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# Dundee Discussion Papers in Economics

## Firing the Old or the Young: A Non-Perpetual Real Options Analysis

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# Firing the Old or the Young:
# A Non-perpetual Real Options Analysis

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## ABSTRACT

We model a firm's choice as to the age composition of dismissed workers for different assumptions about the level of firing costs. We find that with high firing costs (not to mention rising ones), firms will be inclined to fire younger workers while with low costs of firing, the older workers are at risk. Since with high firing costs old workers are better protected against layoffs, an ageing population has the effect of making employment-protection legislation more stringent.

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Young workers are much more likely to be unemployed than their more mature counterparts. In France, Italy and Spain the unemployment rate for workers under the age of 20 has been around 3-5 times as high as for those aged 25-54 (OECD Employment Outlook, 1996). This observation warrants an explanation for its own sake. Moreover, if we can find reasons for such differences, the possibility arises that changes in the age structure of the population affect the aggregate unemployment rate.

The questions that come to mind include: How do firms reach a decision as to whether to hire (fire) a young or a mature worker? How do labour-market institutions such as employment protection affect this choice? And, most importantly, how do changes in the age composition of the labour force affect aggregate outcomes such as the rate of employment and unemployment? In this paper we provide microeconomic foundations to address these issues. We find that employment protection is much more likely to protect the more mature workers. Employment-protection legislation is hence likely to raise the relative unemployment rates among the young. Moreover, the age composition of the labour force is an exogenous determinant of the stringency of any employment-protection legislation.

The low employment-to-population ratios in countries with severe firing restrictions (i.e. France, Italy and Spain) are mainly due to low participation rates of teenagers and women (Nickell (1998). High unemployment rates among prime-aged males are not responsible for this outcome. Thus the impact of such restrictions can be found both in the composition of those nonactive and in the level of employment. To analyse this relationship further we estimated the following equations separately for men and women for a cross section of 18 OECD countries;[1]

$$n^y = \alpha_0 + \beta_0 n^o + \gamma_0 epl + \varepsilon$$

$$u^y = \alpha_1 + \beta_1 u^o + \gamma_1 epl + \varepsilon$$

where $n^y$ and $u^y$ are the employment/population ratio and the unemployment rate of young workers (15-19/20-24 years of age) and $n^o$ and $u^o$ are the corresponding figures for workers aged between 25-54 years. The variable *epl* denotes a measure of employment protection (*epl*) taken from Nickell (1998). The results follow in Table 1 for the year 1994.

**Table 1**. The effect of employment protection on youth employment and unemployment

|          | Men | | | Women | | |
|----------|------|------|------|------|------|------|
|          | $\alpha_i$ | $\beta_i$ | $\gamma_i$ | $\alpha_i$ | $\beta_i$ | $\gamma_i$ |
| u (15-19) | 3.87 | 1.50 | **0.42** | 0.45 | 1.81 | **0.46** |
|          | (0.9) | (3.6) | **(1.4)** | (0.1) | (4.5) | **(1.2)** |
| u (20-24) | 1.31 | 1.25 | **0.53** | -1.30 | 1.48 | **0.42** |
|          | (0.5) | (4.7) | **(2.9)** | (0.5) | (5.9) | **(1.7)** |
| n (15-19) | 60.98 | -0.15 | **-1.79** | 31.59 | 0.23 | **-1.84** |
|          | (1.2) | (0.2) | **(3.8)** | (1.6) | (0.9) | **(3.8)** |
| n (20-24) | 0.56 | 0.87 | **-1.27** | 63.42 | 0.12 | **-1.51** |
|          | (0.0) | (2.5) | **(4.8)** | (3.6) | (0.5) | **(3.4)** |

Data source: Employment Outlook, 1996. t-ratios in parentheses.

The effect of *epl* on the employment/population ratio is very significant for all four groups. The effect on unemployment is less significant but significant at the 5% level for men aged 20-24 and at the 10% level for women in the same age group. We conclude that our measure of employment protection appears to be negatively correlated with employment and positively correlated with unemployment in our sample.

Turning to possible explanations for our empirical observations, Lazear and Freeman (1997) find that in a downturn the young (and also the very old) workers should be the first to be laid off. The young have not been given any firm-specific skills while the old's productivity may have declined relative to their wage. These groups should thus have a higher rate of transition from employment to unemployment than do prime-age workers. The interaction with firing costs is not considered. Layard et al. (1991) find the wage-push factor to be stronger for young workers due to higher turnover and, as a result, their unemployment rate becomes higher. This is because high turnover makes the prospect of unemployment spells less threatening which then makes unions more aggressive. Again, employment protection legislation is not a part of the story.

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[1] The countries are: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, New Zealand, the Netherlands, Norway, Portugal, Spain, Sweden, the U.K. and the U.S.

Other reasons for age discrimination in firing involve the direct and indirect effect of pension schemes. Most pension schemes are defined-benefit schemes, which implies that the benefits increase more rapidly as the age of retirement approaches since they are based on final salary at retirement. This makes employers want to lay off workers with a long tenure. A possible offsetting effect can be found in Orszag et al. (1999). Here old workers have a higher effort level because they have more to lose in the event of a dismissal.

In our model we show how the level of firing costs is important for the (age) structure of unemployment. We assume for simplicity that productivity is independent of age. Workers only differ in their expected remaining tenure and firms take this into account when making hiring and firing decisions. Under these conditions the decision whom to fire first in a downturn depends on the level of firing costs as shown in our model.

We model the hiring and also the firing decision as an intertemporal investment decision. The (sunk) costs of hiring are associated with teaching the worker firm-specific skills while the (sunk) firing costs could be state-mandated redundancy payments. These can be either fixed for all workers or rising in tenure. We use a real-options approach from investment theory to answer these questions.

*Our methodological approach*

The option-valuation approach to investment has been popular since the seminal papers of Black and Scholes (1973) and Merton (1973) on the pricing of stock options. These methods of valuing stocks can be easily applied to *real options*, which denote the option-like characteristics of investment opportunities. The decision to invest (or the decision to exercise real options) becomes important with the existence of uncertainty and sunk costs. McDonald and Siegel (1986) show that the required rate of return on investment in many large industrial projects can be more than doubled by moderate amounts of uncertainty when the investment project is at least in partly irreversible.[2]

In most cases it is assumed that the real options are infinitely lived—the real-life investment opportunities are infinitely lived and never valueless (e.g. McDonald

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[2] For an introduction, see Dixit and Pindyck (1994).

and Siegel; 1984, 1986). However, some research deals with the non-perpetual real options (e.g. Paddock, Siegel and Smith (1988)). But it has been claimed that it is often not possible to solve such non-perpetual options analytically, making numerical methods essential.[3] Generally speaking, it is hard to solve completely for free-boundary time-dependent real options. Part of reason is that the function of time-dependent options has a complicated shape, which might need several analytical functions to simulate with. We will show in the case of real options that *approximate* analytical solutions can exist.[4] The approximate solutions of non-perpetual real options should share the same composite components as perpetual real options. The partial differential equation of non-perpetual real options can then be transformed into a Convection-Diffusion problem,[5] which can be solved for analytically using standard techniques of partial differential equations.

In the following section we will describe the profit-maximisation problem and the underlying stochastic process. We then go on in Sections II and III to describe the hiring- and firing thresholds when they are calculated separately. In Section IV we generalise and calculate the two thresholds simultaneously. Finally, in Section V we discuss the macroeconomic implications.

## I. Basic Framework

There is only one sector in our economy that uses labour as an input to produce a homogenous good. Since our focus is on labour demand, real wages are assumed fixed and their determination is not described. The source of uncertainty is stochastic productivity.

Current profits, measured in units of output, are defined as follows in the absence of firing,

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[3] One of the reasons for the non-existence of analytical solutions is that such options are similar to *American* stock options that can be exercised at any time up to the expiration date. It is well-known that American stock options can only be solved for using analytical approximations or numerical methods such as finite-difference methods. American call options with lump-sum dividends are an exception though in that their terminal and boundary conditions differ (see Roll, 1977; Geske, 1979; Whaley, 1981). The possible analytical solutions to partial differential equations vary greatly when boundary- and terminal (and/or initial) conditions change. Changes in such conditions can result in the non-existence of analytical solutions. The method used here is most similar to Barone-Adesi and Whaley (1987).

[4] '*Approximate*' is in a sense that the solutions are not complete, but still a good proxy for real solutions, compared with the results from explicit finite difference method in the Appendix D.

[5] In physics, *convection* is the movement of the substance by the movement of the medium. Combined with a diffusion problem, it will be like the diffusion of a moving wave.

$$\Pi(g_t, N_t) = g_t N_t^\theta - w N_t, \qquad 0 < \theta < 1, \qquad (1)$$

where $N$ denotes the number of employed workers, $w$ is the real wage and $g$ is a measure of productivity.

It is assumed that each worker has a working life of $T$ years. To simplify the model, we assume that workers die immediately after they retire and that all workers have the same productivity independent of their age. Moreover, we assume that both current and potential employers can observe a worker's age.

It is assumed that $g$ follows a geometric Brownian motion

$$dg = \eta g ds + \sigma g dz; \qquad (2)$$

where $z$ is a Wiener process; $dz = \varepsilon \sqrt{ds}$ since $\varepsilon$ is a normally distributed random variable with mean zero and a standard deviation of unity. Here $\eta$ is the drift parameter (the expected growth rate of labour productivity) and $\sigma$ the variance parameter. It is assumed that this average quit rate per unit time is constant over time and equal to $\lambda$. The probability that a given worker will quit over the interval $ds$ is therefore equal to $\lambda ds$.

The firm's expected marginal value of an employee *without any firing and/or hiring* is

$$v(Y, t; T) \equiv v(Y_t, t; T) = \int_t^T (Y_s - w) e^{-(\rho + \lambda)(s - t)} ds , \qquad (3)$$

where $\rho$ denotes the real interest rate, $v$ is the (intertemporal) marginal value of workers, $Y_s = \theta g_s N_s^{\theta - 1}$ represents the marginal product of labour at time $s$, and $Y_s - w$ denotes instantaneous marginal profits at time $s$. Equation (3) is similar to the expressions in Bentolila and Bertola (1990) except each marginal worker can only work for the firm for a maximum of $T$ periods.[6]

Itô's Lemma gives the following process for marginal labour productivity,

$$dY = \eta_Y Y ds + \sigma Y dz , \qquad (4)$$

where $\eta_Y = \eta + \lambda(1 - \theta)$.

Now consider the effect of the firing and hiring costs on firms' profits decisions. When the marginal profit of a worker is greater than the hiring cost, the firm starts to

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[6] $T$ is the maximum possible tenure since workers might quit or get fired earlier.

hire new workers; when the negative of the marginal profit value of a worker is higher than the firing cost, the firm starts to fire workers. Thus, the process of $Y_s$ or $v(Y,t)$ becomes an *optional stopping problem* or *regulated Itô process*. The firm will hire a marginal worker if

$$v(Y,t;T^h) \geq H \qquad (5.1)$$

and fire a marginal worker if

$$-v(Y,t;T^f) \geq F, \qquad (5.2)$$

where $H$ and $F$ represent hiring and firing costs respectively, $T^h$ denotes $T$ for the worker that the firm hires and $T^f$ for the worker that the firm fires. $T^h$ is different from $T^f$ since hiring and firing decisions cannot happen at the same time.

A standard technique for solving the above dynamic optimisation problem is Bellman's Principle of Optimality (Bellman, 1957). Using Itô's Lemma, we get the following Bellman equation for the marginal value of the firm's stock of workers; $v \equiv v(Y,t;T)$, in the continuation region where the values of future hires and fires are not taken into account,

$$(\rho + \lambda)v = Y - w + \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY} + v_t. \qquad (6)$$

This partial differential equation relates the value of workers to the value of the stochastic variable $Y$ at each point in time.

Equation (6) is different from the expressions in Bentolila and Bertola (1990) and many others. In their setup, the time horizon in equation (4) is set from zero when equation (6) is derived, and thus the function $v$ does not depend on time. Under these conditions, equation (6) becomes a second-order ordinary differential equation in $Y$. As a result, the option values of hiring and firing workers become independent of time. It follows that the options for hires and/or fires do not approach zero when workers age. One of the objectives of this paper is to correct for this and show how important implications arise.

The problem now is to solve for $v$, which is the value of employing a marginal worker. The solution for $v$ consists of the particular integral and the complementary function. A convenient particular solution, $v^P$, for (6) is

$$v^P = aY - bw, \qquad (7)$$

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where $a = \left(e^{-(\rho+\lambda-\eta_Y)t} - e^{-(\rho+\lambda-\eta_Y)T}\right)/(\rho+\lambda-\eta_Y)$, $b = \left(e^{-(\rho+\lambda)t} - e^{-(\rho+\lambda)T}\right)/(\rho+\lambda)$ and it is assumed that the denominator of the parameter $a$ is positive.

The firm takes into account the option value of hiring in the future. There is also the option to fire the worker once he is employed. The two option values are measured by the complementary (or homogenous) solutions to (6). Now only focusing on the homogenous part of equation (6) and letting $v^G$ be the value of the marginal option, we get

$$(\rho+\lambda)v^G = \eta_Y Y v_Y^G + \frac{1}{2}\sigma^2 Y^2 v_{YY}^G + v_t^G. \tag{8}$$

The general solutions of (8) are equal to the value of the options to hire or fire the marginal worker. When $Y$ approaches zero, the value of the option to hire, $v_H^G$, has to go to zero. Similarly, the firing options, $v_F^G$, is equal to zero when $Y$ goes to infinity. Thus, the general solutions for the hiring and firing options have to satisfy the following boundary conditions respectively,

$$\lim_{Y\to 0} v^G(Y,t;T) = 0 \text{ for the hiring option}, \tag{9.1}$$

$$\lim_{Y\to\infty} v^G(Y,t;T) = 0 \text{ for the firing option}. \tag{9.2}$$

A special case of equation (8) is when time is equal to zero ($t=0$) and workers live forever ($T=\infty$). Thus, the term $v_t^G$ in equation (8) disappears and the values of the hiring- and firing options are (see the Appendix A)

$$v_{0H} = A_1 Y^{\beta_1} \text{ for hiring option}, \tag{10.1}$$

$$v_{0F} = A_2 Y^{\beta_2} \text{ for firing option}. \tag{10.2}$$

The unknown parameters of $A_1$ and $A_2$ are determined by the value-matching and smooth-pasting conditions and $\beta$ is determined by equation (11). [see Appendix A]

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \eta_Y\beta - (\rho+\lambda) = 0. \tag{11}$$

The general solutions to (8) are then given by the following equations (see Appendix B):

$$v_H^G(Y,t) = A_1 Y^{\beta_1} N(d_1), \tag{12}$$

$$v_F^G(Y,t) = A_2 Y^{\beta_2} N(-d_2), \tag{13}$$

where $A_1$, $A_2$ are unknown parameters,

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$$d_1 = \frac{\ln Y + \sigma^2(T-t)\sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln Y - \sigma^2(T-t)\sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}},$$

where $\overline{Y}$ is hiring costs or firing costs

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi, \ \ 0 \le N(d) \le 1$$

is the cumulative distribution function for the normal distribution.

Looking at the hiring- and firing options we find two separate cases:

Case 1: $T \rightarrow \infty$

It is easy to show that as $T$ approaches infinity (workers live forever), the cumulative distribution functions of $N(d_1)$ and $N(-d_2)$ become unity. This reduces the firing and hiring options to the case of perpetual options.

Case 2: $T \rightarrow 0$

If $\ln Y > 0$, then $N(d_1) = 1$ and $N(-d_2) = 0$ as $T$ approaches zero. If marginal profitability is high enough, firms mainly focus on the hiring decision. The firing option approaches zero because this marginal worker will retire very soon.

If $\ln Y < 0$, then $N(d_1) = 0$ and $N(-d_2) = 1$ as $T$ approaches zero. If marginal profitability is low, firms mainly consider the firing decision. Since the marginal worker's $T$ is very small when he/she gets fired, the possibility of re-hiring this worker is almost zero. Therefore, the hiring option approaches zero.[7]

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[7] Note that the options of hiring and firing approach zero automatically if $T$ approaches zero since the marginal profits for hiring/firing would be zero in this situation.

## II. The Hiring- and the Firing Decisions

The decision as to hire or fire a worker depends on his value as given by equations (7)-(13) and also on the direct costs of hiring and firing. We assume that the cost of firing takes the form of mandatory redundancy payments. The definition of the firing- and hiring barriers; $Y_F$ and $Y_H$, are given by the value-matching and smooth-pasting conditions:

Value-matching conditions

$$aY_H - bw + v_F^G\left(Y_H, t; T^h, A_2\right) = H + v_H^G\left(Y_H, t; T^h, A_1\right), \qquad (14)$$

$$-\left(aY_F - bw\right) + v_H^G\left(Y_F, t; T^h, A_1\right) = F + v_F^G\left(Y_F, t; T^f, A_2\right). \qquad (15)$$

The left-hand side of (14) has the marginal benefit of hiring which includes the acquired firing option. The right-hand side has the marginal cost of hiring, which includes the sacrificed hiring option. Similarly for equation (15), the left-had side has the marginal benefit and the right-hand side the marginal cost of firing. In our numerical solutions below, we will only include the sacrificed firing option as part of the cost of firing; we will not include the acquired hiring option as a benefit of firing. The reason is that firing one worker is not going to alter a firm's chances at filling a vacancy in the future if there are many firms in the market or if there are many unemployed people to start with.

The smooth-pasting conditions follow.

*Smooth-pasting conditions*

$$a + \frac{\partial v_F^G\left(Y_H, t; T^h, A_2\right)}{\partial Y_H} - \frac{\partial v_H^G\left(Y_H, t; T^h, A_1\right)}{\partial Y_H} = 0, \qquad (16)$$

$$a + \frac{\partial v_F^G\left(Y_F, t; T^f, A_2\right)}{\partial Y_F} - \frac{\partial v_H^G\left(Y_F, t; T^h, A_1\right)}{\partial Y_H} = 0, \qquad (17)$$

where[8]

$$\frac{\partial v_H^G}{\partial Y} = A_1 \beta_1 Y^{\beta_1 - 1} N(d_1) + A_1 Y^{\beta_1} N_Y(d_1), \qquad (18)$$

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[8] For derivation of (20) and (21), see Appendix C.

$$\frac{\partial v_F^G}{\partial Y} = A_2 \beta_2 Y^{\beta_2 - 1} N(-d_2) + A_2 Y^{\beta_2} N_Y(-d_2), \qquad (19)$$

$$N_Y(d_1) = \frac{e^{\frac{-[\ln Y + \sigma^2(T-t)\sqrt{\alpha}]^2}{2\sigma^2(T-t)}}}{\sigma Y \sqrt{2\pi(T-t)}}, \qquad (20)$$

$$N_Y(-d_2) = -\frac{e^{\frac{-[-\ln Y + \sigma^2(T-t)\sqrt{\alpha}]^2}{2\sigma^2(T-t)}}}{\sigma Y \sqrt{2\pi(T-t)}}, \qquad (21)$$

and $\alpha = \left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}$. Equations (14), (15), (16) and (17) are non-linear

systematic equations with four unknown parameters $[Y_H, Y_F, A_1, \text{and } A_2]$ and can be

solved for numerically.

## III. Firing Thresholds under Alternative Institutional Setups

We will now calculate the firing thresholds on the basis of equation (15) and (17)

without taking into account the hiring thresholds. Thus, (15) and (17) become

$$-(aY_F - bw) = F + v_F^G(Y_F, t; T^f, A_2), \qquad (15.1)$$

$$a + \frac{\partial v_F^G(Y_F, t; T^f, A_2)}{\partial Y_F} = 0, \qquad (17.1)$$

We then check the robustness of our results in Section IV for the general case when

both the hiring- and the firing thresholds are calculated simultaneously.

We start with our baseline, which has a fixed level of firing costs that is

independent of age. This is shown in Figure 1 below. Please see the Appendix D for

the comparison of analytical approximations with explicit finite difference method.

As the firing costs rise, the firm becomes more inclined to fire the younger

among its workers. The reason is simple: part of the cost of firing a worker is the

sacrificed option of doing so in the future. This was shown in equation (15). This

firing option is decreasing in both the level of the firing costs and in the worker's age.

For low levels of firing costs, the marginal cost of firing a young person is much

higher than the cost of firing an older one for this reason. But at high firing costs, the

difference is much smaller as the firing option is always very low—both for the young

and the old worker. However, the marginal benefit of firing the young worker is

always higher—that is for all levels of firing costs—because of his longer remaining

tenure. It follows that the firm would choose to fire the young worker first if firing costs are high—the value of the firing option low—but at low firing costs it may choose to fire the older worker first since the marginal cost of doing so is much lower.
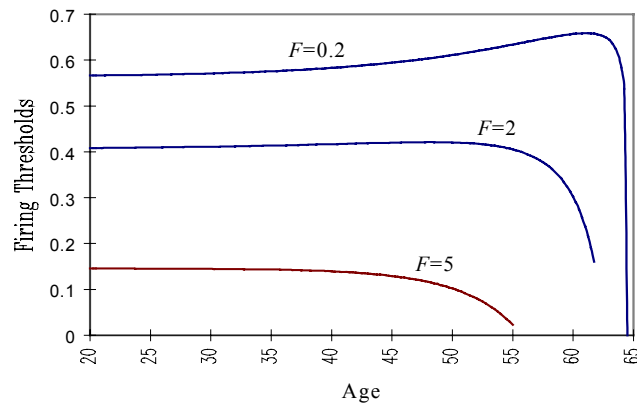


**Figure 1**.The effect of age on firing thresholds with different firing costs. Age is equal to (65-T_f). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, $w$=1, and $t$=0.

It follows from the nature of the stochastic process for productivity shown in equation (2) that the firm does not expect productivity to recover. Therefore, when a young worker is fired the firm is reducing its losses for a much longer period of time than when the older workers are fired. With high firing costs, this is the only difference between the old and the young and the young workers are the first to be fired. However with low firing costs, the firm values the option to be able to fire the workers at a later date when more information about the evolution of productivity is available. This option is worth more in the case of the young workers and can make the firm fire the older ones first.

Note the difference between our setup and that of Lazear and Freeman (1997). They claim that it is optimal to fire the younger workers because they are less productive since the (firm-specific) skill accumulation has not been completed. We find that they should also, if there are significant costs of firing, be the first to go even if their productivity is no lower than that of older workers. Firing a young worker whose productivity is lower than wages is more profitable than firing an older worker since his expected tenure is longer. Note also, that these results do not depend on firing costs rising over tenure. All that is needed is a high and fixed level of firing costs.

We now turn to more realistic scenarios. In Figure 2 we introduce a fixed-term contract at the beginning of employment followed by a permanent contract during

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which redundancy payments are rising in tenure. There is an initial three-year probationary period during which the worker can be fired at no costs. The cost of firing is then an increasing function of tenure. For the flattest firing-cost schedule we find that the old are likely to be fired first. However, as the schedule becomes steeper, the firm resorts to firing younger workers. We conclude that high (as in Figure 1) and rising (as in Figure 2) firing costs affect the age composition of layoffs in a similar manner.
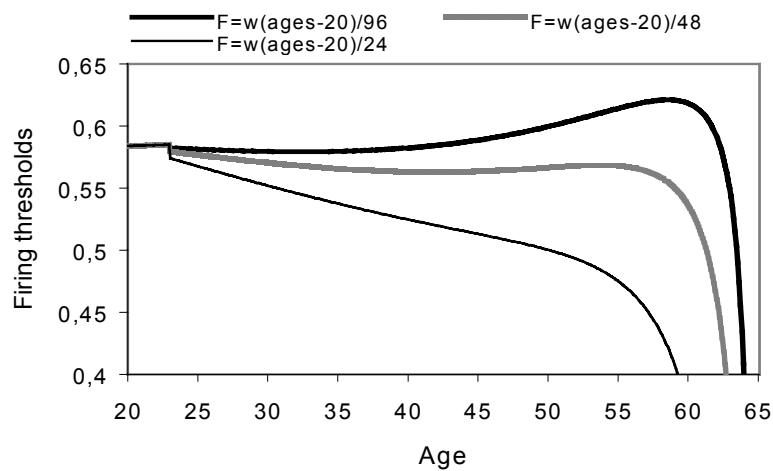


**Figure 2**.The effect of age on firing thresholds with different firing costs and a three-period probationary period. Age is equal to (65-T_f). Other parameters: $\sigma$=0.20, $\rho$=0.10, $\theta$=0.7, $\eta$=0.02, $\lambda$=0.05, $w$=1, and $t$=0.

A discontinuity in the relationship between tenure and firing costs following the completion of the probationary period does not change the results. This is shown in Figure 3. There is a jump in the level of firing costs from zero to a positive number once the probationary period has been completed.
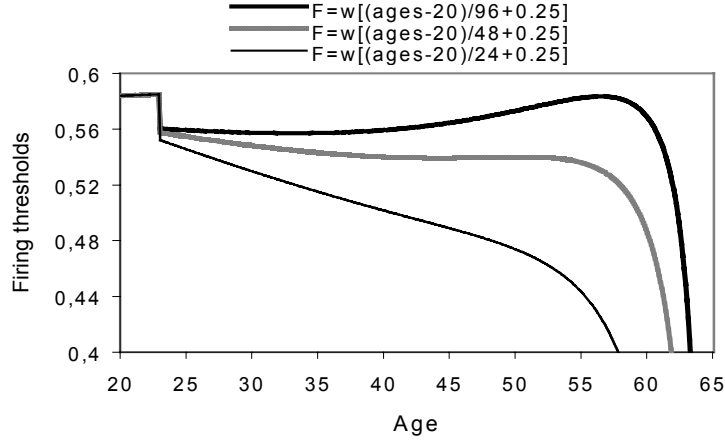
**Figure 3**.The effect of age on firing thresholds with different firing costs and a 3 period probationary period. Age is equal to (65-T_f). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, and t=0.

By comparing Figures 1-3 we find that rising firing costs (such as in Figures 2 and 3) have a bigger effect on the young than a fixed but high level of firing costs Figure 1). Clearly the combination of the two—high firing costs which are rising in tenure—would be the worst combination from the perspective of the young workers.

## IV. Hiring Thresholds in the Two-threshold Case

In order to check the robustness of our results, we calculate the hiring and the firing thresholds for a fixed level of firing costs in the two-threshold case when both the hiring- and the firing thresholds are calculated simultaneously. The results are in Figure 4 below.
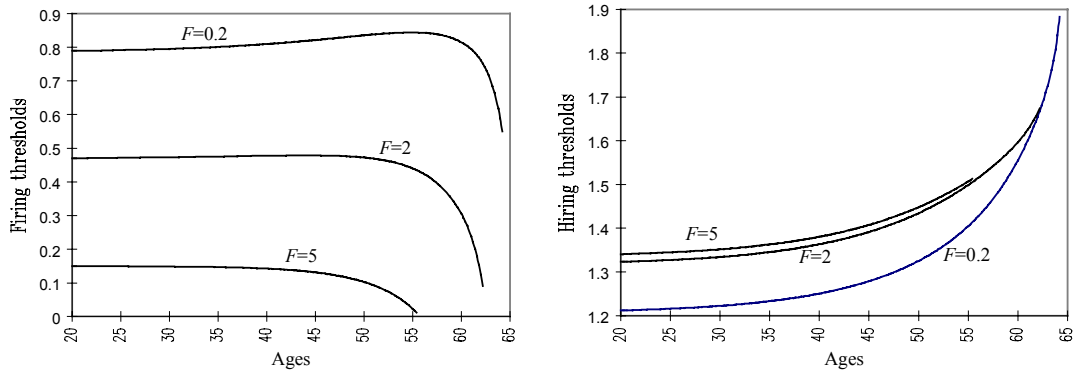


**Figure 4**.The effect of age on the hiring- and firing thresholds with different firing costs. Ages are equal to (65-T_f). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, and t=0.

This figure shows that in this general case, firing thresholds are very similar to the one-barrier case except for a change in the absolute value. Furthermore we find that firms always hire younger workers first no matter what level the firing costs are.

**V. Macroeconomic implications**

We have found that with firing costs (or firing costs that are rising in tenure) provide more protection to the older workers than to the younger ones. It follows that the age structure of the population affects the tightness of employment-protection legislation: the ageing of the workforce has the same effect on the firing thresholds as an increase in the firing costs themselves. This has two implications.

First, when assessing the nature of a country's labour-market institutions one has to normalise for the age structure of the labour force. Two countries with similar legislation can nevertheless have different effective legislation in the sense that firms are more reluctant to lay off (and hire) workers in one of the countries. Second, changes in the age structure of the population over time may have important consequences. The maturing of the baby-boom generation in Europe can be one explanation why a given set of institutions started to generate different labour-market performance in the 1970s and 1980s from that of the 1950s and 1960s. For example, the employment-protection legislation already in place in France, Italy and Spain may have been less restrictive in the 1950s and 1960s than in the 1970s and 80s. We conclude that labour-market rigidity is a function of the age-structure of the population no less than of the nature of labour-market institutions.

Finally, there arises an interaction between the level and steepness of firing costs, on the one hand, and the age of workers, on the other hand, in determining the level of productivity at which firms start firing each worker. With low firing costs (or firing costs that do not rise rapidly with age) the firing threshold is monotonically rising in age—the more mature workers are the first to lose their jobs in a downturn. But as the level of firing costs rises and/or they rise more steeply with age, the sign of this relationship changes and the threshold becomes monotonically falling in age—the young workers are the first to go if labour demand falls. We show the case of different levels of firing costs in Figure 5 and different firing-costs profiles in Figure 6 below.
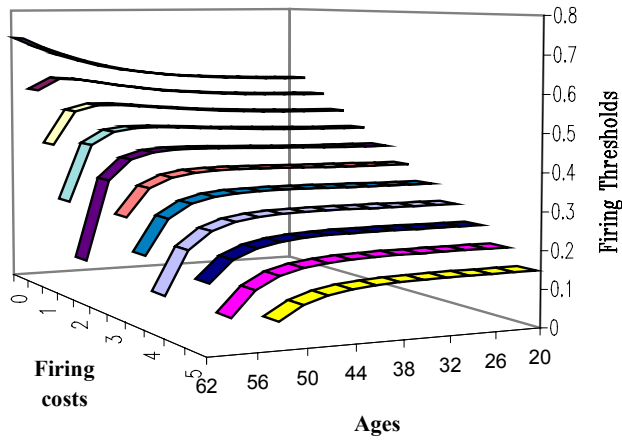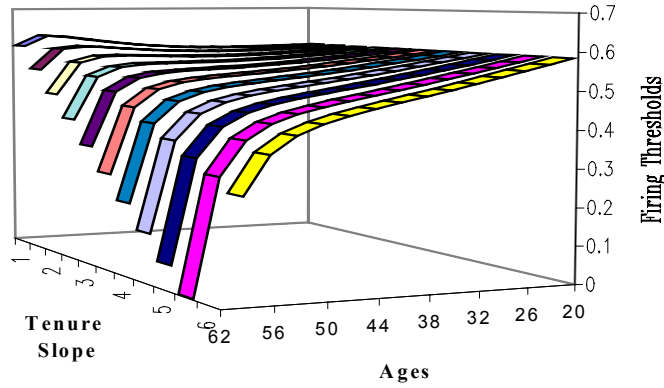
**Figure 5**.The effect of age on the firing threshold with different firing costs. Age is equal to (65-T_f). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, *w*=1, and *t*=0.



Firing costs = (Tenure Slope)*Wage*(45-T f)/96.0

**Figure 6**.The 3-D diagram for the effect of age on the firing thresholds with different tenure slopes (firing costs). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, *w*=1, and *t*=0.

## VI. Conclusions

This paper has shown that the aggregate- and the distributional effects of employment-protection legislation are likely to depend on the age structure of the population and on the age of the workers affected. Such legislation is most effective in deterring the dismissal of mature workers and, as a result, is more likely to lead firms to dismiss the younger ones. Our explanation is independent of the productivity- and wage profiles of workers and also independent of the type of pension schemes they have. The effect arises for the sole reason that the value of the firing option—that is a part of the

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marginal cost of firing—is decreasing in both the level of firing costs and in the age of the worker.

Similarly, we can imagine an economy initially with only young workers. As the workforce ages, the deterrent effect of employment-protection legislation on firms' dismissal decisions (and also hiring decisions due to the expected cost of firing) is likely to rise because the expected return from dismissing a worker is decreasing in his age.[9]

Finally, our results have implications for any empirical work done to test the employment effects of firing costs such as Lazear (1990), Scarpetta (1996), Elmeskov, Martin and Scarpetta (1998), Nickell (1998), and DiTella and MacCulloch (1998). In another paper (Chen and Zoega, 1999) we have shown how the employment effects of firing costs depend on the nature of the stochastic process followed by productivity—trend growth, variance, degree of mean reversion—in addition to the rate of interest and workers' quit rates. Here we have shown that one also has to control for the age-distribution of the workforce when testing for the effect of firing costs on employment or unemployment. In Appendix E we show how this interaction shows up in the data.

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[9] Note that these results would change if productivity had a mean-reverting tendency. We have assumed this to be entirely absent.

# Appendix A:

When equation (8) in the text is set when time is equal to zero ($t=0$) and workers live forever ($T=\infty$), the equation (8) in the text is reduced to

$$(\rho + \lambda)v = \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY}. \tag{A1}$$

(A1) is a homogenous equidimensional linear differential equation and is easily solvable. The solutions to (A1) are:

$$v_0 = A_1 Y^{\beta_1} + A_2 Y^{\beta_2}, \tag{A2}$$

where $A_1$ and $A_2$ are coefficients and $\beta_1$ and $\beta_2$ are the roots of the following characteristic equation,

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \eta_Y\beta - (\rho+\lambda) = 0, \tag{A3}$$

and $\beta_1$ is positive and $\beta_2$ is negative,

$$\beta_1 = \frac{1}{2} - \frac{\eta_Y}{\sigma^2} + \sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}} > 0, \tag{A4}$$

$$\beta_2 = \frac{1}{2} - \frac{\eta_Y}{\sigma^2} - \sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}} < 0. \tag{A5}$$

The hiring and firing solutions for $v_0$ are

$$v_H = A_1 Y^{\beta_1} \quad \text{for hiring option,} \tag{A6}$$

$$v_F = A_2 Y^{\beta_2} \quad \text{for firing option.} \tag{A7}$$

These are equations (10.1) and (10.2) in the text respectively.

# Appendix B:
## Derivation of Equations (12) and (13)

We know that the if workers are expected to have infinite lives, the hiring and firing options are $A_1 Y^{\beta_1}$ and $A_2 Y^{\beta_2}$ respectively. Thus, the first guess for the solutions to equation (8) in the text would be

$$v^G(Y,t) = Y^\beta v(Y,t) \tag{B1}$$

Differentiating (B1) gives

$$v_Y^G = \beta Y^{\beta-1}v + Y^\beta v_Y,$$
$$v_{YY}^G = \beta(\beta-1)Y^{\beta-2}v + 2\beta Y^{\beta-1}v_Y + Y^\beta v_{YY},$$
$$v_t^G = Y^\beta v_t.$$

Substituting into equation (8) in the text gives

$$\frac{1}{2}\sigma^2\left[\beta(\beta-1)v + 2\beta Y v_Y + Y^2 v_{YY}\right] + \eta_Y(\beta v + Y v_Y) + v_t - (\rho+\lambda)v = 0.$$

Rearranging gives

$$\left[\frac{1}{2}\sigma^2\beta(\beta-1) + \eta_Y\beta - (\rho+\lambda)\right]v^G + \frac{1}{2}\sigma^2\left[Y^2 v_{YY} + 2\left(\beta + \frac{\eta_Y}{\sigma^2}\right)Y v_Y + \frac{2}{\sigma^2}v_t\right]Y^\beta = 0. \tag{B2}$$

The first terms in the first bracket are equal to zero automatically due to the characteristic equation of equation (A3) in the Appendix A. With the assumption that the solutions of options have the same components as the ones with infinite maturity, $Y^\beta$, the functions, $v(Y,t)$, then follow a Convection-Diffusion type partial differential equation:

$$Y^2 v_{YY} + 2\left(\beta + \frac{\eta_Y}{\sigma^2}\right)Y v_Y + \frac{2}{\sigma^2}v_t = 0 . \qquad (B3)$$

It is time to get rid of the $Y$ and $Y^2$ terms. Let

$$Y = e^y, \quad -\infty < y < \infty , \quad \frac{1}{2}\sigma^2 t = \frac{1}{2}\sigma^2 T - \tau,$$

where $T$ is a constant and $\overline{Y}$ is the exercise price, firing or hiring costs. Then we get

$$v_y = Y v_Y, \; v_{yy} = Y^2 v_{YY} + Y v_Y, \; \text{and } \frac{1}{2}\sigma^2 v_\tau = -v_t .$$

Substituting into (B3) gives

$$v_{yy} + 2\left(\beta + \frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)v_y - v_\tau = 0 . \qquad (B4)$$

The boundary and conditions, equation (9.1) and (9.2) in the text become

$$v(\infty, \tau) = 0, \; \text{for firing options,} \qquad (B5.1)$$
$$v(-\infty, \tau) = 0, \; \text{for hiring options,} \qquad (B5.2)$$

Substituting the values of betas, $\beta_1$ and $\beta_2$, of equations (A4) and (A5) in the Appendix A into (B4) gives

$$v_{yy} + 2\sqrt{\alpha}\,v_y - v_\tau = 0, \text{ for hiring options,} \qquad (B6)$$
$$v_{yy} - 2\sqrt{\alpha}\,v_y - v_\tau = 0, \text{ for firing options,} \qquad (B7)$$

where $\alpha = \left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho + \lambda)}{\sigma^2}$.

*Hiring options*
We can simplify (B6) by setting

$$x = y + 2\sqrt{\alpha}\,\tau, \; \overline{\tau} = \tau .$$

Note that $\overline{\tau}$ is the same as $\tau$. To rewrite (B6) in terms of $(x, \overline{\tau})$ we use the chain rule

$$v_\tau = v_x x_\tau + v_{\overline{\tau}} \overline{\tau}_\tau = 2\sqrt{\alpha}\,v_x + v_{\overline{\tau}},$$
$$v_y = v_x x_y = v_x, \text{ and } v_{yy} = v_{xx} .$$

Substituting into (B6) gives

$$v_{xx} = v_{\overline{\tau}} . \qquad (B8)$$

A new variable that depends only on $x$ and $\overline{\tau}$ is often used to solve the above partial differential equation:

$$\xi = \frac{x}{\sqrt{\overline{\tau}}}, \qquad (B9)$$

so that $v(x, \overline{\tau}) = u(\xi)$. Differentiating shows that

$$v_{\overline{\tau}} = -\frac{1}{2\overline{\tau}}\xi u'(\xi), \; v_{xx} = \frac{1}{\overline{\tau}}\xi u''(\xi) .$$

Substituting into equation (B8) gives the following second-order ordinary differential equation:

$$u''(\xi) + \frac{1}{2}\xi u' = 0, \quad -\infty < \xi < \infty, \tag{B10}$$

The boundary condition of (B5.1) becomes the following equation:
$$u(-\infty) = 0, \text{ for hiring options,} \tag{B11}$$

Separating the variables, (B10) becomes
$$u'(\xi) = B_1 e^{-\xi^2/4},$$

where $B_1$ is unknown constant. Integrating gives
$$u(\xi) = B_1 \int_{-\infty}^{\xi} e^{-s^2/4} ds + C_1, \tag{B12}$$

where $C_1$ is an unknown constant. Applying the boundary condition for hiring options (B11) gives
$$\lim_{\xi \to -\infty} u(\xi) = C_1 = 0.$$

Substituting into (B12) gives
$$u(\xi) = B_1 \int_{-\infty}^{\xi} e^{-s^2/4} ds.$$

It is convenient to make the change of variable $s = \sqrt{2}\varpi$, so that
$$u(\xi) = B_1 \sqrt{2} \int_{-\infty}^{\sqrt{2}\xi} e^{-\varpi^2/2} d\varpi = A_1 \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{2}\xi} e^{-\varpi^2/2} d\varpi. \tag{B13}$$

where $A_1 = B_1 2\sqrt{\pi}$. Substituting (B13) into (B1) and using the facts of $Y = e^y$,
$$\frac{1}{2}\sigma^2 t = \frac{1}{2}\sigma^2 T - \tau, \quad x = y + 2\sqrt{\alpha}\tau, \text{ and } \bar{\tau} = \tau \text{ gives the hiring options } v_H^G,$$

$$v_H^G(Y,t) = A_1 Y^{\beta_1} N(d_1), \tag{B14}$$

where $d_1 = \dfrac{\ln Y + \sigma^2(T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$ and $N(d) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi,$.

*Firing options*

In a similar way, we can obtain the firing options. We can simplify (B7) by setting $x = y - 2\sqrt{\alpha}\tau$ and $\bar{\tau} = \tau$.

$$v_{xx} = v_{\bar{\tau}}. \tag{B15}$$

A new variable $\xi = \dfrac{x}{\sqrt{\tau}}$ is used to solve the above partial differential equation so that $v(x,\bar{\tau}) = u(\xi)$. Differentiating and substituting into (B15) gives the following simple second order ordinary differential equation:

$$u''(\xi) + \frac{1}{2}\xi u' = 0, \quad -\infty < \xi < \infty.$$

Separating the variables, the above equation becomes
$$u'(\xi) = B_2 e^{-\xi^2/4},$$

where $B_2$ is unknown constant. Integrating gives

$$u(\xi) = B_2 \int_{-\infty}^{\xi} e^{-s^2/4} ds + A_2, \tag{B16}$$

where $A_2$ is an unknown constant. The boundary condition of (B5.2) becomes the following equation:

$$u(\infty) = 0, \quad \text{for firing options,} \tag{B17}$$

Applying the boundary condition for hiring options (B16) gives

$$\lim_{\xi \to \infty} u(\xi) = 2\sqrt{\pi} B_2 + A_2 = 0 \qquad B_2 = \frac{A_2}{2\sqrt{\pi}}.$$

Substituting into (B16) gives

$$u(\xi) = A_2 \left( 1 - \frac{1}{2\sqrt{\pi}} \int_{-\infty}^{\xi} e^{-s^2/4} ds \right).$$

It is convenient to make the change of variable $s = \sqrt{2}\varpi$, so that

$$u(\xi) = A_2 \left( 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\sqrt{2}\xi} e^{-\varpi^2/4} d\varpi \right) = A_2 \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2}\xi}^{\infty} e^{-\varpi^2/2} d\varpi = A_2 \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\sqrt{2}\xi} e^{-\varpi^2/2} d\varpi.$$

Thus, the firing options $v_H^G$ becomes

$$v_F^G(Y,t) = A_2 Y^{\beta_2} N(-d_2), \tag{B18}$$

where $d_2 = \dfrac{\ln Y - \sigma^2(T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$ and $N(d) = \dfrac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi,$ .

## Appendix C:
### Derivation of Equations (20) and (21)

By definition,

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi, \quad 0 \le N(d) \le 1$$

where

$$d_1 = \frac{\ln Y + \sigma^2(T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln Y - \sigma^2(T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}.$$

Differentiation of the integral, $N(d)$, involves a parameter. Suppose a function

$$\varphi(x) = \int_{a(x)}^{b(x)} f(x,s) ds, \tag{C1}$$

where $f$ is such that the integration cannot be effected analytically. Using calculus gives

$$\varphi_x(x) = \int_{a(x)}^{b(x)} \frac{\partial f(x,s)}{\partial x} ds + f(x,b(x))b_x(x) - f(x,a(x))a_x(x). \tag{C2}$$

Applying (C2) to the differentiation of $N(d_1)$ and $N(-d_2)$ gives

$$N_Y(d_1) = \frac{e^{\frac{-[\ln Y + \sigma^2(T-t)\sqrt{\alpha}]^2}{2\sigma^2(T-t)}}}{\sigma Y \sqrt{2\pi(T-t)}}, \tag{C3}$$

$$N_Y(-d_2) = -\frac{e^{\frac{-[-\ln Y + \sigma^2(T-t)\sqrt{\alpha}]^2}{2\sigma^2(T-t)}}}{\sigma Y \sqrt{2\pi(T-t)}}. \tag{C4}$$

where $\alpha = \left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}$.

## Appendix D

Equation (8) in the text can be solved numerically by finite different method. To compare the numerical results of finite different method with the analytical solutions in the Appendix C, we use the simple and robust explicit finite difference method, which is widely used in the pricing of derivatives.

For firing options with maturity $T$, the boundary condition is $v^G(\infty, t) = 0$ and $v^G(0,t) = \max[-(aY - bw) - F, 0]$, where $a = \left(e^{-(\rho+\lambda-\eta_Y)t} - e^{-(\rho+\lambda-\eta_Y)T}\right)/(\rho+\lambda-\eta_Y)$, $b = \left(e^{-(\rho+\lambda)t} - e^{-(\rho+\lambda)T}\right)/(\rho+\lambda)$, and $F$ the firing costs. The terminal condition (in the programme, calculated from $T$ to 0) is $f(Y,T) = 0$. The condition of $v^G(Y,t) = \max[-(aY - bw) - F, v^G]$ is checked for every $t$ since it is a free-boundary condition in a sense that the firing option can be exercised at any time.

Equation (8) in the text,

$$(\rho+\lambda)v^G = \eta_Y Y v_Y^G + \frac{1}{2}\sigma^2 Y^2 v_{YY}^G + v_t^G, \tag{D1}$$

can be approximated by the following grids.[10] Let $v^G(t, Y) \equiv v_{i,j}$,

$$\frac{\partial v^G}{\partial Y} = \frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta Y}$$

$$\frac{\partial^2 v^G}{\partial Y^2} = \frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta Y^2}$$

$$\frac{\partial v^G}{\partial t} = \frac{v_{i+1,j} - v_{i,j}}{\Delta t}$$

Substituting into (D1) gives

$$\frac{v_{i+1,j} - v_{i,j}}{\Delta t} + \eta_Y j\Delta Y\left(\frac{v_{i+1,j+1} - v_{i+1,j-1}}{2\Delta Y}\right)$$
$$+ \frac{1}{2}\sigma^2 j^2 \Delta S^2\left(\frac{v_{i+1,j+1} + v_{i+1,j-1} - 2v_{i+1,j}}{\Delta Y^2}\right) = (\rho+\lambda)v_{i,j} \tag{D2}$$

Rearranging gives

$$v_{i,j} = a_j^* v_{i+1,j-1} + b_j^* v_{i+1,j} + c_j^* v_{i+1,j+1} \tag{D3}$$

where

$$a_j^* = \frac{1}{1+(\rho+\lambda)\Delta t}\left(-\frac{1}{2}\eta_Y j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

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[10] For a similar algorithm in derivative pricing, see Brennan and Schwartz (1978).

$$b_j^* = \frac{1}{1+(\rho+\lambda)\Delta t}\left(1-\sigma^2 j^2 \Delta t\right)$$

$$c_j^* = \frac{1}{1+(\rho+\lambda)\Delta t}\left(\frac{1}{2}\eta_Y j\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

The firing thresholds calculated from above algorithm are shown in figure D1, together with the ones in figure 1. The results show that the analytical solutions are good approximations to the real thresholds.
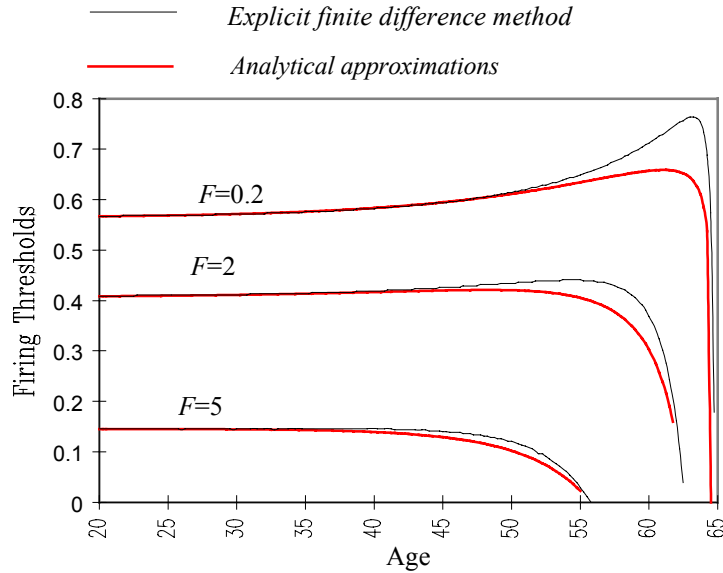


**Figure D1**. The comparison of explicit finite difference method and numerical approximations. All parameters are the same as in figure 1 in the text.

## Appendix E

Following Nickell (1998) we estimate an equation where the dependent variable is average unemployment in 1983-1988 and 1989-1994 for 19 OECD countries.[11] The explanatory variables include the unemployment-benefit replacement ratio (*replace*), the maximum duration of benefits (*duration*), union density (*unden*), union coverage (*uncov*), union coordination (*uncoord*), employer coordination (*emcoord*), a measure of active labour-market policies (*labexp*), the average change in the inflation rate and a measure of employment protection (*epl*).

The first two columns in the table below show the standard results when we do not allow for any interaction with the age structure of the population. The sign and significance of all variables is as expected. We note that *epl* has an insignificant coefficient. We then add the share of the labour force between the ages of 15 and 19 (*age*) as an interaction term with the *epl* in columns 3 and 4. The coefficients are not much affected apart from the constant term and the coefficient of *epl*. The effect of *epl* becomes stronger when we include the share of the labour force between the ages of 15 and 24. This is shown in the last two columns.

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[11] We are grateful to Stephen Nickell for providing us with the data.

**Table 2.** Unemployment equations

| Variable | No interaction terms | | Age (15-19) as interaction term | | Age (15-24) as interaction term | |
|---|---|---|---|---|---|---|
| | Coefficient | t-ratio | Coefficient | t-ratio | Coefficient | t-ratio |
| C | 6.29 | 1.5 | -7.82 | -0.8 | -26.50 | -2.4 |
| Dummy (89-94) | 7.61 | 1.8 | -6.54 | -0.7 | -24.29 | -2.3 |
| Replace | 0.10 | 4.2 | 0.12 | 4.8 | 0.14 | 6.2 |
| Duration | 0.70 | 2.1 | 0.58 | 1.7 | 0.71 | 2.4 |
| Density | 0.03 | 0.8 | 0.03 | 0.6 | 0.06 | 1.7 |
| Uncoord. | -2.12 | 2.0 | -2.21 | -2.1 | -2.74 | -3.0 |
| Emcoord | -3.77 | 4.2 | -4.01 | -4.5 | -4.29 | -5.4 |
| Labexp | -0.07 | 1.9 | -0.10 | -2.4 | -0.11 | -3.0 |
| $\Delta$inflation | -1.01 | 1.5 | -1.45 | -2.1 | -1.47 | -2.4 |
| Uncov | 3.16 | 2.7 | 3.07 | 2.6 | 3.07 | 2.8 |
| Epl | 0.49 | 0.7 | -2.03 | -0.3 | -34.92 | -1.9 |
| Age (15-19) | -0.42 | 1.4 | 0.74 | 1.0 | 1.02 | 2.6 |
| Epl*age | | | 1.09 | 0.5 | 3.62 | 2.3 |
| Epl*age$^2$ | | | -0.07 | -0.7 | -0.09 | -2.5 |

| Observations: 38 | $R^2$ | DW |
|---|---|---|
| Period: 83-88 | 0.87 | 2.03 |
| Period: 89-94 | 0.49 | 1.73 |

In the last two regressions, the effect of *epl* is significant and a function of the age structure. This function is shown in the following figure when we use the age group 15-24.



Effect of Employment Protection
($-34.9 + 3.6*age - 0.09*age^2$)
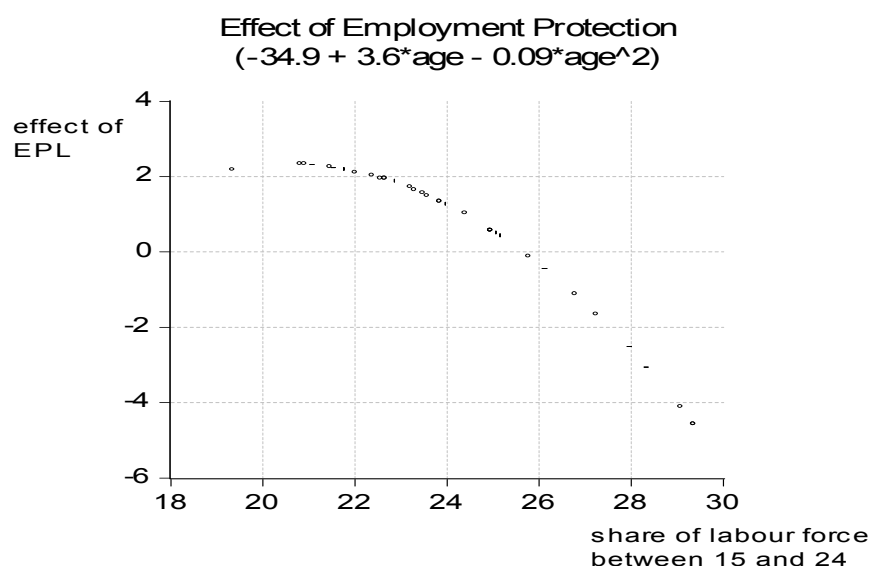
When the share of the labour force between 15 and 24 is less than 26%, the effect of the *epl* on unemployment is positive—greater *epl* gives higher unemployment—while the converse is true when the share is higher than 26%. We conclude that when the effect of *epl* on unemployment is allowed to depend on the age structure of the labour force, its effect becomes statistically significant.

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