Eur. Phys. J. Plus (2017) **132**: 157
DOI 10.1140/epjp/i2017-11452-6

THE EUROPEAN PHYSICAL JOURNAL PLUS

Regular Article

GINGER: A feasibility study

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Received: 20 February 2017
Published online: 10 April 2017

Abstract. GINGER (Gyroscopes IN General Relativity) is a proposal for an Earth-based experiment to measure the Lense-Thirring (LT) and de Sitter effects. GINGER is based on ring lasers, which are the most sensitive inertial sensors to measure the rotation rate of the Earth. We show that two ring lasers, one at maximum signal and the other horizontal, would be the simplest configuration able to retrieve the GR effects. Here, we discuss this configuration in detail showing that it would have the capability to test LT effect at 1%, provided the accuracy of the scale factor of the instrument at the level of 1 part in 10^{12} is reached. In principle, one single ring laser could do the test, but the combination of the two ring lasers gives the necessary redundancy and the possibility to verify that the systematics of the lasers are sufficiently small. The discussion can be generalised to seismology and geodesy and it is possible to say that signals 10–12 orders of magnitude below the Earth rotation rate can be studied; the proposed array can be seen as the basic element of multi-axial systems, and the generalisation to three dimensions is feasible adding one or two devices and monitoring the relative angles between different ring lasers. This simple array can be used to measure with very high precision the amplitude of angular rotation rate (the length of the day, LOD), its short term variations, and the angle between the angular rotation vector and the horizontal ring laser. Finally this experiment could be useful to probe gravity at fundamental level giving indications on violations of Einstein Equivalence Principle and Lorenz Invariance and possible chiral effects in the gravitational field.

1 Introduction

Ring Lasers Gyroscopes (RL) are high-sensitivity devices widely used to measure absolute rotation rates, exploiting the Sagnac effect [1,2]. They are very reliable instruments, with extended bandwidth and very high duty cycle [3–5]. Their sensitivity increases with size to the second power. Small-size RLs are used for inertial navigation. The most advanced RLs, devices with an area of tens of square meters, are used in rotational seismology; in the geodetic community they are considered to measure the fast variations of the Earth rotation rate (daily and sub-daily). RLs are also sensitive to all non-reciprocal effects affecting the two counter-propagating modes of the ring cavity. Following General Relativity (GR), the gravito-magnetic and gravito-electric fields act on the RL cavity as non-reciprocal effects [6], and the RL signal contains additional angular velocity with amplitude of the order of $\sim 10^{-13}$ rad/s, very close to the present best sensitivity in one day of integration time. The purpose of GINGER (Gyroscopes IN GEneral Relativity) is to measure the GR components of the gravito-magnetic field of the Earth at 1% [7,8]. So far, the gravito-magnetic field of Earth has been measured in space with an error at the $\sim 5\%$ [9–11] level, so the experimental objective of measuring the Lense-Thirring angular velocity Ω_{LT} with 1% is still challenging. Different gravity theories (*e.g.*, Chern-Simons gravity [12–14], $f(R)$ theory, and extended gravity models [15–17]) have different time dependence of Lense-Thirring effects to be measured in the experiment; several papers are available in literature, see, for example, [18–21]. GINGER

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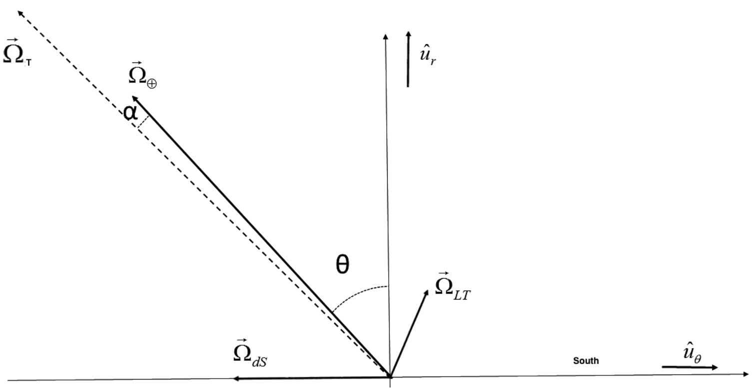

Fig. 1. The three axial vectors $\vec{\Omega}_\oplus$, $\vec{\Omega}_{\rm LT}$, and $\vec{\Omega}_{\rm dS}$ are shown, with the relative orientation at the latitude of the underground laboratory of Gran Sasso (LNGS), following General Relativity. The angle α and Ω_T (dashed line) are shown in the picture. The graph is not to scale and it gives a pictorial view of the relative orientations of the different components. In reality, the modulus of $\vec{\Omega}_\oplus$ is 9 orders of magnitude bigger than the GR terms, and the angle α is of the order of $\sim 3.5 \cdot 10^{-10}$ rad at the latitude of 45°.

with a better accuracy goal will be able to improve the tests and to distinguish the models further. This issue is crucial for a deep understanding of gravity at fundamental level. In fact, the various approaches to gravitational interaction strictly rely on the identification of the true physical variables that can be metric (*i.e.* gravitational potential) and/or affine connections (*i.e.* gravitational force). In general, theories of gravity can be classified as metric, metric-affine and purely affine theories [22,23]. Measuring Lense-Thirring effects, as well as gravito-magnetic effects could have a crucial role in this debate and, in general, to establish if the causal structure (metric) and the geodesic structure (affine connections) are coinciding or not, as assumed in the Einstein Equivalence Principle (EEP). This is nothing but an assumption working at the classical level. Its validity at fundamental and quantum levels is today questionable from several viewpoints (see [24,25] for details).

GINGER would provide *the first measurement* of a GR dynamic effect of the gravitational field on the Earth surface (not considering the gravitational redshift, test of the EEP effect) [26–28]. Though not in free fall condition, it would be a direct local measurement, independent of the global distribution of the gravitational field and not an average value, as in the case of space experiments.

The effect of the gravito-magnetic (Lense-Thirring, LT) and gravito-electric (de Sitter, dS) fields is an angular velocity, $\Omega_{\rm LT}$ and $\Omega_{\rm dS}$, that combines with $\Omega_\oplus$, the Earth angular velocity. GR is extremely predictive, allowing to compute the orientation and the amplitude of $\Omega_{\rm LT}$ and $\Omega_{\rm dS}$, as a function of the latitude of the observer. Note also that several alternative theories of gravitation predict different dependences on latitude; so that a direct observation would, perhaps, discriminate between different theories [16]. Figure 1 shows the mutual orientation of the various vectors for an assumed location of $\simeq 45°$ latitude (as, *e.g.*, at the GranSasso laboratories, LNGS).

An octahedral configuration [7] has been initially considered for GINGER; the GR terms in this scheme would be evaluated by subtracting the kinetic Earth rotation rate, independently measured by the international system IERS (International Earth Rotation System Service) ($\Omega_{\rm IERS}$). To make the 1% test, the octahedral setup requires long-term stability of the apparatus, and a very high accuracy of the scale factor of the instrument (1 part in 10^{12}). The error in the measurement of $\Omega_{\rm IERS}$ at present is $\sim$ 10–15 μs in one day of measurement, this is not a systematic error, this error goes down for longer time measurement, allowing the Lense-Thirring test at 1%.

The complete list of relationships between different RL arrays and GR can be found in the recent paper [6], where the GR terms are expressed using the ratio of Earth's Schwarzschild radius to Earth's radius and the dimensionless moment of inertia of the Earth, called, respectively, a and b. The present paper describes the simplest array that could measure the GR terms; the analysis is done in the most general way in terms of an additional angular rotation vector, ω, which is the sum of the GR terms ($\Omega_{\rm LT}$ and $\Omega_{\rm dS}$). Since the GR terms and the Earth angular velocity are contained inside the meridian plane, the minimum array is composed of 2 RLs with area versors contained in this plane. The analysis is dedicated to the particular choice of one RL, $\rm RL_1$, at maximum signal and the other, $\rm RL_2$, horizontally oriented (vertical area versor). The paper gives a general overview of the mathematical description in a general form, assuming one very large angular velocity ($\Omega_\oplus$) summed to the smaller one (ω). Comparing the measured frequencies, proportional to the total angular rotation Ω_T, with the independently measured Earth angular rotation $\Omega_{\rm IERS}$, it is possible in general to measure the extra term ω with an amplitude below of 9–12 orders of magnitude. It is also shown that it is possible to reconstruct the angle between the horizontal RL and the angular rotation vector with very high precision. This is a major point: each measurement is affected by changes in the orientation of the area versor, the higher the sensitivity the higher the precision required in the relative angles: for example, $2 \cdot 10^{-9}$ rad error in orientation is equivalent to a $\sim 10^{-13}\,\frac{\rm rad}{\rm s}$ error in the measured angular velocity. In this way, the array forms

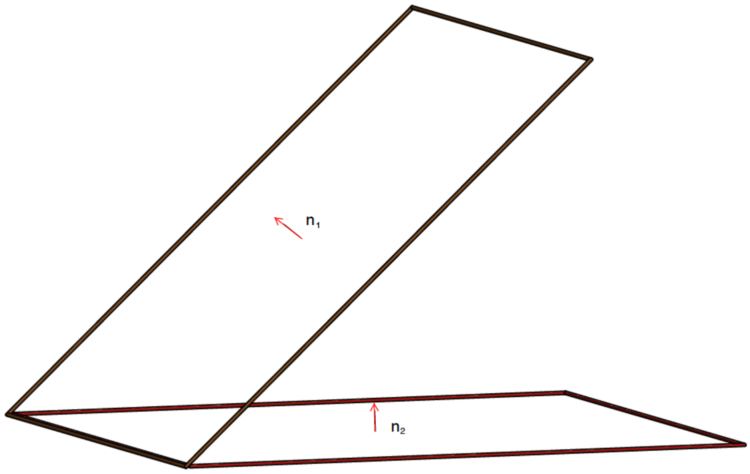

Fig. 2. Sketch of the two RLs. RL_1 is inclined at the maximum signal (latitude 45°); RL_2 is horizontal. This choice allows to have the two versors inside the meridian plane.

a suitable local reference system in the meridian plane, and it is necessary to add the independent measurement of the angle between RL_1 and RL_2, which is certainly feasible. To have the RL_2 horizontal, *i.e.* $\hat{n}_2$ vertically oriented, has two advantages: it guarantees that the two area versors are inside the meridian plane and that the angles with RL_1 is close to the local colatitude. In fig. 2 the two RLs are sketched.

The reference system, in general, is local, but it is important to say that the light beams coming out of each RL follow the orientation of the rings; the comparison of those beams with some local horizontal or vertical reference can allow the determination, with some error, of the absolute orientation.

Comparing the measured frequencies, proportional to the total angular rotation Ω_T, with the independently measured Earth angular rotation $\Omega_{\rm IERS}$, it is in general possible to measure the extra term, ω, with an amplitude below by 9–12 orders of magnitude.

The analysis shows that in this way the projection of ω along the axis of Ω_T is measured. Each RL provides an independent measurement, the comparison of the two results is a suitable check of the test. It is shown that the other component of ω, the one perpendicular to the axis of rotation, can be determined by comparing the orientation with suitable local reference; the GR test necessary precision is rather high, 10^{-9}–10^{-12} rad, and it is not clear, at present, whether such high precision is feasible. The experimental specifications necessary to reach the 1% of the Lense-Thirring are discussed in details for a two-RLs array, each with a perimeter of 28 m, installed inside the Node B of LNGS; the analysis is completed with Monte Carlo study, which compares the precision of the measurement with the experimental specifications.

2 Ring lasers for retrieving a general rotation vector, and evaluation of ζ

The beat frequency f of the RL is proportional to the flux of the total rotation vector $\vec{\Omega}_T$ across the area of the ring. In general we may write

$$f = S\vec{\Omega}_T \cdot \hat{n}, \tag{1}$$

where $\hat{n}$ is the unit vector perpendicular to the plane of the ring, and $S = \frac{4A}{\lambda P}$ is the scale factor of the RL, A and P are the area and perimeter of the ring cavity.

Taking into account the fact that $\Omega_\oplus$ and the GR terms are contained inside the meridian plane, it is possible to restrict the analysis to the case of only two RLs, arranged in such a way that these versors $\hat{n}$ and the $\vec{\Omega}_T$ are all contained in one plane, and the problem becomes bi-dimensional; calling the two, RL_2 and RL_1, making the scalar products explicit, we may write

$$\begin{aligned} f_1 &= S_1\Omega_T\cos(\gamma-\zeta)\\ f_2 &= S_2\Omega_T\cos\zeta. \end{aligned} \tag{2}$$

As shown in fig. 3, the area versors of the two RLs are $\hat{n}_2$ and $\hat{n}_1$, ζ is the angle between $\vec{\Omega}_T$ and $\hat{n}_2$, and γ is the angle between $\hat{n}_2$ and $\hat{n}_1$, which can be independently measured. The two indices can be interchanged. Using eq. (2),

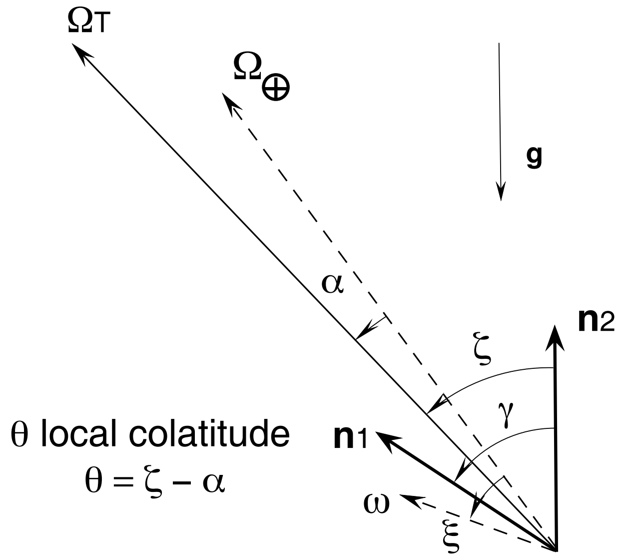

Fig. 3. Pictorial view of the geometrical quantities: RLs versors, the angular rotation vectors, and the main angles. The total angular rotation vector, $\vec{\Omega}_T$, is the sum of two different vectors $\vec{\Omega}_\oplus$, the Earth angular rotation, and $\vec{\omega}$. The angles are positive with the convention expressed by the small arrows, and the RL_2 is horizontal ($\hat{n}_2$ is vertical). The versors of the two RLs, $\hat{n}_1$ and $\hat{n}_2$ are shown with thicker lines, the main angles ζ, γ, α and ξ are shown, the angle α is the angle between $\vec{\Omega}_T$ and $\vec{\Omega}_\oplus$, while ξ is the angle between $\vec{\omega}$ and $\vec{\Omega}_\oplus$.

the quantities Ω_T (the magnitude of $\vec{\Omega}_T$), and ζ can be retrieved

$$\Omega_T = \frac{\sqrt{f_2^2 S_1^2 - 2f_1 f_2 S_1 S_2 \cos(\gamma) + f_1^2 S_2^2}}{S_1 S_2 \sin(\gamma)}$$
$$\zeta = \tan^{-1}\left(\frac{f_1 S_2 - f_2 S_1 \cos(\gamma)}{f_2 S_1 \sin(\gamma)}\right), \tag{3}$$

or, equivalently, as $\zeta = \cos^{-1}(\frac{f_2 S_1 \sin(\gamma)}{\sqrt{(f_2^2 S_1^2 - 2f_1 f_2 S_1 S_2 \cos(\gamma) + f_1^2 S_2^2)}})$. Each scale factor depends on the geometry of the RL; devices with equal scale factors ($S_2 = S_1 = S$) to a certain extent are feasible, the equations simplify if the RL has equal scale factors ($S_2 = S_1 = S$), accordingly,

$$\zeta = \tan^{-1}\frac{f_1 - f_2\cos(\gamma)}{f_2\sin(\gamma)}$$
$$\Omega_T = \frac{\sqrt{f_1^2 + f_2^2 - 2f_1 f_2 \cos(\gamma)}}{S\sin(\gamma)}. \tag{4}$$

The combination of the frequencies of the two RLs determines the measurement of the amplitude of Ω_T, and, if the angle γ between the two RLs is independently measured, it is possible to measure ζ, the angle between the versor $\hat{n}_2$ and the axis of the vector $\vec{\Omega}_T$ ($\gamma - \zeta$ being the angle between $\hat{n}_1$ and $\vec{\Omega}_T$, which in the following will be called ζ_1).

It is extremely important that in this two-RL system it is possible to determine the angle ζ of one of the two RLs with respect to $\vec{\Omega}_T$. It is necessary to estimate the error in the measurement of ζ, which depends on the independent measurement of γ and the indetermination of the frequencies, *i.e.* the shot noise. The error $\delta\zeta_\gamma$ of ζ is proportional to the error $\delta\gamma$ of the measurement of γ,

$$\delta\zeta_\gamma = \frac{f_2(f_2 - f_1\cos(\gamma))}{f_2^2 - 2f_2 f_1\cos(\gamma) + f_1^2} \times \delta\gamma. \tag{5}$$

In general the term multiplying $\delta\gamma$ on the RHS of eq. (5) is not small, and the error in the measured ζ depends on $\delta\gamma$, *i.e.* the error in the measurement of the relative angle between RL_1 and RL_2.

Other noises will affect the evaluation of ζ. The output frequencies of the RLs are shot noise limited, accordingly, assuming that the different contributions are independent, we have

$$\delta\zeta \simeq \sqrt{(\delta\zeta_\gamma)^2 + (\delta\zeta_{f2})^2 + (\delta\zeta_{f1})^2} \tag{6}$$

$$\delta\zeta_{f2} \simeq \frac{f_1 \sin(\gamma)}{-2f_2 f_1 \cos(\gamma) + f_2^2 + f_1^2} \times \delta f_{sn}\,,$$
$$\delta\zeta_{f1} \simeq \frac{f_2 \sin(\gamma)}{-2f_2 f_1 \cos(\gamma) + f_2^2 + f_1^2} \times \delta f_{sn}\,, \tag{7}$$

where δf_{sn} is the shot noise, expressed in frequency; this noise contribution will be equal for both RLs, since they are considered equal in geometry, power, mirrors, etc.

Let us consider the special case in which RL_1 is closely aligned with the total axis of rotation ($\vec{\Omega}_T$); in this case the angle $\zeta_1 \ll 1$, assuming that γ is known with the error $\delta\gamma$ ($\zeta_1 = \gamma + \delta\gamma - \zeta$); substituting in eq. (2), it is possible to show that the error $\delta\zeta$ depends at first order on the product $\zeta_1 \cdot \delta\gamma$,

$$\delta\zeta_\gamma \simeq \cot(\zeta) \cdot \zeta_1 \times \delta\gamma, \tag{8}$$

assuming that $\cot\zeta \leq 2$ ($20° \leq \zeta \geq 140°$). It is straightforward to note that the error is depressed by the value of ζ_1: improving the alignment of RL_1 with the axis of rotation, the error $\delta\zeta_\gamma$ decreases. For example, assuming $\zeta_1 \sim 10^{-6}$ rad, and $\delta\gamma \sim 10^{-6}$ rad, we have $\delta\zeta_\gamma \sim 10^{-12}$ rad. For example, if RL_2 is horizontally aligned with a precision of 10^{-6} rad with respect to the local vertical, and $\zeta_1 \leq 10^{-6}$ rad, it is possible to say that the angle γ is equal to the colatitude with an error $\delta\gamma \simeq 10^{-6}$ rad, and it is not necessary to directly measure the angle γ.

Since a lot of disturbances will affect the apparatus, the orientation of RL_2 will change with time, ζ will change accordingly (*i.e.* RL_2 is a reference system which is not stable with respect to the local reference); it is impossible to distinguish between changes in the orientation of RL_2, or changes of $\vec{\Omega}_T$, but the relative angle is always determined. The absolute orientation of RL_1 does not play a significative role as long as ζ_1 is sufficiently small; in the discussion of the experimental specifications the maximum allowed ζ_1 will be evaluated.

The combination of two RLs with one at maximum signal is very meaningful, as it allows very high precision in the determination of the angle ζ, and the measurement of Ω_T; see eq. (2).

3 Ring lasers attached to the Earth and the composition of different angular velocities

Our main interest is in the RLs system attached to the Earth crust: the mirrors will follow the crust motion and RLs will be sensitive to $\vec{\Omega}_\oplus$, the Earth rotation rate, plus other terms, for example, the GR corrections, tides and terms originated by global or local phenomena. In the present analysis we will consider that all the contributions different from the GR corrections are negligible or off-line subtracted. Let us call $\vec{\omega}$ the sum of all the GR terms affecting the apparatus, so that $\vec{\Omega}_T = \vec{\Omega}_\oplus + \vec{\omega}$. Compared to $\vec{\Omega}_\oplus$ it is clear that $\vec{\Omega}_T$ will be different in amplitude and direction, α being the angle between the two. Using the relation between angles shown in fig. 3, and applying the distributive property of the scalar product, eqs. (2) become:

$$f_1 = S \times (\Omega_\oplus \cos(\gamma - \zeta + \alpha) + \omega \cos(\xi - (\gamma - \zeta + \alpha)))$$
$$f_2 = S \times (\Omega_\oplus \cos(\zeta - \alpha) + \omega \cos(\xi + \zeta - \alpha))\,, \tag{9}$$

where ξ is the angle between $\vec{\Omega}_\oplus$ and $\vec{\omega}$. It is necessary to note that if RL_2 is horizontal, the colatitude θ is $\theta = \zeta - \alpha$.

The RLs, at present operational, have shown that $\vec{\Omega}_\oplus$ is the dominant term by several orders of magnitude; accordingly, it is convenient to write the amplitude of $\vec{\omega}$ as $\omega = \eta\Omega_\oplus$, $\eta \ll 1$. Two different cases will be discussed in the following: the angle ξ between $\vec{\Omega}_\oplus$ and $\vec{\omega}$ is known, and the more general one, in which $\vec{\omega}$ has two components parallel and perpendicular to $\Omega_\oplus$, respectively, $\omega\eta_\parallel$, and $\omega\eta_\perp$. We will always consider that RL_1 is aligned with $\vec{\Omega}_T$ with the angle ζ_1 (angle between $\hat{n}_1$ and $\vec{\Omega}_T$), such that $\zeta_1 \ll 1$ and $\zeta_1 = \gamma - \zeta$. Substituting in eqs. (9) we obtain:

$$f_1 = S\Omega_\oplus \times (\cos(\zeta_1 + \alpha) + \eta \cos(\xi - (\zeta_1 + \alpha)))$$
$$f_2 = S\Omega_\oplus \times (\cos(\zeta - \alpha) + \eta \cos(\xi + \zeta - \alpha))\,. \tag{10}$$

It is possible to evaluate α calculating the derivative in ζ of the generic frequency f, imposing it equal to zero for $\zeta = 0$ (which, by definition, is the position of the maximum), and solving for α, at the first order in η we find:

$$\alpha = \eta \sin(\xi). \tag{11}$$

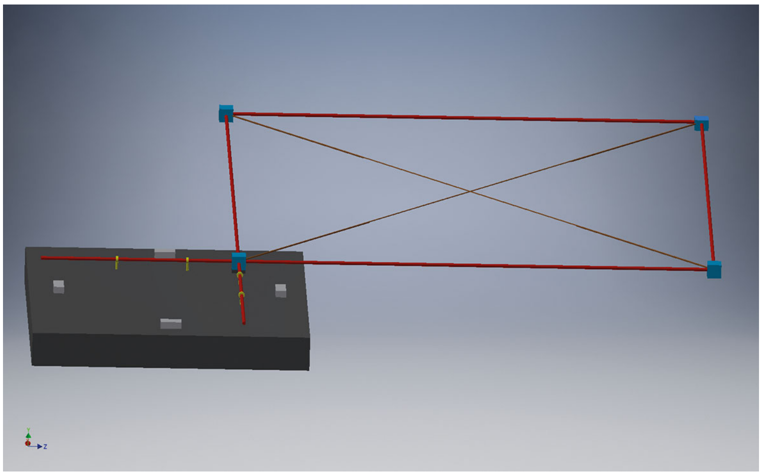

Fig. 4. RL_2 with orientation measured by tilt meters. The output light beams are passing through reference holes rigidly attached to a reference table. High-sensitivity tilt meters are positioned on top of the reference table. In this way, the RL_2 orientation is closely related to the reference table, tilt meters, allowing the monitoring of the orientation changes of RL_2 with respect to the horizontal plane. The 4 grey boxes indicate the 4 tilt meters.

Developing at the first order in α, ζ_1 and η we may write:

$$\begin{aligned} f_1 &= S\Omega_\oplus(1+\eta\cos\xi) \\ f_2 &= S\Omega_\oplus\cos\zeta(1+\eta\cos\xi). \end{aligned} \tag{12}$$

Both the above relations allow to measure η ($\cos\xi \neq 0$, and $\cos\zeta \neq 0$):

$$\begin{aligned} \eta &= \frac{f_1 - S\Omega_\oplus}{S\Omega_\oplus\cos\xi} \\ \eta &= \frac{f_2 - S\Omega_\oplus\cos\zeta}{S\Omega_\oplus\cos\xi\cos\zeta}. \end{aligned} \tag{13}$$

The two solutions of eq. (13) are equivalent, each RL can determine η. This is a consequence of the fact that ξ is considered as known. Accordingly, the comparison of the two measurements will provide a check of ξ (which is considered as known in GR [6], as explained later) and it can be used to verify whether the measurement is affected by systematics of the ring lasers.

Let us consider a more general case, in which ξ is not known; in this case, since $\vec{\omega}$ is a vector, two components for $\vec{\eta}$ are required: $\eta_\perp$ and $\eta_\parallel$, perpendicular and parallel to $\vec{\Omega}_\oplus$, respectively. From fig. 3, it is straightforward to show that $\eta_\perp \simeq \alpha$. In general, α ($\eta_\perp$) cannot be determined with the first-order expansion. Since RL_2 is horizontally oriented, $\hat{n}_2$ is vertical and $\alpha = \zeta - \theta$, where θ is the local colatitude. It is straightforward to say that, in order to measure α, ζ and θ should be measured with an error smaller than α, and RL_2 should be horizontal with and error below α. This last requirement is not really necessary, *i.e.* it is not strictly necessary to have RL_2 horizontal, it would be more appropriate to say that the error with respect to the horizontal direction, δ_{ref}, of RL_2 should be measured with respect to the reference frame of the Earth, which is feasible in practice utilising top-sensitivity tilt meters. Tilt meters are commonly installed on top of our prototypes, they are used to follow the tilts of the monument, but in order to effectively determine δ_{ref}, it is necessary to develop reference structures with reference points in order to link the light beams exiting the RL_2 and the tilt meter. If the square ring geometry is utilised, it is possible to utilise the two diagonals of the square, which are resonant Febry-Perot cavities, as shown in figs. 4 and 5 below.

The total error $\delta\alpha$ of this measurement is the incoherent sum of the error of the measurement of $\delta\zeta$, plus the error δ_{ref}, and the error $\delta\theta$. Considering that α for GR is of the order of $3.5\cdot 10^{-10}$ rad at the latitude of 45°, the 1% test requires each error to be $\sim 10^{-12}$ rad. All the above said can be easily done with micro-radian accuracy: it is necessary to investigate if 10^{-10}–10^{-12} rad is feasible with the present limits on $\delta\theta$, state-of-the-art high-sensitivity tilt meters [29] and the precision reachable in the manufacturing of reference bench.

All the above discussed is very general, it can be used for GR test only if GR terms are present, but it holds also in the case in which the additional term is unknown. It is possible to distinguish two different cases: $\vec{\omega}$ (η) is a function of time, it is a perturbation with limited duration (AC), or it is constant with time (DC). In the AC case, the information before and after the event allows determination of $\vec{\omega}$, since $S\Omega_\oplus$ can be evaluated by the averaged value before and after; in this case $\eta_\perp$ is measured by the variation of ζ, and $\eta_\parallel$ by the first eq. (13), assuming $\cos\xi$

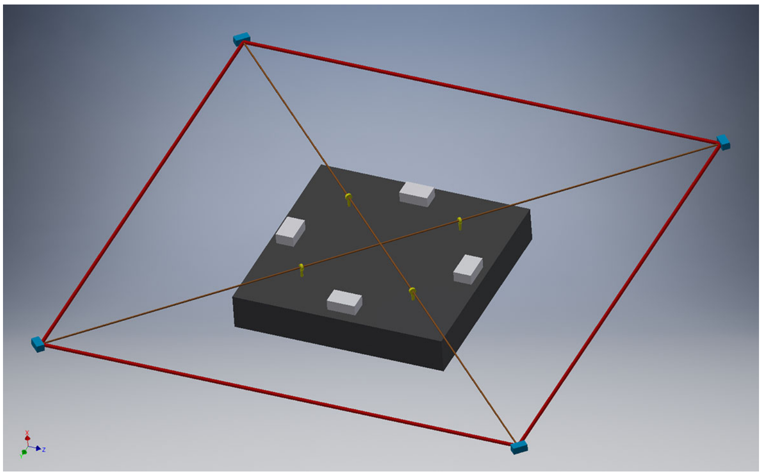

Fig. 5. RL_2 with orientation controlled by tilt meters. The geometry of RL_2 is controlled by controlling the length of the two diagonals. The injected light is passing through reference holes rigidly attached to a reference table. High-sensitivity tilt meters are positioned on top of the reference table. In this way the RL orientation is closely related to the reference table, which is monitored by the tilt meters, allowing the monitoring of the orientation changes of RL_2 with respect to the horizontal plane. The 4 grey boxes indicate the 4 tilt meters.

as known. In the AC case, it not required to have $\Omega_L = \Omega_\oplus$. The DC case is extremely challenging, since the General Relativity effects are DC terms. In the following, the experimental requirements for GR test will be discussed.

3.1 The RL signal expressed with the GR terms

The GR components depend on the ratio of the Schwarzschild radius to the radius of Earth (parameter $a \equiv \frac{2GM_\oplus}{R_\oplus c^2} \simeq 1.3918082245(20) \cdot 10^{-9}$) and on its dimensionless moment of inertia (parameter $b \equiv \frac{GI_\oplus}{R_\oplus^3 c^2} \simeq 2.301326(700) \cdot 10^{-10}$); accordingly, each RL output frequency f is expressed as a function of a, and b as

$$f = S\Omega_\oplus |\cos(\beta) - (a-b)\sin\theta\sin(\beta-\theta) + 2b\cos\theta\cos(\beta-\theta)|, \tag{14}$$

where the absolute value bars | have been introduced just to remember that the frequency is always a positive quantity; β is the angle between $\vec{n}$ and $\vec{\Omega}_\oplus$, $\beta = \zeta - \alpha$ following fig. 3. The angle α is defined in GR [6] and is $\alpha = (a - 3 \cdot b)\cos\theta\sin\theta$, considering that $\alpha = \eta_\perp$, and, evaluating $\eta_\parallel$ from eq. (14), it is possible to write

$$\begin{aligned} \eta_\parallel &= (a-b)\sin^2\theta + 2b\cos^2\theta \\ \eta_\perp &= (a - 3\cdot b)\cos\theta\sin\theta, \end{aligned} \tag{15}$$

so, it is possible to evaluate a and b, if $\eta_\perp$ and $\eta_\parallel$ are evaluated from the experiment, obtaining

$$\begin{aligned} a &= \frac{(3\eta_\parallel\sin(2\theta) + 3\eta_\perp\cos(2\theta) + \eta_\perp)}{4\sin\theta\cos\theta} \\ b &= \frac{1}{2}(\eta_\parallel - \eta_\perp\tan(\theta)). \end{aligned} \tag{16}$$

From the numerical value of a and b, it is possible to show that the GR terms give $\eta_\parallel \sim 8.1 \cdot 10^{-10}$ and $\eta_\perp \sim -3.5 \cdot 10^{-10}$, or equivalently $\eta \sim 8.82 \cdot 10^{-10}$ and $\xi \sim 23.37°$. If the only LT effect is taken into account, we obtain $\eta_\parallel \sim 1.15 \cdot 10^{-10}$ and $\eta_\perp \sim 3.45 \cdot 10^{-10}$, or equivalently $\eta \sim 3.64 \cdot 10^{-10}$ and $\xi \sim 71.56°$. The procedure is valid if only the LT test is required; in this case the de Sitter term, which is in principle well known [10], should be summed to the measurement of $\Omega_\oplus$ done by IERS.

4 Error budget and specifications of the experimental apparatus

In the following the experimental requirements, *i.e.* the errors in the measurement of η, will be discussed. The analysis will be restricted to the 2-RLs system with one at maximum signal, and systematics of the ring lasers will not be considered. Considering that each measured frequency is shot noise limited, the general formulas (12) become

$$
\begin{aligned}
f_1 &= S\Omega_\oplus(1+\eta\cos\xi) + S\omega_{sn} \\
f_2 &= S\Omega_\oplus\cos\zeta(1+\eta\cos\xi) + S\omega_{sn},
\end{aligned}
\tag{17}
$$

where ω_{sn} is the shot noise contribution (expressed in frequency $\delta f_{sn} = S\omega_{sn}$); ω_{sn} is a white noise, with rms value expressed in $\frac{\text{rad}}{\text{s}}$,

$$
\omega_{sn} = \frac{c}{2LQ}\sqrt{\frac{h\nu}{P_{\text{out}}t}}\,,
\tag{18}
$$

where c is the speed of light, ν the frequency of the light, h the Plank constant, P_{out} the output power, t the integration time of the measurement, L the side of the RL, and Q the quality factor. Q depends on the shape of the ring and on the optical quality of the mirrors; for triangular and square rings we have

$$
Q = \frac{2\pi\nu L}{c\cdot\text{Losses}}\,,
\tag{19}
$$

where Losses indicates the total losses of each mirror; the mirrors are considered equal, the transmission is equal to the scattered light, and negligible absorptions.

The shot noise is only one of the contributions to the noise; in the following the experimental specifications will be investigated. It is necessary to evaluate the influence of δS, the error in the accuracy of the scale factor S, $\delta\Omega_\oplus$, the error in the measurement of $\Omega_\oplus$ (which should be independently done), and $\delta\zeta$, the error in the measurement of ζ, which depends on the shot noise and on ζ_1 and $\delta\gamma$, see eqs. (6) and (7). Evaluating the variations in the general equation, $f = f_2 = S\Omega_\oplus(\eta\cos\xi\cos\zeta + \cos\zeta) + S\omega_{sn}$, the errors are

$$
\delta f_{\delta S} = \Omega_\oplus\cos\zeta(1+\eta\cos\xi)\times\delta S
\tag{20}
$$

$$
\delta f_{\delta\Omega_\oplus} = S\cos\zeta(1+\eta\cos\xi)\times\delta\Omega_\oplus
\tag{21}
$$

$$
\delta f_{\delta\zeta} = 2S\Omega_\oplus(1+\eta\cos\xi)\sin\zeta\times\delta\zeta.
\tag{22}
$$

Please note that $\delta f_{\delta S}$ and $\delta f_{\delta\Omega}$ are higher at $\zeta = 0$, while the contribution of $\delta f_{\delta\zeta}$ is zero at $\zeta = 0$ and higher at $\pi/2$. In short, the RL at maximum signal is not affected by angular variations, but it is more sensitive to variations of the scale factor S and of $\Omega_\oplus$.

For a given $\eta\cos\xi$ ($\eta_\parallel$), imposing $\frac{\omega_{sn}}{\Omega_\oplus} \leq \rho\eta$ (ρ is the test relative precision, $\rho = 1\%$ is the aim of GINGER), the size of the apparatus and the duration of the measurement can be determined.

As far as the evaluation of η is concerned, see eq. (13), there are no advantages in using one or the other RL; the requirements for δS and the error in $\delta\Omega$ are the same: in the general case, it is straightforward to show that the relative variations of the errors, $\delta\eta_{\delta\Omega}$, $\delta\eta_{\delta S}$ and $\delta\eta_{\delta\zeta}$, should all be below the quantity $\rho\eta$.

The condition to be inside the meridian plane has to be taken into account. RL_1 is at maximum with an angle ζ_1 with variations $\Delta\zeta_1$. In order to be indistinguishable from the one at maximum signal, it is necessary to have $2\zeta_1\Delta\zeta_1 < \rho\eta_\parallel$. The angle ϕ is the misalignment of $\hat{n}_1$ and $\hat{n}_2$, ϕ and its variations $\Delta\phi$ should be $2\phi\Delta\phi \leq \rho\eta_\parallel$.

In the paper, S_1 and S_2 have been considered equal, in practice they will have a small difference, which can be expressed with the parameter ϵ ($S_1 = S$ and $S_2 = (1+\epsilon)S$). In general the requirement for the parameter ϵ is

$$
\epsilon < \rho\eta_\parallel.
\tag{23}
$$

If the requirement on accuracy is fulfilled it is not necessary to have equal scale factors, then this is enough to reconstruct the angle ζ. S depends basically on the geometry, the quality of the mirrors, the pressure and composition of the gas mixture, and the plasma. Suitable control strategy for the geometry of the square ring laser has been studied and it is at present under study [30,31], two colocated devices will have equal gas mixture and pressure, each mirror can be characterised and they can be interchanged, accordingly the realisation of two RLs with equal scale factors is feasible.

In order to estimate the requirements for the Lense-Thirring test at $\rho = 1\%$, we have studied the specifications for $\eta_\parallel \sim 10^{-10}$ and $\eta_\perp \sim 3.510^{-10}$ (LT test). Table 1 summarises the main experimental specifications as a function of the relative precision of the GR test (in order to barely provide the evidence of GR corrections, or to provide a 10% or 1% test).

In the calculation equal scale factors have been chosen; this is not strictly necessary if the scale factors are known with high accuracy, and it is important to remind that to have equal scale factors is a less stringent requirement compared with high accuracy, and the measurement of ζ requires to have equal scale factors only. However the most difficult part of this test is the accuracy requirement, particularly the scale factor, since the measurement of $\Omega_\oplus$ done by IERS is adequate for that purpose for long-time measurements. It has been shown that the RL G of the Geodetic

Table 1. Experimental requirements.

$\eta_{\parallel} \sim 10^{-10}$	$\sim$	10%	1% test
Accuracy of S	10^{-10}	10^{-11}	10^{-12}
Error of $\Omega_{\oplus}$	10^{-10}	10^{-11}	10^{-12}
Alignment with the meridian plane (rad)	10^{-5}	3×10^{-5}	10^{-6}
Indetermination of ζ_1, alignment at the maximum signal (rad)	10^{-5}	3×10^{-5}	10^{-6}
Precision of γ, RL horizontal(rad)	10^{-5}	10^{-6}	10^{-7}
ϵ inequality of scale factors, relative	10^{-10}	10^{-11}	10^{-12}
Error of θ colatitue (rad)	10^{-4}	10^{-5}	10^{-6}
Angle $\alpha \sim 3.6 \cdot 10^{-10}$ ($\eta_{\perp}$)	$\sim$	10%	1% test
Error of β (rad)	10^{-10}	10^{-11}	10^{-12}
Error of ζ (rad)	10^{-10}	10^{-11}	10^{-12}
δ_{ref} (rad)	10^{-10}	10^{-11}	10^{-12}
Error of θ colatitude (rad)	10^{-10}	10^{-11}	10^{-12}

observatory of Wettzell is accurate to 1 part in 10^8 [32]; the main limitations come from the index of refraction of the gas mixture, the plasma, and the effective propagation of the photons inside the mirrors; the proposed test requires an improvement of a factor 100–1000; it is important to remember that there are not fundamental limitations, the accuracy of G is based on models. Since there are not fundamental limitations, the scale factor accuracy would improve by measuring the index of the refraction of the gas mixture, by characterising each single mirror, and the plasma with dedicated test benches.

As already said, the test could be done measuring $\eta_{\perp}$, which is feasible if the angle β can be independently measured. In this case very high accuracy for S and the measurement of $\Omega_{\oplus}$ is not necessary, but it is necessary to develop a suitable experimental apparatus in order to find the orientation of RL_2 with respect to the local reference frame, and link it to the Earth reference frame, see figs. 4 and 5. However it is important to say that so far this kind of upgradings of RL has never been studied or proposed.

4.1 Reconstruction of η and ζ

The final result contains the contribution of the whole set of errors in the various parameters. We have controlled with a Monte Carlo simulation the consistency of all the specification listed in table 1 with the model. The data set of the Monte Carlo simulation, the frequencies f_1 and f_2, have been generated with the following procedure:

- the frequencies have been evaluated using eq. (2);
- RL_1 is aligned at maximum signal with the angle $\zeta_1 = 1\,\mu$rad;
- RL_2 is the horizontal RL ($\hat{n}_2$ vertical);
- all versors are inside the meridian plane with and angle $\phi = 1\,\mu$rad;
- RLs are squared rings with a 7 m side;
- the wavelength of the light is 633 nm.

Each second the following variations are included:

- the shot noise, δf_{sn} (expressed as Hz), is added independently to each frequency, it is extracted from a normal distribution with standard deviation $\sigma = S\omega_{sn}$ (see eq. (18)),
- the 4 mirrors are identical, Losses = 2 ppm, and $P_{\text{out}} = 500$ nW;
- each scale factor has been variated independently with $\sigma = 10^{-12}$ (normally distributed);
- extra noise with $\sigma = 1\,\mu$rad is added to ζ each second.

η is evaluated using relationships (13) using the following:

- the angle ζ is evaluated using the ratio of $\frac{f_2}{f_1}$;
- each day a different value of $\Omega_{\oplus}$ is used, its noise is extracted from a normal distribution with $\sigma = 10\,\mu$s;
- the scale factor S is variated with respect to the one used for the generation of the frequencies, using normal distribution with $\sigma = 10^{-12}$.

It has been checked that the average value of the reconstructed η reaches the 1% level by using 500 day statistics. It has been checked that the error of the evaluated ζ is shot noise limited down to the pico-rad level.

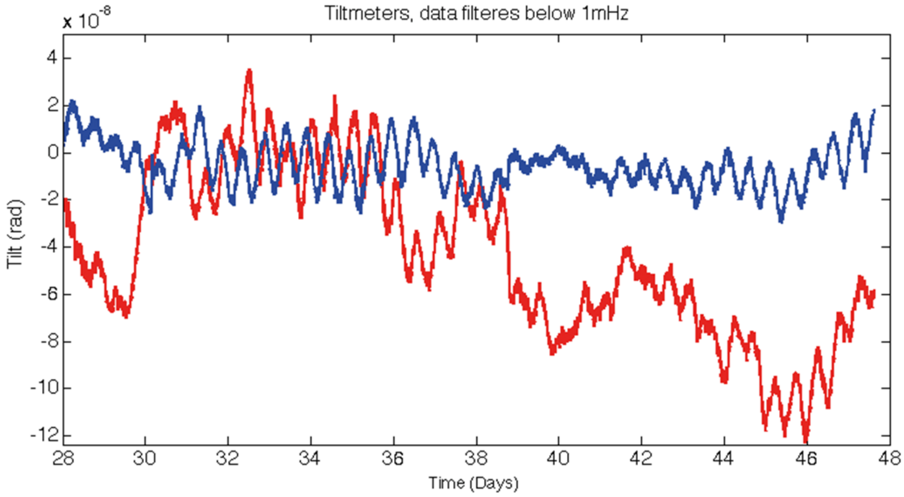

Fig. 6. Long-term stability of the monument of GINGERino. The effect of thermal drift of the room has been subtracted with a fit.

4.2 Alignment with the meridian plane and at maximum Sagnac frequency

RL_1 must be oriented as much as possible close to the maximum signal, *i.e.* its area versor should have $\vec{\Omega}_T$ and angle ζ_1 small enough. It has been shown that the 1% LT test requires $\zeta_1 \sim \mu$rad. It is necessary to find the position of the mirrors of RL with this precision, and, since the measurement requires quite a long time, the orientation of the monument which supports the apparatus must be stable at a level such that the variations of ζ_1 remain lower than μrad. From our measurement at LNGS, it has been shown that the monument of GINGERino is stable at a better level than tens of nano-rad. One two-channel high-sensitivity tilt meter is located on top of the monument. Figure 6 shows the inclination of the monument of GINGERino during several days, measured by the tilt meter; the room of GINGERino has drifts in temperature of fractions of degrees, which affect the monument and the tilt meter. In the measurement, the effect of the temperature has been subtracted with a fit using the temperature probe installed on top of the monument. It must be taken into account that the tilt meter is affected by random walk for time of measurements longer than a few days. For this reason this measurement must be considered an upper limit, the real stability of the monument should be better.

The hetero-lithic RL has mirrors movable with piezoelectric transducers, which can be equipped with encoders in order to know the position of each mirror with respect to the monument; precision of fractions of microns, or even nano meter, is feasible. The alignment can be done moving the mirrors and looking at the measured angle ζ or maximising the Sagnac frequency. It is necessary to find a suitable procedure to effectively find the right orientation: the apparatus cannot be constructed with such high mechanical precision. To that purpose, if RL_2 is horizontally oriented with μrad precision, it could be enough to optimise the measured angle ζ and bring it close to the local colatitude moving the mirrors with the piezoelectric transducers. There are other ways to align RL_1. The Sagnac frequency itself can be maximised; in this case, it is necessary to know the minimum angle that can be discriminated considering that the frequency resolution is limited by the shot noise. Let us consider a RL with side L; the precision in the variation $\Delta\zeta_1$ of ζ_1 depends on the variation that can be discriminated from the shot noise keeping the integration time T ($\omega_{sn}(T)$) fixed. For example, integrating the signal for one day, in a device like G, which has obtained 10^{-13} rad/s in one day, $\Delta\zeta_1 = 50\,\mu$rad can be obtained. In order to reach the $1\,\mu$rad precision it is necessary to increase the output power or the size of the instrument: for example, with $0.5\,\mu$W of power (at present G output power is $\sim 10\,$nW), and a device with size 7 or 10 m, the angular resolution of $\Delta\zeta_1 \sim 2.5\,\mu$rad or $\Delta\zeta_1 \sim 1\,\mu$rad can be respectively obtained.

The stroke of piezoelectric transducers are usually of the order of 0.1 mm; a rough alignment system is required, able to cover the distance between the stroke of the piezoes and the limitation imposed by the construction. Since it is not possible to cover large distances by steps of few microns, at the beginning a guess of the optimal position can be found comparing the response of RL in the extreme positions of the rough alignment system. Let us assume RL is being built with $\zeta_1 \sim 10\,$mrad (1 cm error in the position of the mirrors from the optimal one); the Sagnac frequency changes are or the order of 5 part 10^5, a quantity that can be determined in less than one hour measurement. The Earth angular rotation rate is perfectly stable at this level, it is modulated by daily and sub-daily polar motions, the Earth axis vibrates in the North-South direction in the meridian plane by tens of nano-rad, with a very well-known frequency, those signals can be easily handled at this level of precision. In summary: a RL of side larger than 7 m with output power of $0.5\,\mu$W can be aligned close to $\zeta_1 \sim 2$–$3\,\mu$rad, better precision requires larger RL. The long-time stability of the orientation of the monument at LNGS is at least by a factor 10 better than μrad.

5 Comparing installations at different latitudes

The array composed of 2 RLs at maximum signal and located at two separated latitudes has been discussed [6]. Here the main considerations on the noise of the measurement are reported; for simplicity, the two scale factors are considered to be equal. In the paper, it is shown that the test of the Lense-Thirring effect consists in measuring with the RLs frequency the Schwarzschild radius and the momentum of inertia of the Earth, called, respectively, a and b. Combining the two measured frequencies f_1, at coatitude θ_1, and f_2, at colatitude θ_2, a and b can be measured:

$$a = \frac{f_1 - f_2 - 3\cos(2\theta_1)(f_2 - S\Omega_\oplus) + 3\cos(2\theta_2)(f_1 - S\Omega_\oplus)}{2S\Omega_\oplus(\cos(2\theta_2) - \cos(2\theta_1))} \tag{24}$$

$$b = \frac{\sin^2(\theta_1)(f_2 - S\Omega_\oplus) + \sin^2(\theta_1)(S\Omega_\oplus - f_1)}{S\Omega_\oplus(\cos(2\theta_2) - \cos(2\theta_1))}. \tag{25}$$

It is straightforward to show that the above measurement requires an accuracy of 1 part in 10^{12} of the scale factor S, while the colatitudes, θ_1 and θ_2, do not require too stringent requirements. From these two measurements it is possible to build the combination $(a - 3 \cdot b)$:

$$(a - 3 \cdot b) = \frac{2f_2 - 2f_1}{S\Omega_\oplus(\cos(2\theta_1) - \cos(2\theta_2))}. \tag{26}$$

This combination does not require a high accuracy for the scale factors, since the product $S \times \Omega_\oplus$ acts as a multiplication factor.

Since the proposed 2-RL array provides the measurement of the angle ζ, all said above for the two RLs at the maximum, can be generalised to any RL. To have equal RL simplifies the problem, but it is not strictly necessary. It is important to say that the test can be considered complete if both a and b are measured, and for that a high accuracy of 1 in 10^{12} for the scale factor is required.

6 Discussion and conclusion

In a recent paper [6] the use of a ring laser to test the Lense-Thirring effect has been discussed in detail, providing a full list of the relationships between the physics and the instrument, using one or more RLs either co-located or at different latitudes. In this paper, we investigate the experimental requirements focusing the discussion on the properties of the array composed of 2 co-located RLs, one aligned at the maximum signal (RL_1) and the other horizontal (RL_2). The GR test requires to align the area versors inside the meridian plane. It is shown that the output frequencies of both RLs can measure the component of the LT parallel to the Earth axis. Each of the two devices provides the same measurement, see eq. (13), giving the possibility to check whether other disturbances, generated from the environment or the systematics of the laser as the nul-shift, are present. The experimental requirements for the LT test can be divided into 3 steps with improving precision: some first evidence $\sim$, 10% and 1%. It is shown that this array can measure the absolute angle ζ between RL_2 and Ω_T with high precision. The procedure to align the RL_1 at maximum signal is discussed. The requirement for RL_2 is to be horizontal with an error of μrad, which is certainly feasible with present tilt meters. Recent measurements taken at the GINGERino installation [4] show that the underground location is stable at a level below fractions of μrad, giving the indication that LNGS is a suitable location. It is shown that using the output frequencies of RLs the component of ω parallel to $\Omega_\oplus$ is measured; however, improvement of at least 2 orders of magnitude in the accuracy of the scale factors is required. The duration of the measurement depends on the size of the rings; 1% can be obtained in less than 2 years, with square rings with a 7 m side, top-quality mirrors and 500 nW of output power. The experimental apparatus is rather simple, it is mainly composed of two RLs, and it does not require a large space; several arrays could be located at different latitudes, giving the possibility to investigate the response of the RL as a function of the latitude.

It is shown that the component of ω perpendicular to $\Omega_\oplus$ can be measured if the orientation of RL_2 can be measured in the Earth reference frame; in this case the scale factor accuracy is not necessary and the measurement is based on the confrontation of the angles and the local latitude. This is, in principle, feasible using the light coming out of the RL or the diagonals, if the RL is based on a squared ring. Further investigation is necessary to study the real feasibility of this measurement and its limits.

The above discussion can be generalized to the problem of detecting small variation of the local angular rotation rate, not only GR effects, but also geophysical and geodetic signals. As discussed in the introduction, these tiny effects could be extremely important to discriminate among theories of gravity because these could contribute in discriminating between geodesic and metric sracture. The 2-RLs array is the basic element, adding one or two RLs with different orientations it is possible to extend to 3D; in this case, it is necessary to measure the relative angles between RLs, and the use of external metrology to well reconstruct the geometry of the apparatus becomes necessary.

Two separated arrays at different latitudes can make the test; the interesting fact is that, in this case, high accuracy of the scale factor is not strictly necessary. To have several devices allows the confrontation of different results, which can be used to validate the test.

It is important to remark that the ring lasers at maximum signal provide fast variations of the Earth rotation rate, and the LOD, while the combination of the two rings could give information on the angular variation of the horizontal RL_2 with respect to the Earth axis.

We are thankful to Angelo Tartaglia for his continuous assistance and suggestions, to Salvatore Capozziello and Matteo Luca Ruggiero for stimulating discussions. A special thanks goes to Antonello Ortolan for having recognised from the beginning the necessity of accuracy.

References

1. G. Sagnac, D.R. Acad. Sci. Paris **157**, 708710 (1913).
2. Gauguet Alexandre (Editor), C. R. Phys. **15**, 787 (2014).
3. U. Schreiber, J.P. Wells, Rev. Sci. Instrum. **84**, 041101 (2013).
4. J. Belfi, N. Beverini, Filippo Bosi *et al.*, Rev. Sci. Instrum. **88**, 034502 (2017).
5. A. Di Virgilio *et al.*, C. R. Phys. **15**, 866 (2014).
6. A. Tartaglia, A. Di Virgilio, J. Belfi, N. Beverini, M.L. Ruggiero, Eur. Phys. J. Plus **132**, 73 (2017).
7. F. Bosi *et al.*, Phys. Rev. D **84**, 122002 (2011).
8. A. Di Virgilio *et al.*, Int. J. Mod. Phys. D **19**, 2331 (2010).
9. I. Ciufolini *et al.*, Eur. Phys. J. C **76**, 120 (2016).
10. C.W.F. Everitt, D.B. DeBra, B.W. Parkinson *et al.*, Phys. Rev. Lett. **106**, 221101 (2011).
11. D. Lucchesi *et al.*, Class. Quantum Grav. **32**, 155012 (2015).
12. S. Alexander, N. Yunes, Phys. Rev. Lett. **99**, 241101 (2007).
13. Daiki Kikuchi, Kei Yamada, Hideki Asada, *Gyroscope precession for gravity Probe B by Chern-Simons gravity*, Hirosaki University report (February 9, 2015).
14. Wei-Ping Pan, E. Koptelova, Ruolan Jin, Wei-Tou Ni, *Report on fitting the parameters of Chern-Simons theory from the GP-B data*, National Tsing Hua University Report (March 26, 2015).
15. S. Capozziello, G. Lambiase, M. Sakellariadou, A. Stabile, Phys. Rev. D **91**, 044012 (2015) arXiv:1410.8316 [gr-qc].
16. S. Capozziello, M. De Laurentis, Phys. Rep. **509**, 167 (2011) arXiv:1108.6266 [gr-qc].
17. M.L. Ruggiero, Galaxies **2015**, 84 (2015).
18. N. Radicella, G. Lambiase, L. Parisi, G. Vilasi, JCAP **12**, 014 (2014) arXiv:1408.1247 [gr-qc].
19. Q.G. Bailey, V.A. Kostelecky, Phys. Rev. D **74**, 045001 (2006) gr-qc/0603030.
20. Q.G. Bailey, Phys. Rev. D **82**, 065012 (2010) arXiv:1005.1435 [gr-qc].
21. D. Kikuchi, N. Omoto, K. Yamada, H. Asada, Phys. Rev. D **90**, 064036 (2014).
22. S. Capozziello, M. Francaviglia, Gen. Relativ. Gravit. **40**, 357 (2008) arXiv:0706.1146 [astro-ph].
23. Y.F. Cai, S. Capozziello, M. De Laurentis, E.N. Saridakis, Rep. Prog. Phys. **79**, 106901 (2016) arXiv:1511.07586 [gr-qc].
24. B. Altschul *et al.*, Adv. Space Res. **55**, 501 (2015) arXiv:1404.4307 [gr-qc].
25. V. Faraon, S. Capozziello, *Beyond Einstein Gravity*, in *Fundamental Theories of Physics*, Vol. **170** (Springer Netherlands, 2011) DOI: 10.1007/978-94-007-0165-6.
26. C.M. Will, Living Rev. Relativ. **17**, 4 (2014).
27. W.-T. Ni, Int. J. Mod. Phys. D **25**, 1630003 (2016).
28. C. Laemmerzahl, A. Di Virgilio, Int. J. Mod. Phys. D **25**, 1630022 (2016).
29. Jan Harms, Krishna Venkateswara, Class. Quantum Grav. **33**, 234001 (2016).
30. R. Santagata, Class. Quantum Grav. **32**, 055013 (2015).
31. J. Belfi *et al.*, Class. Quantum Grav. **31**, 225003 (2014).
32. R. Hurst, M. Mayerbacher, A. Gebauer, K. Schreiber, J. Wells, Appl. Opt. **56**, 1124 (2017).