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Link to publication in Discovery Research Portal

Citation for published version (APA):
Bhattacharjee, A., Sun, Q., & Chadha, J. S. (2008). Productivity, Preferences and UIP deviations in an Open Economy Business Cycle Model. (SIRE Discussion Papers; No. 2008-53). St. Andrews: Scottish Institute for Research in Economics .

scottish institute for research in economics

SIRE DISCUSSION PAPER

SIRE-DP-2008-53

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Open Economy Business Cycle Model

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Productivity, Preferences and UIP deviations in an Open Economy Business Cycle Model[*]

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ABSTRACT

We show that a flex-price two-sector open economy DSGE model can explain the poor degree of international risk sharing and exchange rate disconnect. We use a suite of model evaluation measures and examine the role of (i) traded and non-traded sectors; (ii) financial market incompleteness; (iii) preference shocks; (iv) deviations from UIP condition for the exchange rates; and (v) creditor status in net foreign assets. We find that there is a good case for both traded and non-traded productivity shocks as well as UIP deviations in explaining the puzzles.

JEL Classification: E32; F32; F41.

Keywords: Current account dynamics, real exchange rates, incomplete markets, financial frictions.

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1. Introduction[1]

It is well documented that international risk sharing and the real exchange rate seem to divert far from the levels that would be associated with their complete market allocations. Many authors, originating with Backus and Smith (1993) and Backus, Kehoe and Kydland (1995),[2] have pointed to a lack of aggregate risk sharing across open economies and as an analogue many have also commented on the disconnect between the relative price of goods and their relative consumption, see, for example, Obstfeld and Rogoff (2000) for a summary. We concentrate on a flexible price solution to the problem in the vein on Baxter and Crucini (1995) and Stockman and Tesar (1995) but also allow for financial market imperfections, following Devereux and Engel (2002). We find, within the context of a new methodology for model evaluation of calibrated models, that a two-sector open economy replete with financial market imperfections and driven by productivity, preference and exchange rates that are allowed to deviate stochastically from UIP may provide a reasonably satisfactory contribution to the solution of these puzzles.

To understand the puzzles, we use a two-sector version of Chari, Kehoe and McGrattan (2002), developed by Benigno and Thoenissen (2008), in which there are infinitely-lived representative optimizing households, a two-sector production sector for traded and non-traded goods, where the law of one price holds but where there are also incomplete financial markets. As is well known, under a complete markets environment, cross-country holdings of assets should be sufficient to ensure that consumption rather than income is highly correlated in open economies and that relative consumption responds to changes in relative prices.[3] Because considerable evidence has suggested that international portfolios

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[2]Simply put the Backus-Kehoe-Kydland puzzle is that it is income rather than consumption that is more closely correlated across open economies, which suggests that payoffs from idiosyncratic foreign (domestic) income shocks are not being used to smooth domestic (foreign) consumption. The Backus-Smith puzzle is the analogous puzzle that relative consumption across open economies does not arbitrage relative price (real exchange rate) differences.

[3]Baxter and Jermann (1997) conclude, under a wealth holding model with a production sector, that domestic individuals should hold only foreign shares against loss caused for labour

are home-biased (Tesar and Werner, 1995) and imply that an important channel for risk sharing may be impeded, to some extent, a popular treatment is to introduce incomplete markets by assuming that portfolio diversification relies only on non-state contingent bonds, as in Kehoe and Perri (2002), and accordingly we adopt this feature.[4]

Full price flexibility is maintained in the model but real rigidities are present in the form of a home bias in both consumption and the use of both traded and non-traded goods in output. The model we adopt also allows for costly capital accumulation, an interest rate spread and the possibility of a country being a net creditor (or debtor).[5] The model is driven by three types of shocks: to both traded and non-traded sector productivity; to preferences in the allocation of time between work and leisure of the representative household, and by deviations of the exchange rate from the path expected by relative interest rates (see, Frankel, 1996, and Sarno and Taylor, 2002).

A further innovation of this paper is the development of summary statistics on the distance of each model simulation to the data in the sense of Geweke's (1999) 'weak' interface with the data. We define a model as a structural set of equations, which are parameterized, and simulated with forcing variables defined over a given variance-covariance matrix (VCM) of shocks. The model then produces an artificial economy which can be thought of as lying some distance from our systematic observations on real-world economies (Watson, 1993). In this sense, the open-economy puzzles drive a large wedge between theory and observation and so we construct a number of empirical measures of this wedge across models and choice of forcing variables to understand which models provide a more satisfactory resolution of the puzzles; see Bhattacharjee and Thoenissen (2007) for related work.

Our results suggest that some form of financial market incompleteness will probably be required to solve the open-economy puzzles (as suggested by Engel, 2000). A key result is that price stickiness may not necessarily be required to resolve the puzzles. It turns out that reasonable answers can be found with reference to traded and non-traded forcing processes and by allowing the exchange

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income by a domestic negative shock.

[4]Recently authors such as Sorensen *et al.* (2007) have documented a reduction in home bias but continue to draw a clear link between home bias and risk sharing. Our set-up is sufficiently flexible to allow us to alter the cost of borrowing from abroad.

[5]The importance of these creditor or debtor positions have been explored comprehensively by Lane and Milesi-Ferretti (2002).

rate to deviate from the UIP condition. In the former case, with a dominant role for traded over non-traded productivity shocks, in an incomplete financial market, domestic households raise consumption for traded and non-traded goods compared to overseas but the real exchange rate depreciates if the terms of trade effect outweighs the Harrod-Balassa-Samuelson effect (Corsetti *et al.*, 2004). In the case of preference (for work over leisure) shocks, the labor supply curve shifts out and hence demand for goods increases (Hall, 1997) but with an elastic investment supply schedule, and hence output, there is little response in the real exchange rate. And deviations from the uncovered interest rate parity equation for the exchange rate can operate to drive the exchange rate to appreciate even if domestic interest rates fall. Consumption increases in response to the fall in real rates and investment also increases, with wage growth attenuated by the exchange rate appreciation and this results in a reduction in net foreign assets (a current account deficit). Finally, it can also be shown that a combination of these shocks seems to explain the puzzles best.

1.1. Some simple observations

We examine open economy data from 24 OECD and emerging country economies. Figure 1 gives the descriptive statistics of HP filtered cyclical data and illustrates some clues that the behavior of the current account over the cycle is likely to help explain the puzzles. We note that (i) the real exchange rate is considerably more volatile than relative consumption; (ii) that relative output still seems more correlated than relative consumption; (iii) that current and trade account dynamics follow each other closely and (iv) that the current account is (mostly) countercyclical.

Figure 1 is set over four panels. The top left hand panel of Figure 1 shows the extent to which the real exchange rate seems noisy and significantly more volatile than its fundamentals would imply. The range for observed volatility of the real exchange rate is between 1-9, with an average, over this dataset of nearly 4. Researchers have explained this high volatility from many dimensions in the literature.[6] And certainly, we find that compared to relative consumption, which ranges from 0.5 to just under 3, the real exchange rate does look 'disconnected'. The top right hand side panel of Figure 1 scatters the correlation of national consumption of the economies with US consumption against the correlation of

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[6]These explanations include price stickiness and the famous case of exchange rate overshooting (Dornbusch, 1976).

output with US output and suggests in general that output is more closely related across countries than consumption, which implies somewhat less than perfect risk sharing.

The left hand lower panel of Figure 1 shows the close correspondence between the business cycle dynamics of the current account and the trade balance over the business cycle across these economies - suggesting a strong role for intertemporal trade over the business cycle with some deviation from complete markets as the balance on the trade account is not offset by returns from assets held overseas.[7]

Finally, the lower right hand side panel of Figure 1 suggests that the current account tends to be countercyclical (with a deficit under an economic expansion). But that the real exchange rate looks as likely to appreciate or depreciate over the same economic cycle. Put alternatively, there is a higher demand for foreign assets during an expansion (with current account output correlations negative) but that the real exchange rate plays a limited role in choking off that higher demand.

A second modelling question concerns whether price stickiness is required for the resolution of the puzzles. Figure 2 shows the forecast error correlation of up to 25 quarters of US and UK current account and real exchange rate and relative consumption and the real exchange rate (den Haan, 2000). The panels show that over the long run, these quantities are countercyclical but over the short term, all three measures somewhat less so. As price stickiness can be expected to play a less important role in long run dynamics, than in short run, there is some initial motivation for excluding this feature from our model.

The rest of the paper is organized as follows. Section 2 describes the model, section 3 outlines the solution technique and model calibration, section 4 offers the model results, section 5 compares the model to the data VCM and section 6 concludes. Appendices A and B offer more detail on model, shock selection and the evaluation methodology.

2. The Model

This section describes the baseline model. Essentially, we take the flexible price two-country, two sector model derived by Benigno and Thoenissen (2008) and

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[7]The finding that the current account is likely to play an important role in the resolution of puzzles has two implications for our work, we will want to adopt a model where current account dynamics play an important role and assess the fit of any models we develop with, inter alia, their match to current account data.

emphasize the specification of driving forces as in Chadha, Janssen and Nolan (2001). The model is driven variously by forcing variables in domestic and overseas traded and non-traded productivity shocks, domestic and overseas preference shocks and by deviations from the UIP condition for the exchange rate.

2.1. Consumer behavior

We adopt a two-country model. Consumers are infinitely lived. The world economy is populated by a continuum of agents on the interval $[0, 1]$, with the segment $[0, n)$ belonging to the country H (Home) and the population on segment $[n, 1]$ belonging to the F (Foreign) country. Preferences for the Home consumer (with an identical set-up for the foreign consumer) are described by the utility function:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s^j, \xi_{C,s}) V(l_s^j) \right], \tag{2.1}$$

where E_t denotes the expectation conditional on the information set at date t, and β is the intertemporal discount factor, with $0 < \beta < 1$. The Home consumer obtains utility from consumption, C^j, and receives disutility from supplying labor, l^j. $\xi_{C,s}$ is a stochastic disturbance affecting the utility the agent receives from a unit of consumption.

The asset market structure in the model is standard and is described in detail in Benigno (2001) and Benigno and Thoenissen (2008). Home individuals are able to trade two nominal bonds denominated in the domestic and foreign currency. The bonds are issued by residents in both countries in order to finance their consumption expenditure. Foreign residents, on the other hand, can allocate their wealth only in bonds denominated in the foreign currency. Home households face a cost when they take a position in the foreign bond market. As in Benigno (2001), this transaction cost depends on the net foreign asset position of the home economy.[8]

The Home consumer maximizes utility subject to the following budget constraint:

$$P_t C_t^j + \frac{B_{H,t}^j}{(1 + i_t)} + \frac{S_t B_{F,t}^j}{(1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)} = B_{H,t-1}^j + S_t B_{F,t-1}^j + P_t w_t l_t^j + \Pi_t^j \tag{2.2}$$

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[8]Alternative ways of closing open economy models are discussed in Schmitt-Grohe and Uribe (2003).

where P_t is the price index corresponding to the basket of final goods C, w is the real wage earned by agent in return for supplying labor and Π are dividends received by the agent from holding an equal share of the economy's intermediate goods producing firms.

Home agents can hold two types of nominal, non-state contingent bonds. B_H^j denotes agent j's holdings of Home-currency denominated bonds. The one-period return from these bonds is denoted by $(1 + i_t)$. S denotes the nominal exchange rate, defined as Home currency price of a unit of foreign currency. B_F^j denotes agent j's holdings of Foreign-currency denominated bonds. The one-period return from foreign-currency denominated bonds is $(1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)$, where $(1 + i_t^*)$ is the gross rate of return and $\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)$ is a proportional cost associated with foreign currency-denominated bond holding that depends on the economy-wide holdings of foreign-currency denominated bonds.[9]

The first order condition of the representative consumer can be summarized as follows:

$$U_{c,t} = (1 + i_t)\beta E_t\left[U_{c,t+1}\frac{P_t}{P_{t+1}}\right] \tag{2.3}$$

$$U_{c,t+1} = (1 + i_t^*)\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)\beta E_t\left[U_{c,t+1}\frac{S_{t+1}P_t}{S_t P_{t+1}}\right]. \tag{2.4}$$

$$U_{c,s} w_t = V_l(l_s) \tag{2.5}$$

where $U_{c,t} \equiv U_c(C_t, \xi_{C,t}, 1 - l_t)$ and where there is an analogous intertemporal condition to (2.3) for the Foreign consumer. As in Benigno (2001), we assume that all individuals belonging to the same country have the same level of initial wealth. This assumption, along with the fact that all individuals face the same labor demand and own an equal share of all firms, implies that within the same country all individuals face the same budget constraint and so they will choose identical paths for consumption. As a result, we are able to drop the j superscript and focus on a representative individual for each country.

2.2. The supply side

There are three layers of production in this economy. Final goods are produced by a competitive final goods producing sector using Home traded and non-traded

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[9] The factor of proportionality $\Theta\left(\frac{S_t B_{F,t}}{P_t}\right)$ is equal to unity only when economy-wide bond holdings are at their initial steady state level, thus ensuring that in the long-run the economy returns to its initial steady state level of bond holdings.

intermediate goods as well as foreign-produced traded intermediate-goods. Final goods are non-traded and are either consumed or used as investment goods to augment the domestic capital stock. Intermediate goods producers combine labor and capital according to a constant returns to scale production technology. Each country produces two types of intermediate goods, a differentiated traded good and a non-traded good.

2.2.1. Final good producers

Let Y be the output of final goods produced in the home country. Final goods producers combine domestic and foreign-produced intermediate goods to produce Y in a two-step process. The final good Y is made up of traded, y_T, and non-traded inputs, y_{NT}, combined in the following manner:

$$Y = \left[\omega^{\frac{1}{\kappa}} y_T^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} y_N^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{2.6}$$

where ω is the share of traded goods in the final good, and κ is the intratemporal elasticity of substitution between traded and non-traded intermediate goods. The traded component, y_T, is, in turn, produced using home and foreign-produced traded goods (y_H and y_F respectively) in the following manner:

$$y_T = \left[v^{\frac{1}{\theta}} y_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} y_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{2.7}$$

where v is the domestic share of home produced traded intermediate goods in total traded intermediate goods and θ is the elasticity of substitution between home and foreign-produced traded goods. Final goods producers are competitive and maximize profits, where P is the aggregate or sectoral price index and Y the aggregate output; therefore, they maximise the profits

$$\max_{y_N, y_H, y_F} PY - P_T y_T - P_N y_N, \tag{2.8}$$

subject to (2.7), where traded goods' output is maximized subject to the value of home and foreign traded goods.

This maximization yields the following input demand functions for the home

and foreign (not shown but identical) firm:

$$y_N = (1-\omega)\left(\frac{P_N}{P}\right)^{-\kappa} Y \tag{2.9}$$

$$y_H = \omega v \left(\frac{P_H}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\kappa} Y$$

$$y_F = \omega(1-v)\left(\frac{P_F}{P_T}\right)^{-\theta} \left(\frac{P_T}{P}\right)^{-\kappa} Y.$$

The price index that corresponds to the above maximization problem is:

$$P_T^{1-\theta} = [vP_H^{1-\theta} + (1-v)P_F^{1-\theta}] \tag{2.10}$$
$$P^{1-\kappa} = [\omega P_T^{1-\kappa} + (1-\omega)P_N^{1-\kappa}],$$

And the goods produced in the final goods sector are only used domestically, either for consumption or investment, x_t, for home and overseas:

$$Y_t = C_t + x_t. \tag{2.11}$$

2.2.2. Traded-intermediate goods sector

Firms in the traded intermediate goods sector produce goods using capital and labor services. The typical firm maximizes the following profit function:

$$\max P_{H_t} y_{H_t} + S_t P_{H_t}^* y_H^* - P_t w_t l_{H,t} - P_t x_{H,t}, \tag{2.12}$$

or because the law of one price holds at the wholesale level,

$$\max_{H_t} P_{H_t}\left(y_{H_t} + y_H^*\right) - P_t w_t l_{H,t} - P_t x_{H,t}.$$

This maximization is subject to:

$$y_{H_t} + y_{H_t}^* = F(k_{H,t-1}, l_{H,t}) = (A_t l_{H,t})^\alpha k_{H,t-1}^{1-\alpha} \tag{2.13}$$

$$k_{H,t} = (1-\delta)k_{H,t-1} + x_{H,t} - \phi\left(\frac{x_{Ht}}{k_{Ht-1}}\right) k_{Ht-1},$$

where $\phi(.)$ denotes the cost for installing investment goods.[10]

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[10]Following Benigno and Thoenissen (2008), in steady state: $\phi(.) = x/k, \phi'(.) = 1, \phi''(.) = b < 0$.

Then, the stochastic maximization problem of the domestic intermediate goods firm is given by:

$$L = \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{U_{c,t}}{P_t} \left\{ \begin{array}{c} \left[P_{H,t} \left(A_t l_t \right)^{\alpha} (k_{H,t-1})^{1-\alpha} - P_t w_t l_{H,t} - P_t x_{H,t} \right] \\ + \lambda_t \left[\begin{array}{c} (1-\delta) k_{H,t-1} + x_{H,t} \\ -\phi \left(\frac{x_{H,t}}{k_{Ht-1}} \right) k_{H,t-1} - k_{H,t} \end{array} \right] \end{array} \right\}. \quad (2.14)$$

The first order conditions with respect to the labor input, investment and capital

are given by:

$$P_t w_t = \alpha P_{H,t} (A_t)^{\alpha} \left(\frac{k_{H,t-1}}{l_{H,t}} \right)^{1-\alpha}, \quad (2.15)$$

$$P_t = \lambda_t - \phi' \left(\frac{x_{H,t}}{k_{H,t-1}} \right) \lambda_t, \quad (2.16)$$

$$\lambda_t = \mathrm{E}_t \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \left\{ \begin{array}{c} P_{H_{t+1}} (1-\alpha) \left(\frac{A_{t+1} l_{H,t+1}}{k_{Ht}} \right)^{\alpha} + \\ \lambda_{t+1} \left[(1-\delta) - \phi \left(\frac{x_{Ht+1}}{k_{H,t}} \right) + \phi' \left(\frac{x_{H,t+1}}{k_{H,t}} \right) \frac{x_{H,t+1}}{k_{H,t}} \right] \end{array} \right\}. \quad (2.17)$$

And using the expression for $P_{H,t}$ from the wage equation yields:

$$\lambda_t = \mathrm{E}_t \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \left\{ \begin{array}{c} \frac{(1-\alpha)}{\alpha} \left(\frac{l_{t+1}}{k_t} \right) P_{t+1} w_{t+1} + \\ \lambda_{t+1} \left[(1-\delta) - \phi \left(\frac{x_{H,t+1}}{k_{H,t}} \right) + \phi' \left(\frac{x_{H,t+1}}{k_{H,t}} \right) \frac{x_{H,t+1}}{k_{H,t}} \right] \end{array} \right\}.$$

Next, we substitute in the expression for λ to obtain:

$$U_{c,t} = \left[1 - \phi' \left(\frac{x_t}{k_{t-1}} \right) \right] \mathrm{E}_t \beta U_{c,t+1} w_{t+1} \frac{f_{k_{t+1}}}{f_{l_{t+1}}} + \quad (2.18)$$

$$\mathrm{E}_t \beta \frac{1 - \phi' \left(\frac{x_t}{k_{t-1}} \right)}{1 - \phi' \left(\frac{x_{t+1}}{k_t} \right)} U_{c,t+1} \left[(1-\delta) - \phi \left(\frac{x_{H,t+1}}{k_{H,t}} \right) + \phi' \left(\frac{x_{H,t+1}}{k_{H,t}} \right) \frac{x_{H,t+1}}{k_{H,t}} \right],$$

where f_{k_t} is the marginal product of capital and $f_{l_{t+1}}$ the marginal product of labor and w_{t+1} is the real wage, $U_{c,t} \equiv U_c(C_t, \xi_{C,t}, 1 - l_t)$.

2.2.3. Non-traded-intermediate goods sector

The non-traded intermediate goods producer has the similar maximization problem:

$$\max P_{N_t} y_{N_t} - P_t w_t l_{N,t} - P_t x_{N,t}, \tag{2.19}$$

which is subject to

$$y_{N_t} = F(k_{t-1}, l_{N,t}) \tag{2.20}$$

$$k_{N,t} = (1-\delta)k_{N,t-1} + x_t - \phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right)k_{N,t-1},$$

and where $\psi y_{H,t} + \psi y_{F,t}$ are demands for non-traded goods coming from the distribution sector. If we now set up the stochastic maximization problem of the domestic intermediate goods firm:

$$L = \mathrm{E}_t \sum_{t=0}^{\infty} \beta^t \frac{U_{c,t}}{P_t} \left\{ \begin{bmatrix} P_{N,t}\left(A_{N,t}l_{N,t}\right)^{\alpha}(k_{N,t-1})^{1-\alpha} + P_{N,t}\left(\psi y_{H,t} + \psi y_{F,t}\right) \\ -P_t w_t l_{Nt} - P_t x_{N,t} \\ +\lambda_t \begin{bmatrix} (1-\delta)k_{N,t-1} + x_{N,t} \\ -\phi\left(\frac{x_{N,t}}{k_{N,t-1}}\right)k_{N,t-1} - k_{N,t} \end{bmatrix} \end{bmatrix} \right\}. \tag{2.21}$$

The first order condition with respect to labor input is then given by:

$$P_t w_t = \alpha P_{N,t}(A_{N,t})^{\alpha}\left(\frac{k_{N,t-1}}{l_{N,t}}\right)^{1-\alpha}.$$

The first order condition with respect to investment is:

$$P_t = \lambda_t - \phi'\left(\frac{x_{N,t}}{k_{N,t-1}}\right)\lambda_t.$$

The first order condition with respect to capital is:

$$\lambda_t = \mathrm{E}_t \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \left\{ \begin{matrix} P_{N_{t+1}}(1-\alpha)\left(\frac{A_{t+1}l_{N,t+1}}{k_{N,t}}\right)^{\alpha} + \\ \lambda_{t+1}\left[(1-\delta) - \phi\left(\frac{x_{N,t+1}}{k_{N,t}}\right) + \phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\frac{x_{N,t+1}}{k_{N,t}}\right] \end{matrix} \right\}, \tag{2.22}$$

and using the expression for P_N from the wage equation yields:

$$\lambda_t = \mathrm{E}_t \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \left\{ \begin{matrix} \frac{(1-\alpha)}{\alpha}\left(\frac{l_{N_{t+1}}}{k_{N_t}}\right)P_{t+1}w_{t+1} + \\ \lambda_{t+1}\left[(1-\delta) - \phi\left(\frac{x_{N,t+1}}{k_{N,t}}\right) + \phi'\left(\frac{x_{N,t+1}}{k_{N,t}}\right)\frac{x_{N,t+1}}{k_{N,t}}\right] \end{matrix} \right\}.$$

As before, substituting in the expression for λ, we have

$$U_{c,t} = \left[1 - \phi' \left(\frac{x_{N,t}}{k_{Nt-1}} \right) \right] \mathrm{E}\beta U_{c,t+1} w_{t+1} \frac{f_{k_{t+1}}}{f_{l_{t+1}}} + \tag{2.23}$$

$$\mathrm{E}_t \beta \frac{1 - \phi' \left(\frac{x_{N,t}}{k_{N,t-1}} \right)}{1 - \phi' \left(\frac{x_{N,t+1}}{k_{N,t}} \right)} U_{c,t+1} \left[(1 - \delta) - \phi \left(\frac{x_{N.t+1}}{k_{N,t}} \right) + \phi' \left(\frac{x_{N,t+1}}{k_{N,t}} \right) \frac{x_{N,t+1}}{k_{N,t}} \right].$$

2.3. The real exchange rate

In this model, the real exchange rate is defined as:

$$RS_t = \frac{S_t P_t^*}{P_t} \tag{2.24}$$

and can deviate from purchasing power parity (PPP) as a result of three channels. As in Benigno and Thoenissen (2008), allowing for the possibility of home bias in consumption ($v > v^*$), via the terms of trade channel (because of home bias) and via the internal real exchange rate channel (because of non-traded goods), (2.24) can be expanded to give:

$$\frac{S_t P_t^*}{P_t} = \frac{S_t P_{H,t}^*}{P_{H,t}} \frac{P_{H,t}}{P_{T,t}} \frac{P_{T,t}^*}{P_{H,t}^*} \frac{P_{T,t}}{P_t} \frac{P_t^*}{P_{T,t}^*},$$

which when linearized around the steady state, where $\frac{SP^*}{P}$ equals unity, can be shown to be equal to:

$$\widehat{RS}_t = (v - v^*)\hat{T}_t + (\omega - 1)\,\hat{R}_t + (1 - \omega^*)\,\hat{R}_t^*. \tag{2.25}$$

The deviation of the real exchange rate around its steady state depends on deviations of the home and foreign retail to wholesale price ratios, the terms of trade, T, defined as $\frac{P_F}{P_H}$, and the relative price of non-traded to traded goods, R.

2.4. The current account

The current account is defined as changes in foreign asset holding, within the incomplete financial market. Home and foreign agents trade intermediate goods and the trade balance is used to buy foreign bonds and so the flow budget

constraint shows the current account dynamics below. The left hand side is the changes in foreign asset holding. The right hand side shows the total production (first two terms) minus consumption and investment, yielding adjustment of bond wealth:

$$\frac{S_t B_t^F}{P_t \left(1 + i_t^*\right)} \frac{1}{\Theta\left(\frac{S_t B_t^F}{P_t}\right)} - \frac{S_t B_{t-1}^F}{P_t} = \frac{P_{H_t}}{P_t}\left(y_{H_t} + y_H^*\right) + \frac{P_{N_t}}{P_t} y_{N_t} - C_t - x_t. \quad (2.26)$$

2.5. Forcing variables

We adopt the specification of Stockman and Tesar (1995) and Chadha, Janssen and Nolan (2001) by investigating the role of both productivity and preference shocks for an open economy. We use both traded sector and non-traded sector productivity, which drive the input and hence product price, shocks to the allocation of time spent in work over leisure, which affects labor supply, and to stochastic deviations in the UIP condition, which directly affects the terms of trade. Each shock originates from a different sector but allows us to attribute exchange rate volatility to more than one exogenous factor. In total, we enable seven shocks (two sectoral and a preference shock in each of two countries, plus UIP deviations) and try to locate the importance in explaining open economy business cycles. The construction of each shock process is explained in Appendix A.

3. Solution and Model Calibration

3.1. Solution method

Before solving the model, it is log-linearized around the steady state to obtain a set of equations describing the equilibrium fluctuations of the model. The log-linearization yields a system of linear difference equations which we list in an appendix and can be expressed as a singular dynamic system of the following form:

$$\mathbf{A}\mathrm{E}_t\mathbf{y}(t+1 \mid t) = \mathbf{B}\mathbf{y}(t) + \mathbf{C}\mathbf{x}(t)$$

where $\mathbf{y}(t)$ is ordered so that the non-predetermined variables appear first and the predetermined variables appear last, and $\mathbf{x}(t)$ is a martingale difference sequence. There are up to seven shocks in $\mathbf{C}$. The variance-covariance as well as the autocorrelation matrices associated with these shocks are described in Table 1.

Given an initial parametrization of the model, which we describe in the next section, we solve this system using the King and Watson (1998) solution algorithm.

3.2. Data and calibration

Table 1 summarizes the calibration parameters for the baseline simulation of the model. We collect both quarterly and annual data and calibrate the model for the pair of countries – the UK and the US. Values of parameters are either estimated from US or UK data or taken from extant literature. An annual risk free rate of 4% and depreciation at 10% is assumed. Labor share is calibrated at 0.67 for the UK and the US. We take the consumption and leisure curvature of 2 (Corsetti *et al.*, 2004) and 4 (Chadha *et al.*, 2001). The elasticity of substitution between home and foreign goods in UK is 1.5 as in Chari *et al.* (2002). For the trade-off between traded and non-traded goods we adopt the elasticity suggested by Corsetti *et al.* (2004) of 0.74. UK and US trade data reveals the shares of UK produced goods in UK and US production to be 0.73 and 0.0157. Traded goods weights in all household consumption are estimated to be 0.3 and 0.24, smaller than that of Corsetti *et al.* (2004), 0.45 to 0.5. Cost of financial intermediation is 70bp as in Selaive and Tuesta (2003). The cost of investment, $b = 2$, is chosen to match the relative volatility of investment.[11] Steady state of net foreign asset is set to be 0 or 0.5 which means, respectively, that the UK has a balanced current account or is a creditor.

We have at most seven exogenous shocks in our experiments. The vector of shocks Π_t are assumed to follow a VAR(1) process:

$$\Pi_{t+1} = A\Pi_t + U_{t+1},$$

$$U_{t+1} \sim N\left(0, \Sigma\right).$$

4. Model Results

We now turn to the evaluation of the structural linear model by its simulation and comparison to our observations on the economy. As well as standard matching of moments, we develop a new approach for model evaluation and model selection.

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[11]However we also run experiments with $b = 5$, which are available on request and covered in the robustness exercise of Figure 7.

4.1. Methodology

Conventional tools such as the impulse response function and variance decomposition help us understand the dynamics of an artificial economy. The standard practice is also to assess models against some selected second moments of the data. But in this paper we introduce criteria that takes into account all the second moments and evaluate model performance based on formal statistical measures. We define a better model, as one that can render a better match between VCM of the data and the VCM simulated by the model. In order to pin down some parameter value or decide on certain features of a model, we work on a class of candidate models (or calibrations). By examining the corresponding match for candidate models, we call any improvement towards the criterion a gain in marginal information. We can also evaluate the gain on a particular parameter, by which we can signal the importance of any one feature of the model. Strictly speaking, we cannot guarantee the marginal information gain is reliable, or nearer to the 'true' model, unless we are quite certain about the rest of the model. The proposition of a marginal information gain we make is therefore a 'weak-form' of model selection (see Geweke, 1999).

The criteria we use involve the statistical divergence of the two VCMs. We develop formal and also intuitive distance measures elsewhere but some details are available in Appendix B. A higher value of distance denotes a model that is further from our measure on 'true' data process.[12] The data required to evaluate the open economy model is of high dimension and a relatively short sample, which tends to make model evaluation and selection very challenging problems. We calculate for each candidate model a distance and compare across each measure. We are cautious in making a proposition of model selection, especially for a particular parameter constellation, but feel able to make some statements on the validity of the joint choices on model and shock processes.

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[12]In developing this approach, we use Monte Carlo simulations on some artificial models. We find: (1) this approach works very well , particularly if the multivariate normality is approximately tenable; (2) our approach helps overcome small-sample bias, and (3) the model selection outcome depends quite strongly on the sub-block of the full VCM chosen for comparison. In other words, the choice of state variables is very crucial, and has to be made carefully based on the specifics of the application considered.

4.2. Impulse responses

The impulse response functions are based on the seven-shock model.[13] In this calibration, the foreign country has the same properties as home, such as shares of traded and home goods on market. As an alternate specification, we consider $v = 0.85$ and $v^* = 0.15$, the home produced share of tradeables in intermediate goods production at home and overseas respectively, in order to highlight the effect of foreign sector.

4.2.1. Traded productivity shocks

Figure 3 plots the response of quantities and relative prices to a traded productivity shock in the home country. The response of real exchange rate depends on two effects: the terms of trade and the Harrod-Balassa-Samuelson (HBS) effect. The former requires an adjustment in relative traded prices, which requires a depreciation in the real exchange rate in the long run. However, the latter effect drives up wages in both the traded and non-traded sector but with no productivity improvement in the non-traded sector, non-traded prices will rise and hence so will the real exchange rate. This effect is especially strong, see section 2.3, when there is a home bias in consumption, which acts to accentuate the real exchange rate change. Finally, the lack of complete risk sharing means that consumption is more elastic to a productivity shock than under a complete markets allocation. The combination of forward-looking domestic consumption responding to higher productivity (income) but an attenuated overall investment response - where traded sector investment rises but non-traded sector investment falls - leads to the accumulation of foreign debt to finance current demand.

4.2.2. Non-traded productivity shocks

Following a non-traded productivity shock (Figure 4), investment and labor increase. Home households enjoy somewhat higher consumption in this case, more so than in the case of traded sector productivity shock. In this case, the terms of trade effect and HBS effect are the same, causing the real exchange rate to depreciate. Although the response of relative consumption is positive, it is not large enough to bring about a current account deficit, because there is a larger

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[13]The construction and estimation of shocks is outlined in appendix A: traded and non-traded productivity shocks in A.1, preference shocks in A.2 and stochastic deviations from UIP in A.3.

response from the labor input, and hence there is net lending overseas. In general the impulse responses suggest that strong traded-sector productivity shocks can lead to the matching of some elements of the open economy. A lack of complete risk sharing raises consumption at home compared to abroad and a strong preference for home goods consumption also amplifies the extent to which output increases.

4.2.3. Preference shocks

In principle, preference shocks might be thought to contribute a solution to the Backus-Smith puzzle simply as marginal utility is now, *inter alia*, a function of the preference shocks rather than just consumption growth:

$$RS = \frac{U_C^*}{U_C},$$

where we note that the real exchange rate can be thought of as related to the ratio of marginal utilities in consumption (in a complete markets set-up). But these preference shocks by themselves may not provide a resolution as they seems to imply relatively acyclical current account dynamics and a reduction of real exchange rates along with higher domestic supply (see Chadha *et al.*, 2001). This is because they alter the equilibrium point in the household trade-off between leisure and consumption. Following Hall (1997) such shocks simply suggest that the household decides to allocate more (or less) time to work, which finances consumption, rather than leisure. As one would expect preference shocks help increase the volatility of the labor input by introducing exogenous shifts in work and may act to solve the puzzle of the Backus-Smith correlation (Figure 5). A home preference shock drives up labor input and consumption and reduces relative prices, if the supply response is elastic. So unless home agents become elastic in the substitution of leisure across periods, increased consumption is also met by an increase in investment and the current account remains acyclical.

4.2.4. Stochastic deviations from UIP

Following the suggestion of Devereux and Engel (2002), we explore the implication of stochastic deviations from the uncovered interest rate parity (UIP) condition for the determination of exchange rate changes. These shocks, motivated by the poor empirical performance of UIP equations, (see Sarno and Taylor, 2002 for an indicative survey) imply that the exchange rate does not move equiproportionately to interest rate differentials and in fact it often moves in the opposite direction.

These stochastic deviations, which can be thought of as excess returns in a particularly currency mean that the exchange rate can disconnect from the relative interest rates. The impulse responses show that a shock that brings about an initial exchange rate appreciation is similar to a demand shock in that it depresses traded and non-traded wages via competition with overseas traded-sector wages. To deal with the temporary fall in wages, consumption - which is tilted up by the fall in domestic interest rates - is maintained by overseas borrowing and investment is stimulated by the fall in wages.

4.3. Variance decomposition

Table 2 shows the decomposition of unconditional variances for relative consumption, the real exchange rate and the current account from the model simulation. The first four columns show the contribution from each of the seven shocks in explaining the variance of these three key variables in the case of persistent, temporary UIP deviations and when the home economy is a creditor or debtor. The final three columns then exclude one type of shock in turn and shows the resulting contribution by the remaining shocks. Table 2 illustrates that both sets of productivity shocks and UIP deviations are likely to play a dominant role in explaining the variance of the key open economy variables, the former for relative consumption and the latter for the real exchange rate and the current account.

The Table shows the dominant role that UIP deviations play under the baseline calibration in explaining the variance of the current account and real exchange rate over the business cycle. It also suggests that productivity shocks, particularly in the non-traded sector, might play an important role in explaining fluctuations in relative consumption and also for the real exchange rate and the current account when UIP deviations are excluded. Preference shocks play a negligible role in explaining the variances of these key variables unless we exclude productivity shocks altogether in which case they can explain over 20% of the variance in relative consumption. The finding that productivity shocks are important for quantities and relative prices even in the presence of exchange rate volatility is similar to other studies, such as Straub and Tchakarov (2004).

4.4. Simulated moments

In Table 3, we present second moments of the artificial simulated model for the benchmark calibration. The first column gives the moments from the UK data

over the period 1980-2006. The next four columns correspond to the cases of persistent UIP deviations, temporary UIP and for the persistent UIP case also when the economy is a steady-state creditor or debtor - with assets or debts at 50% of GDP in each case, respectively. In the final three columns, we remove one set of shocks from the baseline calibration in order to understand how the artificial model data changes.[14]

The baseline calibration captures well the main moments of the data: consumption, labor inputs and wages are smooth relative to output and investment, the real exchange rate and the terms of trade are markedly volatile. The correlations of the main quantities and relative prices with output are all correctly signed (apart from interest rates). The model produces the positive relationship between the terms of trade and the real exchange rate found in the data, as well as the exchange rate disconnect, with relative consumption negatively correlated. Finally, although higher than the correlation observed in data (0.16), the model does not predict that relative consumption will be perfectly correlated (with estimates in the range 0.5 to 0.7) and thus goes some distance towards understanding the lack of complete risk sharing.

This is because the non-state contingent bond is used to smooth investment and consumption following a shock.[15] In the event of a temporary productivity shock, which has little impact on permanent income, the home country consumer borrows from abroad, which raises overseas interest rates and lowers overseas consumption as well, which leads to a correlation in relative consumption. But when there are persistent productivity shocks, permanent income falls somewhat and so there is not as strong a need to borrow from abroad to smooth consumption or investment, which then means that overseas interest rates do not rise and lower overseas consumption. Hence there is something of a fall in the consumption correlation when there are non-state contingent bonds and persistent productivity shocks.

The persistence of the UIP shocks plays an important role in explaining both the relative variance of the real exchange rate and to a lessor extent that of relative consumption, which falls from 5.2% to 2.5% and from 1.1 to 0.9, respectively when we reduce the AR(1) persistence of UIP deviations from 0.88 to 0.38. Note also that the relative consumption becomes nearly acyclical (-0.02) when the UIP

[14]In earlier versions of this paper we also presented results for the estimated spill-over of productivity and preference shocks but as we found that these do not change the moments qualitatively we have removed them from this version.

[15]Unlike an asset that can be bought to insure prior to shocks.

shocks fall in persistence. Moving towards a model where the steady-state level of net foreign assets is not zero does not alter the basic picture but when the home country is treated as a debtor investment, the real exchange rate and the terms of trade become more volatile and the current account becomes considerably less volatile.

If we examine the model with or without UIP deviations (compare column 2 to the final column), it appears that UIP deviations play a clear role in helping to explain the exchange rate disconnect. This is because the exchange rate can be driven whether there are movements in relative interest rates or not, which in turn depend mostly on planned relative consumption levels. An absence of UIP deviations from the model thus drives the correlation of relative consumption with real exchange rate to 0.76 rather than the data estimate of -0.61 or the benchmark model estimate of -0.65. Note also that in the model without UIP deviations, consumption, investment, labor inputs, real exchange rates and the terms of trade are somewhat too volatile. The main role of preference shocks it to raise the volatility of the labor input and lower that of the wage rate.

The overall performance of baseline calibrated model in terms of explaining the puzzles is reasonable. Specifically, we find that: (1) the model enables different shocks to interact and seems to solve the Backus-Smith puzzle in that it does not forecast perfect consumption correlation across the two economies with the help of a non-traded sector and incomplete financial markets; (2) this model stresses the HBS effect and therefore generates volatile real exchange rates; and (3) countercyclical current account is a robust result, as the current account moves together with real exchange rate. In other words it seems to match the OECD and emerging economy experience suggested in Figure 1.

5. Model-data comparison

A typical business cycle exercise examines the volatility of key economic variables and their correlation with output - as a measure of their business cycle behavior. At the very least such an examination neglects the cross-correlations in other elements in the VCM that may matter to us, which in this case is the relationship between exchange rates, relative consumption and the current account. Our model selection is thus based on the comparison of the VCM of seven key endogenous variables simulated by our model to the actual data, see Appendix B for some further details. To illustrate our point, we consider the open economy sub-set of the variables for this exercise. In this section we obtain six statistical measures of

distance of the model-generated data from the sample observations and the results are given in Table 4. The smaller statistics indicate a better fit of data to model and we find for the main model selection criterion the models with persistent UIP deviations with debtor status are closest to the observed data.

5.1. Model selection based on VCM

If we choose to define a preferred model as that with the least deviation from the data, there may be a number of possible metrics we can employ. Our model selection from a class of candidate models is based on the comparison of the VCM of endogenous variables simulated by our model to that of the actual data, see Appendix B for further details on the distance measures used.[16] We consider a sub-set of the model variables that are closely related to the open economy puzzles highlighted in Figure 1: relative consumption, real exchange rate, relative output, home current account and home trade balance.

Specifically, as well as basic criteria such as root mean squared error (RMSE) and mean absolute error (MAE), we use two likelihood based methods to determine how different the two matrices are: (1) the Box-Bartlett test (Bartlett, 1937; Box, 1949); (2) the distance measure flowing from the Kullback-Leibler (Kullback and Leibler, 1951) Information Criteria (KLIC) method. As shown in Bhattacharjee and Thoenissen (2007), we can also use the hypothesis testing method of Nagao (1973) and a revised test by Ledoit and Wolf (2002), which are designed to test an equality hypothesis of VCMs.[17] The key differences between these classes of approach are explained in the Appendix B but essentially the basic criteria of RMSE and MAE are akin to an approximate eyeballing of the data whereas the Box-Barlett test, KLIC methods, Nagao and Ledoit-Wolf allow for sampling variability and the KLIC also allows sampling variability in the simulated model.

For each case, we obtain six statistical measures of distance of the simulated

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[16] A copy of the procedures written in MATLAB will be made available on request.

[17] The original Nagao's (1973) test is also an LR type test. The Ledoit and Wolf (2002) method aims to deal with the special cases where data dimension is larger than number of observations (or relatively small sample data). Such a property makes the data VCMs rank-deficient. Although we have rank-deficient VCMs in DSGE models for a different reason, where variables are greater in number than shocks and predetermined variables taken together, we utilize this method to deal with rank-deficiency problem. Note that canonical LR methods cannot be directly applied to rank-deficient VCMs; see Bhattacharjee and Thoenissen (2007) for further discussion. We outline our distance metrics in Appendix B.

model from the sample VCM for our 7 key variables. The results are given in Table 4. We assess the distance with different degrees of persistence in the UIP deviations and varying the NFA position. The smaller statistics indicate a better fit of data to the model. The best calibration according to each of the six criteria is therefore marked with an asterisk. If we examine the first three columns of results we will note that simple eyeballing of the data might lead us to prefer models with less persistent UIP shocks. But, when sampling and model uncertainty is accounted for, the other tests suggest we should prefer more persistence in the UIP deviations. We find models with persistent UIP deviations are closest to the observed data. Furthermore when we allow the steady-state debt position to move from creditor to debtor status we find that the best fit - smallest distance - occurs when the home economy is a debtor.

There are two main findings that stand out. Firstly, the distance measures suggest that persistent UIP deviations are helpful in generating a VCM similar to that of UK/US open economy data. We have shown in the impulse responses that deviations from UIP are the only forcing variable which helps resolve the Backus-Smith puzzle, by driving up large swings of real exchange rate and generating a volatile and countercyclical current account. More dominant UIP deviations are required to replicate the observed data. Secondly, we find that a non-zero NFA position is also helpful for improved goodness of fit, with net debtor calibration for the UK being slightly better than the net creditor case. However, negative or positive NFA position improve the model fit quite differently. A net debtor calibration mainly contributes to a better fit associated with current account dynamics. A net creditor calibration improves the goodness of fit for UK and US output and consumption data. In a two-country model, a net creditor UK means a net debtor US (as in the real world). This realistic calibration can better explain relative output and consumption but also generate a volatile current account on both sides and thus increase distance measures for the overall fit. Therefore, based on the VCM distance approach, we highlight a net creditor and persistent UIP deviation calibration for the UK/US small open economy model.

5.2. Sensitivity analysis

Sensitivity analysis is shown in Figures 7-12 and is based on the seven-shock model with the basic calibration given in Table 1. We simulate the model and allow some deep parameters to change and check the sensitivity of some key moments with respect to several main statistical measures: the Backus-Smith correlation,

the extent of exchange rate disconnect, the correlation between the trade and current account and the cyclicality of the current account. The vertical solid line(s) denotes the initial calibration.

First, we consider frictions in the model: costly investment and costly foreign asset holding. In Figure 7, although higher cost of investment alters volatility of open economy variables, it does not change the basic correlation structure. In Figure 8, costly foreign asset holding make the channel of risk sharing smaller, therefore the Backus-Smith correlation tends to zero. However, this will happen when the cost is extremely high. As the model has very simple assumption for financial markets, we emphasize its qualitative implication instead of its value denoted by basis points.

Secondly, we discuss the characteristics of the market and production. Steady state NFA does not alter real exchange rate dynamics significantly but it is crucial for current account dynamics. For a net debtor, a positive traded TFP shock leads to current account deficit. For example, upon a positive traded productivity shock, output increases, the real exchange rate appreciates, Home country borrows and a current account deficit results. But as a debtor there is requirement for paying interest, making the borrowing incentive lower and thus the extent to which the current account is countercyclical is mitigated, as shown in Figure 11.

Thirdly, we consider varying sources of dynamics - the exogenous forcing variables. The UIP shock in the baseline calibration is highly persistent and by examining different degrees of persistence in UIP deviations as in Figure 10, we find real effects only in the case of highly persistent shocks. Adding UIP deviations reinforces the pattern of correlation we find in the data. When we vary the relative magnitude of non-traded productivity shocks in Figure 11, it leads to changes in the key correlations. Relatively strong traded compared to non-traded productivity shocks contribute to negative Backus-Smith correlation and countercyclical current account. Turning to Figure 12, as preference shocks are strengthened, the negative correlation on both counts is weakened.

5.3. From model selection to parameter estimation

We can also replicate the sensitivity analysis for each of these key parameters but in terms of the distance measures rather than the base correlations in the data as in the previous section.[18] The diagnostics can be used to obtain estimates of the

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[18]These Figures are excluded from this version of the paper in the interest of space but are available on request.

parameters that provide the best match to the data and essentially support the results outlined in the previous section; see also Bhattacharjee and Thoenissen (2007).

It is clear that the minimum distance is achieved when treating the cost of investment is in the neighborhood of 2. The required costs of financial intermediation (ε, the spread between return on foreign and domestic bond) do not seem to have to be especially high when we examine the sensitivity analysis. This parameter affects the trade-off between home and foreign bonds. But all the four criteria considered here suggest that model fit will improve when the spread increases somewhat. We attribute this result to strong home bias in asset holding. The adoption of the assumption that the home economy is a net debtor also seems to enhance model fit.[19]

Finally we examine the level of persistence for the shock processes. In general, we find that more persistent UIP deviations increase the relative volatility of traded to non-traded shocks, and therefore improve model fit. However, like previous results (see Chadha *et al.*, 2001), increasing the volatility of preference shocks does not seem to improve the fit of the model markedly.

6. Conclusion

Open-economy general equilibrium models offer an attractive laboratory in which to examine the stylised facts, whether empirical regularities or puzzles, observed in the data. We examine the properties of a two-sector real business cycle model with incomplete financial markets. The model is driven by a number of driving forces or shocks: in both domestic and overseas traded and non-traded productivity, to the work-leisure margin at home and overseas and to deviations in the exchange rate from the level suggested by the UIP equation.

We find evidence to support the proposition that when all these shocks perturb the model economy there is some move towards resolution of the Backus-Smith puzzle, and the model also stresses the Harrod-Balassa-Samuelson effect. The most important modelling choices - over and above a standard one-sector small economy RBC model - involve the adoption of a two sector model, allowing for shocks to non-traded as well as traded sector productivity, the employment of

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[19] Although the UK has a steady-state level of debt near zero, Lane and Milesi-Ferretti (2002) document the mean net foreign asset position to GDP at 6% i.e., UK is a small net creditor. In comparison, our approach seems to locate the correct approximate region for the level of steady-state debt at a level not very far from zero.

incomplete markets with the existence of a non-state contingent bond and of stochastic deviations from the UIP equation for the exchange rate. The aspects of the model induce greater real exchange rate variability and yet alongside the absence of complete risk sharing ensure that consumption need not simultaneously jump to arbitrage price differentials.

Finally we note that the modelling approach we use is flexible and simple to use. Our methodology facilitates examination of deep parameters and shock processes for small open economies, and allows the researcher to examine some simple summary statistics when assessing model fit. The proposed distance measures might usefully be applied more generally to the question of the fit of data to DSGE models.

Appendix

A. Measurement of Exogenous Shocks

A.1. Productivity Measurement

Sectoral productivity is calculated as total factor productivity (TFP) in traded (manufacturing) or non-traded (services) sector. We use OECD STAN database 2005 Release (OECD, 2004) to construct sectoral TFP series for the UK and the US. Incomplete data on total hours and gross capital stock are complemented by total employment and capital formation data. Based on these data, TFP is measured as:

$$TFP_t^A = \log\left(\frac{Y_t^A}{(K_t^A)^{1-\alpha}(N_t^A)^\alpha}\right), \tag{A.1}$$

where $A = \{T \text{ (traded)}, NT \text{ (non-traded)}\}$, and α denotes the labor share in production calibrated at 0.67.

A.2. Measuring the preference shock

We follow Holland and Scott (1998) for measuring preference deviations, $\widehat{\xi}_t$. Specifically, we use the Euler equation describing the leisure-consumption trade-off to find an expression for the preference variable ξ_t.[20] In the above equilibrium

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[20] A positive preference deviation, such as $\xi_t = 1\%$, is said to be biased to leisure time.

the real wage must equals $\frac{U_L}{U_C}$, the marginal rate of substitution between leisure and consumption, to clear labor market. The time endowment and the utility non-separable to leisure are:

$$L_t = 1 - l_t \tag{A.2}$$

$$U = \frac{1}{1 - \rho} C_t^{1-\rho} \frac{L_t^{\eta \xi_t}}{\eta} \tag{A.3}$$

Then, the leisure-consumption trade-off yields:

$$w_t = \frac{U_L}{U_C} = \frac{\eta C_t \xi_t}{(1 - \rho)(1 - l_t)} \tag{A.4}$$

Calibration for the parameters are taken from Table 1. Then, the stochastic preference shocks can be measured by using US and UK aggregate data on w_t, C_t and l_t.

Note we could also enforce the equilibrium condition for flexible wage setting:

$$w_t = F_l(K_{t-1}, l_t). \tag{A.5}$$

In this case, we can replace w_t by $\frac{\alpha Y_t}{l_t}$ to avoid the sticky wage setting which may undermine the basic assumption of the flexible price model. This measure also captures very well the idea of preference shocks in Hall (1997), since data shows an association of recession years with increase in leisure-biased preference ξ_t.

Finally, our preference shocks are the detrended series of ξ_t in logarithm:

$$\ln \xi_t = \ln \frac{\alpha Y_t (1 - \rho)(1 - l_t)}{\eta l_t C_t} \tag{A.6}$$

A.3. Expectational Errors in Exchange Rates

Furthermore, we allow a random shock in the UIP condition, making exchange rate volatility attributable to more factors. We have a simple model for the UIP shocks with participants in foreign exchange market allowed to let exchange rate deviate from theoretical value in the short run. The nominal exchange rate adjustment follows an UIP condition of foreign bond holding but is also subject to a shock $x_{u,t}$:

$$E_t \Delta s_{t+1} = i_t - i_t^* + \varepsilon \widehat{B}_t + x_{u,t} \tag{A.7}$$

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Attemps to estimate the above relationship can be problemetic due to potential omitted variables. This partly contributes to the dominant role of expectational errors in foreign exchange market, such as the UIP deviations in our set-up. Instead of estimation, we refer to Selaive and Tuesta's (2003) finding on foreign bond holding costs, and adopt their calibration of $\varepsilon = 0.007$ (based on quarterly data). The UIP deviations are therefore computed by substituting calibrated parameters and historical data into equation (A.7), where we measure net foreign asset adjustment $\widehat{B}_t$ (bond holding) by the detrended series of Net Foreign Asset to GDP ratio: $\frac{S_t B_t}{Y_t}$.

We find the resulting UIP deviation is highly volatile but not highly persistent. By contrast, Kollmann (2003) uses a two-part UIP shock $x_{u,t} = a_t + \omega_t$ and finds UIP shocks to be quite persistent. We conduct sensitivity check by allowing persistent UIP deviations versus the base case scenario.

B. Evaluating Model Fit

Canova and Ortega (2000) discuss four possible approaches in evaluating DSGE model fit. The variety of approaches arise from the different treatments of model uncertainties and data sampling uncertainties: (a) an informal approach, which ignores both sampling variability in the data and uncertainty regarding model parameters, (b) methods that consider model uncertainty but not sampling variability in the data, and (c) methods that consider sampling variability in the data but not uncertainty in model; and (d) approaches that account for both sampling variability in the data and model uncertainty.

Bhattacharjee and Thoenissen (2007) propose the modified Nagao test which belongs to the class of methods (c):

> "... we consider an approach that uses sampling variability of actual data to provide a measure of the distance between model and the data, holding the model VCM fixed. This approach is explicitly based on the context of dynamic general equilibrium macroeconomic models, where given specific calibrated or estimated values for the parameters, the model can be simulated for as many periods of time as desired. Thus, for given parameter values, the asymptotic VCM of the state variables obtained from such simulation has no sampling variability. On the other hand, the data VCM is based on a data for a finite sample period. In most applications, this period would be from 1960 or later

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to the most recent period for which data are available. Thus, there is substantial sampling variation in the data VCM, while the model VCM can be considered fixed for a given combination of parameter values. By computing distances for distinct combinations of possible parameter values across all the competing models, we can ignore the uncertainty regarding calibration or estimation of parameters, while taking account of sampling variability in the actual data."

Using approach (c), we propose similar methods derived from the Box-Bartlett test (Bartlett, 1937; Box, 1949) and its variant based on the Ledoit-Wolf test (Ledoit and Wolf, 2002). In addition we explore the possibility of using parallel approaches following Canova and Ortega's (2000) guideline: eyeballing approach such as RMSE and MAE are implementations of approach (a), while Kullback-Leibler is an implementation of approach (d).

Since most DSGE models are driven by only a limited number of shocks and predetermined state variables, the model VCM is usually rank-deficient. Except for RMSE and MAE, we use a projection of both data and model VCM to lower dimensional subspace introduced by Bhattacharjee and Thoenissen (2007) to deal with the rank-deficient problem.

The methods developed here will also take into account two other common features of model selection in the stated context. First, as emphasized earlier, DSGE models are intended to be abstractions of reality and are often driven by a lesser number of shocks than the number of state variables. In other words, while actual data VCMs would be full-rank, simulated data VCMs may often have a lower rank. Our methods will explicitly take into account this possibility. Second, the metrics will be developed in such a way that enables model selection when the candidate DSGE models may be non-nested. This feature of our methodology is obviously important and enhances the applicability of the methods.

B.1. Distance metrics

We denote by $[\Sigma_0]_{m \times m}$ the full-rank data VCM estimated using n_0 data points $(\rho(\Sigma_0) = m)$, where ρ is the rank of VCM. $[\Sigma_{M_1}]_{m \times m}, [\Sigma_{M_2}]_{m \times m}, [\Sigma_{M_3}]_{m \times m}, \ldots$ denote estimated VCMs using simulated data from a countable collection of competing models $M_1, M_2, M_3, \ldots$ and based on $n_1, n_2, n_3, \ldots$ simulated observations respectively. Some of these matrices may be rank deficient; that is, $\rho(\Sigma_{M_j}) \leq \rho(\Sigma_0) = m$.

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We shall propose several alternate metrics, denoted $d(\Sigma_0, \Sigma_{M_j})$, that give scalar measures of how different any of the simulated VCMs are from Σ_0, where d is a metric measuring the distance between Σ_0 and Σ_{M_j}. These measures can then be used to select an appropriate model from all the competing ones. In the following, we focus on one competing model VCM, say Σ_M and elaborate on different possible approaches and corresponding metrics.

B.1.1. Naive, or Eyeballing, approach

This is not based on any distributional assumption. Root Mean Squared Errors (RMSE) and Mean Absolute Errors (MAE) are defined as:

$$RMSE = \sqrt{MSE}; MSE = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} \widetilde{\sigma}_{i,j}^2$$

$$MAE = \frac{1}{m^2} \sum_{i=1}^{m} \sum_{j=1}^{m} |\widetilde{\sigma}_{i,j}|$$

where $\widetilde{\Sigma} = ((\widetilde{\sigma}_{ij}))_{m \times m} = \Sigma_M - \Sigma_0$. In terms of the typology developed in Canova and Ortega (2000), the above two metrics ignore sampling variability in both data and model VCM.

B.1.2. Testing approach

This approach is based on a multivariate normality assumption underlying both the estimated VCMs, Σ_0 and Σ_M. However, we consider the possibility that the model VCM may not be full rank. The idea here is to pretend that we are conducting a test of the hypothesis $H_0 : \Sigma_0 = \Sigma_M$ against the omnibus alternative $H_1 : \Sigma_0 \neq \Sigma_M$. We are not as such interested in the outcome of the test, since we do not strongly believe that any of the models will generate simulated VCMs that are statistically indistinguishable from the data VCM. However, we can still use the p-values of the tests (or the values of the test statistic itself, adjusted for degrees of freedom) to give us a metric to compare between competing models. Note that the testing approach considers sampling variation in the data VCM, but the comparison is made with a simulated VCM based on large data where sampling variability may be negligible. We consider the following cases:

Σ_M **is full-rank** Here we can use a whole battery of tests developed in the multivariate statistics literature. The most popular of these tests are the Box (1949) modification to the test proposed by Bartlett (1937), and the test proposed by Nagao (1973).

Bartlett (1937) proposed the test statistic:

$$M = \sum \left(n_0 + n_M\right) \ln |\Sigma| - n_0 \ln |\Sigma_0| - n_M \ln |\Sigma_M|$$

where the pooled estimate of the common covariance matrix under the null hypothesis is

$$\Sigma = \frac{1}{n_0 + n_M} \left[n_0 \Sigma_0 + n_M \Sigma_M\right].$$

When multiplied by a scaler C^{-1} (Box, 1949):

$$C^{-1} = 1 - \frac{2m^2 + 3m - 1}{6\left(m + 1\right)} \left(\frac{1}{n_0} + \frac{1}{n_M} - \frac{1}{n_0 + n_M}\right),$$

the Box's M test statistic MC^{-1} has a Chi-square distribution (df $= m\left(m + 1\right)/2$) under the null hypothesis and multivariate normality assumption.

Nagao (1973) proposed a test for the null hypothesis $H_0 : \Sigma_M^* = I$ against the omnibus alternative (where I is the identity matrix) given by the test statistic:

$$N = \frac{n_M}{2} tr \left(\Sigma_M^* - I\right)^2,$$

where $tr(.)$ denotes trace of a square matrix. The test statistic has a Chi-square distribution (df $= m\left(m + 1\right)/2$) under the null hypothesis and multivariate normality assumption. This test can be adopted to our situation by using the Cholesky decomposition of Σ_0, as follows:

$$
\begin{aligned}
\Sigma_0 &= P'P \\
\Sigma_M^* &= P'^{-1}\Sigma_M P^{-1} \\
I &= P'^{-1}\Sigma_0 P^{-1}
\end{aligned}
$$

so that testing $H_0 : \Sigma_0 = \Sigma_M$ is now equivalent to testing $H_0 : \Sigma_M^* = I$ against the omnibus alternative. This is equivalent to premultiplying the actual and simulated data vectors by P'^{-1}. Both the Box's M-test and Nagao's test are known to be very conservative even in small samples (seldom accept the null hypothesis); this is, however, not of any major consequence for our work since we are not interested in the exact results of the test.

Σ_M is rank deficient $(\rho(\Sigma_M) < \rho(\Sigma_0) = m)$ This is the usual case. The model here is clearly an abstraction driven by only a limited number of shocks and predetermined variables. In fact, this abstraction can also represent reality to a high degree, in the sense that often only a small number of shocks can explain a substantial part of the variation in actual data on a larger number of state variables. In most applications, only a limited number of leading eigenvalues (and their corresponding eigenvectors) account for most of the variation in the data VCM, the remaining eigenvalues are small in comparison.

While the Box-Bartlett and Nagao tests do not directly apply to this situation, we propose two simple modifications. First, we adapt an extension of Nagao's test to the rank deficient case proposed by Ledoit and Wolf (2002). Ledoit and Wolf (2002) have recently considered a situation where the number of variables is large and higher than the sample size. They modify the Nagao (1973) test to this situation and derive asymptotic theory when both the dimension of the VCM and sample size increase to ∞ at the same asymptotic rate. In particular, their test statistic is given by:

$$W = \frac{1}{m} tr \left(\Sigma_M^* - I \right)^2 - \frac{m}{\rho(\Sigma_M^*)} \left[\frac{1}{m} tr \left(\Sigma_M^* \right) \right]^2 + \frac{m}{\rho(\Sigma_M^*)}.$$

Under the null hypothesis and multivariate normality, $\frac{1}{2}\rho(\Sigma_M^*).m.W$ has a Chi-squared distribution with $m(m+1)/2$ degrees of freedom. This extension is based on an asymptotic setup where, as sample size (time periods under study) increases, the set of state variables under comparison is also augmented; this assumption is reasonable in many practical situations.

Second, following Bhattacharjee and Thoenissen (2007), we project the data VCM onto a lower dimensional subspace spanned by the shocks and free predetermined variables driving the model. The usual Box-Bartlett and Nagao tests are then employed for VCM comparisons over this lower dimensional subspace; see Bhattacharjee and Thoenissen (2007) for further details.

B.1.3. Measures based on distance between distributions

One possible limitation of the above testing based approach is that it ignores sampling variation in the model VCM, and therefore its applicability for moment comparison specific to known time periods may be tenuous. An alternative is the approach, indicated in Watson (1993), based on computing the Kullback-Leibler Information Criteria (KLIC) between the distributions given by the data (mean

zero, VCM Σ_0) and the model (mean zero, VCM Σ_M) and choosing the best model based on this measure. The KLIC is given by:

$$I\left(\Sigma_0, \Sigma_M\right) = E_{f(.;0,\Sigma_0)} \ln \frac{f(Y;0,\Sigma_M)}{f(Y;0,\Sigma_0)} = \int_{-\infty}^{\infty} \ln \frac{f(y;0,\Sigma_M)}{f(y;0,\Sigma_0)} f(y;0,\Sigma_0) dy,$$

where $f(.;0;\Sigma)$ denotes the density of the multivariate Gaussian distribution with mean vector zero and VCM Σ, and the expectation is taken with respect to the distribution of the data (mean zero and VCM Σ_0).

While Watson (1993) suggests use of the KLIC in full-rank situations, we extend the method to models with lower number of shocks by using density functions for singular normal distributions. Specifically, we consider the singular value decomposition (SVD) of the simulated model VCM : $\Sigma_M = \lambda_1 e_1 e_1' + \lambda_2 e_2 e_2' + ... + \lambda_p e_p e_p' + 0 e_{p+1} e_{p+1}' + ... + \lambda_m e_m e_m'$, where $p = \rho(\Sigma_M) < m$ is the rank of the model VCM. The density function of this rank-deficient model (mean zero, VCM Σ_M, $\rho(\Sigma_M) = p < \rho(\Sigma_0) = m$) on the subspace spanned by only the p leading eigenvectors is:

$$f\left(\underline{y}_{m \times 1}; 0, \Sigma_M\right) = \frac{1}{(2\pi.\lambda_1.\lambda_2....\lambda_p)^{m/2}} . \exp\left(-\frac{1}{2}\underline{y}'\Sigma_M^-\underline{y}\right),$$

where a generalized inverse (g-inverse) of Σ_M is given by $\Sigma_M^- = 1/\lambda_1.\underline{e_1}.\underline{e_1}' + 1/\lambda_2.\underline{e_2}.\underline{e_2}' + ... + 1/\lambda_p.\underline{e_p}.\underline{e_p}'$. The density of the data VCM (full-rank) is computed in the usual way.

The KLIC approach, however, has a few features that are of importance. First and most importantly, KLIC does not give a strict distance metric, since it is not symmetric in its arguments. One can use symmetric versions of KLIC reported in the literature and besides this may not be a major issue in our case, since we are interested only in finding distances of different models from the data VCM, which is held constant throughout the exercise, and to this extent our approach is consistent. Second, KLIC is of course based on an assumed parametric distribution. We may assume multivariate normality or if appropriate, some other parametric distribution. Third, the KLIC is often difficult to compute particularly in a multi-dimensional case because this involves numerical integration in high dimensions. We address this issue by taking a Monte Carlo or bootstrap approach as follows.

We note that the KLIC is the expected value of difference of log-likelihoods under the two alternative distributions (given by Σ_0 and Σ_M) for samples from the

distribution given by the data VCM. Empirically we can either generate a Monte Carlo sample (sample size N_0^{MC}) with data VCM, or take bootstrap resamples (bootstrap sample size N_0^{BS}) from the actual data, and then calculate the sample mean of log likelihood ratios. By the weak law of large numbers, both these approaches will give consistent estimates of the KLIC. However, the Monte Carlo method will depend more specifically on the validity of the multivariate normality assumption, hence the bootstrap approach may be preferable in practice:

$$\widehat{I}_{Monte\ Carlo}\left(\Sigma_0, \Sigma_M\right) = \frac{1}{N_0^{MC}} \sum_{i=1}^{N_0^{MC}} \ln \frac{f(y_i; 0, \Sigma_M)}{f(y_i; 0, \Sigma_0)},$$

$$\widehat{I}_{Bootstrap}\left(\Sigma_0, \Sigma_M\right) = \frac{1}{N_0^{BS}} \sum_{i=1}^{N_0^{BS}} \ln \frac{f(y_i; 0, \Sigma_M)}{f(y_i; 0, \Sigma_0)}.$$

MATLAB codes for the implementation of the metrics used in this paper are available from the authors on request.

References

[1] Backus, David K., Patrick J. Kehoe, and Finn E. Kydland, (1995). International business cycles: theory and evidence. In T.F. Cooley (ed.) *Frontiers of Business Cycle Research*, Princeton University Press, Princeton: 331-56.

[2] Backus, David K. and Gregor W. Smith, (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, vol. 35: 297-316.

[3] Bartlett, Maurice, (1937). Properties of sufficiency and statistical tests. *Proceedings of the Royal Society of London Series A* 160: 268-82.

[4] Baxter, Marianne and Mario J. Crucini, (1995). Business cycles and the asset structure of foreign trade. *International Economic Review*, vol. 36: 821-54.

[5] Baxter, Marianne and Urban J. Jermann, (1997). The international diversification puzzle is worse than you think. *American Economic Review*, vol. 87: 170-80.

[6] Benigno, Pierpoalo. (2001). Price stability with imperfect financial integration. New York University, *Mimeo*.

[7] Benigno, Gianluca and Christoph Thoenissen, (2008). Consumption and real exchange rates with incomplete financial markets and non-traded goods. *Journal of International Money and Finance*, vol. 27: 926-48.

[8] Bhattacharjee, Arnab and Christoph Thoenissen, (2007). Money and monetary policy in dynamic stochastic general equilibrium models. *Manchester School*, vol. 75(S1): 88-122.

[9] Box, George E. P., (1949). A general distribution theory for a class of likelihood criteria. *Biometrika*, vol. 36: 317-46.

[10] Canova, Fabio and Eva Ortega, (2000). Testing calibrated general equilibrium models. In R. Mariano, T. Shuermann and M. Weeks (eds.) *Inference using Simulation Techniques*, Cambridge University Press.

[11] Chadha, Jagjit. S, Norbert Janssen and Charles Nolan, (2001). Productivity and preferences in a small open economy. *Manchester School,* vol. 69: 57-80.

[12] Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R. McGrattan, (2002). Can sticky price models generates volatile and persistent real exchange rate?. *Review of Economic Studies*, vol. 69: 633-63.

[13] Corsetti, Giancarlo, Luca Dedola and Sylvain Leduc, (2004). International risk-sharing and the transmission of productivity shocks. *ECB Working Paper Series* No.308. Forthcoming in the *Review of Economic Studies.*

[14] den Haan, Wouter, (2000). The comovement between real activities and price at different business cycle frequencies. *Journal of Monetary Economics*, vol. 46: 3-30.

[15] Devereux, Michael B. and Charles Engel, (2002). Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect. *Journal of Monetary Economics*, vol. 49: 913-40.

[16] Dornbusch, Rudiger, (1976). Expectations and exchange rate dynamics. *Journal of Political Economy*, vol. 84: 1161-76.

[17] Engel, Charles, (2000). Comment on Obstfeld and Rogoff. *NBER Macroeconomics Annual.* National Bureau of Economic Research.

[18] Fitzgerald, Doireann, (2003). Terms-of-trade effects, interdependence and cross-country differences in price levels. UC-Santa Cruz, *Mimeo.*

[19] Frankel, Jeffrey, (1996). How well do foreign exchange markets work: might a Tobin tax help?. In M. ul Haq, I. Kaul and I. Grunberg (eds.) *The Tobin Tax: Coping with Financial Volatility*, Oxford: Oxford University Press.

[20] Geweke, John, (1999). Computational experiments and reality. *Mimeo.*

[21] Hall, Robert. E., (1997). Macroeconomic fluctuations and the allocation of time. *Journal of Labor Economics*, vol. 15(S1): 223-8.

[22] Holland, Allison and Andrew Scott, (1998). The determinants of UK business cycles. *Economic Journal*, vol. 108: 1067-92.

[23] Kakkar, Vikas, (2003). The relative price of non-traded goods and sectoral total factor productivity: an empirical investigation. *Review of Economics and Statistics*, vol. 85: 444-52.

[24] Kehoe, Patrick J. and Fabrizio Perri, (2002). International business cycles with endogenous incomplete markets. *Econometrica*, vol. 70: 907-28.

[25] King, Robert G. and Mark W. Watson, (1998). The solution of singular linear difference systems under rational expectations. *International Economic Review*, vol. 39: 1015-26.

[26] Kollmann, Robert, (2003). Monetary policy rules in an interdependent world. CEPR Discussion Paper No. 4012.

[27] Kullback, Solomon, and Richard A. Leibler, (1951). On information and sufficiency. *Annals of Mathematical Statistics*, vol. 22. pp.79-86.

[28] Lane, Philip R. and Gian Maria Milesi-Ferretti, (2002). External wealth, the trade balance, and the real exchange rate. *European Economic Review*, vol. 46: 1049-71.

[29] Ledoit, Oliver and Michael Wolf, (2002). Some hypothesis tests for the covariance matrix when the dimension is large compared to the sample size. *Annals of Statistics*, vol. 30: 1081-102.

[30] Nagao, Hisao, (1973). On some test criteria for covariance matrix. *Annals of Statistics*, vol. 1: 700-9.

[31] Obstfeld, Maurice and Kenneth Rogoff, (2000). The six major puzzles in international macroeconomics: is there a common cause?. *NBER Macroeconomics Annual.* National Bureau of Economic Research.

[32] OECD (2004). *STAN database.*

[33] Sarno, Lucio and Mark P. Taylor, (2002). *The Economics of Exchange Rates*, Cambridge University Press.

[34] Schmitt-Grohe, Stephanie and Martin Uribe, (2003). Closing small open economy models. *Journal of International Economics,* vol. 61: 163-85.

[35] Selaive, Jorge D. and Tuesta, Vincente (2003). Net foreign assets and imperfect pass-through: the consumption-real exchange rate anomaly. Board of Governors of the Federal Reserve System, International Finance Discussion Papers, No. 764.

[36] Sorensen, Bent E., Yi-Tsung Wu, Oved Yosha and Yu Zhu (2007). Home bias and international risk sharing: twin puzzles separated at birth. *Journal of International Money and Finance*, vol. 26: 587-605.

[37] Stockman, Alan C and Linda L. Tesar, (1995). Tastes and technology in a two-country model of the business cycle: explaining international comovements. *American Economic Review*, vol. 85: 168-85.

[38] Straub, Roland and Ivan Tchakarov (2004). Non-fundamental exchange rate volatility and welfare. *ECB Working Paper Series* No. 328.

[39] Tesar, Linda L. (1993). International risk sharing and non-traded goods. *Journal of International Economics*, vol. 35 (1-2): 69-89.

[40] Tesar, Linda L. and Ingrid M. Werner (1995). Home bias and high turnover. *Journal of International Money and Finance*, vol. 14: 467-92.

[41] Watson, Mark W. (1993). Measures of fit for calibrated models. *Journal of Political Economy*, vol. 101: 1011-41.

Table 1 - Quarterly Calibration for Small Open Economy Model

Parameter	Values	Description
β	0.99	Discount factor
δ	0.025	Depreciation factor
α	0.67	Labor share
ρ	2	CRRA
η	-4	Elasticity of marginal value of time
θ	1.5	Elasticity: Home/Foreign traded goods
κ	0.74	Elasticity: Traded/Non-traded goods
(υ, υ^*)	$(0.73, 0.02)$	Home prod. share of tradeables (home, overseas)
(ω, ω^*)	$(0.45, 0.45)$	Share of tradeables in output (home, overseas)
ε	70 basis points	Interest spread (quarterly)
$\overline{a}$	0	Steady state Net Foreign Asset
b	10	Cost of capital adjustment
(ρ_A, ρ_{A^*})	0.918	Persistence of traded productivity shocks
(σ_A, σ_{A^*})	$(1.17\%, 1.41\%)$	Volatility of traded productivity shocks
(ρ_{AN}, ρ_{AN^*})	0.945	Persistence of non-traded productivity shocks
$(\sigma_{AN}, \sigma_{AN^*})$	$(0.51\%, 0.56\%)$	Volatility of non-traded productivity shocks
(ρ_ξ, ρ_{ξ^*})	0.937	Persistence of preference shocks
$(\sigma_\xi, \sigma_{\xi^*})$	$(0.82\%, 0.82\%)$	Volatility of preference shocks
$(\rho_{UIP^H}, \rho_{UIP^L})$	0.88 or 0.38	Persistence of UIP deviations (high or low)

Note: We have an utility function similar to Chadha *et al.* (2001). The elasticity of intertemporal substitution in leisure $\frac{1}{\eta-1}$ is -0.2; the elasticity of labor supply in this model is around 4; the discount factor β, CRRA ρ, depreciation coefficient δ and labor share α are taken from standard open economy and real business cycle literature such as Corsetti *et al.* (2005), Chari *et al.* (2002); we take elasticity of substitution among consumables θ, κ from Corsetti *et al.*; the share of traded goods ω, ω^* are taken as 0.45 in accordance with the literature; for home bias feature in traded goods, we take average value share of UK produced goods in UK and US GDP, υ and υ^*, respectively; interest spread ε is a yield discount when holding foreign bond and is calibrated as 280 base points annually by Selaive and Tuesta (2003); the cost of capital adjustment b is calibrated to match UK output volatility; we set Net Foreign Asset position $\overline{a}$ as zero in benchmark case; the persistence and volatility of shocks are estimated on UK data.

Table 2 - Variance Decomposition of Current Account, Real Exchange Rate and Relative Consumption

	(a) All shocks				(b) Excluding shocks:		
	Persistent UIP Dev.	Temporary UIP Dev.	Net Creditor[1]	Net Debtor[1]	Productivity	Preference	UIP
Current Account (CA/Y)							
Traded productivity, Home	0.2%	1.6%	0.1%	0.4%	—	0.2%	28.6%
Non-Traded productivity, Home	0.4%	3.1%	1.0%	3.1%	—	0.4%	55.6%
Traded productivity, Foreign	0.0%	0.1%	0.0%	0.0%	—	0.0%	1.5%
Non-Traded productivity, Foreign	0.0%	0.0%	0.0%	0.0%	—	0.0%	0.1%
Preference, Home	0.1%	0.5%	0.1%	0.1%	0.1%	—	9.0%
Preference, Foreign	0.0%	0.3%	0.0%	0.1%	0.0%	—	5.2%
UIP	99.3%	94.4%	98.8%	96.3%	99.9%	99.4%	—
Real Exchange Rate (RER)							
Traded productivity, Home	0.8%	5.6%	0.9%	0.8%	—	0.8%	35.9%
Non-Traded productivity, Home	0.0%	0.1%	0.0%	0.0%	—	0.0%	0.7%
Traded productivity, Foreign	0.1%	0.6%	0.1%	0.1%	—	0.1%	3.8%
Non-Traded productivity, Foreign	0.5%	3.2%	0.5%	0.4%	—	0.5%	20.1%
Preference, Home	0.4%	2.7%	0.4%	0.4%	0.4%	—	17.4%
Preference, Foreign	0.5%	3.5%	0.5%	0.5%	0.5%	—	5.2%
UIP	97.7%	84.3%	97.6%	97.8%	99.1%	98.6%	—
Relative Consumption (CC*)							
Traded productivity, Home	14.4%	16.0%	14.5%	14.3%	—	15.8%	16.2%
Non-Traded productivity, Home	62.8%	70.0%	62.6%	63.0%	—	69.2%	71.0%
Traded productivity, Foreign	0.8%	0.9%	0.8%	0.8%	—	0.9%	0.9%
Non-Traded productivity, Foreign	1.3%	1.4%	1.3%	1.3%	—	1.4%	1.4%
Preference, Home	4.7%	5.3%	4.8%	4.7%	22.8%	—	5.3%
Preference, Foreign	4.5%	5.0%	4.5%	4.5%	21.9%	—	5.1%
UIP	11.5%	1.3%	11.6%	11.4%	55.4%	12.6%	—

Table 3 - Results of benchmark UK/US calibration

	UK Data	(a) All shocks				(b) Excluding shocks:		
		Persistent UIP Dev.	Temporary UIP Dev.	Net Creditor[1]	Net Debtor[1]	Productivity	Preference	UIP
Relative volatility to output (interest rate and CA/Y take raw value)								
Consumption	*0.78*	0.92	0.64	0.91	0.93	1.09	1.04	0.59
Investment	*2.30*	3.99	2.84	3.91	4.08	4.62	4.58	2.60
Interest rate	*1.01*	0.42	0.37	0.40	0.43	0.42	0.42	0.05
Labour	*0.95*	0.90	1.00	0.89	0.90	1.13	0.36	1.02
Wage	*0.86*	0.89	0.65	0.88	0.90	1.01	1.02	0.61
RER	*4.89*	5.16	1.91	5.13	5.20	6.71	6.21	0.67
ToT	*1.66*	5.88	2.47	5.85	5.91	7.50	7.08	1.46
CA/Y	*1.06*	1.99	0.61	4.12	2.80	1.99	1.99	0.12
CC*	*1.27*	1.13	0.94	1.12	1.14	1.22	1.24	0.92
Correlation with output								
Consumption	*0.79*	0.90	0.97	0.90	0.89	0.91	0.90	1.00
Investment	*0.79*	0.88	0.93	0.89	0.88	0.89	0.90	0.96
Interest rate	*0.19*	−0.33	−0.08	−0.33	−0.33	−0.51	−0.43	0.90
Labour	*0.78*	0.56	0.79	0.57	0.56	0.55	0.14	0.82
Wage	*0.13*	0.56	0.32	0.56	0.55	0.37	0.94	0.27
RER	*-0.10*	−0.46	0.00	−0.46	−0.45	−0.63	−0.58	0.57
ToT	*-0.13*	−0.47	−0.05	−0.47	−0.46	−0.64	−0.59	0.19
CA/Y	*-0.30*	−0.52	−0.22	−0.36	−0.22	−0.65	−0.64	−0.13
CC*	*0.19*	0.64	0.48	0.65	0.64	0.83	0.65	0.46
Correlation with RER								
ToT	*0.10*	0.98	0.81	0.98	0.98	1.00	0.98	0.21
CC*	*-0.61*	−0.65	−0.02	−0.65	−0.66	−0.83	−0.75	0.76

Table 4 - Model Selection: Distance Measures between Data and Model VCMs

VCM Distance Calculation Method	All Shocks				
	Persistent UIP Dev.	Temporary UIP Dev.	i.i.d UIP Dev.	Net Creditor	Net Debtor
RMSE	0.0462%	0.0431%	0.0444%	0.0620%	0.0388%∗
MAE	0.0241%	0.0122%	0.0121%∗	0.0313%	0.0204%
Box-Bartlett	92053	99787	99841	44379	43947∗
Kullback-Leibler	432	468	469	208	206∗
Nagao	2.74×10^7	3.18×10^7	3.18×10^7	5.12×10^6	4.95×10^6∗
Ledoit-Wolf	2.70×10^7	3.14×10^7	3.14×10^7	5.02×10^6	4.85×10^6∗

Notes to Tables 2 to 4: The quarterly data is the HP filtered series of OECD MEI, 1980-2006: RER is real exchange rate; CA/Y is current account to GDP ratio; ToT is terms of trade and is import price over export price; CC* is relative consumption to US. The base case calibration is as Table 1. The UK is the home country. Net creditor calibration is set at $\overline{a} = 0.5$ while net debtor sets $\overline{a} = -0.5$ for a small open economy; the UIP shock has an AR(1) coefficient of $\rho_{UIP} = 0.88$ in the persistent case as in Kollmann (2003), whereas the temporary case and i.i.d cases correspond to $\rho_{UIP} = 0.38$ and $\rho_{UIP} = 0$ respectively; in both net creditor and net debtor case the UIP deviations are persistent; RMSE denotes root mean squared errors; MAE denotes mean squared errors; each of the distance metrics is discussed in Appendix B; the asterisk (*) denotes minimum distance measure across all the five calibrations.
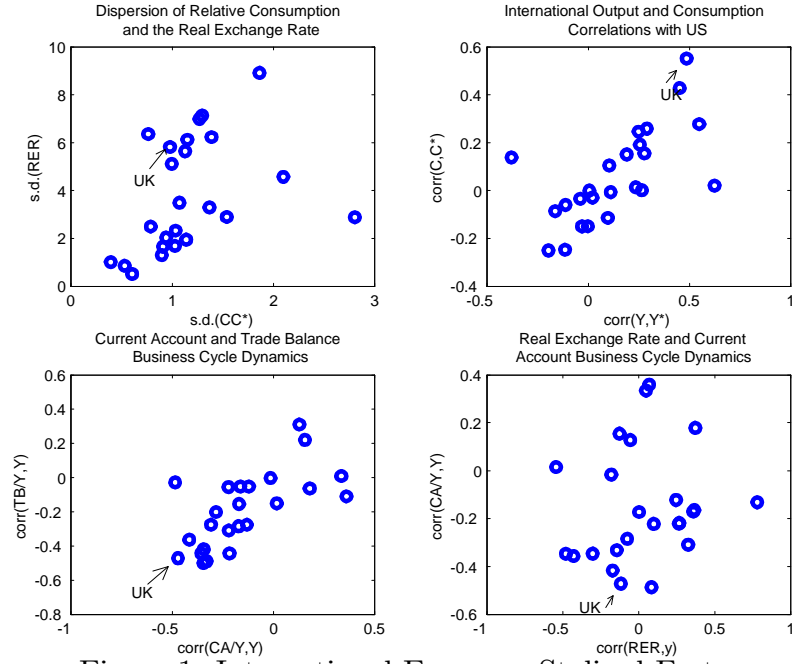
Figure 1: International Economy Stylised Facts

Note: Quarterly data from 1980 to 1998 for 24 OECD and emerging market economies is obtained from the IMF IFS database. s.d. denotes standard deviation of HP-filtered series of the variables. corr denotes the correlation coefficient between two HP-filtered series. RER denotes bilateral real exchange rate. C, C*, Y, Y* are household consumption and real GDP of small open economy and US respectively. CC* is the relative consumption to US. TB/Y is the ratio of trade balance to output and CA/Y the ratio of current account to output.

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UK Current Account and Output

US Current Account and Output

UK Relative Consumption to US and Bilateral Real Exchange Rate
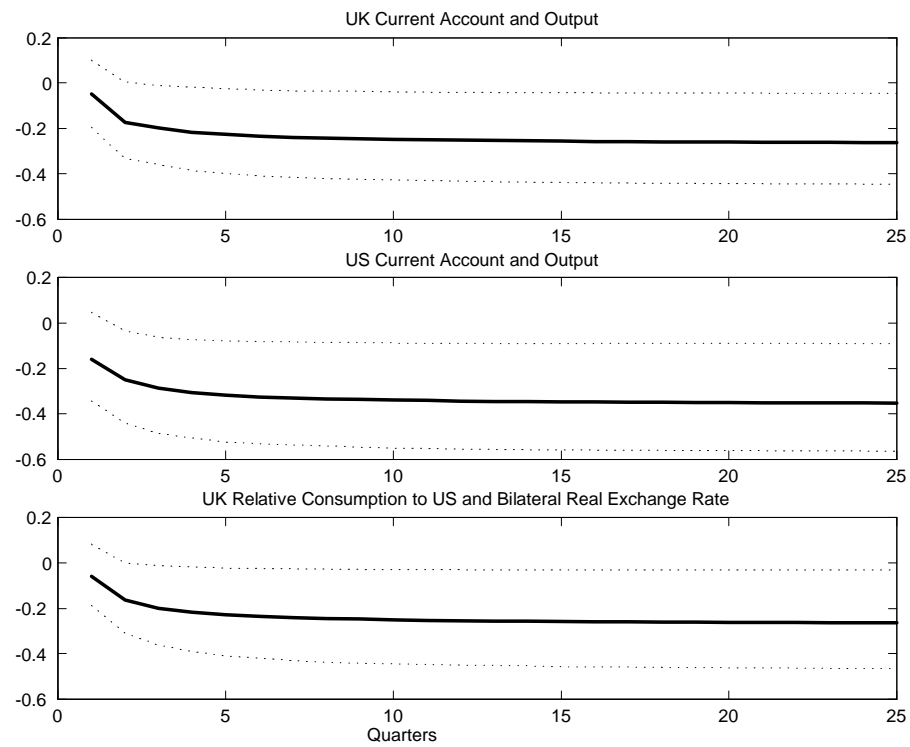
Note: Forecast error correlation calculated with Den Haan (1996) code. Solid lines denote correlations and the dotted lines represent a 90% confidence interval of point estimate.

Figure 2: Price Stickiness

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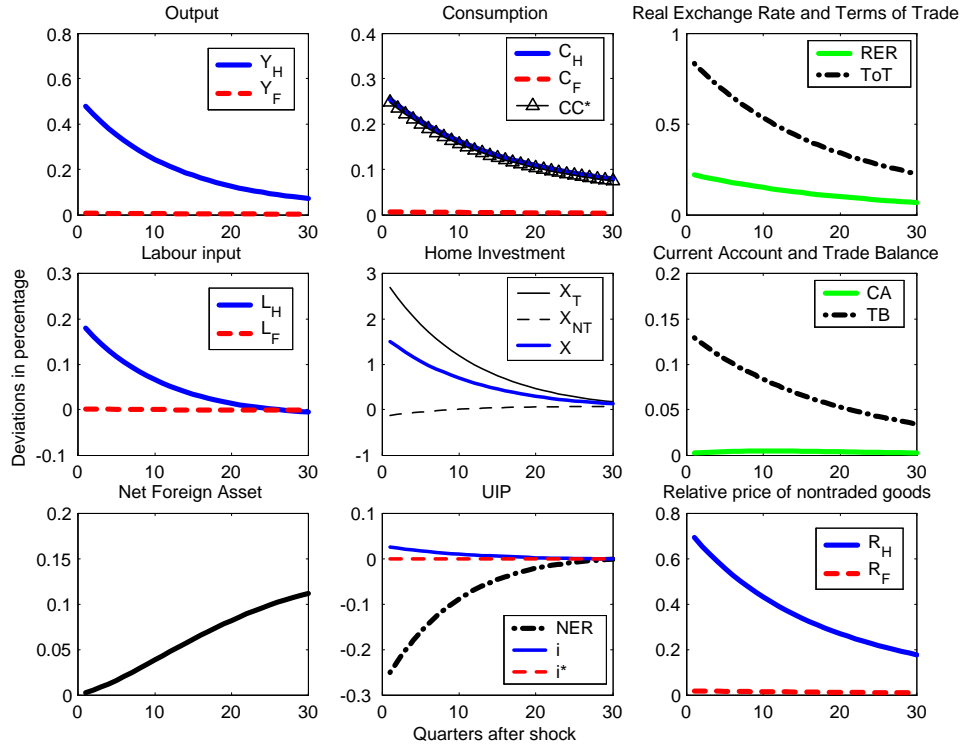
Figure 3: Response to Traded Productivity Shock

Notes for Figures 3 to 6: The impulse responses show percentage deviation from steady state from period 1 when there is a 1% shock to traded productivity: RER - real exchange rate; CA, TB - current account and trade balance measured as percentage of output; NER - nominal exchange rate; i, i* - interest rate of small open economy and US; CC* - relative consumption to US; subscript H denotes home country whereas F denotes foreign country; subscript T denotes traded sector and NT denotes non-traded sector.
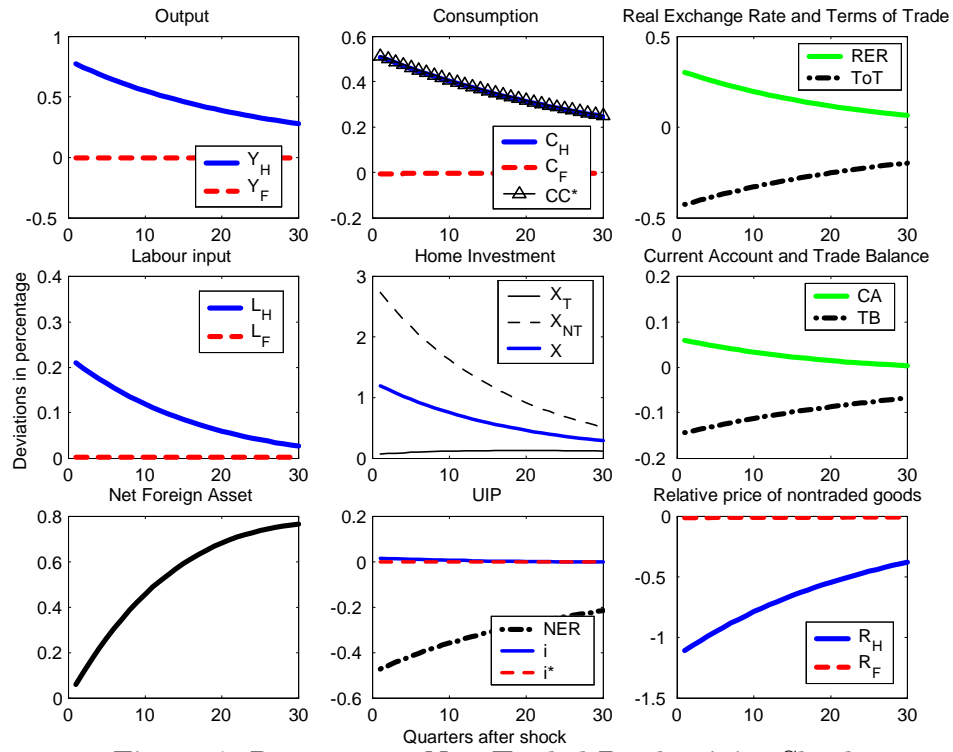
Figure 4: Response to Non-Traded Productivity Shock
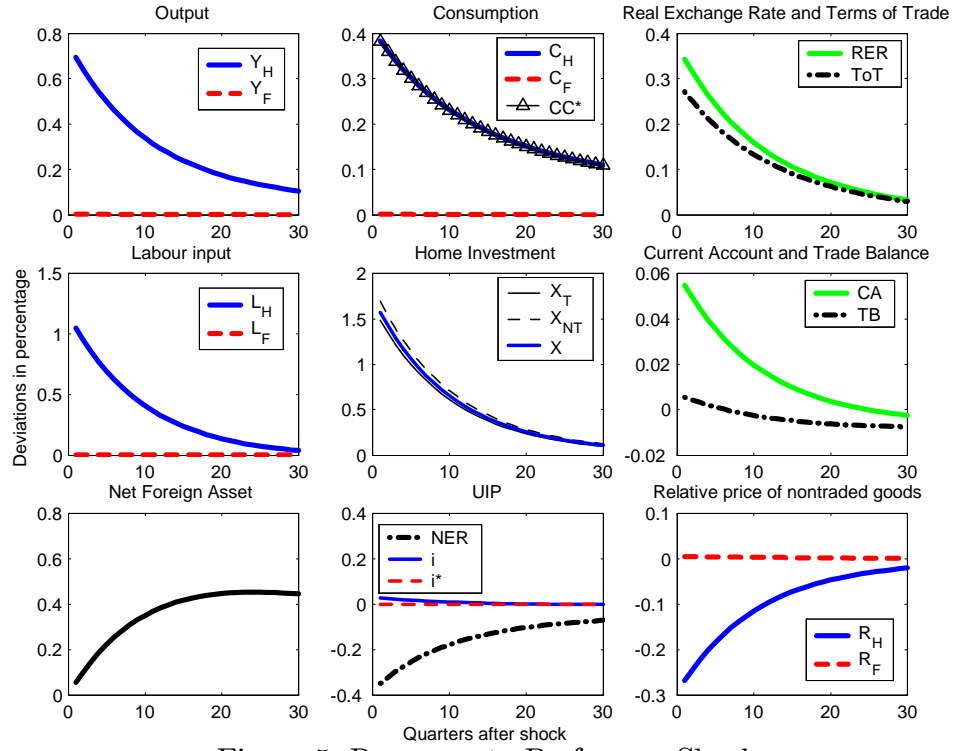
Figure 5: Response to Preference Shock
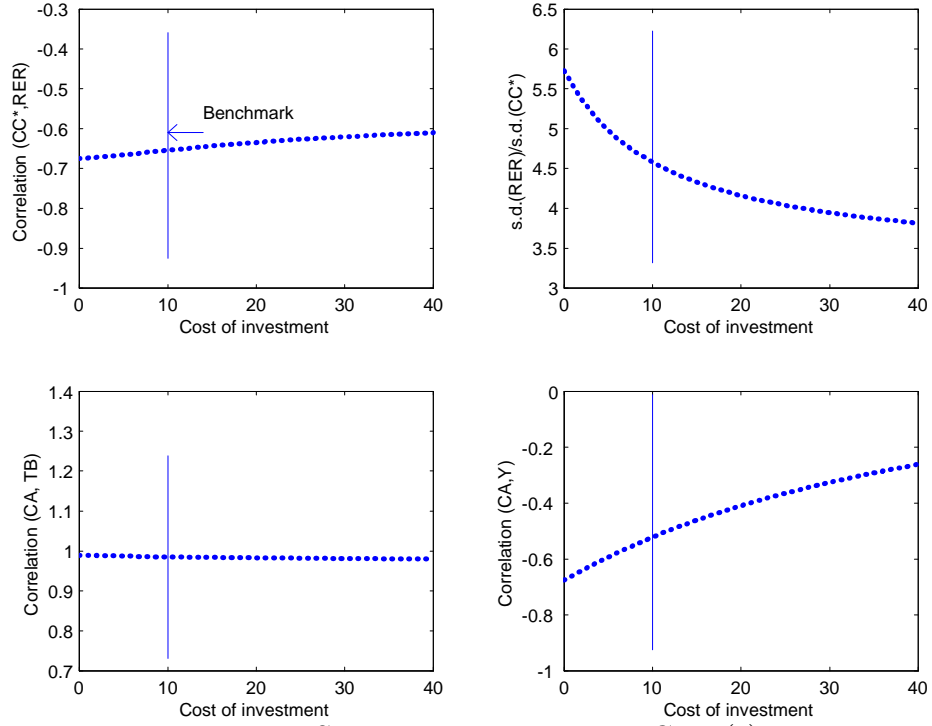
45
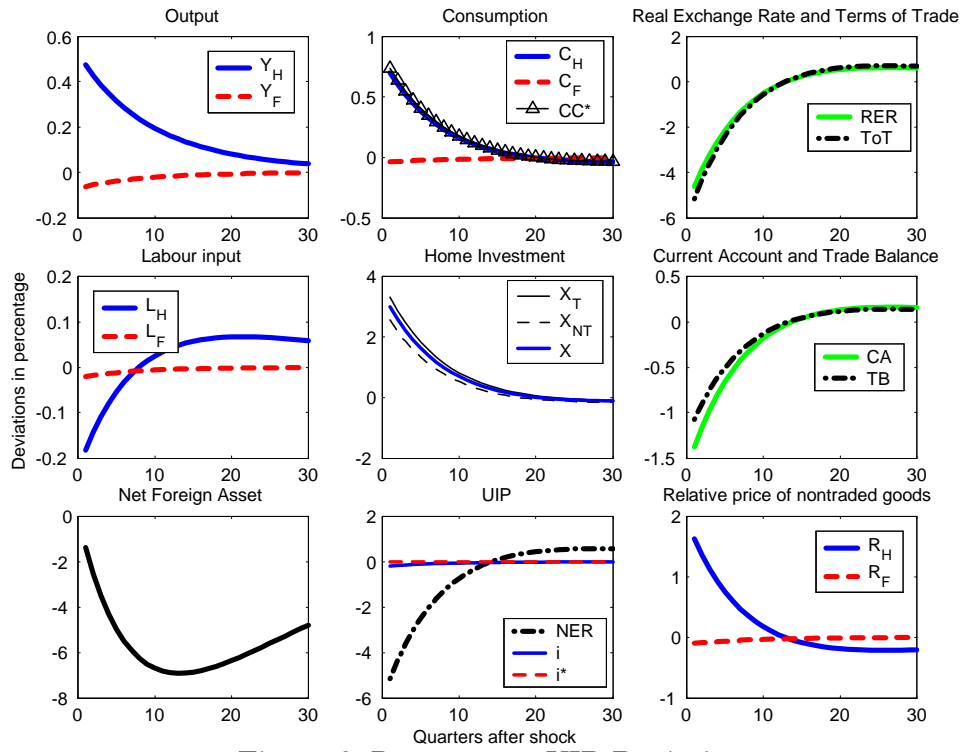
Figure 6: Response to UIP Deviation

Figure 7: Sensitivity - Investment Cost (b)

Note: The charts show the sensitivity to the investment cost coefficient. The vertical line denotes the benchmark calibration of Table 1. The four charts show the key correlation and relative volatility statistics from calibrated model. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.
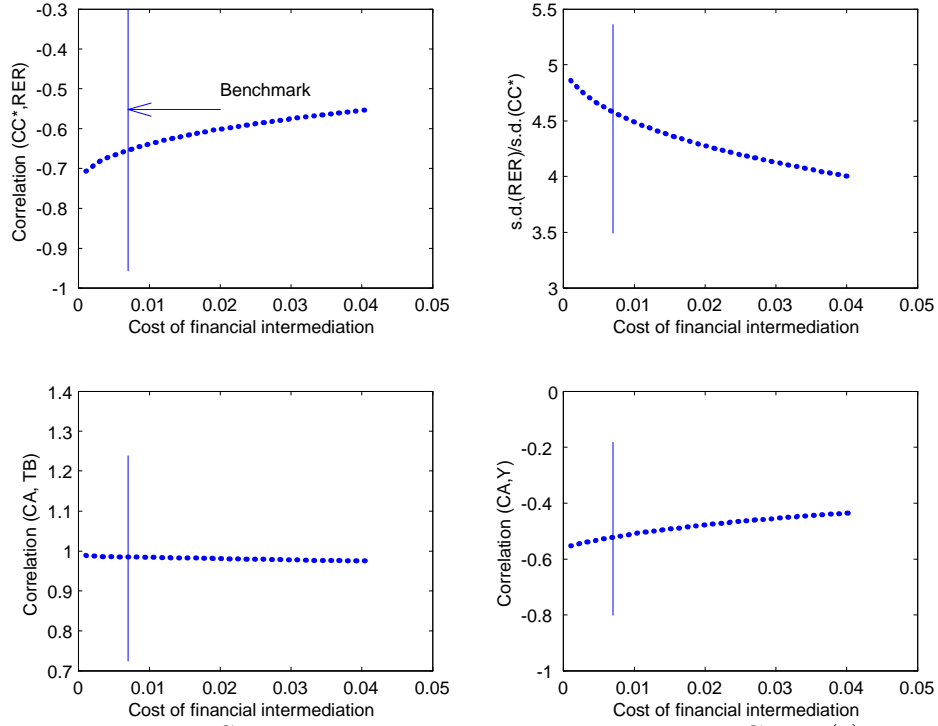
Figure 8: Sensitivity - Financial Intermediation Costs (ϵ)

Note: The charts show the sensitivity to the bond holding cost coefficient. The vertical line denotes the benchmark calibration of Table 1. The four charts show the key correlation and relative volatility statistics from calibrated model. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.
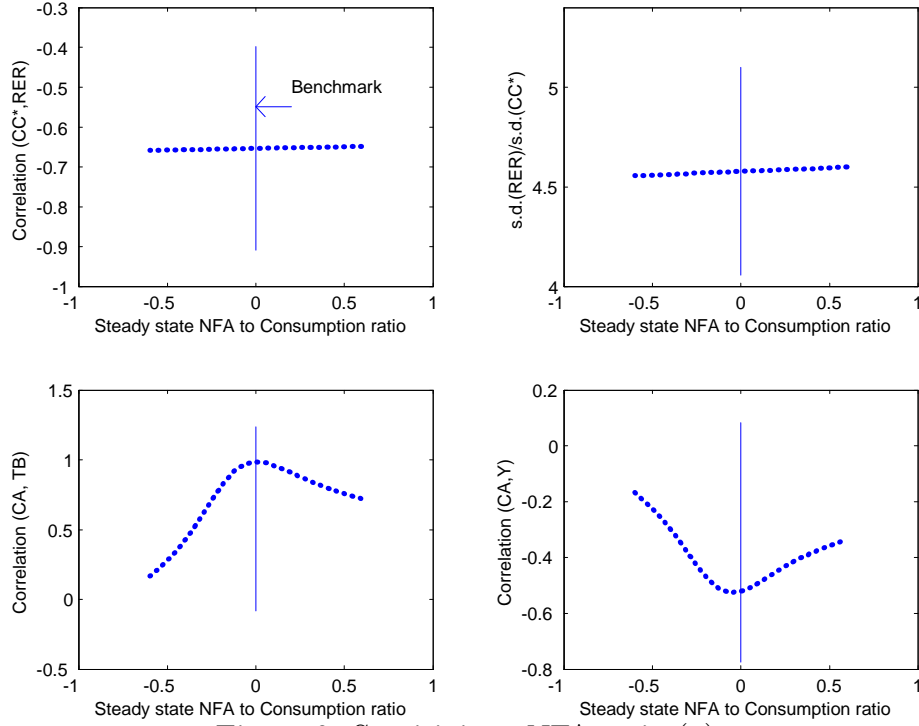
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Figure 9: Sensitivity - NFA ratio $(\overline{a})$

Note: The charts show the sensitivity to the steady state of net foreign asset position coefficient. The vertical line denotes the benchmark calibration of Table 1. The four charts show the key correlation and relative volatility statistics from calibrated model. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.
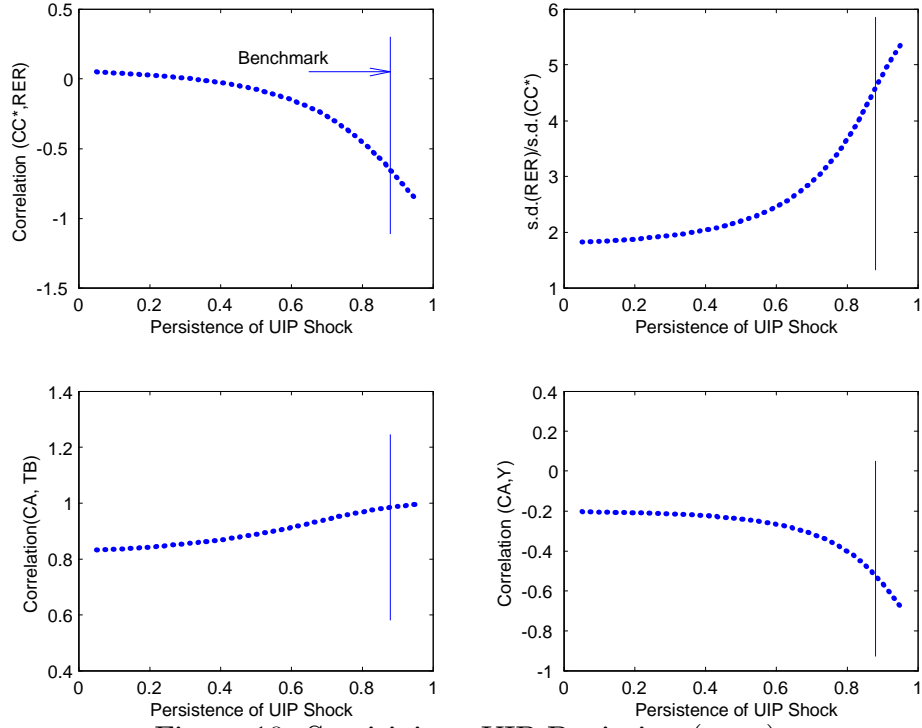
Figure 10: Sensitivity - UIP Deviation (ρ_{UIP})

Note: The charts show the sensitivity to the UIP shock persistence coefficient. The vertical line denotes the benchmark calibration of Table 1. The four charts show the key correlation and relative volatility statistics from calibrated model. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.
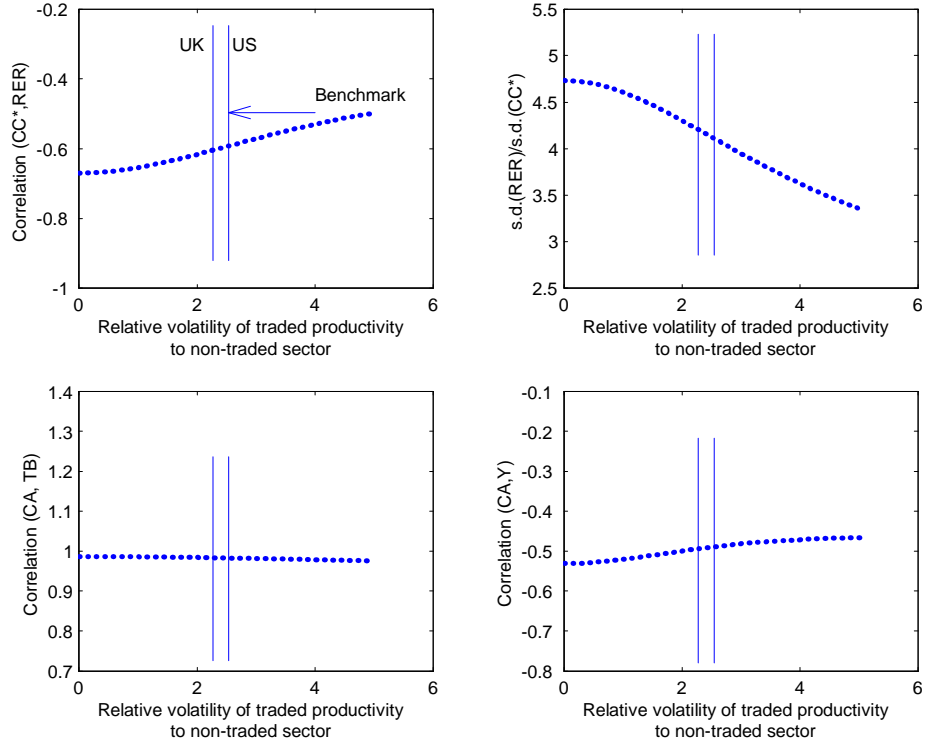
Figure 11: Sensitivity - Traded Sector Volatility (σ_A/σ_{AN})

Note: The charts show the sensitivity to the relative volatility coefficient. The vertical line denotes the benchmark calibration of Table 1. The four charts show the key correlation and relative volatility statistics from calibrated model. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.

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Figure 12: Sensitivity - Preference Shock Volatility (σ_P)

Note: The charts show the sensitivity to the volatility of preference shocks. The vertical line denotes the benchmark calibration of Table 1. The four charts show the key correlation and relative volatility statistics from calibrated model. CC* denotes relative consumption to US; RER: real exchange rate; CA: current account; TB: trade balance; Y: real output. CA and TB are measured as ratio to GDP.
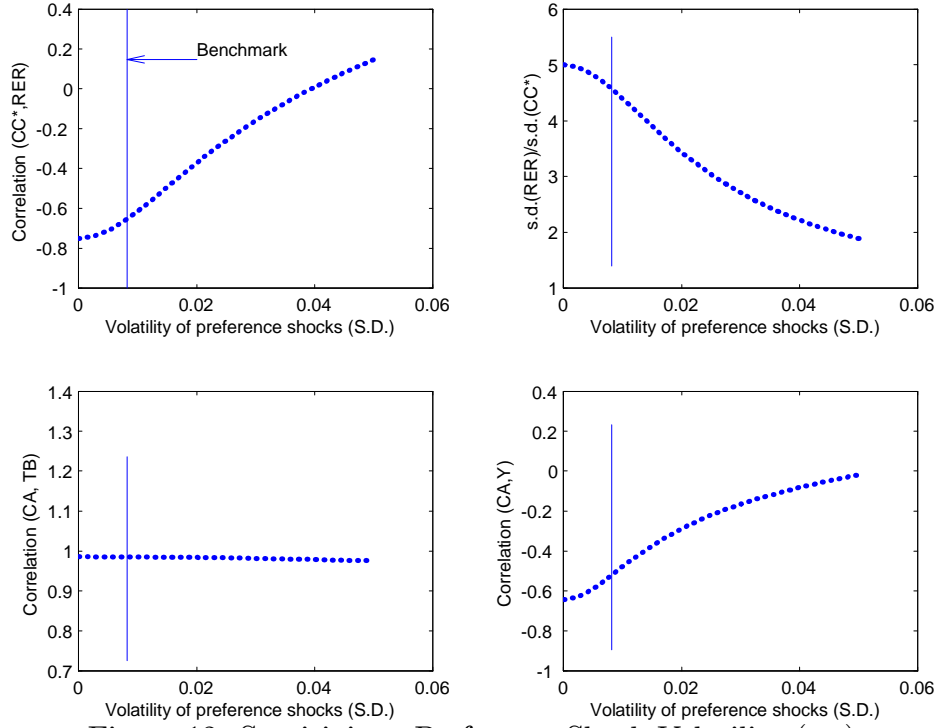
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