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Research Article

Dynamics of a Stage Structured Pest Control Model in a Polluted Environment with Pulse Pollution Input

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By using pollution model and impulsive delay differential equation, we formulate a pest control model with stage structure for natural enemy in a polluted environment by introducing a constant periodic pollutant input and killing pest at different fixed moments and investigate the dynamics of such a system. We assume only that the natural enemies are affected by pollution, and we choose the method to kill the pest without harming natural enemies. Sufficient conditions for global attractivity of the natural enemy-extinction periodic solution and permanence of the system are obtained. Numerical simulations are presented to confirm our theoretical results.

1. Introduction

Nowadays, the problem of the world's environmental pollution is serious, which has a frustrating effect on the ecosystem damage in the direct or indirect ways. Pollution leads to the living environmental change and gene mutation. It results in not only birth defects and deformities but also population variability, which decreases the number of the population in the nature and even makes them extinct. In order to assess the risk of the populations exposed to a polluted environment, in recent years, mathematical models concerning this topic have been studied extensively including continuous pollution input and impulsive pollution input [1–11].

As we all know, the predator-prey system can be used to model the process of controlling the pests by spraying pesticides, as well as relying on their natural enemies. However, in a polluted environment, some natural enemies are affected by pollution seriously and pests almost are not affected. For example, frogs are the natural enemies of beetles, locusts, and mole cricket, but some chemical plants discard waste products into rivers for their convenience, which cause severe water contamination, seriously injures frog's reproductive system, and significantly decreases their fertility. Moreover, water pollution also causes large quantities of the fertilized eggs and tadpoles to die, resulting in the decrease of frogs. It is shown in a Sweden's new study that male tadpoles can eventually grow into female frogs only in the environment similar to the nature but full of pollutants with estrogen. However, some male frogs have ovaries but no fallopian tubes, and they finally turn into lifelong infertile frogs, which are called "Yin and Yang frog", and nearly one-third of the world's frog species may be extinct because of the environmental pollution. People must control the period and quantity of emission of pollution to prevent natural enemy from extinction. In addition, too much pesticide spraying will reduce pests significantly; meanwhile, it also causes serious environmental pollution. Therefore, when controlling pests, we had better choose the method to kill the pests without polluting the environment and harming natural enemies at regular intervals.

The predator-prey models with stage structure for the predator were introduced or investigated by Hastings and Wang [12–14]. Since the immature predator takes τ (which is called maturation time delay) units of time to mature, the death toll during the juvenile period should be considered, and time delays have important biological meanings in stage structured models. Recently, many models with time delay were extensively studied [15–22].

According to the above biological background, in this paper, we suggest an impulsive predator-prey pollution model with stage structured for predator by introducing a constant periodic pollutant input and proportional killing pest at different fixed moments to model the process of pest control and polluted environment. Recently, there has been quite a lot of literatures on the applications of impulsive differential equations on population [1, 2, 8, 10, 11, 20–31]. To our knowledge, there have been no results on this topic in the literature. The questions that arise here are as follows: how do we control the emission of pollution to prevent the extinction of natural enemies? Under what condition can the system be permanent? How can we control pests effectively?

The organization of this paper is as follows. In the next section, we formulate our model and give several lemmas which are useful for our main results. In Section 3 and Section 4, the sufficient conditions for the global attractivity of the "natural enemy-extinction" periodic solution and permanence of the system are obtained. We give a brief discussion of our results in Section 5. Numerical simulations are presented to illustrate our theoretical results.

2. Model Formulation and Preliminaries

In this paper, we assume only that the natural enemies are affected by pollution and we choose the method to kill the pest without harming natural enemies. Then a pest control model with stage structure for natural enemy in a polluted environment by introducing a constant periodic pollutant input and killing pests at different fixed moment is formulated as follows:

$$\begin{aligned}
\frac{dx(t)}{dt} &= \beta x(t)\left(1-\frac{x(t)}{K}\right)-\frac{qx(t)y_2(t)}{1+\alpha x(t)},\\
\frac{dy_1(t)}{dt} &= \lambda\frac{qx(t)y_2(t)}{1+\alpha x(t)}\\
&\quad -\lambda e^{-d\tau}\frac{qx(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)}\\
&\quad -dy_1(t)-f_1c_0(t)y_1(t),\\
\frac{dy_2(t)}{dt} &= \lambda e^{-d\tau}\frac{qx(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)}\\
&\quad -\gamma y_2(t)-f_2c_0(t)y_2(t),\\
\frac{dc_0(t)}{dt} &= kc_e(t)-gc_0(t)-mc_0(t),\\
\frac{dc_e(t)}{dt} &= -hc_e(t),\\
&\qquad t\neq(n+l-1)T,\ t\neq nT,\\
\Delta x(t) &= 0,\quad \Delta y_1(t)=0,\quad \Delta y_2(t)=0,\\
\Delta c_0(t) &= 0,\quad \Delta c_e(t)=b,\\
&\qquad t=(n+l-1)T,\\
\Delta x(t) &= -\delta x(t),\quad \Delta y_1(t)=0,\quad \Delta y_2(t)=0,\\
\Delta c_0(t) &= 0,\quad \Delta c_e(t)=0,\\
&\qquad t=nT,
\end{aligned}\tag{1}$$

where $0\le l\le 1$, $\Delta x(t)=x(t^+)-x(t)$, $\Delta y_i(t)=y_i(t^+)-y_i(t)$ $(i=1,2)$, $\Delta c_0(t)=c_0(t^+)-c_0(t)$, $\Delta c_e(t)=c_e(t^+)-c_e(t)$. $x(t)$, $y_1(t)$, and $y_2(t)$ represent the densities of prey (pest), immature, and mature predator (natural enemy) at time t, respectively; $c_e(t)$, $c_0(t)$ represent the concentration of pollution in the environment and organism at time t, respectively; β is intrinsic growth rate of the pests in the absence of natural enemies; $K>0$ is the pest capacity of environment; q is the predation rate of natural enemy and λ represents the conversion rate at which ingested pest in excess of what is needed for maintenance is translated into natural enemy increase; α is the saturation which represents that a certain amount of natural enemies can prey on a limited amount of pests, though the pests are numerous; d and γ are the death rate of immature and mature natural enemies, respectively; in addition, we assume that juveniles suffer a mortality rate of d (the through-stage death rate) and take τ units of time to mature. f_1 and f_2 are the dose-response parameters of species to the pollution in the immature and mature natural enemies, respectively; the exogenous quantity of impulsive input of pollutant into the environment at time $t=(n+l-1)T$ is represented by b; δ $(0\le\delta<1)$ represents a proportional decrease of pest because of being harvested at time $t=nT$. The other parameters can be seen in [1].

The initial conditions for (2) are

$$\begin{aligned}
&(x(t),y_1(t),y_2(t),c_0(t),c_e(t))\\
&\quad=(\varphi_1(t),\varphi_2(t),\varphi_3(t),\varphi_4(t),\varphi_5(t))\in C_+\\
&\quad=C([-\tau,0],R_+^5),\quad \varphi_i(0)>0,\ i=1,2,3,4,5.
\end{aligned}\tag{2}$$

Note that the variable $y_1(t)$ does not appear in the first, third, forth, and fifth equations of system (2); hence, we only need to consider the subsystem of (2) as follows:

$$\begin{aligned}
\frac{dx(t)}{dt} &= \beta x(t)\left(1-\frac{x(t)}{K}\right)-\frac{qx(t)y_2(t)}{1+\alpha x(t)},\\
\frac{dy_2(t)}{dt} &= \lambda e^{-d\tau}\frac{qx(t-\tau)y_2(t-\tau)}{1+\alpha x(t-\tau)}\\
&\quad -\gamma y_2(t)-f_2c_0(t)y_2(t),\\
\frac{dc_0(t)}{dt} &= kc_e(t)-gc_0(t)-mc_0(t),\\
\frac{dc_e(t)}{dt} &= -hc_e(t),\\
&\qquad t\neq(n+l-1)T,\ t\neq nT,
\end{aligned}$$

$$\begin{aligned}
&\Delta x(t)=0, \quad \Delta y_2(t)=0,\\
&\Delta c_0(t)=0, \quad \Delta c_e(t)=b,\\
&\qquad t=(n+l-1)T\\
&\Delta x(t)=-\delta x(t), \quad \Delta y_2(t)=0,\\
&\Delta c_0(t)=0, \quad \Delta c_e(t)=0,\\
&\qquad t=nT.
\end{aligned} \tag{3}$$

Lemma 1 (see [32]). *Consider the following delay differential equation:*

$$\frac{dx(t)}{dt}=r_1x(t-\tau)-r_2x(t), \tag{4}$$

where r_1, r_2, and τ are all positive constants and $x(t)>0$ for $-\tau\le t\le 0$; we have

(1) *if $r_1<r_2$, then $\lim_{t\to+\infty}x(t)=0$;*

(2) *if $r_1>r_2$, then $\lim_{t\to+\infty}x(t)=+\infty$.*

Lemma 2 (see [1]). *Consider the following subsystem of* (2)*:*

$$\begin{aligned}
&\frac{dc_0(t)}{dt}=kc_e(t)-gc_0(t)-mc_0(t),\\
&\frac{dc_e(t)}{dt}=-hc_e(t),\\
&\qquad t\neq(n+l-1)T,\\
&\Delta c_0(t)=0, \quad \Delta c_e(t)=b, \quad t=(n+l-1)T.
\end{aligned} \tag{5}$$

Then system (5) *has a unique positive T-periodic solution $(c_0^*(t),c_e^*(t))$, which is globally asymptotically stable, where*

$$\begin{aligned}
c_0^*(t)&=c_0^*(0)\,e^{-(g+m)(t-(n+l-1)T)}\\
&\quad+\frac{kb\left(e^{-(g+m)(t-(n+l-1)T}-e^{-h(t-(n+l-1)T}\right)}{(h-g-m)\left(1-e^{-hT}\right)},\\
c_e^*(t)&=\frac{be^{-h(t-(n+l-1)T)}}{1-e^{-hT}},\\
c_0^*(0)&=\frac{kb\left(e^{-(g+m)T}-e^{-hT}\right)}{(h-g-m)\left(1-e^{-(g+m)T}\right)\left(1-e^{-hT}\right)},\\
c_e^*(0)&=\frac{b}{1-e^{-hT}},
\end{aligned} \tag{6}$$

and $t\in((n+l-1)T,(n+l)T]$.

Lemma 3. *There exists a constant $L>0$ such that $x(t)\le K$, $y_1(t)\le L$, $y_2(t)\le L$, $c_0(t)\le L$, and $c_e(t)\le L$.*

Proof. Define $V(t)=\lambda x(t)+y_1(t)+y_2(t)+c_0(t)+c_e(t)$. Since $dx(t)/dt\le\beta x(t)(1-(x/K))$, $\dot{x}|_{x=K}\le 0$, in addition, $0\le\delta<1$, $x(nT^+)\le x(nT)$, thus $x(t)\le K$ for t large enough. Define $d^*=\min\{d,\gamma,g+m,h-k\}$; then, for $t\neq(n+l-1)T$, $t\neq nT$; we have

$$\frac{dV(t)}{dt}<L_0-d^*V(t), \tag{7}$$

where $L_0=\lambda K(\beta+d^*)^2/4\beta$. Consider the following impulse differential inequalities:

$$\begin{aligned}
&\frac{dV(t)}{dt}\le -d^*V(t)+L_0, \quad t\neq(n+l-1)T,\ t\neq nT,\\
&V\left((n+l-1)T^+\right)=V((n+l-1)T)+b,\\
&\qquad t=(n+l-1)T,\\
&V\left(nT^+\right)\le V(nT), \quad t=nT.
\end{aligned} \tag{8}$$

We have

$$\begin{aligned}
V(t)&\le V\left(0^+\right)e^{-d^*t}+\int_0^t L_0e^{-d^*(t-s)}ds\\
&\quad+\sum_{0<(n+l-1)T<l} be^{-d^*(t-(n+l-1)T)}.
\end{aligned} \tag{9}$$

Hence

$$\lim_{t\to+\infty}V(t)\le\frac{L_0}{d^*}+\frac{be^{d^*lT}}{e^{d^*T}-1}=L, \tag{10}$$

so $V(t)$ is uniformly ultimately bounded. Therefore, by the definition of $V(t)$, system (2) is uniformly ultimately bounded. The proof is completed. □

Lemma 4 (see [29]). *If $\delta<1-e^{-\beta T}$ holds, system*

$$\begin{aligned}
&\frac{dx(t)}{dt}=\beta x(t)\left(1-\frac{x(t)}{K}\right), \quad t\neq nT,\\
&x\left(t^+\right)=(1-\delta)x(t), \quad t=nT
\end{aligned} \tag{11}$$

has a unique positive globally asymptotically stable periodic solution $x^*(t)=K(1-\delta-e^{-\beta T})/(1-\delta-e^{-\beta T}+\delta e^{-\beta(t-nT)})$, $t\in(nT,(n+1)T]$.

Therefore, if $\delta<1-e^{-\beta T}$ holds, the system (2) has a natural enemy-extinction periodic solution $(x^*(t),0,c_0^*(t),c_e^*(t))$. In this paper, we assume that $\delta<1-e^{-\beta T}$ always holds.

Remark 5 (see [1]). $c_0(t)$ and $c_e(t)$ are the concentration of pollution. To assure $0\le c_0(t)\le 1$ and $0\le c_e(t)\le 1$, it is necessary that $g\le k\le g+m$, $b\le 1-e^{-hT}$.

Remark 6 (see [1]). According to the biological significance, we assume $k<h$.

3. Global Attractivity of the "Natural Enemy-Extinction" Periodic Solution

In this section, we discuss under what condition the natural enemies will go extinct.

Denote

$$R_1 = \frac{\lambda q K e^{-d\tau}}{(\gamma + f_2\rho)(\alpha K + g_1(\delta, T))}, \tag{12}$$

where

$$\rho = \frac{kb\left(e^{-(g+m)T} - e^{-hT}\right)e^{-(g+m)T}}{(h-g-m)\left(1-e^{-(g+m)T}\right)\left(1-e^{-hT}\right)},$$
$$g_1(\delta, T) = \frac{(1-\delta)\left(1-e^{-\beta T}\right)}{1-\delta-e^{-\beta T}}. \tag{13}$$

Theorem 7. *If $R_1 < 1$, then the "mature natural enemy-extinction" periodic solution $(x^*(t), 0, c_0^*(t), c_e^*(t))$ of system (2) is globally attractive.*

Proof. Since $R_1 < 1$, we have

$$e^{-d\tau}\frac{\lambda q K\left(1-\delta-e^{-\beta T}\right)}{\left(1-e^{-\beta T}\right)(1-\delta)+\alpha K\left(1-\delta-e^{-\beta T}\right)} < \gamma + f_2\rho. \tag{14}$$

By Lemma 2, for sufficiently small enough $\varepsilon_1 > 0$, there exists a positive constant N_1 such that

$$c_0(t) > c_0^*(t) - \varepsilon_1 > \rho - \varepsilon_1 > 0. \tag{15}$$

holds for $t \geq N_1 T$.

Note that

$$\frac{dx(t)}{dt} \leq \beta x(t)\left(1 - \frac{x(t)}{K}\right), \quad t \neq nT,\ n \in Z^+,$$
$$\Delta x(t) = -\delta x(t), \quad t = nT,\ n \in Z^+. \tag{16}$$

Then we consider the following comparison system:

$$\frac{dz(t)}{dt} = \beta z(t)\left(1 - \frac{z(t)}{K}\right), \quad t \neq nT,\ n \in Z^+,$$
$$\Delta z(t) = -\delta z(t), \quad t = nT,\ n \in Z^+. \tag{17}$$

According to Lemma 4, we know that

$$z^*(t) = x^*(t) = \frac{K\left(1-\delta-e^{-\beta T}\right)}{1-\delta-e^{-\beta T}+\delta e^{-\beta(t-nT)}}, \quad t \in (nT, (n+1)T] \tag{18}$$

is a unique globally asymptotically stable positive T-periodic solution of system (17).

By using comparison theorem of impulsive differential equation, there exist a positive integer N_2 and a sufficiently small positive constant ε_2 such that for all $nT < t \leq (n+1)T$, $n > N_2$,

$$x(t) \leq x^*(t) + \varepsilon_2 \leq \frac{K\left(1-\delta-e^{-\beta T}\right)}{\left(1-e^{-\beta T}\right)(1-\delta)} + \varepsilon_2 = \eta \tag{19}$$

holds. From (15), (19), and the second equation of (2), we obtain that for $t > NT + \tau$,

$$\frac{dy_2(t)}{dt} \leq \lambda e^{-d\tau}\frac{q\eta}{1+\alpha\eta}y_2(t-\tau) - (\gamma + f_2(\rho - \varepsilon_1))y_2(t) \tag{20}$$

holds.

Consider the following comparison equation:

$$\frac{du(t)}{dt} = \lambda e^{-d\tau}\frac{q\eta}{1+\alpha\eta}u(t-\tau) - (\gamma + f_2(\rho - \varepsilon_1))u(t). \tag{21}$$

By inequality (14), we have that $\lambda e^{-d\tau}q\eta/(1+\alpha\eta) < \gamma + f_2(\rho - \varepsilon_1)$ holds; then, according to Lemma 1, we obtain that

$$\lim_{t\to+\infty} u(t) = 0. \tag{22}$$

By the comparison theorem of delay differential equation, we have $\lim_{t\to+\infty} y_2(t) = 0$.

Without loss of generality, we may assume that $0 < y_2(t) < \varepsilon_3$ (ε_3 is sufficiently small positive constant such that $\delta < 1 - e^{-(\beta - q\varepsilon_3)T}$) for all $t \geq 0$; by the first equation of system (2), we have

$$\frac{dx(t)}{dt} \geq \beta x(t)\left(1 - \frac{x(t)}{K}\right) - \frac{qx(t)\varepsilon_3}{1+\alpha x(t)} \geq \beta x(t)\left(1 - \frac{x(t)}{K}\right) - qx(t)\varepsilon_3. \tag{23}$$

Consider the following comparison equation:

$$\frac{dz_1(t)}{dt} = \beta z_1(t)\left(1 - \frac{z_1(t)}{K}\right) - qz_1(t)\varepsilon_3 = (\beta - q\varepsilon_3)z_1(t)\left(1 - \frac{z_1(t)}{K(\beta - q\varepsilon_3)/\beta}\right), \quad t \neq nT,\ n \in Z^+,$$
$$\Delta z_1(t) = -\delta z_1(t), \quad t = nT,\ n \in Z^+. \tag{24}$$

By Lemma 4,

$$z_1^*(t) = \frac{(K(\beta - q\varepsilon_3)/\beta)\left(1-\delta-e^{-(\beta - q\varepsilon_3)T}\right)}{1-\delta-e^{-(\beta - q\varepsilon_3)T}+\delta e^{-(\beta - q\varepsilon_3)(t-nT)}}, \quad t \in (nT, (n+1)T] \tag{25}$$

is a unique globally asymptotically stable positive T-periodic solution of system (24). By using comparison theorem of impulsive differential equation, for above ε_2 and t large enough, we have

$$x(t) > z_1^*(t) - \varepsilon_2. \tag{26}$$

It follows from (19) and (26) that

$$z_1^*(t) - \varepsilon_2 < x(t) < x^*(t) + \varepsilon_2, \tag{27}$$

holds for t large enough. Let $\varepsilon_3 \to 0$; we can get $z_1^*(t) \to x^*(t)$, so

$$x^*(t) - \varepsilon_2 < x(t) < x^*(t) + \varepsilon_2 \tag{28}$$

holds for t large enough, which implies $x(t) \to x^*(t)$ as $t \to +\infty$. According to Lemma 2, $c_0(t) \to c_0^*(t)$, $c_e(t) \to c_e^*(t)$ as $t \to +\infty$. This completes the proof. □

4. Permanence

Definition 8. System (2) is said to be permanent if there are positive constants m, M, and a finite time T_0 such that for all solutions $(x(t), y_1(t), y_2(t), c_0(t), c_e(t))$ with initial conditions (2), $m \le x(t) \le M$, $m \le y_i(t) \le M$, $m \le c_0(t) \le M$, $m \le c_e(t) \le M$ holds for all $t \ge T_0$, $i = 1, 2$.

Denote

$$R_2 = \frac{\lambda q K e^{-d\tau}}{(\gamma + f_2 B)(\alpha K + g_2(\delta, T))}, \tag{29}$$

where

$$\begin{aligned} B &= \frac{kb\left(e^{-(g+m)T} - e^{-hT}\right)}{(h-g-m)\left(1 - e^{-(g+m)T}\right)\left(1 - e^{-hT}\right)} \\ &\quad + \frac{kb}{|h-g-m|\left(1 - e^{-hT}\right)}, \\ g_2(\delta, T) &= \frac{1 - e^{-\beta T}}{1 - \delta - e^{-\beta T}}. \end{aligned} \tag{30}$$

Theorem 9. *If $R_2 > 1$, there exists a positive constant m^* such that $y_2(t) \ge m^*$ for any solution $(x(t), y_2(t), c_0(t), c_e(t))$ of system (2).*

Proof. Since $R_2 > 1$, we can choose positive constants m_1^*, ϵ_1, and ϵ_2 such that

$$\lambda q e^{-d\tau} \frac{\sigma}{1 + \alpha\sigma} > \gamma + f_2(B + \epsilon_2) \tag{31}$$

and $\delta < 1 - e^{-(\beta - q m_1^*)T}$ hold, where

$$\begin{aligned} \sigma = {} & \frac{K\left(\beta - q m_2^*\right)\left(1 - \delta - e^{-(\beta - q m_1^*)T}\right)}{\beta\left(1 - e^{-(\beta - q m_2^*)T}\right)} \\ & - \epsilon_1 > 0. \end{aligned} \tag{32}$$

The second equation of system (2) can be written as

$$\begin{aligned} \frac{dy_2(t)}{dt} &= \left(\lambda q e^{-d\tau} \frac{x(t)}{1 + \alpha x(t)} - \gamma - f_2 c_0(t)\right) y_2(t) \\ &\quad - \lambda q e^{-d\tau} \frac{d}{dt} \int_{t-\tau}^{t} \frac{x(\theta)}{1 + \alpha x(\theta)} y_2(\theta)\, d\theta. \end{aligned} \tag{33}$$

Define

$$V(t) = y_2(t) + \lambda q e^{-d\tau} \frac{d}{dt} \int_{t-\tau}^{t} \frac{x(\theta)}{1 + \alpha x(\theta)} y_2(\theta)\, d\theta. \tag{34}$$

Calculating the derivative of $V(t)$ along the solution of (2), we have

$$\frac{dV(t)}{dt} = \left(\lambda q e^{-d\tau} \frac{x(t)}{1 + \alpha x(t)} - \gamma - f_2 c_0(t)\right) y_2(t). \tag{35}$$

We claim that the inequality $y_2(t) < m_1^*$ cannot hold for all $t \ge t_0$. Otherwise, there is a positive constant t_0 such that $y_2(t) < m_1^*$ for all $t \ge t_0$. From the first equation of system (2), we have

$$\begin{aligned} \frac{dx(t)}{dt} &> \beta x(t)\left(1 - \frac{x(t)}{K}\right) - q x(t) m_2^*, \\ &\qquad t \ne nT,\ n \in Z^+, \\ \Delta x(t) &= -\delta x(t), \qquad t = nT,\ n \in Z^+. \end{aligned} \tag{36}$$

Consider the following comparison system:

$$\begin{aligned} \frac{dz(t)}{dt} &= \beta z(t)\left(1 - \frac{z(t)}{K}\right) - q z(t) m_2^*, \\ &\qquad t \ne nT,\ n \in Z^+, \\ \Delta z(t) &= -\delta z(t), \qquad t = nT,\ n \in Z^+. \end{aligned} \tag{37}$$

Then

$$\begin{aligned} z^*(t) &= \frac{\left(K\left(\beta - q m_2^*\right)/\beta\right)\left(1 - \delta - e^{-(\beta - q m_2^*)T}\right)}{1 - \delta - e^{-(\beta - q m_2^*)T} + \delta e^{-(\beta - q m_2^*)(t - nT)}}, \\ &\qquad t \in (nT, (n+1)T] \end{aligned} \tag{38}$$

is a unique globally asymptotically stable positive T-periodic solution of system (37). By using comparison theorem of impulsive differential equation, for $\epsilon_1 > 0$, there exists a $T_1 > t_0$ such that for $t > T_1$,

$$\begin{aligned} x(t) &> z^*(t) - \epsilon_1 \\ &\ge \frac{K\left(\beta - q m_2^*\right)\left(1 - \delta - e^{-(\beta - q m_2^*)T}\right)}{\beta\left(1 - e^{-(\beta - q m_2^*)T}\right)} - \epsilon_1 \\ &= \sigma \end{aligned} \tag{39}$$

holds.

By Lemma 2, for $\epsilon_2 > 0$, there exists a $T_2 > 0$ such that for $t > T_2$,

$$c_0^*(t) - \varepsilon_2 < c_0(t) < c_0^*(t) + \varepsilon_2 < B + \varepsilon_2. \tag{40}$$

Let $T_0 = \max\{T_1, T_2\}$, and from (39) and (40), we have

$$\frac{dV(t)}{dt} > \left(\lambda q e^{-d\tau}\frac{\sigma}{1+\alpha\sigma} - \gamma - f_2(B+\epsilon_2)\right) y_2(t), \quad t > T_0. \tag{41}$$

Let

$$y_2^l = \min_{t\in[T_0, T_0+\tau]}\{y_2(t)\}. \tag{42}$$

We will show that $y_2(t) \ge y_2^l$ for all $t \ge T_0$. Otherwise, there exists a nonnegative constant T_3 such that $y_2(t) \ge y_2^l$ for $t \in [T_0, T_0+T_3+\tau]$, $y_2(T_0+T_3+\tau) = y_2^l$ and $dy_2(T_0+T_3+\tau)/dt < 0$. Thus from the second equation of (2), (31), and (41), we easily see that

$$\begin{aligned}\frac{dy_2(T_0+T_3+\tau)}{dt} &\ge y_2^l\left[\lambda q e^{-d\tau}\frac{\sigma}{1+\alpha\sigma} - \gamma - f_2(B+\epsilon_2)\right] \\ &> 0,\end{aligned} \tag{43}$$

which is a contradiction. Hence we get that $y_2^l \ge m_1^*$ for all $t > T_0$. Then we have

$$\begin{aligned}\frac{dV(t)}{dt} &> y_2^l\left(\lambda q e^{-d\tau}\frac{\sigma}{1+\alpha\sigma} - \gamma - f_2(B+\epsilon_2)\right) \\ &> 0, \quad t > T_0,\end{aligned} \tag{44}$$

which implies $V(t) \to +\infty$ as $t \to +\infty$. This is a contradiction to $V(t) \le L + KL\lambda q e^{-d\tau}\tau$. Therefore, for any positive constant t_0, the inequality $y_2(t) < m_1^*$ cannot hold for all $t \ge t_0$. If $y_2(t) \ge m_1^*$ holds true for all t large enough, then our aim is obtained; otherwise, $y_2(t)$ is oscillatory about m_1^*. Let

$$m^* = \min\left\{\frac{m_1^*}{2}, m_1^* e^{-(\gamma+f_2L)\tau}\right\}. \tag{45}$$

In the following, we will show that $y_2(t) \ge m^*$. There exist two positive constants t^* and θ^* such that

$$\begin{aligned}y_2(t^*) &= y_2(t^*+\theta^*) = m_1^*, \\ y_2(t) &< m_1^*, \quad t^* < t < t^*+\theta^*.\end{aligned} \tag{46}$$

When t^* is large enough, the inequality $x(t) > \sigma$ holds true for $t^* < t < t^* + \theta^*$. Since $y_2(t)$ is continuous and bounded and not affected by impulses, we conclude that $y_2(t)$ is uniformly continuous. Hence there exists a constant T_4 ($0 < T_4 < \tau$, and T_4 is independent of the choice of t^*) such that $y_2(t) > m_1 * /2$ for all $t^* \le t \le t^* + T_4$.

If $\theta^* \le T_4$, our aim is obtained.

If $T_4 < \theta^* \le \tau$, from the second equation of (2), we have that

$$\frac{dy_2(t)}{dt} \ge -(\gamma + f_2L)\, y_2(t), \quad t^* < t \le t^*+\theta^*. \tag{47}$$

Then we have $y_2(t) \ge m_1^* e^{-(\gamma+f_2L)\tau}$ for $t^* < t \le t^*+\theta^* \le t^*+\tau$. It is clear that $y_2(t) \ge m^*$ for $t^* < t \le t^* + \theta^*$.

If $\theta^* > \tau$, by the second equation of (2), then we have that $y_2(t) \ge m^*$ for $t^* < t \le t^* + \tau$. The same arguments can be continued, and we can obtain $y_2(t) \ge m^*$ for $t^*+\tau < t \le t^* + \theta^*$. Since the interval $[t^*, t^*+\theta^*]$ is arbitrarily chosen, we get that $y_2(t) \ge m^*$ for t large enough. In view of our arguments above, the choice of m^* is independent of the positive solution of (2). This completes the proof. □

Theorem 10. *If $R_2 > 1$, the system (2) is permanent.*

Proof. Suppose that $(x(t), y_1(t), y_2(t), c_0(t), c_e(t))$ is any positive solution of system (2) with initial conditions (2). By (39), we have $x(t) \ge \sigma$ for t large enough. By Theorem 9, we have $y_2(t) \ge m^*$ for t large enough. From the second equation of system (2), we obtain

$$\begin{aligned}\frac{dy_1(t)}{dt} &\ge \lambda\frac{q\sigma m^*}{1+\alpha\sigma} - \lambda e^{-d\tau}\frac{q\sigma m^*}{1+\alpha\sigma} \\ &\quad - dy_1(t) - f_1y_1(t) \\ &= \lambda\left(1-e^{-d\tau}\right)\frac{q\sigma m^*}{1+\alpha\sigma} \\ &\quad - (d+f_1)\,y_1(t) \\ &\doteq H - (d+f_1)\,y_1(t).\end{aligned} \tag{48}$$

So,

$$\begin{aligned}y_1(t) &\ge \frac{H}{d+f_1} + \left(y(0) - \frac{H}{d+f_1}\right)e^{-(d+f_1)t} \\ &\longrightarrow \frac{H}{d+f_1}, \quad \text{as } t \longrightarrow \infty.\end{aligned} \tag{49}$$

Thus $y_1(t) \ge H/(d+f_1)$ for t large enough. By Lemma 2, we know for a sufficiently small positive ϵ_3, $c_e(t) > c_e^*(t) - \epsilon_3 \ge (be^{-hT})/(1-e^{-hT}) - \epsilon_3 > 0$. Then from (15), Lemma 3, and Definition 8, we have that system (2) is permanent. The proof is completed. □

5. Discussion

In this paper, we discuss a pest control model with stage structure for natural enemy in a polluted environment by introducing a constant periodic pollutant input and killing pest at different fixed moments. From Theorems 7, 9, and 10, we can observe that the extinction and permanence of the population are very much dependent on b, T, and δ.

To verify the theoretical results obtained in this paper, in the following we will give some numerical simulations and take $\beta = 0.9$; $K = 0.8$; $q = 0.8$; $d = 0.4$; $\tau = 0.5$; $\lambda = 0.8$; $\gamma = 0.3$; $b = 0.1$; $k = 0.5$; $f_2 = 0.2$; $f_1 = 0.1$; $\delta = 0.1$; $\alpha = 0.2$; $h = 2$; $g = 0.2$; $m = 0.4$; $T = 1$ (see Figure 1), and here

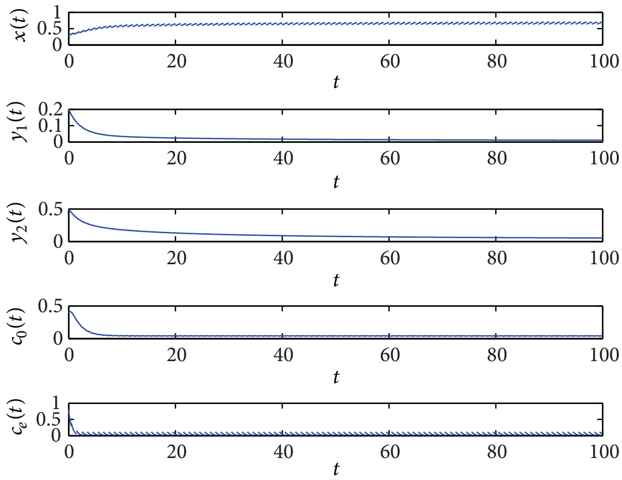

Figure 1: Time series of system (2) with parameters $\beta = 0.9$, $K = 0.8$, $q = 0.8$, $d = 0.4$, $\tau = 0.5$, $\lambda = 0.8$, $\gamma = 0.3$, $b = 0.1$, $k = 0.5$, $f_2 = 0.2$, $f_1 = 0.1$, $\delta = 0.1$, $\alpha = 0.2$, $h = 2$, $g = 0.2$, $m = 0.4$, and $T = 1$.

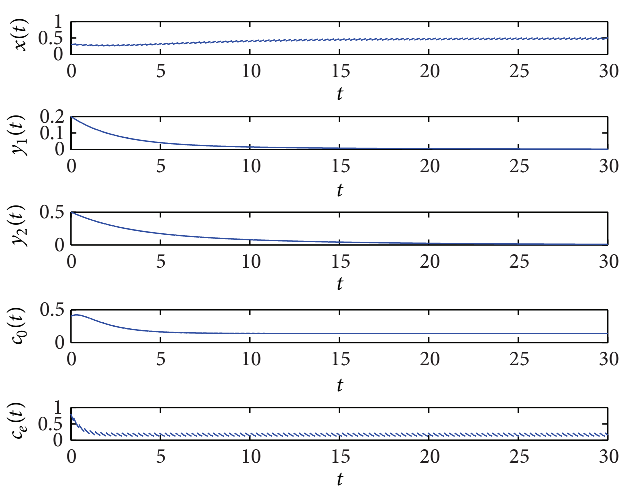

Figure 2: Time series of system (2) with parameters $\beta = 0.9$, $K = 0.8$, $q = 0.8$, $d = 0.4$, $\tau = 0.5$, $\lambda = 0.8$, $\gamma = 0.3$, $b = 0.1$, $k = 0.5$, $f_2 = 0.2$, $f_1 = 0.1$, $\delta = 0.1$, $\alpha = 0.2$, $h = 2$, $g = 0.2$, $m = 0.4$, and $T = 0.3$.

we can compute $R_2 = 1.005252 > 1$, and from Theorem 10 we know the system (2) is permanent. If we decrease the period of pulsing $T = 0.3$ ($R_1 = 0.755095 < 1$) or increase the pollution input amount to $b = 0.8$ ($R_1 = 0.996079 < 1$), and other parameters are the same with those in Figure 1, the natural enemy will be extinct (see Figures 2 and 3). If we increase the harvesting rate of pests to $\delta = 0.5$, and other parameters are the same with those in Figure 1, then $R_1 = 0.413158 < 1$, and the natural enemies will also be extinct (see Figure 4). Our results indicate that if impulsive period T is short or b or δ is too large, the natural enemy will go extinct, but we wish to protect natural enemy from extinction, so we should harvest the pests reasonably and control the period and quantity of emission of pollution into the environment efficiently. This offers us some reasonable suggestions for pest management.

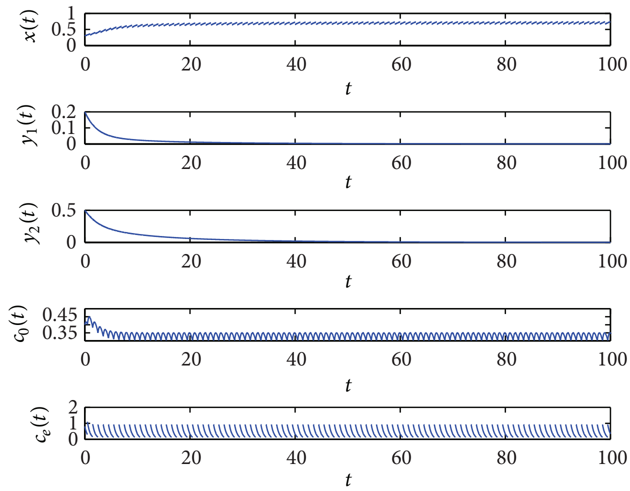

Figure 3: Time series of system (2) with parameters $\beta = 0.9$, $K = 0.8$, $q = 0.8$, $d = 0.4$, $\tau = 0.5$, $\lambda = 0.8$, $\gamma = 0.3$, $b = 0.8$, $k = 0.5$, $f_2 = 0.2$, $f_1 = 0.1$, $\delta = 0.1$, $\alpha = 0.2$, $h = 2$, $g = 0.2$, $m = 0.4$, and $T = 1$.

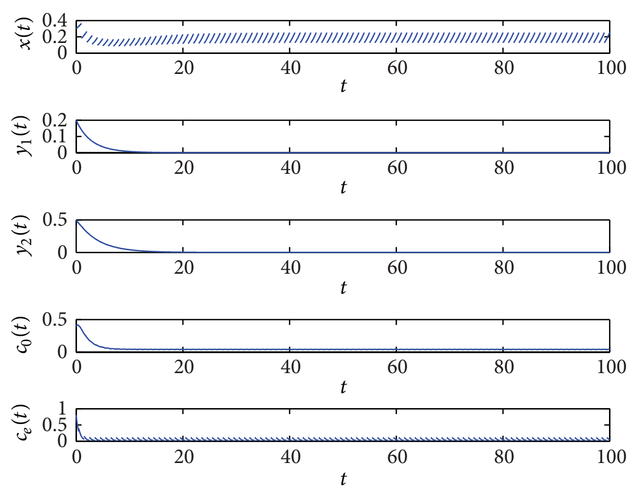

Figure 4: Time series of system (2) with parameters $\beta = 0.9$, $K = 0.8$, $q = 0.8$, $d = 0.4$, $\tau = 0.5$, $\lambda = 0.8$, $\gamma = 0.3$, $b = 0.1$, $k = 0.5$, $f_2 = 0.2$, $f_1 = 0.1$, $\delta = 0.5$, $\alpha = 0.2$, $h = 2$, $g = 0.2$, $m = 0.4$, and $T = 1$.

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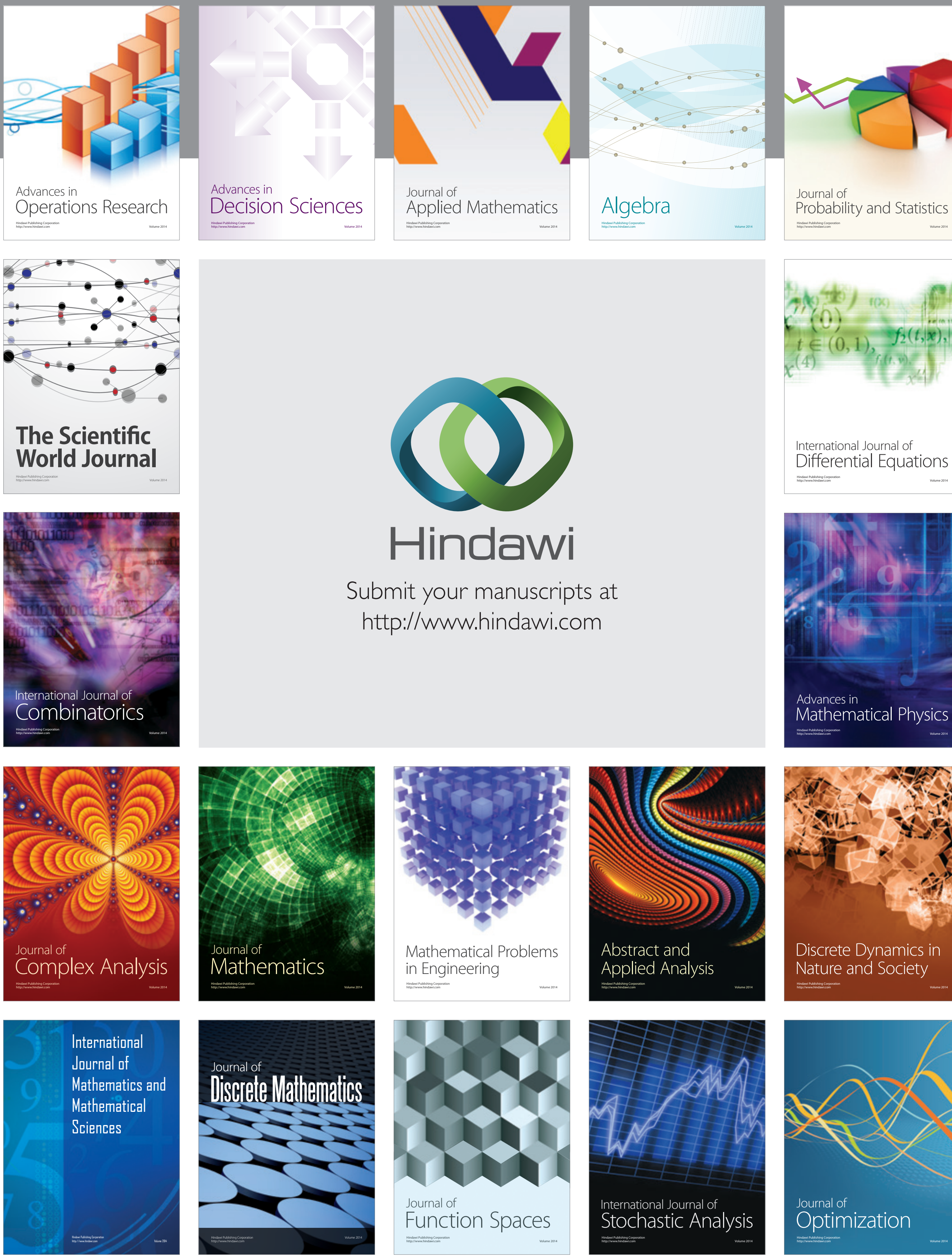
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