Temporal and Transverse Coherence of Self-Amplified Spontaneous Emission [*]

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Abstract. We review the coherence properties of the self-amplified spontaneous emission (SASE). Temporally, SASE is similar to the spontaneous undulator radiation except that the spectral bandwidth is about ten times narrower compared with typical undulator radiation. The situation is quite different in the transverse dimension, where SASE is fully coherent.

INTRODUCTION

Several laboratories are pursuing the R and D of the linac-based light source based on the SASE principle [1], [2] for its promise of extreme high brightness and time resolution [3]. It therefore is important to clearly understand the coherence properties of SASE, in particular, compared to the current generation of synchrotron radiation sources. This paper is a review of this topic.

In the temporal (longitudinal) dimension, the first order coherence of SASE sources is improved about ten times compared with typical undulator sources in current generation synchrotron radiation facilities. The second order coherence has to do with the intensity fluctuation, and SASE in this regard is very similar to undulator radiation, consisting of a random superposition of wave packets. The fluctuation property of such light, known as thermal or chaotic light, is well-known [4]: the fluctuation of intensity at a given time or frequency is 100%. However, the fluctuation for an integrated intensity is smaller by a factor $\sqrt{m_c}$, where m_c is the number of modes in the integration interval. The probability distribution of the integrated intensity is given by

1

the Gamma probability distribution. The fluctuation properties of undulator radiation was discussed in [5], [6], and of SASE in [7], [8], [9].

In the transverse dimension, SASE is a linear superposition of a set of transverse eigenmodes, each having a different growth rate [10]. If the growth rate of one or more higher order modes were the same as the fundamental mode (degenerate), then SASE would be partially coherent transversely. However, it turned out that the higher order modes have significantly smaller growth rates than the fundamental mode [11]. Therefore, the transverse behavior of SASE is determined entirely by the fundamental mode, and SASE becomes fully coherent transversely. This situation is markedly different from the usual undulator radiation which is partially coherent [12], the degree of coherence being determined by the ratio of the coherent phase space area $\lambda/2$ and the electron beam phase space area ($2\pi \times$ emittance).

TEMPORAL COHERENCE

The temporal coherence characteristics of self-amplified spontaneous emission (SASE) is based on the 1-D analysis of Maxwell-Vlasov Equations [13], [14]. In the frequency domain, the evolution of the electric field in the exponential gain regime is given by

$$E_\omega (z) = G_\omega (z) \ E_\omega (0) \quad . \tag{1}$$

Here $E_\omega(0)$ is the initial amplitude

$$E_\omega (0) = \sum_{i=1}^{N_e} e^{-i2\pi\omega t_i^0} \quad , \tag{2}$$

where ct_i^0 is the initial coordinate of ith electron with respect to the bunch center, and N_e is the number of the electrons. The gain function $G_\omega (z)$, neglecting electron beam energy spread, is given by

$$G_\omega (z) = \frac{1}{3} e^{\sqrt{3}\rho k_u z} \ e^{-i\left(\frac{2}{3}\frac{\Delta\omega}{\omega_0} + \rho\right) k_u z} \ e^{-\frac{(\Delta\omega)^2}{4\sigma_\omega^2}\left(1+\frac{i}{\sqrt{3}}\right)} \tag{3}$$

Here ρ is the FEL scale parameter [1], $k_u = 2\pi/\lambda_u$, λ_u = undulator period length, z = distance along the undulator, $\Delta\omega = \omega - \omega_0$, ω_0 = central frequency, and σ_ω is the gain bandwidth

$$\sigma_\omega = \omega_0 \sqrt{\frac{9\rho}{2\pi\sqrt{3}z/\lambda_u}} \approx \omega_0 \sqrt{\frac{\rho}{z/\lambda_u}} \quad . \tag{4}$$

The field in the time domain is obtained by the Fourier transform:

$$E\left(z,t\right) = \frac{1}{\sqrt{2\pi}} \int d\omega E_\omega\left(z\right) e^{i\omega\left(t\ -\ z/c\right)}$$

$$= \frac{1}{9\sqrt{z}} e^{\sqrt{3}\rho k_u z} \sum_{i=1}^{N_e} e^{i\omega_0\left(t-z/c\left(1+\rho\Delta\beta\right)-t_i\right)}$$

$$\times\ e^{-\frac{\left(t-\frac{z}{c}\left(1+\frac{2}{3}\Delta\beta\right)-t_i^0\right)^2}{4\sigma_\tau^2}\left(1-\frac{i}{\sqrt{3}}\right)} \tag{5}$$

Here $\Delta\beta = 1 - \beta$, β = electrons average longitudinal speed, and

$$\sigma_\tau = \frac{1}{2\sigma_\omega} \approx \frac{1}{2\omega_0}\sqrt{\frac{z/\lambda_\mu}{\rho}} \tag{6}$$

is the coherence length. Equation (5) describes a sum of N_e wave packets of rms pulse length σ_τ, propagating with a group velocity $v_g = c/\left(1 + \frac{2}{3}\Delta\beta\right)$, slower by $\frac{2c}{3}\Delta\beta$ than the speed of light in a vacuum [7]. As it propagates, the wave amplitude grows exponentially. The wave packets are randomly distributed with relative positions ct_i^0. The time domain picture was emphasized by Bonifacio, et al [7].

For our purpose here, the relevant features of the results in the above are the coherence length of the individual wave packet, which characterizes the first order coherence, and the random distribution of the center of the wave packets, which refers to the second order coherence.

The coherence length increases as given by Equation (6), or equivalently, the radiation bandwidth decreases, as given by Equation (4). The saturation takes place at about $z/\lambda_u = 1/\rho$, so that the bandwidth of SASE is about $\sigma_\omega/\omega_0 \approx \rho$. Since ρ is typically about 10^{-3}, the SASE bandwidth is about ten times narrower than the typical bandwidth of undulator radiation in current synchrotron radiation sources.

The random distribution of wave packets of SASE is similar to the undulator radiation. Figure 1 shows an example of random distribution of wave packets. This kind of light wave is very common in nature and is referred to as"chaotic light"; almost all light encountered in daily life, such as sunlight, as well as synchrotron radiation, is this kind. The properties of chaotic light have been discussed extensively in the literature, for example in [4]. The following is a brief summary of these results as applied to undulator radiation [5] [6] and SASE [7], [8], [9].

A simplified model of chaotic light (observed at a fixed position) can be represented in the time domain as a super position of Gaussian pulses

$$E\left(t\right) = E_0 \sum_{i=1}^{E_e} e^{-\frac{\left(t-t_i\right)^2}{4\sigma_\tau^2}\ -\ i\omega_0\left(t-t_i\right)}\ , \tag{7}$$

or in the frequency domain (In fact, the undulator radiation is a truncated sinusoidal pulse. However, the Gaussian pulse is easier to analyze.)

FIGURE 1. An example of chaotic light given by a random superposition of 100 sinusoidal wave packets each with six periods long. The total length of the pulse is T.

$$E\left(\omega\right) = E_0 \sum_{i=1}^{N_e} e^{-\frac{(\omega-\omega_0)^2}{4\sigma_\omega^2} - i\omega t} \quad . \tag{8}$$

Figure 2 shows an example of explicitly adding wave packets with $\lambda = 1$, $\sigma_\tau = 2$ ($\sigma_\omega = 0.25$), $N_e = 100$, assuming that t_i's are randomly distributed with equal probability in a bunch length $T = 100$. The remarkable feature of this figure is that the resultant wave is a relatively regular oscillation interrupted only a few times, much less than one might have naïvely expected from the fact that it is a superposition of 100 waves. In fact, the number of the regular regions has nothing to do with the number of wave packets. Each regular region is a coherent mode, the length of which is the coherent length. Therefore, the number of the regular region is the number of the coherent modes, which is the ratio of the bunch length to the coherence length,

$$m_c = T/2\sqrt{\pi}\sigma_\tau \approx T/4\sigma_\tau \tag{9}$$

The situation in the frequency domain is similar. Figure 3 shows the intensity spectrum $dn/d\omega \propto |E\left(\omega\right)|^2$ (n = number of photons), with $E(\omega)$ computed by Equation 8 with the same wave parameters as in Figure 2. The spectrum consists of sharp peaks of width $\Delta\omega_p \sim 2/T$ randomly distributed within the radiation bandwidth $\sigma_\omega \sim 1/2\sigma_\tau$. Thus the number of the spectral peaks is the same as the number of the coherent modes in the time domain.

The intensity fluctuation described here is a simple consequence of the central limit theorem: for a random superposition of a large number of waves, the probability distribution of the field amplitude in either frequency domain or time domain is given by a Gaussian function. This implies that the intensity distribution is given by the inverse exponential function with a variance the same as the average.

In the above, we have discussed the case in which the radiation is observed within a narrow interval. If the whole pulse is integrated, the fluctuation will be smoothed out, and the variance reduced roughly by $\sqrt{m_c}$. The probability distribution in this case is the so-called gamma probability distribution [4].
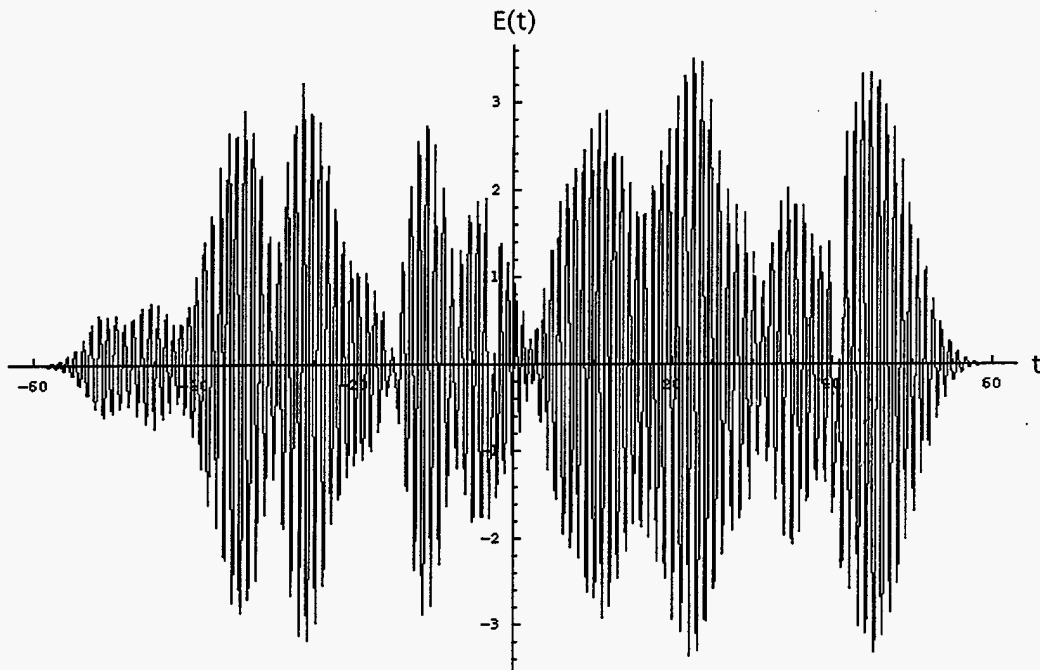
FIGURE 2. The resultant wave of a random superposition of Gaussian wave packets as in Eq. (8). The parameters of each wave packet are $\lambda = 1$, $\sigma_\tau = 1/2\sigma_\omega = 2$. A total of 100 wave packets are distributed randomly with uniform probability in a length of 100.

This statement can be generalized for the partial intensity integrated over a finite frequency interval $\Delta\omega$. Thus, let us consider

$$n_\Delta\left(\omega\right) = \int_{\omega-\Delta\omega/2}^{\omega+\Delta\omega/2} d\omega' \; dP/d\omega' \quad . \tag{10}$$

As long as $\Delta\omega > \Delta\omega_p$, we define the number of the coherent modes within the integration (observation) interval
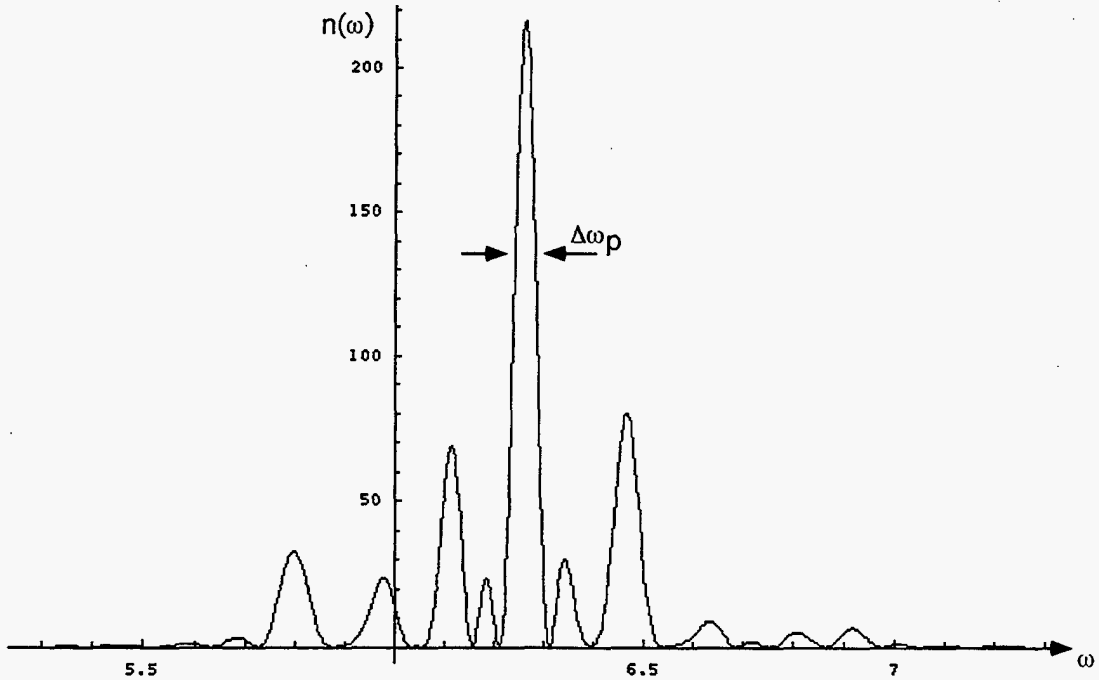
$$m_c = \Delta\omega/\Delta\omega_p \quad . \tag{11}$$

The probability distribution for $n_\Delta\left(\omega\right)$ is then given by the gamma distribution, peaked at $\langle n_\Delta \rangle$ with a variance $\sigma_{n_\Delta} = \sqrt{\langle n_\Delta^2 \rangle - \langle n_\Delta \rangle^2} \approx \frac{1}{\sqrt{m_c}} \langle n_\Delta \rangle$. Figure 4 illustrates the reduction of fluctuation for the integrated intensity.

Taking into account the quantum fluctuation of photons, we have

$$\sigma_{n_\Delta}{}^2 = \frac{\langle n_\Delta \rangle^2}{m_c} + \langle n_\Delta \rangle \quad . \tag{12}$$

The second term in Equation (12), due to the quantum fluctuation, can be written as

FIGURE 3. The intensity spectrum corresponding to Fig. (2). The spectrum consisits of sharp peaks of width $\Delta\omega_p \sim 1/T$, which are distributed within a Gaussian envelope or rms width σ_ω. The height of the spectral peaks fluctuates 100%.

$$\langle n_\Delta \rangle = \frac{\langle n_\Delta \rangle^2}{m_c \delta}, \quad \delta = \frac{\langle n_\Delta \rangle}{m_c} \quad . \tag{13}$$

Here δ is the number of photons per mode, known as the degeneracy parameter. Comparing Equation (13) with the first term of Equation (12), we find that the quantum fluctuation is negligible for $\delta \gg 1$. This is the case in SASE.

TRANSVERSE COHERENCE

Taking into account the 3-D effects, the field amplitude for SASE as a function of z can be written as

$$E_\omega(\mathbf{x}, \mathbf{z}) = \sum_n e^{\mu_n z} C_n(z) E_n(\mathbf{x}) + \text{continuum modes} \tag{14}$$

Here $\mathbf{x}$ is the transverse coordinate, and μ_n and E_n are the discrete solutions for the eigen value and the eigen mode of the Maxwell-Vlasov equations. An example of the intensity profiles for the fundamental and second order mode are shown in Figure 5 [11]. The simplest way to determine the expansion coefficient C_n seems to be the Van Kampen method [10], [15].

As z increases, the field amplitude is dominated by the fundamental mode with the largest growth rate, the real part of μ_1, if there is no degeneracy, i.e.,

if the growth rate of the higher order modes is sufficiently smaller than the fundamental.

The growth rates of the fundamental, and the first and second higher order modes obtained by solving the FEL eigenvalue equation [11] are shown as a function of the ratio of the 1-D gain length to the Rayleigh length in Fig. 6. All SASE projects currently under discussion are in the region far away from the degenerate limit.

The fact that SASE is dominated by a single transverse mode means that it is completely coherent transversely. This is in marked contrast to the spontaneous emission in which the coherence criteria is, under the optimal matching condition, determined by the ratio of the electron beam emittance to the coherent radiation emittance $(\lambda/4\pi)$ [12]. In the case of SASE, the modes are non-degenerate even if this ratio is large, and therefore is transversely coherent. For temporal coherence, the case of SASE was similar to the spontaneous emission. For transverse coherence, the SASE is almost always fully coherent.

The difference of the transverse coherence characteristics of SASE from those of the spontaneous emission may be understood as follows [16]. In the case of the spontaneous radiation the total radiation phase space area is the incoherent sum of the electron phase space area. In the case of an electron beam bunched by the FEL action, on the other hand, the radiation angular

$n_\Delta(\omega)$
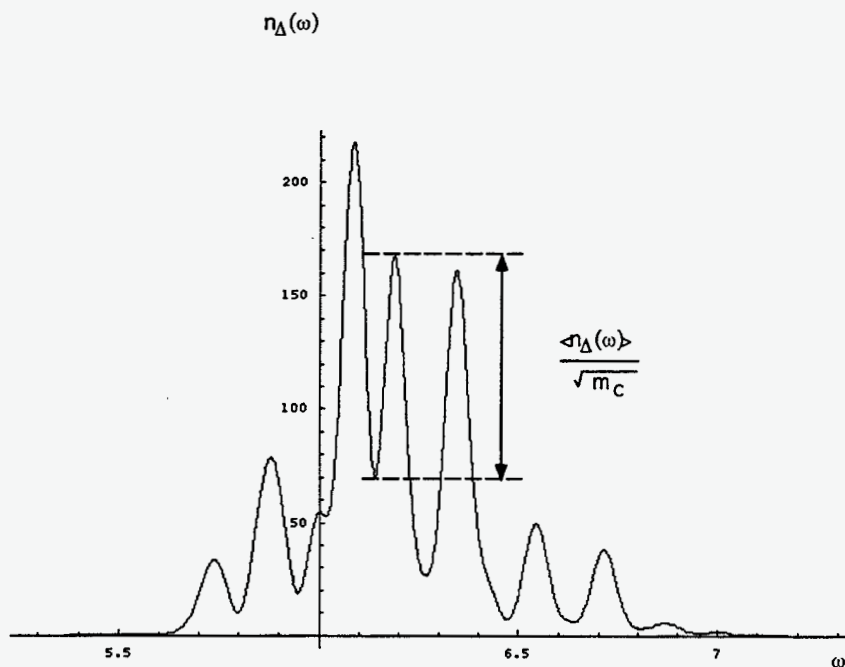
FIGURE 4. The spectrum of the integrated intensity defined by Eq. (10). Notice that the spectrum is more smooth, the fluctuation being reduced by $\sqrt{m_c}$.
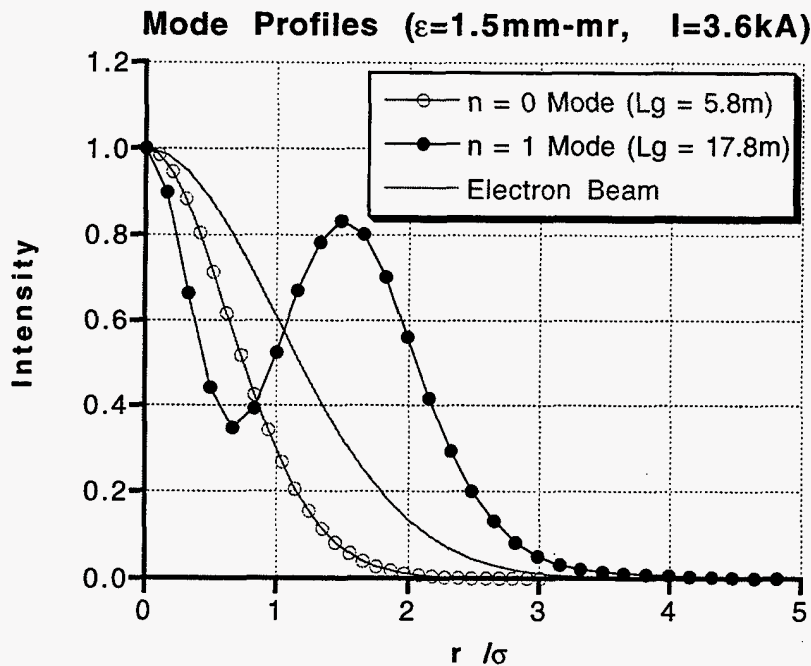
Mode Profiles (ε=1.5mm-mr, I=3.6kA)

FIGURE 5. An example of mode profiles. (Courtesy of M. Xie)

divergence is the diffraction angle determined by the electron beam transverse size, implying that the radiation is fully coherent transversely. Although the bunching in SASE is not complete, the SASE radiation is dominated by the bunched part, and is therefore transversely coherent.

QUANTUM EFFECTS IN SASE

So far we have treated electrons as classical. There are two corrections due to the quantum nature of electrons. The first is a correction on the gain, which is small if the recoil is small compared with the electrons' energy spread [17]:

$$\frac{\hbar\omega}{E_e/N_u} \approx \frac{\hbar\omega}{\Delta E_e} << 1 \quad . \tag{15}$$

Here N_u is the number of the undulator periods (which for SASE is about ρ^{-1}) and ΔE_e is the e-beam energy spread.

The second is the effect on the statistics, such as the effective input signal in SASE. The quantum correction on SASE noise was first considered in [18]. Continuing the analysis in that paper, and taking into account the correct multi-particle electron wave functions, it is found that the correction is small [19] when

Growth Rates of Transverse Modes
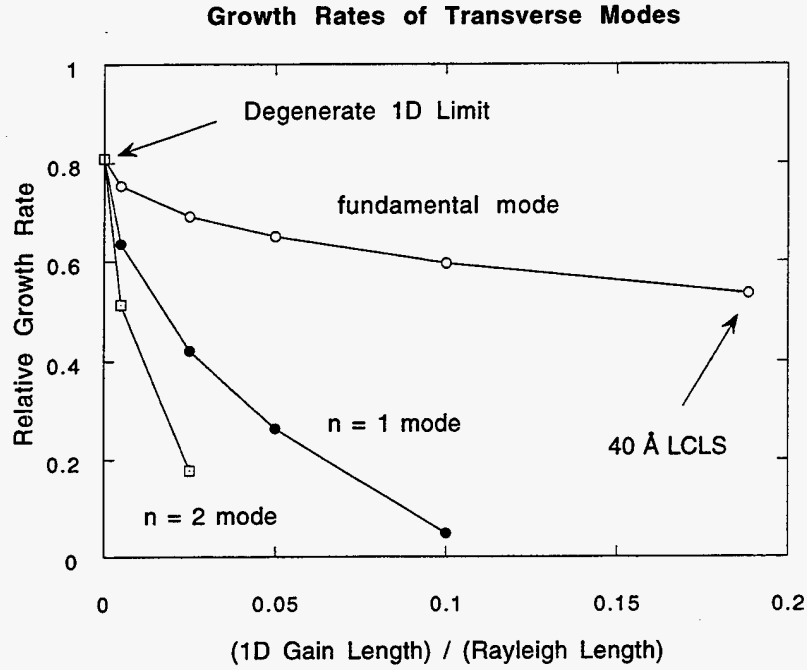
FIGURE 6. The growth rates for fundamental, and first and second higher modes, illustrating the dominance of the fundamental mode in most of the parameter space. (Courtesy of M. Xie)

$$\varepsilon_{\perp n}{}^2 \, \varepsilon_{\| \, n} < N_e (\lambda_c)^3 \quad . \tag{16}$$

Here $\varepsilon_{\perp n}$ is the normalized transverse emittance and $\varepsilon_{\| n}$ is the normalized longitudinal emittance ($\sigma_{\Delta\gamma} \, \sigma_z$). The inequality (16) could have been expected from quantum statistical mechanics.

In all cases of SASE under discussion at present times, the inequalities (15) and (16) are well satisfied.

We have remarked that quantum corrections in SASE are negligible. This may also be seen from the comparison of the effective noises in atomic lasers and in SASE. For this purpose, it is convenient to compare the brightness, i.e., photons per unit six dimensional phase space volume:

$$\frac{d^6 \, n_{ph}}{d^2 \mathrm{x} d^2 \phi \, (d\omega/\omega) \, dz} = B_{\text{noise}} \, e^{z/L_G} \quad . \tag{17}$$

Here L_G is one gain length.

For the atomic laser, it is well known that the effective noise is due to the vacuum fluctuation given by one photon per mode with phase space volume λ^3 [20]. Thus

$$B_{noise} \approx \frac{1}{\lambda^3} \quad . \tag{18}$$

The effective noise for SASE derived in Refs. [13] [14] [10] can be written approximately as

$$B_{noise} \cong \frac{\alpha K^2 N_G^e}{\lambda^3} \quad , \qquad (19)$$

where α = fine structure constant, K = the undulator parameter, N_G^e is the number of electrons in a distance λ (L_G/λ_u). The number of the noise photons per mode in the case of SASE is therefore $\alpha K^2 N_G^e$, which is much larger than one. This is because the start-up noise of SASE is the classical shot noise and not from quantum fluctuation.

ACKNOWLEDGMENTS

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Report Number (14) _LBNL--40672_

CONF-9706211--

Publ. Date (11) _199706_

Sponsor Code (18) _DOE/ER, XF_

UC Category (19) _UC-414, DOE/ER_

19980707 068

DOE