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An Investigation of the Effects of Analogy on Matching Games

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An Investigation of the Effects of Analogy on Matching Games

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Abstract: Analogy is now an active field of research in psychology with many experimental results available and some theoretical work developed. Within the economics literature analogy has made little headway and there has been no attempt to empirically investigate the phenomenon. A new experiment is reported which investigates the role of analogy in picking a strategy in a matching game. It is found that analogy does exist and its effects are consistent with the use of current theories of choice in pure coordination games.

Keywords: Analogy, Matching games

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Analogy has been little studied in the economics literature in spite of its acknowledged role in the reasoning process. However, in spite of increasing empirical research in the psychology literature and the increased interest in behavioural explanations in economics, little empirical work has been done on analogy as a reasoning process. This paper intends to make an initial contribution to the analogy literature by defining analogy within game theory and then testing to see what influence analogy has on the selection of equilibria in games.

Examples of analogical reasoning abound. One example is in the formation of stereotypes. When one meets someone from another country then one may make judgements about them based on what one knows about the characteristics of people from that country. One would make an analogy to extrapolate how one would behave when meeting such a person. Another use of analogy would be when making moral judgements. If, for example, one is wondering whether to give money to a beggar then one's behaviour depends on one's previous similar experiences.

Another area in which analogical reasoning may be useful would be when firms decide to start up a branch in another country. In a different country they would be faced with different business cultures and so may use analogies in order to inform their behaviour. Such an analogy could either be an analogy with the company's own past behaviour in its home country or it could be an analogy with other companies' current behaviour in the new country.

One thing to notice here is that these analogies are not always correct. In the case of firm behaviour, for example, making an analogy with one's previous experience could be disastrously wrong. Likewise, copying the behaviour of the wrong firms could also result in an error. This implies that analogies do not necessarily act as "corrective" tools but only as a method of solving a problem whether it is done correctly or not. Another point is that analogies need not emerge from one's own behaviour but could come from how other people behave in the same circumstances. Thirdly analogies can be created in a wide range of circumstances and form a very general class of reasoning. Finally, as shown in the last example, there are many potential economic applications of analogical reasoning. For this reason it is worth investigating further.

1- Discussion of analogy in games and decisions

There has been limited discussion of analogical and similarity reasoning in the economics literature. Kreps (1990) discusses analogical reasoning informally when analysing how people interpret games which are not identical. Suppose, for example, that a game has the same strategies and number of players as a previous game but varies slightly in the payoffs while preserving ordinal payoff rankings. The game might very well be understood by looking at the similarities between the two games and so similar strategies may be played. In fact Kreps argues that this is precisely what people do when they are learning game theory. The solution used in one game is carried over to a similar but not identical game. Furthermore, as Kreps claims, it is certainly true that the use of subgames within game theory uses an analogy between the solution of a small game and that of a subgame to help with the solution of the whole of the larger game.

There is also the possibility that analogy may be invoked when choosing an equilibrium in a multiple equilibrium game. Such a possibility has been put forward informally by Sugden (2004) as a means of deciding how to solve coordination games. This could be seen as a "benign" use of analogy. A more "malignant" use of analogy was suggested by Binmore (1999). According to Binmore, social conventions may be used by experimental subjects in order to interpret games in experiments. As a result of these conventions the subjects may not behave rationally and it requires a period of trial- and error learning to eliminate these biases. It seems reasonable to suppose that the reason that these social conventions are invoked by subjects is because the game seems analogous to the situation in which the convention is usually applied.

While there have been informal discussions of analogy such as these in the literature there have also been some more formal attempts to theorise about analogy and similarity. A theory of case based reasoning has been proposed by Gilboa and Schmeidler (1995). This theory builds up a theory of choice derived from revealed preferences constrained by axioms. A value function for an outcome in a particular case depends on a measure for the utility of the payoff multiplied by a measure for the similarity of the case to other previous cases in the agent's memory. Choices are then made by maximising the value function This is a subjectivist theory of case- based reasoning as there is no attempt to define whether there are objective similarities between actions in each case. Similarity is assumed to be a perceptual, subjective concept. A similar, although less detailed, account of a similarity relation and its relationship with preference is given by Rubinstein (1988).

Another theoretical investigation of similarity and analogy is that of Jehiel (2005). Jehiel's aim is to investigate the ways in which analogy limits human reasoning within game theory. Nodes played by opponents in games are bundled into "analogy classes" from which an analogical expectation is derived from the average behaviour in those classes. From these expectations an equilibrium can be derived from the best replies to one's analogical expectations of how one's opponents are going to behave. One important point here is that in Jehiel's theory analogy classes are exogenously imposed so there is no determination *inside the model* of how these analogy classes are formed.

This means that there is no theoretical guidance to experimenters as to how analogy should be tested in an economic experiment. It may be possible to test Gilboa and Schmeidler's axioms in the same way that, for example Savage's axioms have been tested. However, there is no guide in an *objective* sense as to what object could be considered to be similar to another object. Jehiel's theory is even worse in this respect in that his analogical classes are simply assumed. It follows that there is a need to demonstrate the existence of analogy within a game theoretic setting.

This latter point is particularly crucial as it is not immediately obvious that all potential analogies can be used in a particular decision context. It may be the case that some analogies are not noticed or not used. It may be the case that the decision problem is too complex for analogical reasoning to be that effective. It is insufficient therefore to simply assume that analogies exist. One has to discover the conditions in which they can be used.

A further objection against these theoretical analyses of similarity and analogy is that, in fact, they only, strictly, deal with similarity. To take an example, in Jehiel's theory, all the members of an analogy class must have the same numbers of actions. There is no means of varying the number of actions and still perceiving similarity. The Gilboa- Schmeidler theory has a similar limitation. However, this would seem to go against the whole idea of analogy which is to find *partial* similarity between the cases. It does not specify that the problems need be identical. There is certainly no need for the strategic framework (as opposed to the context) to be identical. Indeed

Kreps (1990) ideas on how people actually learn to solve games relies on a notion of analogy in which there is only partial correspondence between games.

2- Discussion of analogy in psychology

There has been a large amount of work done in recent years on analogy from both theoretical and experimental points of view (See Gentner 2002 for an overview). Analogical mapping is the crucial part of the analogical mechanism which we shall be interested in this paper. An analogical mapping consists of matching a familiar situation or a *base* description with a less familiar situation or a *target* description. It requires a correspondence between elements of the two descriptions and then inferring how one should behave in the less familiar situation from how one behaved in the more familiar situation.

The main theory of analogy in the psychological literature is Gentner's *structure- mapping theory* (See Gentner 1983, 2001) where the aim is for subjects to find an alignment between elements of the base and target representations. This alignment has to have a one- to- one correspondence between some elements of the representations. Such an alignment binds the target and base representations together and can then be used to project further alignments from the base to the target.

Not everything between analogues can or should be aligned and it is a problem as to which elements are important in a particular analogy. There seem to be three factors involved; relevance to the goal of the problem being solved in the target situation; generality of the alignment between the base and target problems and the ease with which the base representation fits into the target representation.

One aspect of analogy which is of importance is that it has been shown to play a role in decision making (Markman & Moreau 2001). In fact analogy applies to many areas of decision making. Analogies can be used to frame a decision (as suggested by Binmore1999). They can also be used to deal with "affective uncertainty" where one is uncertain about the utility which can be derived from certain outcomes (See Cubitt et al. 2001). Analogy can also be used to form a comparison set of possible options by using a similar better known problem as an analogue.

The final use of analogy in decision making is when subjects use problems as analogues when making a decision between options. In particular, decision makers will form an analogy between one decision problem and another and concentrate on those alternatives which hold for both analogues. Subjects then transfer the method used to solve the base decision problem over to the target problem (Gick & Holyoak 1983). This form is the type of analogy which will be studied in this paper. The other forms of analogy used in decision theory, such as that used to resolve affective uncertainty or to create an option set, will be assumed not to hold as all the options will be known and well- specified. There will also be controls over the framing of a given problem so that it can only be framed in particular ways.

It follows that analogy could profitably applied to a decision theory context. Given a common goal in decision making (to maximise utility) between problems then an analogy might be expected to work by isolating the common set of alternatives between two analogues and then using the method of solution in the base problem to solve the target problem.

3- A Definition of Analogy within game theory

As can be seen from the above discussion, analogy has been discussed in game theory both within a formal and an informal framework. However, in all cases, there has been little attempt to seriously study analogy in the manner outlined by psychologists. A game, in the psychological view, should be seen as a problem which needs to be solved by the participants. An analogy, on this viewpoint, would be a partial mapping between one game and another so that the strategy used to find an equilibrium in one game would also be used to find an equilibrium in the second game. This differs considerably from the ideas of analogy put forward by Jehiel or Gilboa and Schmeidler where analogy is simply a symptom of bounded rationality and so acts as a *constraint* on the subjects rationality.

Analogy is seen here as an attempt by the subject to learn. However, it should be realised that this does not mean that current theories of learning in economics can be applied to the games. Learning theories rely on there being a *full* mapping between the structure and payoffs of the game so that the stage games are effectively identical to each other. Analogy, by contrast, relies on information transmitted as part of a partial mapping.

An example of analogy in game theory would be, for example, if a person played a stag- hunt game as if it was a prisoner's dilemma and as a result continuously played the risk- dominant strategy. In this case the player would be using an analogy between the prisoner's dilemma as the base game and the stag hunt as the target game in order to play the latter. In this case there is a partial mapping of some of the payoffs (or the *ordering* of some of the payoffs) in the prisoner's dilemma to the stag game.

Logically there are a variety of ways in which analogy could be used within game theory. Given that a game is composed of players, strategies, information sets and payoffs, in theory one could have a mapping between games which varied on any of these aspects. One could have, for example, a mapping between games which varied in payoffs but had the same game structure and number of players. Indeed one experiment which tests this has been done by Rankin et al. (2000)[2] which is a variant on the comment made by Kreps (1990) outlined in the first section.

As was mentioned in the previous section, there have been a variety of psychological investigations of analogy in decision making (Markman & Morceau 2001). It can be seen that each of the possible ways mentioned above (apart from the number of players) has been considered in the psychology literature. As we mentioned before, we will be ignoring the effects on payoffs and information sets (as well as players) and instead concentrate our analysis on strategies and framing.

In this paper we will focus on analogy between games which have different numbers of strategies but the same number of players and identical payoffs. In particular we will be looking at normal form games which have two players and are symmetric both for the base and the target games. Symmetry is used here to encourage players to use analogies both from their past play but also from their opponent's play. The payoffs remain the same for those strategies which are extant in both the base and target games. This is not to say that other results cannot be obtained through other restrictions but this one seemed one of the most intuitive ways of representing analogy.

In order to tightly define an analogy we will use the notion of a *substructure* (Harsanyi and Selten 1993). Roughly, a substructure is a game G' which results from reducing the option set Φ of the game G to Φ' by eliminating pure strategies[3]. It will

[2] Rankin et al.'s experiment is strictly a limited test in that the ordering of the payoffs is preserved.
[3] Note that it is assumed that we are looking at the agent normal form so it is assumed that agents and players are identical and there is no need to "fix" agents.

be assumed in this paper that the base game of an analogy is identical to the substructure of the target game. The analogy therefore is a full mapping between the base game and the substructure of the target game. This becomes a partial mapping by adding strategies to the option set in the target game.

It may be objected that this is a peculiar definition of an analogy since it seems to eliminate a variety of other possibilities, in particular the possibility that the target game is a substructure of the base game. This is possible as analogy merely requires a partial mapping between the two and does not specify which should be the most complicated. However it could be argued that there is an informational asymmetry in that the base game is more easily solved if it is less complicated than the target game. It may then be argued that the aim of an analogy should be to make the finding of a solution in the target game comparatively more easy than it would otherwise be. There would be *no point* in using a complicated analogue to find the solution for an easy target game. This possibility should therefore be eliminated on pragmatic grounds.

An analogy therefore will be *defined* in this paper as a mapping between a base game and a substructure of the target game. Furthermore it will be assumed, for the sake of this paper, that the substructures are formed by eliminating symmetric strategies from the target game.

Definition: An analogy in two player symmetric games is where there is a mapping Θ: $L(G^*) \rightarrow L(G')$ where L is a solution in game G* (the base game) and G' (a substructure of G) given that G* and G' are identical in structure, players and payoffs.

The mapping Θ therefore takes any solution to the base game G* and transfers it over to the substructure G' of game G where it acts as a solution to the game G. The function L denotes a generic solution to the games G* and G' which need not be a Nash Equilibrium.

The most plausible candidate for a solution based on analogy in game theory is Correlated Equilibrium (Aumann 1987). This is because Correlated Equilibrium is a method by which an external signal can be used by players in a game to create a self-enforcing equilibrium. This is done by the players either privately or publicly observing a random event and then basing their choice of action on it. The contention here is that an analogy in the form of a correlated equilibrium from a base game is a method by which the players in the target game may be able to correlate their strategies and so find a solution for the latter.

The question is: why should correlated equilibria be a better solution concept than any other for analogy? Why not use Nash Equilibrium? The answer to this is that if an analogy is available publicly (as is assumed in this paper[4]) then it is always available as a possible correlating device. This means that a correlated equilibrium is always possible in the presence of an analogy and so it should be modelled as such.

It should be noted that this is simply a mechanism by which a simpler solution to a problem can be used to solve a more complex problem. This does not state how the correlated equilibrium in the base game was originally achieved but rather shows how this can be transferred to the target game. As an equilibrating device, analogy is incomplete but it does allow a step approach. The correlated equilibrium in the base game therefore may derive from some other source.

This implies that any theory of analogy should be able to transfer a correlated equilibrium from the base game to the target game. It can easily be seen that a correlated equilibrium in a target game which has as supports the strategies in the substructure is also a correlated equilibrium in the substructure. If a base game is identical to this substructure then a correlated equilibrium in the base game is also a correlated equilibrium in the target game.

To see this take the definition of a correlated equilibrium of a target game:

$$\sum_{\omega \in \Omega} \pi(\omega) u_i(\sigma_{-i}(\omega), \sigma_i(\omega)) \geq \sum_{\omega \in \Omega} \pi(\omega) u_i(\sigma_{-i}(\omega), \tau_i(\omega))$$

where ω is a member of a partition of the set of states Ω (describing the outcome of a random event) and π is a probability measure over the ω in Ω. The subscript $i \in \{1,2\}$ refers to one of the two players. σ_i and τ_i are strategies which map the set Ω onto the set of actions. A substructure simply restricts the number of strategies which can be used. Suppose that the set of states Ω is partitioned into two subsets: $\{\Omega^*, \Omega'\}$ We can rewrite the above inequality as follows:

[4] Naturally, if it was not available publicly then only subjective correlated equilibrium would be possible.

$$\sum_{\omega \in \Omega^*} \pi(\omega) u_i(\sigma_{-i}(\omega), \sigma_i(\omega)) + \sum_{\omega \in \Omega'} \pi(\omega) u_i(\sigma_{-i}(\omega), \sigma_i(\omega)) \geq$$
$$\sum_{\omega \in \Omega^*} \pi(\omega) u_i(\sigma_{-i}(\omega), \tau_i(\omega)) + \sum_{\omega \in \Omega'} \pi(\omega) u_i(\sigma_{-i}(\omega), \tau_i(\omega))$$

It can easily be seen that if the second term on each side of the inequality (i.e. where $\omega \in \Omega'$) has zero probabilities (i.e. the supports of the correlated equilibrium are in the substructure) then these terms will be equal to zero. If the resulting inequality still holds then it would also be a correlated equilibrium of the target game. However if the second terms on each side are completely ignored (rather than the probabilities being set to zero) then this is also a correlated equilibrium of the substructure. This in turn would be a correlated equilibrium in the base game.

There are a wide number of possible analogies between games which could be studied in an experimental setting. However, there is an issue of control in trying to disentangle the effects of analogy from other factors (Payoffs, the structure of the game, uncertainty etc.) in some of the more complicated games. There may also be a problem in inducing a correlated equilibrium for the base game which can be transferred to the target game. This is particularly problematic as there has been little experimental research done on correlated equilibrium and correlative devices. It is not known with certainty which correlative devices would work to set up a correlated equilibrium in the base game.

For these reasons it was decided to focus on matching games when undertaking an experimental test of analogy. Matching games have a variety of advantages over other games in this respect. First of all the games are very simple and do not have many complications as regards payoffs or strategies. Secondly the role of non- strategic methods (i.e. salience) to coordinate in matching games is well known in the theoretical and experimental literature (see Schelling 1960, Bacharach 1993, Sugden 1995, Janssen 2001, Casajus 2000). This gives a resource of theories to analyse the effects of non- analogical choice in matching games as well as the effects of analogy.

Matching games also have some useful theoretical properties which mean that they are particularly well- suited to the experimental analysis of analogy. Matching games are symmetric and if symmetric strategies are eliminated then the resulting substructure is also a matching game. Furthermore, a correlated equilibrium of such a substructure must be a convex combination of the Nash equilibrium strategies of that substructure. In addition any such substructure of the matching game will be a *formation* of the target game (Harsanyi and Selten 1993). This means that the Nash Equilibria in the base game will also be Nash Equilibria in the target game. It follows that there are substantial rational reasons not to deviate from choosing an equilibrium in both the base and target games, although there may be ambiguity as to which one is chosen.

In effect the theories of coordination given above analyse how correlated strategies can be formed in matching games using salient characteristics as the correlative device. Furthermore these theories can accommodate analogy quite easily. Analogy simply becomes another form of salience. A strategy is chosen in a target game simply because it stands out as a result of it having being used before in a base game. In addition they also give theoretical reasons for choices in the base game and so allow prediction at each stage of the analogical process.

There are many extant theories of choice using salience although there has been little attempt to test to see which theory is the best for analysis[5]. In a way this is inevitable as the theories tend to have different foci from each other (see the contrast between Sugden's and Bacharach's theories). Furthermore these theories all intend to explain the same phenomena. A choice between theories of salience will, to a certain extent, be based on convenience and, in the case of this paper, on the closeness to the theoretical concerns of the paper. For this reason the closest theory to this framework is that of Casajus (2000).

4- Experimental design

The experiment has three purposes. First of all it is designed to test for the existence of analogy in matching games. Secondly the experiment is done in a manner which allows a comparative test between theories of salience and analogy. Finally, the experiment reflects the best practice in the psychological analogy literature while remaining loyal to the principles of game theory.

The experiment consisted of four treatments with one of the treatments being a "control". Each treatment is constructed in the same way with two base games and a

[5] This is not to say that there are no attempts at testing- see Bacharach and Bernasconi (1997)for an example. However, this does not attempt to test against other theories.

final target game. This reflects the practice in psychological experiments of allowing two attempts for the subjects to "notice" the analogy before it had to be used in the target game (Gick & Holyoak 1983). This helps to overcome the "analogical paradox" (Dunbar 2001) whereby it has been found that analogies are easy to use in natural settings but difficult in the experimental laboratory. The games were all matching games which used shapes to indicate strategies. An example of the target game can be seen in figure 1. This target game was answered by subjects in all four of the treatment groups. As can be seen the strategies are denoted by eight pairs of symbols. Most of these pairs are simply both squares. However three of them have one square and another symbol; a cross, a triangle or a circle. This allows for a comparison between saliencies over different treatments.

It will be useful to define the term "oddity". An oddity is a symbol pair which stands out in some way from the other symbol pairs in the game being played. An example of this can be seen in the game in figure 1 where the cross symbol pair stands out from the other non- cross symbol pairs. Of course the circle symbol pair as well as the triangle symbol pair are also oddities.

The four treatments in the experiment are the circle treatment, the triangle treatment , the combined treatment and the control treatment. All of these treatments vary in the base games but are identical for the target game. Each base game had six symbols and were constructed to be substructures (and indeed formations) of the target game. In the circle treatment the subjects were given base tasks each of which had one symbol pair out of six which had a circle. Examples of the two circle treatment base games are given in figure 2. The triangle treatment was similar to the circle treatment except that the base tasks had triangles as the oddity rather than circles. The combined treatment had base tasks where only four of the symbol pairs were both squares. The other two had a circle in one symbol pair and a triangle in the other.

While this will be analysed in more depth in the next section, it can be seen that the aim of the first two treatments is to induce an analogy between the first two tasks and the third task. The aim of this analogy is to encourage the subject to choose the relevant symbol such as the circle in the circle treatment and the triangle in the triangle treatment. The combined treatment is more complicated in that there is no unique analogy which can be transferred from the base tasks to the target task. The aim here is to see whether there is an attempt to transfer such an analogy across.

The control treatment differs from the other three in that the base tasks do not aim to create an analogy with the target task. To facilitate this the base tasks were given a set of symbols of which none were similar to those in the target class. Furthermore the symbols were different between the two base tasks thus disrupting any analogues formed as a result of repeating the base task. It may be the case that some symbols are privately salient to individuals who may use them to form analogies. However, these analogies will not be accessible to the rest of the people in the treatment so there will be no large scale effect as a result of this in the data.

5.Analysis of analogy and salience in the experimental task

The aim of the experimental design outlined above is twofold: first of all the task aims to isolate analogy as a phenomenon, controlling for a variety of confounding factors; second it aims to ground this within Casajus' theory of salience to demonstrate that analogy has relevance to game theory. In order to demonstrate that analogy is a genuine phenomenon a variety of factors have been taken into account in the experimental design:

i) The choices in the final task may simply be the result of the selected symbol pair being more salient than those symbols which are not "oddities" . This is controlled for by having three "oddities" in the last task. In this case the choices should be spread amongst them in equal proportions.

ii) The Choices in the final task may be the result of one oddity being more attractive than the other oddities. This is controlled for by having two "oddity" treatments which target different shapes.

iii) The choices may be the result of "nuisance attributes" within the target game. This is controlled for by using the control group. While it is possible that there are nuisance attributes in the structure of the game, separate from the shapes used, these should also exist in the control treatment. Any effect from analogy would therefore have to be significantly different from the control group.

The main analysis of the results of this experiment will be derived from Casajus' (2000) theory of salience. Casajus put forward a theory of salience which built on previous work by Bacharach (1993) and Janssen (2001). His basic idea was to focus on Harsanyi and Selten's notion of *Symmetry Invariance* when applied to strategic form games. Symmetry invariance implies that any equilibrium in a game should not be affected by the labelling or order of the pure strategies. Furthermore, any two such pure strategies which have the same payoffs can be treated as symmetric and should be assigned equal probabilities.

Casajus extends this notion of symmetry invariance to framing in order to apply it to matching games. He defines a frame as a set of attributes from which labels can be applied to strategies. Examples of attributes could be "Shape" or "Colour" while the labels applied to strategies could be "Cube" and "Red" respectively. It is assumed that labels within frames are exhausting of all possible labels within their attributes and that attributes are exhausting of all possible attributes. This is done by placing any unknown labels or attributes into "default" labels or attributes. The frame defined in this way is assumed to be the players' characterisation of the game rather than that of the theorist.

A game which incorporates such frames is known as a Framed Strategic Form. A Framed Strategic Form (FSF) therefore is an extended game where an equilibrium is also an equilibrium of the underlying strategic game. However, the idea of the FSF is that this reduces the number of equilibria from the original game. Such an FSF should have the same solutions if the frames are translated into another language or some other isomorphic mapping is imposed on it. FSFs therefore are *language invariant*. This means that it is possible to relabel some of the strategies in a one- to one translation which will not affect the structure of the game. This will effectively reduce the number of symmetries in a matching game as an FSF's labelling will not have as many symmetries as an unlabelled game.

In a similar way to Harsanyi and Selten, a notion of symmetry invariance is defined whereby the strategies are symmetry invariant as previously but there is also symmetry invariance between labels as well as a result of language invariance between labels in the same game. To take an example (Casajus 2001) if two people are playing a matching game and both are faced with three pure strategies labelled by a red ball, a white ball and a white cube which should one choose? If colour is an attribute salient to both players then one would expect "red" to be chosen as the two whites are symmetric. By contrast, if "shape" is salient then the cube would be chosen as the other two objects are symmetric in being balls. If, however, both colour and shape attributes are salient then one would choose the white ball. The reason for this is that the attributes "Shape" and "Colour" can be mutually translated because (in this limited universe) they only have two labels each and so are symmetric. Between these attributes the labels can be paired so that they exhaust the labels available within the problem. The best way of doing it within this example would be to pair "white" with "ball" and "red" with "cube". Other ways of pairing will not pair all the labels.

However, on the next level, there is not a full symmetry between the red-cube label symmetry and the white- ball symmetry as there are more examples of the latter than the former. This means that the labels of "white cube" and "red ball" can be fully mutually translated using these symmetries but "white ball" cannot be fully mutually translated into either. Picking the white ball is the same as choosing the symmetric invariant equilibrium.

From this it would be interesting to ask how Casajus' theory of salience would apply to the analogy experiment. The analogy would in this case operate as a method of identifying salient strategies. The strategy identified would be labelled as the analogue while the other strategies would be non- analogues As such these salient strategies would enter into the theory in much the same way as any other attribute.

However, this does not answer the question of what should be counted as an attribute within this particular experiment. As has been pointed out before (Bacharach & Bernasconi 1997), there is a perennial problem of "Nuisance Attributes" emerging as a result of the structure of the game. While there have been attempts to solve these problems it is unlikely that all such oddities can be eliminated. It follows that in the experiment there will be some "Nuisance Attributes". However, it is unlikely that many of them will be noticeable to all of the subjects and so they will not be picked up in a significant way by the data. What is of interest is isolating attributes from the point of view of the goals of the experiment and the objective salience of those attributes.

It was decided that the attributes analysed would be those which are salient to the subjects and seem generalisable. This means that, in this experiment, any given player would be faced with just two attributes:

i) Within the target game there are the three oddities (the cross, circle and triangle). The labels would be “Oddity” or “Non- oddity”. These have been united together by the fact that they are all individually oddities but there is no objective way of distinguishing one as more salient as the others.

ii) From the analogy there is a unique analogy . The labels would be (for example) “Circle” or “Non- circle” with labels varying according to the analogy.

This assumption of just two attributes may seem too radical. Why just these two attributes rather than the wide range of possible other attributes? To answer this, first of all note that all effects outside the base games or target game are excluded from the domain of the experiment. It is possible that these exist but they cannot be controlled for. At best we can assume that they are private to the individual. This assumption can be challenged but it is incumbent on the critic to show that these attributes exist and whether they are important. However there are also the controls outlined above so if there is a “nuisance attribute” within the target game then this will be picked up by the control treatment.

We assume that just one analogue is picked out from the base games. This is because, for an analogy to work it must present a solution to the game. Two analogies will not do this so a rational player will just transfer one analogy across. Which analogy is transferred across will be established empirically. Whether a person has played a strategy in the base game will be taken as an indicator of whether they treat it as an analogy or not. Only when a strategy is played in the second base game and is also played in the target game will it then be considered as an analogue. The reason for picking a choice in the second base game as the crucial game is that this reflects the psychology literature where it is only the addition of a second task which fixes an analogy. This will give a unique attribute for testing. In the case of the combined treatment where there are several possible analogues, this method allows us to pick up several “types” of people according to their analogues.

Although there are many controls on this experiment it was decided that there is one, known, “nuisance attribute” which is too prominent to ignore. This is the “Topness” heuristic (e.g. Bacharach and Bernasconi 1997) where subjects tend to focus on symbols at the top of the page irrespective of their shape. While this is controlled for in terms of the target game alone, it may still emerge as an analogy from the base games. Since it may usefully be used as an analogue (because it is quite "public" and easily noticed) it will be included in the analysis here.

The next question concerns what one should expect to see as the results of the experiment if the subjects are following Casajus' theory of salience and the theoretical outline of analogy given above. For the "Circle" and "Triangle" treatments it should be fairly straightforward. First, one would expect that, given the simplicity of the base games, most would choose the circle and triangle respectively in each base game. This would then be used to form an analogy. Taking the "circle" treatment first, the analogy attribute would simply label the circle in the target game as the "analogue" and the other strategies as "non- analogues". Since the Oddity analogue would already highlight the oddity symbols this means that the symbol pair with a circle will be doubly marked and so will be the unique oddity which would be selected. The same argument applies to the "Triangle" treatment.

Another possibility is that the "Topness" analogue may function instead. The analysis is rather more complicated in this case. Both of the topmost symbol pairs are selected out as analogues. The circle is selected out as the only oddity which is topmost while the squares are selected out as the only topmost square. The circle is most likely to be selected because the there are fewer symmetries between the circle and the other oddities compared to the topmost square and the other squares.

The Control treatment is straightforward. The only meaningful analogy to come through would be the "Topness" heuristic so this might mean that the two symbol pairs (and particularly the circle) at the top may be highlighted. However, it may be that this heuristic may not work (as it is a "nuisance" oddity not designed into the experiment) in which case only the Oddity attribute will hold and one would expect subjects' choices to be equally divided between the three oddities. The Combined treatment is more complicated as there are potentially three different analogues which may come through from the base games: Topness, Circle and Triangle. It follows that each of these needs to be studied individually. The analysis for each of these is much the same as before given that analogues are assumed to be unique.

Until now we have referred to the analogies as coming from a player's own previous play. This has been done in strict comparison with the single- player problems undertaken in the psychology literature. However, as was mentioned earlier, there is another source of information in the form of the other player. Since a player has access to an opponent's play because of feedback in the experiment it follows that they can learn from this. Since they are aiming to coordinate with the opponent and the game is symmetric then this forms a good source of analogical information. In this case the analogical reasoning is more complex in that, instead of looking to see what worked in one's own past performance and forming an analogy from that, one is looking at one opponent's behaviour and forming an analogy to one's own behaviour in the target game.

6 Results

Table 1 gives an overview of the raw data for the target game over all four treatments. The labels along the top give the position of the symbol- pairs. These can be understood by a comparison with figure 1 and the abbreviations are explained in footnote 6. Those which have "oddities" attached to them are given descriptive labels (e.g. "Triangle"). Down the right hand side are χ^2 statistics comparing the distributions of the row with that of treatment 4.

Table 1

	BL[6]	BR	Cross	Triangle	TL	Circle	LU	LLo	χ^2
1	0	2	1	2	2	27	0	0	26.8**
2	0	0	4	21	4	3	0	0	29.9**
3	0	0	6	6	6	11	2	0	10.5
4	2	1	7	1	6	7	1	0	N/A

** Indicates significance at 5% level

Looking first at treatment 4 we can see what is the "base state" without analogy. There does seem to be some clustering around the two "oddities" of the cross and the circle but there does not seem to be any clustering around the triangle. Instead there is some clustering around the top left square pair symbol, suggesting that the "Topness" heuristic has influenced some of the subjects. The lack of clustering on the "triangle" symbol is a mystery. Maybe the triangle was less distinguished from the squares than the properties of topness or being a circle or a cross. Even so, it is

[6] Labelling: BL= Bottom Left; BR = Bottom Right; TL = Top Left; LU = Left Upper; LL = Left Lower

noticeable that the choices are not random and that there is some evidence of influence from focal points in the diagram. It may also be noted that here, as to a lesser extent in other treatments, there is some variation in the choices. This would reflect both the effects of error and also of "nuisance attributes" many of which will be private to the individual making the choices.

Treatments 1 and 2 have similar outcomes in that the subjects choices cluster respectively round the circle and the triangle.. What deviation there is seems to go towards the oddities. A test against treatment 4 in both cases shows highly significant differences. This suggests that there is a definite effect as a result of analogous reasoning. Furthermore, these differences can be seen to be definite effects as a result of having either circles or triangles as analogues beforehand. One additional effect which may be noted is that it seems that, as with treatment 4, the triangle is marginally less popular in treatment 2 than the circle in treatment 1.

Treatment 3 is more complicated. This treatment, as has been mentioned, does not have an obvious analogue. There are the two shapes, the circle and the triangle, while there is also the nuisance attribute of topness. A look at table 1 shows that there is a definite clustering of the choices around the "oddities"- including topness (which includes both "TL" and "Circle". There is also evidence that some are preferred to others- circle for example has more choices than any other. However, one possible reason for this could be that the circle is at the top and may attract because of a topness heuristic. In addition, a non- analogue- the cross- attracted the same number of choices as the triangle and "TL". Furthermore when one looks at the χ^2 statistic on the right then it can easily be seen that there is no significant difference between this and the control treatment.

At face value then, it may seem that Treatment 3 is the weakest of the three analogous treatments in that there is no obvious effect of analogy at all. Instead it may be the case that the subjects are simply influenced by the structure of the target game and simply ignored the base games. However this would be a premature conclusion as we have not examined what the subjects chose in the base games. This will indicate how much analogy was involved in this choice.

Tables 2 and 3 give the raw data for choices in the two base games. In each case the symbol pairs are labelled by position. Each oddity is also labelled in the cells in the table, either as circle or triangle depending on the treatment.

Table 2- second base game

	B[7]	LLo	RLo	T	LU	RU
1	30 (C[8])	0	0	3	0	0
2	0	0	0	6	1	25 (T)
3	8 (C)	1	0	13	1	9 (T)
4	5	4	2	7	4	8

Table 3- first base game

	B	LLo	RLo	T	LU	RU
1	0	1	29 (C)	3	1	0
2	0	23 (T)	5	5	0	3
3	1	0	4(C)	23 (T)	2	2
4	4	9	0	13	0	4

Looking at Table 3 one can see that there is evidence that the oddities are being picked out in treatment 1 and 2, although again there seems to be the case that a marginally higher proportion of people chose circle than triangle. This might seem to be reversed in treatment 3 where there are both oddities. However, this is misleading as it can be seen that in treatment 3 the triangle oddity coincides with the "nuisance attribute" of topness. According to Casajus' theory this would mean that the triangle *in this case* would become salient. Looking at Treatment 4 we can see that there is indeed some clustering around certain symbol pairs and particularly around the top symbol pair. However, these symbol pair selections are not continued in the next base game or the target game so any clustering here is incidental and does not affect the analysis.

Looking at Table 2 and the second base game we can see a similar story for treatments 1 and 2 with the salience of circle and triangle dominating respectively. For treatment 3 we can see that the dominance of "triangle" in base game 1 has faded although "triangle" and indeed "circle" still have a cluster of choices. Noticeably

[7] Same labelling as before except that T=Top; B = Bottom
[8] C = Circle; T = Triangle

however the “topness” heuristic still seems to be operating and the largest cluster of choices is on the topness symbol pair. This suggests that the topness heuristic has already become an analogue. Looking at treatment 4 one can see that the previous pattern in base game 1 has not been replicated so there is no sign of any analogy forming. It seems that most choices are either random or use private saliencies.

The issue then becomes that of how much the choices in the target game are influenced either by an analogy from one’s own play or by an analogy from one’s opponent’s play. Table 4 shows, for each treatment, the proportions of subjects whose choices in the target game are similar to those in the second base game. It should be noted that this “similarity” covers both the oddities and the “Topness” heuristic. This has significance for the target game because the propensity to choose the circle symbol could be the result both of a circle analogy and a topness analogy. For this reason the circle symbol in the target game will be counted as being caused by an analogy for either depending on what the subject (or their opponents) chose in the second base game.

Table 4

	Treatment 1	Treatment 2	Treatment 3
Self	79.4%	75%	37.5%
Opponent	82.4%	62.5%	59.4%

Another point to note in connection with this table is that this is not making any claims as to whether the subject is being influenced by her opponent’s choices or her own. Indeed there could be a mix of influences and this experiment cannot separate them out. The comparatively high levels of repetition between base and target games whether from the player or the player’s opponent is suggestive that some kind of analogy is taking place whatever the source of the analogy.

It will be noticed that treatment 3 has the lowest prevalence of analogy, especially from a person’s own choices in the second base game. This should not be a surprise as there is a substantial possibility of confusion in this treatment. In fact only 10 out of the 32 subjects in treatment 3 chose using the same heuristic consistently all through the three games. It is noticeable however that the opponent’s choices are used by the majority of subjects as analogues for how to choose in the target game. This suggests that analogy is at work even in the complex environment of treatment 3 and that this is not purely the result of salience in the target game.

6. Discussion and conclusion

This paper set out to demonstrate that the notion of analogy which has been widely tested within psychology could also be applied and tested in an experimental economics environment. Despite the fact that some economic theories have purported to use analogy it has been shown that these theories are limited in their scope and effectively assume the existence of analogical reasoning without investigating the conditions which result in its emergence. Instead it was shown that analogy can be analysed as a theory of learning based on partial mappings between analogues. This can then be modelled as a correlated equilibrium in the target game.

This experiment was constructed in the form of a series of matching games. The design of the experiment controlled for confounding factors and isolated analogy in very simple games. However it was shown that analogy worked in more complicated situations where there was more than one potential analogue. Although there is evidence that analogy was used to a lesser extent in the latter case this suggests that, given more opportunity for the players to settle on one analogue, the incidence of analogy use could have been higher. The high level of use of the opposing player's previous base game play suggests that part of the problem was that the players were trying to coordinate as well as use analogies and the existence of two analogies caused confusion.

This experiment and its results link in very closely with previous experiments and also the various theories of matching. Indeed this experiment shows that there is no real contradiction between the various theories of matching put forward, especially that of Casajus, and the theory of analogy. It seems that analogy can therefore be accommodated simply as a different type of salience within matching games. However Kreps (1990) and other authors seem to believe that analogy can be applied in a far wider set of circumstances than simply in matching games. If they are right then there is a need for further experiments in analogy to find out how it works. The idea of analogy as a form of mapping which creates a correlating device in a correlated equilibrium is a good starting point for experiments on non- matching games.

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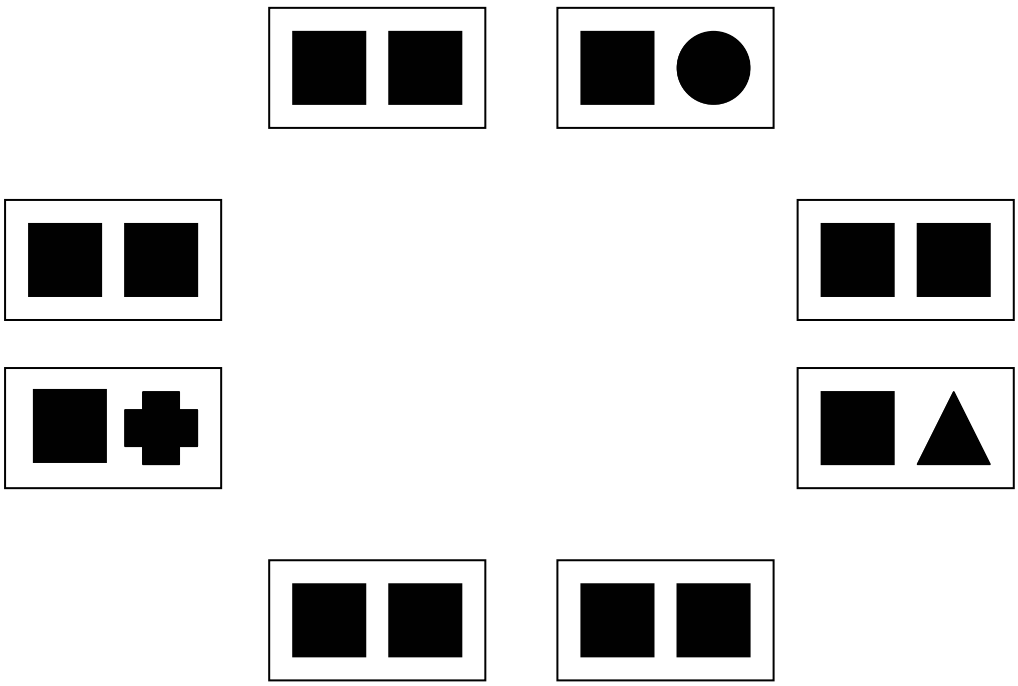
Appendix 1: Figure 1

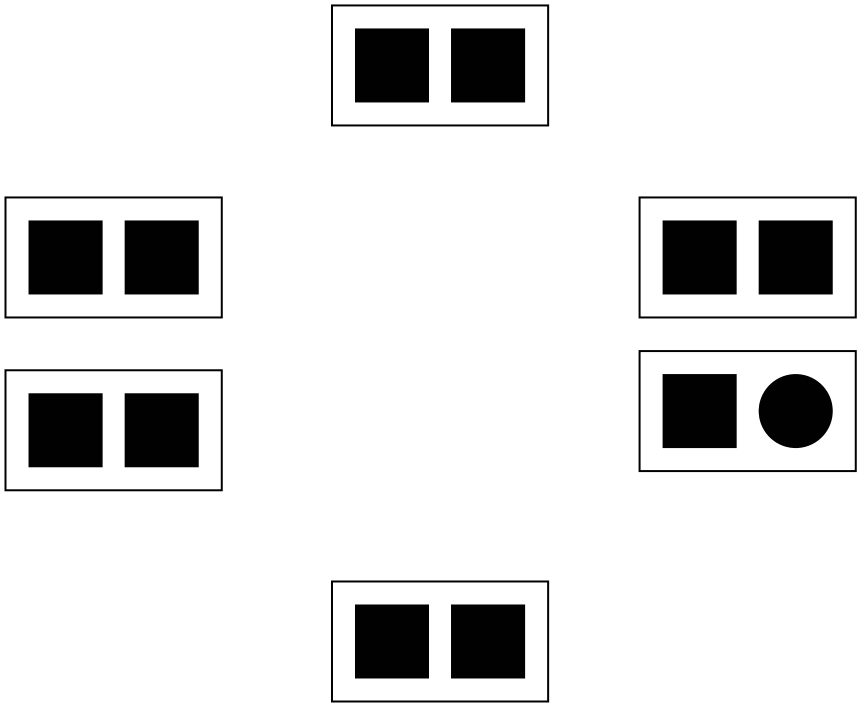
You are paired with the same person in this room as you were paired with for the last two games. Ring ONE of the rectangles below. If both you and your partner ring the same rectangle then you will get £4.



When you have ringed one of the rectangles please wait for the experimenter to come around.

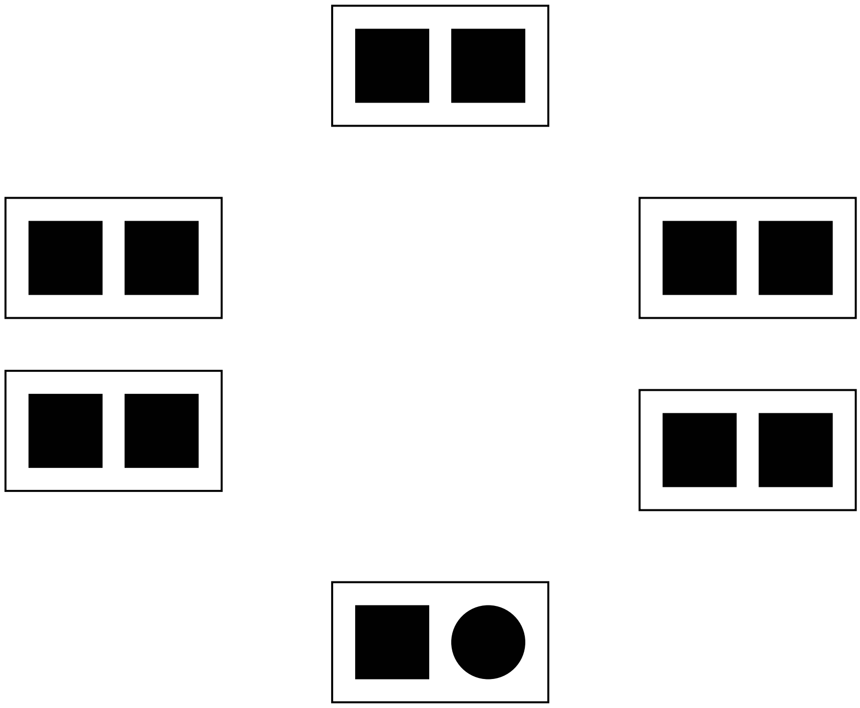
Appendix 2: Figure 2

You have been paired with one other person in this room who you will be playing with throughout this session. Ring ONE of the rectangles below. If both you and your partner ring the same rectangle then you will get £4.



When you have ringed one of the rectangles please wait for the experimenter to come around.

You are paired with the same person in this room as you were paired with in the last game. Ring ONE of the rectangles below. If both you and your partner ring the same rectangle then you will get £4.



When you have ringed one of the rectangles please wait for the experimenter to come around.