{ai}^T \\ * & -\widehat{Q} - \tau_m (\widehat{T}_2 + \widehat{T}_2^T) & 0 & \tau_m \widehat{T}_2 & Y_j^T \widehat{B}_i^T & \tau_m Y_j^T \widehat{B}_i^T & Y_j^T E_{bi}^T \\ * & * & -\varepsilon I & 0 & D_i^T & \tau_m D_i^T & 0 \\ * & * & * & -\widehat{R} & 0 & 0 & 0 \\ * & * & * & * & -X & 0 & 0 \\ * & * & * & * & * & -X\widehat{R}^{-1}X & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0, \tag{39}$$

where $\Gamma_1 = -X + (\tau_m - \tau_0 + 1)\widehat{Q} + \tau_m (\widehat{T}_1 + \widehat{T}_1^T)$.

Due to the nonlinear term $-X\widehat{R}^{-1}X$ in the matrix inequality (39), it cannot be solved directly by MATLAB LMI TOOLBOX. So the work at hand is to find an approach to transform the nonlinear term to be linear. The developed cone complementarily linearization type algorithms are an alternative scheme to solve this problem. However, a more directly approach described in [25, 27] is also efficient and convenient to deal with the nonlinear term. Note that $(X - \alpha\widehat{R})\widehat{R}^{-1}(X - \alpha\widehat{R}) \geq 0$ can be obtained if $\alpha > 0$, which implies $-X\widehat{R}^{-1}X \leq \alpha^2\widehat{R} - 2\alpha X$, and the matrix inequality (38) can be guaranteed. Hence, the proof procedure is completed. □

Because of the introduction of the parameter $\alpha$, the first work before finding the feasible solution of the matrix inequality (38) is to choose an appropriate value of the parameter $\alpha$. In view of that, a searching algorithm is presented to find an appropriate value of the parameter $\alpha$. Inspired by [18], the procedure of the searching algorithm is given below.

*Step 1.* For the given $\tau_0$ and sufficiently big $\alpha_m$, choose a sufficiently small initial value of the upper bound as $\tau_m = \tau_{m0} \geq \tau_0$. Generally, $2\alpha X - \alpha^2\widehat{R} \leq X\widehat{R}^{-1}X$ can be reduced to $2X - \widehat{R} \leq X\widehat{R}^{-1}X$ when $\alpha = 1$, which is widely used in [25, 27]. Hence, $\alpha$ is initially set to 1. If it needs, the initial value of $\alpha$ can also be a positive number less than 1.

*Step 2.* For the parameters $\alpha$ and $\tau_m$, if there exists a feasible solution satisfying LMIs described in (38), go to Step 3; otherwise, go to Step 4.

*Step 3.* Set $\tau_m = \tau_m + \Delta\tau_m$, where $\Delta\tau_m$ is the step increment of $\tau_m$ and go to Step 2.

*Step 4.* Set $\alpha = \alpha + \Delta\alpha$, where $\Delta\alpha$ is the step increment of $\alpha$. If $\alpha < \alpha_m$, go to Step 2; otherwise, denote the current $\alpha$ by $\alpha_0$ and go to Step 5.

*Step 5.* Output the value of parameter $\alpha = \alpha_0 + \Delta\alpha$ and the corresponding value of $\tau_m$.

## 4. Simulations and Experimental Results

In this section, considering the full-scale single bogie of CMS-04 maglev train developed by National University of Defense Technology as the controlled object, we illustrate the effectiveness of the proposed approach by numerical simulations and physical experiments. The parameters are shown in Table 1.

Due to the fact that the magnetic suspension control system needs quick response to the variation of the levitation gap, the sampling time $T$ of the sensors message is set to be 0.25 ms in the simulations and experiments. The transmission speed of the CAN bus is set at 1 Mbps. Data frame with eight bytes can contain the total sensors sampling values, and it will be transmitted within the interval of 100 $\mu$s. Considering that there exist network conflicts and packets lost, the actual random time varying delay will be much greater than this theoretical value. So, $0.25 < \tau < 3$ ms is assumed. With the parameters in Table 1, the local linear models of the magnetic suspension system are obtained by using (5) when the equilibrium suspension gap is supposed at 3 mm, 9 mm, and 20 mm accordingly. Then, we discrete the system with $T = 0.25$ ms, and the corresponding state matrixes are obtained as follows:

$$\widehat{A}_1 = \begin{bmatrix} 1 & 0.0002 & 0 \\ 1.635 & 1 & -0.0005 \\ 0.6012 & 0.7353 & 0.9995 \end{bmatrix}, \qquad \widehat{B}_1 = \begin{bmatrix} 0 \\ 0 \\ 0.0003 \end{bmatrix},$$
$$\widehat{A}_2 = \begin{bmatrix} 1 & 0.0002 & 0 \\ 0.545 & 1 & -0.0002 \\ 0.2003 & 0.7351 & 0.9990 \end{bmatrix}, \qquad \widehat{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0.0009 \end{bmatrix},$$
$$\widehat{A}_3 = \begin{bmatrix} 1 & 0.0002 & 0 \\ 0.2453 & 1 & 0.0001 \\ 0.0901 & 0.7347 & 0.9979 \end{bmatrix}, \qquad \widehat{B}_3 = \begin{bmatrix} 0 \\ 0 \\ 0.0021 \end{bmatrix}. \tag{40}$$

As for the magnetic suspension system, the parametric uncertainties are mainly affected by the parameter $k_e$. This paper considers an additive uncertainty on the parameter $k_e$ which can be described as $|\Delta k_e| \le 0.2k_e$. By this way, one obtains the matrixes in (10) approximately as follows:

$$D_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$
$$E_{ai} = \begin{bmatrix} 0 & 0 & 0 \\ 0.536 & 0 & 0 \\ 0.2027 & 0 & -0.0007 \end{bmatrix}, \tag{41}$$
$$E_{bi} = \begin{bmatrix} 0 \\ 0 \\ 0.0002 \end{bmatrix},$$
$$F_i(k) = \operatorname{diag}\{\operatorname{rand}(k), \operatorname{rand}(k), \operatorname{rand}(k)\}, \quad i = 1, 2, 3,$$

where rand($k$) represents the random number in the range of $[-1, 1]$.

Table 1: Parameters of single suspension control point.

| Parameter | Value |
|---|---|
| $m$ | 1020 kg |
| $A_s$ | 0.0186 m$^2$ |
| $g$ | 9.81 m/s$^2$ |
| $N$ | 320 |
| $R$ | 0.5 Ω |
| $\mu_0$ | $4\pi \times 10^{-7}$ H/m |

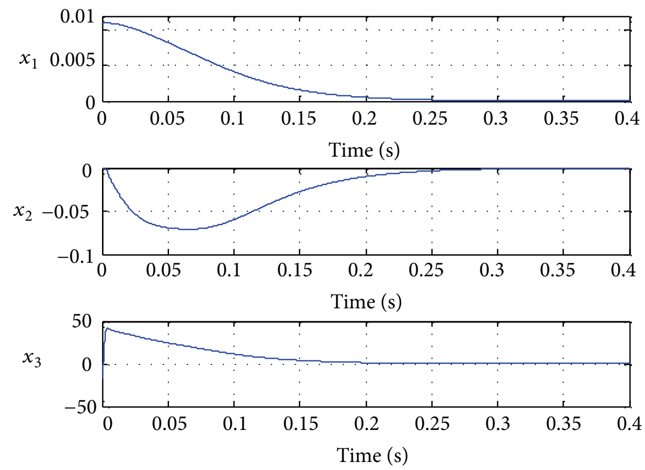


Figure 3: Initial state response of the networked suspension control system.

The aim of this paper is to design paralleled state feedback controllers to make the networked suspension system asymptotically stable. Based upon Theorem 7, one can obtain the feedback gains as $K_1 = [622000, 5639.5, -177.3]$, $K_2 = [184800, 3230.6, -49.57]$, and $K_3 = [66180, 1779, -17.54]$ with $\alpha = 10$. In the simulations, the levitation procedure that the levitation gap changes from 20 mm to 9 mm and the current changes form 0 A to 26.48 A is shown. For normalization, we assume the initial condition to be $x_e = [0.011, 0, -26.48]$. The state responses of the closed loop suspension control system are shown in Figure 3. And Figure 4 shows the distribution of the random transmission time delay and the control input of the closed loop system. The results illustrate that the presented method can guarantee that the networked suspension control system is asymptotically stable.

To show the effectiveness of the proposed method on dealing with the random time delays in the networked suspension control system, the simulations with a common state feedback controller designed in [28] have also been finished. Firstly, considering no time delay in the suspension control system, the response of the state of $x_1$ is given in Figure 5, from which it is shown that the suspension control system is stable. Then, a random time delay $\tau < 1.5$ ms is added and the response of the state of $x_1$ is given in Figure 6. The curve shows that the system becomes unstable as a consequence of random time varying delay. From the comparison between the proposed method and the common

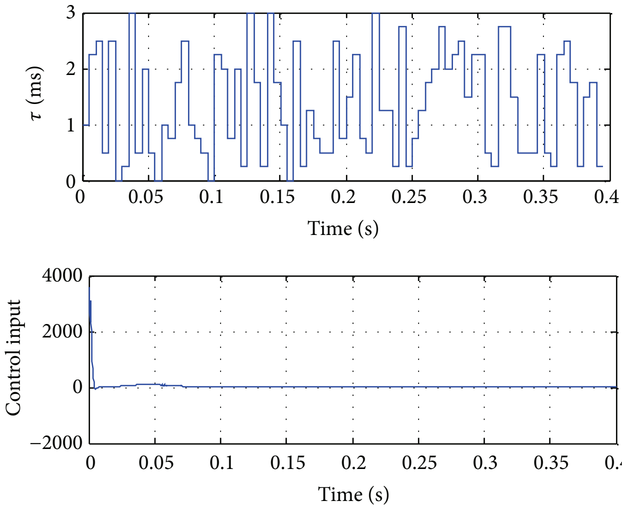


Figure 4: Distribution of the time delay and control input of the closed loop system.

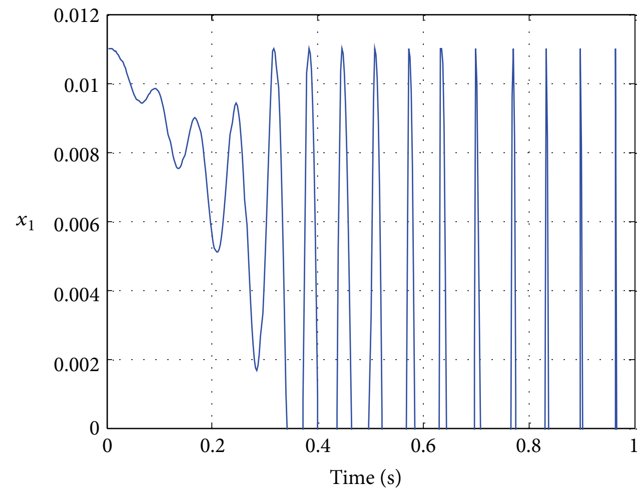


Figure 6: Initial state response of the suspension control system with time delay under a common state feedback controller.

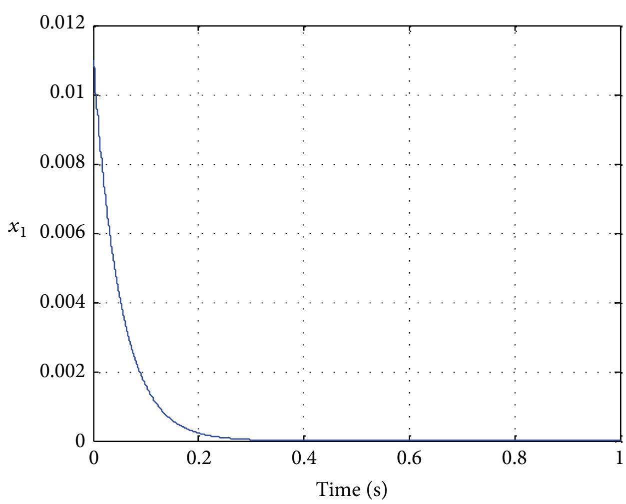


Figure 5: Initial state response of the suspension control system without time delay under a common state feedback controller.

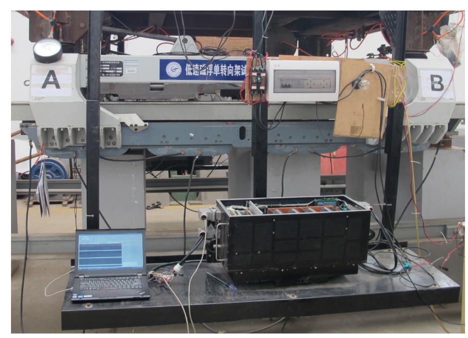


Figure 7: Full-scale single bogie of CMS-04 maglev train.

state feedback controller designed in [28], it is illustrated that the proposed method can deal with the random time delays in the networked suspension control system effectively.

Besides, experiments also have been carried out on the full-scale single bogie of CMS-04 maglev train in our lab which is shown in Figure 7. In the experiments, the controller adopts the T-S fuzzy PDC state feedback controller designed in this paper. The experiment focuses on the procedure that the vehicle is levitated from the initial levitation gap to 9 mm. Because of the track limit in our lab, the initial levitation gap is 15.7 mm. Firstly, without the time delay, the vehicle can be levitated steadily with good performance. Then, by introducing the random time varying delay of $0.25 < \tau < 3$ ms into the sensors message, the levitation procedure is repeated. The plot of levitation gap is given in Figure 8, from which it can be seen that the levitation procedure is still stable with good performance. Here, the longer time of the levitation procedure than the simulations turns up because slow levitation technology is adopted in the controller to make the levitation procedure more comfortable. Besides, to test the capacity of the presented method on coping with the random time delays, the attempts to increase the upper limit of the random time delay $\tau_m$ have been done. And the results show that the vehicle can be levitated steadily until the upper limit of the random time delay $\tau_m$ is set to be 5 ms. When $\tau_m > 5$ ms vibrations occur in the levitation procedure and the system becomes unstable when $\tau_m > 8$ ms. Figure 9 gives the curve of the levitation gap in the levitation procedure when $\tau_m = 6$ ms.

From the experimental results, it can be illustrated that the proposed T-S fuzzy control approach can guarantee the stability of the networked suspension control system with a bounded random induced time delay and meet the control need of the networked suspension system.

## 5. Conclusions

In this paper, we have addressed the stability and control synthesis of the networked suspension control system with induced time delays and packet dropouts. The nonlinear

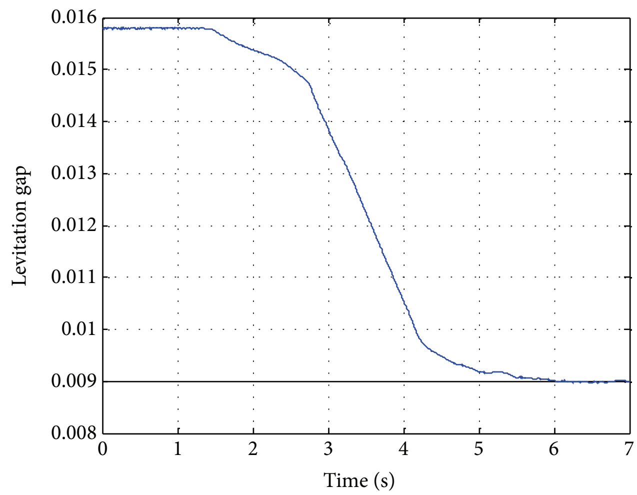


Figure 8: The curve of levitation gap when the random time delay is $0.25 < \tau < 3$ ms.

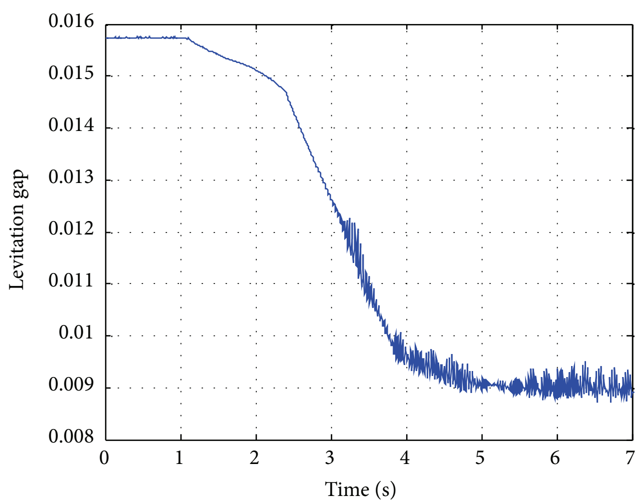


Figure 9: The curve of levitation gap when $\tau_m = 6$ ms.

networked suspension control system is modeled as discrete T-S fuzzy control models with random input time delay. Then, by using Lyapunov-Krasovskii functional, delay dependent stability conditions for the existence of fuzzy controllers have been derived. The final control gains are given in terms of strict LMIs, which can be solved by MATLAB LMI Toolbox conveniently. Finally, simulation and experimental results indicate that the proposed method is effective on the application of the networked suspension control system in maglev train.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

## Acknowledgment

This work is supported by the National Natural Science Foundation of China under Grant no. 11202230.

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