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Life-Cycle, Effort and Academic Deadwood

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Abstract

It has been observed that university professors sometimes become less research active in their mature years. This paper models the decision to become inactive as a utility maximising problem under conditions of uncertainty and derives an age-dependent inactivity condition for the level of research productivity. The economic analysis is applicable to other professions as well were work effort is difficult to observe along some dimensions.

Keywords: Deadwood, aging, optimal stopping, salary schemes.

JEL Classification: J44, J22

1. Introduction

Young university lecturers frequently complain about colleagues who are not engaged in research, who have become “deadwood” in common parlance. The reasons why university teachers may end up as deadwood are seldom explained; sometimes one might conclude from the complaints that they were infected by unexpected spells of slothfulness. Workers in other professions may also gradually reduce their effort and become less active at work. Some workers may also choose to move to professions where opportunities for on-the job leisure are greater because it is more difficult to measure effort – politicians sometimes end up as diplomats; football players as celebrities and movie stars may take on fewer roles and end up enjoying leisure and sometimes fame. In some cases the decision is driven by physical deterioration, such as in sports, but in other

cases it is for other less well-defined reasons such as when academics stop spending his time doing research.¹ The question that then arises is whether this is due to declining mental abilities or results from changing incentives. The objective of this paper is to show how older intertemporal-utility-maximising workers may face incentives to become less active at work in spite of undiminished physical and mental strength.

We will explain our argument using the university workplace as an example but it applies also to other professions where workers' level of exertion is at least partly up to their own discretion and can only be imperfectly observed by employers. Our model shows why academics may face reduced incentives to do research as they get closer to retirement. One insight coming from our model is that older workers are less threatened by the possibility of being dismissed and future research successes are also less important for them when compared to current sacrifices brought about by strenuous research effort.² The model also helps distinguish those individuals who are more likely to become inactive. Those who remain research active are the ones who enjoy research or those whose productivity is sufficiently great to offset any incentives to slow down. It follows that while there are many inactive older professors the active professors tend to be quite productive. This suggests that the degree of heterogeneity in terms of research productivity increases with age.

There is statistical evidence showing that research productivity is declining in age. Oster and Hamermesh (1998) find that economists' productivity measured by publications in leading journals declines with age although the probability of acceptance, once an article has been submitted to a leading journal, is independent of age. Moreover they find that the median age of authors of articles in leading economics journals was 36 in the 1980s and the 1990s and that a very small minority of authors are over 50 in spite of a

¹ Even in the case of athletes, the retirement decision is to some extent up to the individual's discretion because the rate of deterioration of physical ability tends to be quite small. This has been demonstrated in many studies, such as Fair (1994, 2007) who fails to find a strong effect of aging on physical abilities.

² The model resembles the model of Lazear and Rosen (1981) on rank-order tournaments. In their model workers exert effort in the hope of being promoted in the future while in the current model they exert effort in the hope of enjoying an unexpected productivity surge.

substantial percentage of AEA members being over the age of 50.³ However, they cannot discriminate between the two possible reasons for this observation; whether the falling frequency of publications is due to deteriorating mental faculties or, alternatively, reflects rational decisions to devote less time to research. In a recent paper, Jones (2010) analyses the age of individuals at the time of their greatest achievements in science using data on research that leads to the Nobel Prize in physics, chemistry, medicine and economics and also data on research that leads to great technological achievements as shown in the almanacs of the history of technology. He finds that the greatest concentration of innovations in the life of a scientist comes in the 30s but a substantial amount also comes in the 40s, while scientists in their 50s, and even more so in the 60, generate far fewer discoveries.

2. A Model of Academic Deadwood

In this section we model the decision by a tenured professor whether to remain research active. The representative professor devotes his time to teaching, administration and research. Of these tasks, his efforts at teaching and administration are observable by the university and justify paying him a fixed salary w_0 . In contrast, there is asymmetric information about research effort. Low observable output in the form of published papers and books can have many possible explanations, such as excessive attention to detail by the professor, the research projects being very ambitious and time consuming, bad luck when it comes to the choice of journals and publishers for submitting research results or simply that the professor is engaged in the type of research that the profession does not value at the moment due to fads and fashions. There is also, of course the possibility that the professor is simply not spending his time doing research.

We first explore the case when it is impossible to monitor research effort but then extend the analysis in Section 3 to the case of imperfect monitoring of effort and finally to include tenure effects in Section 4.

³ Similar results are reached by Lehman (1953), Diamond (1986), McDowell (1982) and Levin and Stephan (1992) for other disciplines. However, Jan van Ours (2009) finds no relationship between the quality-adjusted rate of publication and age among his colleagues at Tilburg University.

2.1 Assumptions

We assume that a representative professor faces a decision whether or not to continue doing research in the future. The level of effort, f , has two possible values, zero and one as in Shapiro and Stiglitz (1984); when f is equal to one the professor is doing research while f takes the value zero when he is inactive in research although still teaching and performing administrative duties. When doing research the professor suffers disutility γ – caused by the constant exertion needed to get results – but generates measurable research output g which the university uses to calculate his performance-related pay. The variable γ can take a negative value if the professor enjoys doing research – in which case he will never choose to become inactive. When a professor decides to become inactive in research, that is f becomes zero, he faces the one-off wrath of his colleagues which we measure by the variable W .⁴ However, continuing research effort gives an uncertain return in the form of research output that is used to calculate performance-related pay.

The professor's pay is a linear function of observable research output g ;

$$(1) \quad w = w_0 + w_b g f .$$

It follows that a worker not doing research would receive the basic professorial pay w_0 and a worker engaged in research would receive $w_0 + w_b g$, while having disutility γ from doing the work.⁵

Future research output is uncertain because of the possibility that research effort will not generate sufficiently interesting and innovative research results, because of uncertainty about how quickly results will be achieved and also because of uncertainty about the reception by editors of professional journals whose preferences are difficult to predict.

⁴ One can also model the reaction of colleagues as a constant expression of disapproval when not doing research. In this case one can define the variable γ as the disutility of effort net of the approval of colleagues when doing research.

⁵ The model can be easily modified to take into account wage compression – differences in productivity between two workers exceeding differences in wages – as in Frank (1984) by raising g in equation (1) to a power which is less than one in numerical value. See also Booth and Zoega (2004).

Each individual can also expect his productivity to change in the future, depending on the environment in which he finds himself, the extent of learning by doing, personal circumstances, the productivity of colleagues and collaborators and so forth. To capture these dynamics, it is assumed that the level of g follows a geometric drifted Brownian motion,

$$(2) \quad dg = \eta g dt + \sigma g dz,$$

where η is the drift parameter of research output, σ is the uncertainty parameter about future research output and z is the standard Wiener process – a normalised Gaussian process with independent increments.

The professor has to make up his mind whether to do research or not, keeping in mind that although not doing research yields utility in the form of leisure at work, it also reduces the level of research output and hence also the amount of performance-related pay. The professor thus faces an optimal stopping problem when he decides whether to shirk his research duties. We make the assumption the professor cannot resume researching once he has decided to shirk.⁶

2.2 The research decision

The professor has utility which is linear in wages and the disutility of doing research; $w_0 + w_b g_t - \gamma$ when doing research and w_0 when not doing research. This gives the following intertemporal utility function;

$$(3) \quad V = E \left[\int_t^T (w_0 + w_b g_s f - \gamma f) e^{-\rho(s-t)} ds \right],$$

⁶ Implicitly, it means that there are large sunk costs involved in resuming research such that the option to resume research approaches null value for the decision not to do research; for example, the laboratory is gone forever or the human capital has depreciated.

subject to (2), where $E[\]$ is the expectation operator, ρ is the discount rate of the professor and $T - t$ is the time remaining until the professor retires. If the professor chooses not to do research, the intertemporal utility for not being research active V^S is

$$(4) \quad V^S = E \left[\int_t^T w_0 e^{-\rho(s-t)} ds \right] = \frac{w_0 (1 - e^{-\rho(T-t)})}{\rho},$$

which is obtained by integrating the integral directly as there is no way back to research in the future. In contrast, if he chooses to do research the professor has the following intertemporal utility V^R

$$(5) \quad V^R = E \left[\int_t^T (w_0 + w_b g_s - \gamma) e^{-\rho(s-t)} dt \right],$$

where the difference between (4) and (5) lies in the research-active professor expecting performance-related pay but also enduring the disutility of more effort γ , which can be either positive or negative, negative if the professor enjoys the effort and challenges of research effort, otherwise positive.

The decision by the professor whether to discontinue research depends on whether the discounted utility from not being research active for the remainder of his tenure V^S exceeds the sum of the discounted utility from being research active V^R and the expected discomfort from the reaction of colleagues when stopping research W . The equality of the two generates an inactivity condition which we call the *non-deadwood condition*

$$(6) \quad V^S = V^R + W$$

Equation (6) is analogous to the non-shirking condition of Shapiro and Stiglitz (1984). The condition determines the research productivity level g – hence the wages $w_0 + w_b g$ – that is needed to convince the professor to continue doing research for a given

performance-related pay w_b , a system of measuring performance g , the disutility from doing research γ , peer pressure W and, as we will show, age.

2.3 The non-deadwood condition

We need to solve the non-deadwood condition. While the solution for V^S is given by equation (4), we still need to solve for V^R . The Bellman equation for equation (5) is the following,

$$(7) \quad \rho V^R = w_0 + w_b g - \gamma + \eta g V_g^R + \frac{1}{2} \sigma^2 g^2 V_{gg}^R + V_t^R,$$

where $w_0 + w_b g - \gamma$ represents the net utility from working at the university, $\eta g V_g$ shows changes in V^R due to a drift in research productivity, and the last two terms denote changes in V^R due to diffusion.

The solution to equation (7) comprises a particular solution, representing the net benefits from doing research for the rest of one's career, and a homogenous solution, which is equivalent to the value of the real option to discontinue research later. Therefore, we have the following solutions for V^R (see Appendix A for details),

$$(8) \quad V^R = \frac{(w_0 - \gamma)(1 - e^{-\rho(T-t)})}{\rho} + \frac{w_b g (1 - e^{-(\rho-\eta)(T-t)})}{\rho - \eta} + A_2 g^{\beta_2} N(-d_2),$$

where the first two terms of the right-hand side are obtained from directly integrating the intertemporal utility; the last term on the right-hand side denotes the real option to discontinue research and A_2 is unknown parameter to be determined by the value-matching condition of the optimal stopping problem.⁷ The parameter β_2 is the negative root of the following characteristic equation

⁷ For readers who would like to study the rapidly developing literature of real options and optimal stopping applications in economics for the past two decades, see Dixit and Pindyck (1994) and Stokey (2008) for further references.

$$(9) \quad \frac{1}{2} \sigma^2 \beta (\beta - 1) + \eta \beta - \rho = 0,$$

$$\text{and } d_2 = \frac{\ln g - \sigma^2 (T - t) \sqrt{\left(\frac{\eta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}}{\sigma \sqrt{T - t}}, \quad N(-d_2) = \left(1/\sqrt{2\pi}\right) \int_{-\infty}^{-d_2} e^{-\varpi^2/2} d\varpi$$

and $0 \leq N(d) \leq 1$ is the cumulative normal distribution function. Note that as T approaches infinity, $N(-d_2) = 1$ and the option to discontinue research becomes a perpetual option case; however, as T approaches zero, $N(-d_2) = 0$ if $g > 1$. As the professor is near retirement, the value of the option to discontinue research approaches zero because he is going to retire soon.⁸

We can now write an equation for the non-deadwood condition $V^S - V^R = W$ where \underline{g} is the productivity threshold at which the professor decides to discontinue research:

$$(10) \quad \frac{\gamma(1 - e^{-\rho(T-t)})}{\rho} = \frac{w_b \underline{g} (1 - e^{-(\rho-\eta)(T-t)})}{\rho - \eta} + A_2 \underline{g}^{\beta_2} N(-d_2(\underline{g})) + W,$$

The left-hand side of equation (10) shows the benefits of discontinuing research – becoming a deadwood – and the right-hand side the cost. The benefits consist of the expected discounted disutility of doing research, which the professor avoids by not doing research. The costs consist of the sum of the sacrificed expected discounted utility of the performance-related pay, the value of the real option to discontinue research in the future, and the one-off no-pecuniary penalty W – or peer pressure – imposed by colleagues when a professor stops doing research.⁹

⁸ Note that the A_2 parameter is also a function of $(1 - e^{-(\rho-\eta)(T-t)})$ and comes from the value-matching/smooth-pasting conditions. The option therefore also approaches zero as $T \rightarrow t$. See equation (13) for details.

⁹ See footnote 4 on the case of a constant peer pressure.

A professor may continue doing research even when the performance-related pay no longer compensates for the disutility from doing research. For someone who either dislikes doing research – γ is positive and high – or is not very productive – g is low – or is not paid very much for his research output – w_b is low – or expects his research output to decline – η is negative – it may nevertheless be optimal to continue doing research because of the possibility that productivity may improve in the future, the real option value is large (β_2 is negative so that g^{β_2} is higher the lower is the value of g). However, as the professor nears retirement, both the benefits from continuing research and the real option approach zero, as does the discounted disutility of research effort.

Equation (10) has several intuitive implications. Clearly, when productivity g is sufficiently high, the professor will continue doing research. The critical productivity level \underline{g} is higher

- a) the lower is the performance-related pay w_b ,
- b) the greater is the disutility of doing research γ ,
- c) the weaker is peer pressure from colleagues W ,
- d) the smaller is the rate of growth of research productivity η , and
- e) the lower is the level of uncertainty σ .

More importantly, the critical productivity threshold depends on the age of the workers;

- f) the older the worker the higher is the productivity threshold at which he becomes inactive for $\eta > 0$, except for workers who are to retire very soon.

The intuition for the age effect is the following: Due to expected growth in research productivity η the performance-related pay is expected to rise, as well as the real option, while the disutility from doing research stays constant over time. Hence, from the current perspective, the final year's expected performance pay and possible research successes (the real option) count more in the professor's decision than the expected disutility of work. It follows that as the professor gets older the cost of giving up research declines

more rapidly than the benefits and he is more likely to decide to become inactive in research for a given slump in research productivity. The young, in contrast, may decide to continue doing research because they have more time to enjoy the fruits of higher productivity and unexpected successes and this justifies current effort.

2.4 Numerical simulations

In order to further analyse the properties of the non-deadwood condition we run some numerical simulations. Before doing so, we need to determine the value of A_2 from the smooth-pasting condition,

$$(11) \quad -\frac{w_b(1-e^{-(\rho-\eta)(T-t)})}{\rho-\eta} = A_2 \frac{\partial(\underline{g}^{\beta_2} N(-d_2))}{\partial \underline{g}} = A_2 \left[\beta_2 \underline{g}^{\beta_2-1} N(-d_2) - \frac{1}{\sqrt{2\pi}} \frac{\underline{g}^{\beta_2-1}}{\sigma\sqrt{T-t}} e^{-d_2^2/2} \right].$$

The equation yields a solution for A_2 , which is the following equation,

$$(12) \quad A_2 = \frac{w_b(1-e^{-(\rho-\eta)(T-t)})}{\rho-\eta} \frac{1}{\left[-\beta_2 \underline{g}^{\beta_2-1} N(-d_2(\underline{g})) + \frac{1}{\sqrt{2\pi}} \frac{\underline{g}^{\beta_2-1}}{\sigma\sqrt{T-t}} e^{-d_2^2/2} \right]}.$$

And the value of the real option to discontinue research is then as follows:

$$(13) \quad A_2 \underline{g}^{\beta_2} N(-d_2(\underline{g})) = \frac{w_b \underline{g} (1-e^{-(\rho-\eta)(T-t)}) N(-d_2(\underline{g}))}{(\rho-\eta) \left(-\beta_2 N(-d_2(\underline{g})) + \frac{1}{\sqrt{2\pi}} \frac{e^{-d_2^2/2}}{\sigma\sqrt{T-t}} \right)}.$$

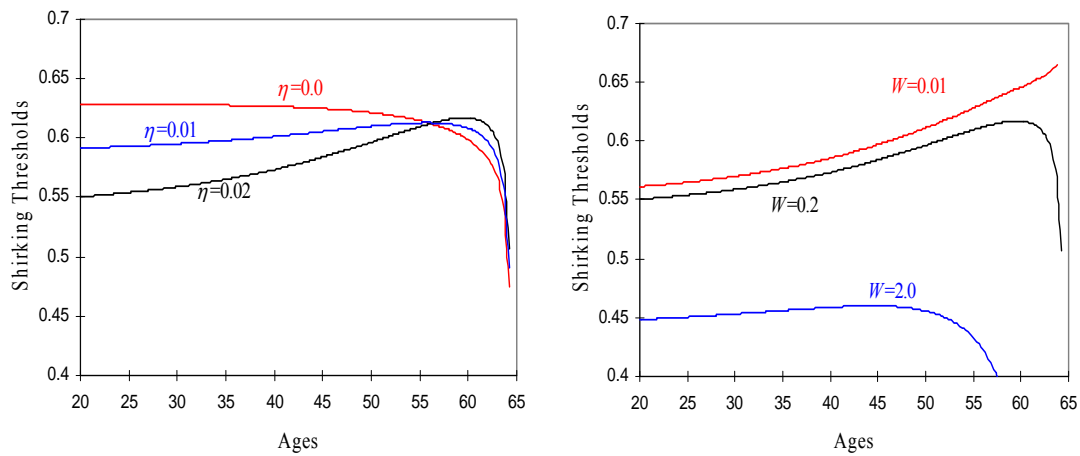
To further analyse the relationship between research effort and age we will perform numerical simulations on equations (10) to (13).

The figures below show the non-deadwood threshold derived from simulation results using some benchmark values listed at the bottom of the figures. The left-hand panel of

Figure 1 shows that the non-deadwood productivity threshold is increasing in age when $\eta > 0$ until the professor is just about to retire when it falls abruptly.¹⁰ This implies that as the professor gets older he needs a higher level of research productivity to justify continued research. A slump in research productivity – perhaps a sequence of rejections from academic journals – is hence more likely to convince the older workers to discontinue research and become inactive. The right-hand panel of Figure 1 shows how an increase in peer pressure W lowers the threshold so that it takes a greater slump in research productivity to convince the professor to stop research. Figure 2 shows that increased uncertainty about future research output σ has the same effect of shifting the thresholds downwards, as does an increase in the rate of performance-related pay w_b .

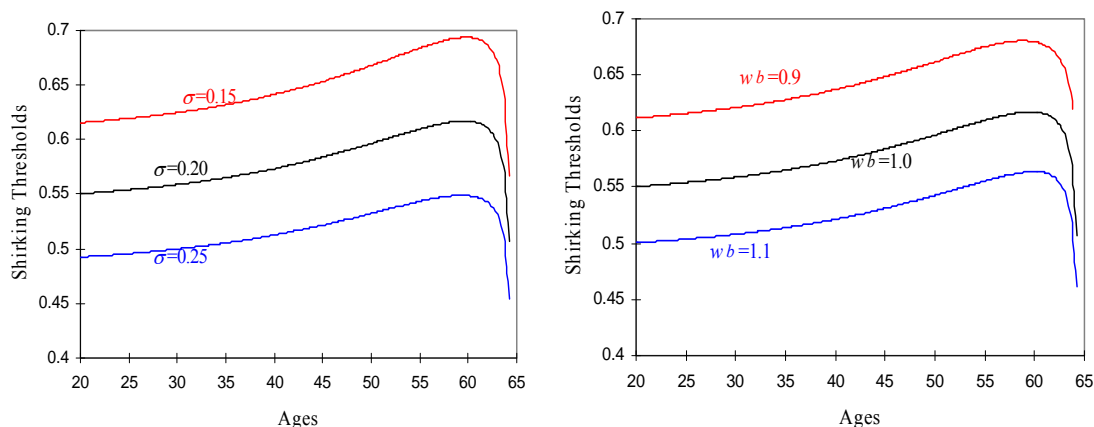
The fall in the threshold close to retirement is caused by the non-zero cost of discontinuing research W . A professor will not want to attract the scorn of his colleagues for stopping research for a very short period of time. This is apparent in the right-hand panel of Figure 1.

Figure 1. Productivity growth, peer pressure and the inactivity threshold



¹⁰ The effect is reversed when growth η is negative, the younger professors may then decide to become inactive at higher levels of productivity than the older ones.

Figure 2. Uncertainty, performance-related pay and the inactivity threshold



Parameter values: $\sigma = 0.2$, $\rho = 0.1$, $\eta = 0.02$, $w_b = 1$, $W = 0.2$, $t = 0$, $\gamma = 1.0$, and age $= 65 - T$. Note that for a professor with ten years to retirement at the age of 55 the value of T is equal to 10.

3. Monitoring

We now change the model in order to allow for monitoring of research effort. As in the efficiency wage literature, we assume that the departmental chair can observe research effort, in addition to research output, but only at a cost. Research effort can thus be checked regularly and the professor gets fired if caught shirking his research duties.

Following Shapiro and Stiglitz (1984) we assume a Poisson detection technology and let professors who do not do research face a constant probability q of being fired. This makes the research inactive professor discount his future wages w_0 at rate $\rho + q$ when not doing research, while the discount rate remains unchanged at ρ when doing research. This addition to the model changes equation (4) to

$$(4') \quad V^S = E \left[\int_t^T w_0 e^{-(\rho+q)(s-t)} ds \right] = \frac{w_0 (1 - e^{-(\rho+q)(T-t)})}{\rho + q}$$

and we get two new terms on the right-hand side of equation (10);

$$(10') \quad \frac{\gamma(1 - e^{-\rho(T-t)})}{\rho} = \frac{w_b \underline{g}(1 - e^{-(\rho-\eta)(T-t)})}{\rho - \eta} + A_2 \underline{g}^{\beta_2} N(-d_2(\underline{g})) + W + \frac{w_0(1 - e^{-\rho(T-t)})}{\rho} - \frac{w_0(1 - e^{-(\rho+q)(T-t)})}{\rho + q}.$$

Those who shirk their duties discount future wages w_0 at a higher discount rate because they face the probability q per unit time of being fired by their departmental chair. Hence the difference between the last two terms on the right-hand side of the equation is positive and measures the value of job security, which is lost when a professor decides to stop doing research.

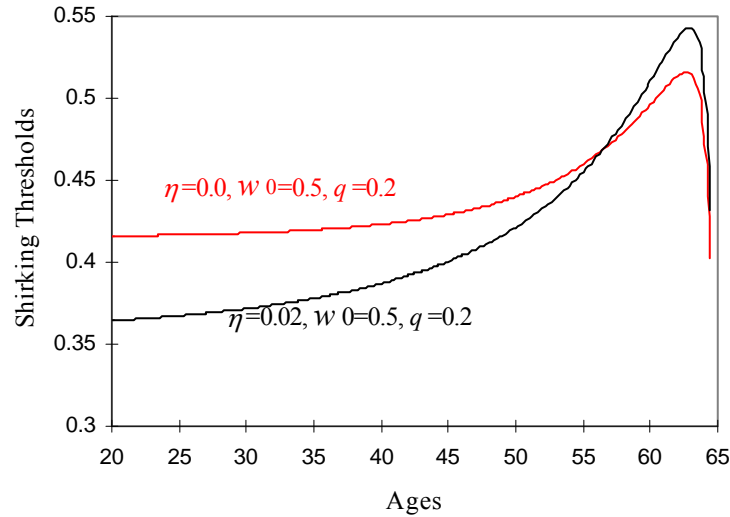
The cost of not doing research becomes the sum of four terms: The sacrificed performance-related pay; the sacrificed option to stop research at a later time; the negative response of colleagues; and reduced job security.

Figure 3 shows the non-deadwood thresholds that have become steeper with monitoring and remain upward sloping even for the case of $\eta = 0$. The slope of the threshold is steeper because the value of job security is falling in age – workers close to retirement have less to lose from being fired since they would have quit their job soon anyway.¹¹

An older professor who has suffered setbacks in research – experienced a lower level of g – would hence be more likely to become inactive than a younger professor because he has less time left to recover his productivity and enjoy unexpected research results, as described in Section 2, and, moreover, he has less to fear from his employer since he is going to retire soon anyway.

¹¹ This leaves out any reputational effects that could have an offsetting effect and also any adverse effect on pension rights.

Figure 3. Non-deadwood thresholds with monitoring



It follows from the analysis that older research-active professors either enjoy doing research better than their younger colleagues or are more productive on average. It follows that while there are many inactive older professors the active professors tend to be quite productive. This is in accordance with the empirical results of Oster and Hamermesh (1998) who find that, comparing authors age 36-50 to those over 50, the degree of heterogeneity in terms of research productivity increases with age.

4. Tenure

Finally, we allow for a tenure effect by letting q be declining in age. In this case workers face increased job security – a falling probability of detection and dismissal – the older they get. We assume the time profile for q shown in Figure 4 and captured by the logistic function where job security increases initially at an increasing rate but then stabilizes around the age of 45.

The probability of firing, q , is assumed to be highest at age 20 and equal to 0.25, and lowest when the worker approaches retirement and equal to 0.05. The reflection on the

second derivative happens at age 40. The time profile is described by the following logistic function:

$$(14) \quad q = 0.25 - \frac{0.2}{1 + \exp(-0.3 \times (\text{age} - 40))}$$

According to this function, q is near 0.25 for young workers, and 0.05 for old workers who are close to retirement.

Figure 4. The effect of age on job security

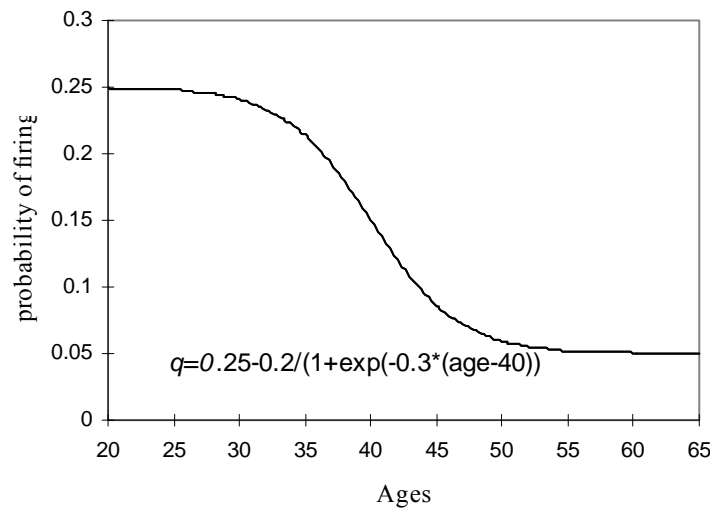
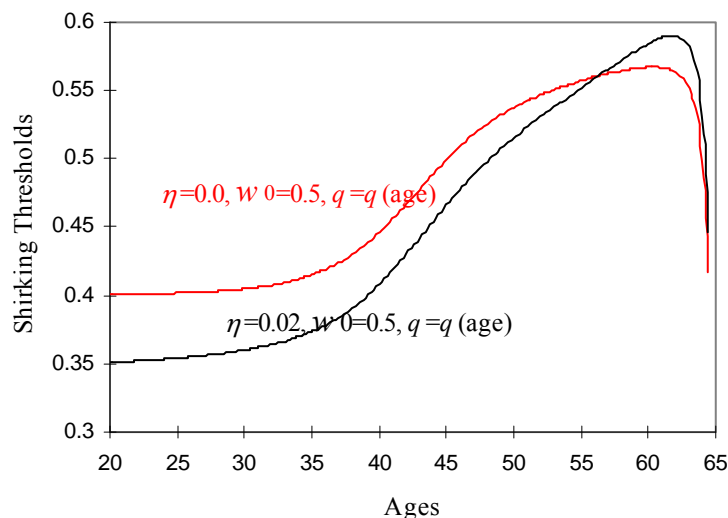


Figure 5 then shows the non-deadwood thresholds for both $\eta=0.0$ and $\eta=0.02$. Note that the threshold is upward-sloping in both cases, more so when $\eta=0.02$. Compared to the thresholds in Figure 3, the difference between the value of the new threshold for the young and the old workers is much greater in this case. The tenure effect weakens the incentive for the old workers to continue with research further. The old, unproductive professor may now decide to enjoy leisure on the job by discontinuing research because he has little time left to attain higher productivity; it does not matter much if he is found out and dismissed; and the chances that he be dismissed are low because of his tenured position.

Figure 5. Non-deadwood thresholds with a tenure effect



5. Conclusions

Following a string of setbacks in research, it takes a higher level of productivity to convince an older professor to discontinue research than a younger one. The young can look forward to a long career that may generate rising productivity and unexpected successes while the older ones are closer to retirement and have less to lose from discontinuing research. The older professor may also be less threatened by the prospect of being dismissed in the light of low research effort because he has little time left before retirement and because of his tenured position. Professors who are not active in research in the later stages of their careers may for this reason need additional financial incentives, a greater support network or other amenities that make their research effort more enjoyable. Those professors who remain active throughout their careers are, on average, either very successful or enjoy doing research, or both.

The intuition for our results can be explained as follows: When the expected growth in research productivity η is positive, the performance-related pay is expected to rise as well as the value of the real option to discontinue research at a later date while the disutility

from doing research stays constant overtime. From the current perspective, the final year's expected performance pay and possibly research successes (as captured by the current value of the real option) count more in the professor's decision than the expected disutility of work. Hence, as the professor gets older the cost of giving up research declines more rapidly than the benefits and he is more likely to decide to become inactive in research for a given level of current productivity. Moreover, the older professor is less threatened by the prospects of being dismissed because of his more secure, tenured, position and the short time left before retirement.

The intuition of the model is also applicable for other professions where a part of a worker's effort is not observable by the employer – workers require a higher real wage the older they become to deter them from shirking their duties. The model can thus be extended to give an explanation for age profiles in wages and productivity in addition to giving a rationale for mandatory retirement. In this way the model complements the work of Lazear (1979) who showed that rising wage profiles could be used to solve the agency problem by inducing workers to produce a greater lifetime output while holding the present value of the lifetime wage bill constant.

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Appendix: Derivation of Equation (8)

The corresponding integral for equation (7) in the text is denoted by

$$(A1) \quad V^R = E \left[\int_t^T (w_0 + w_b g_s - \gamma) e^{-\rho(s-t)} ds \right].$$

Directly integrating (A1) without considering the possibility of shirking in research gives the following particular solution to V^R

$$(A2) \quad V^R = \frac{(w_0 - \gamma)(1 - e^{-\rho(T-t)})}{\rho} + \frac{w_b g (1 - e^{-(\rho-\eta)(T-t)})}{\rho - \eta}.$$

Substituting (A2) back into equation (7) in the text shows (A2) is the correct particular solutions. The homogenous part of equation (7) in the text has the following form:

$$(A3) \quad \rho V^R = \eta g V_g^R + \frac{1}{2} \sigma^2 g^2 V_{gg}^R + V_t^R$$

Chen and Zoega (2010) have shown the detailed derivations of the solution for an equation similar to (A3). We use another way to show how to obtain real options for discontinuing research. It is commonly known that the perpetual real options have the functional form of component g^β and the corresponding characteristic equation without considering V^R is as follows,

$$(A4) \quad \frac{1}{2} \sigma^2 \beta(\beta-1) + \eta\beta - \rho = 0.$$

As we only consider the opt-out (shirking) options, we only need to choose the negative root of equation (A4),

$$(A5) \quad \beta_2 = \frac{1}{2} - \frac{\eta}{\sigma^2} - \sqrt{\left(\frac{\eta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0.$$

Positive root for beta is not chosen in order for the real options not to approach infinity when productivity becomes very big. It is then natural to guess that real options to equation (A3) have the following functional form

$$(A6) \quad V^R(g, t; T) = A_2 g^{\beta_2} N(-d_2(g, t; T)),$$

where A_2 is the unknown parameter, and

$$(A7) \quad d_2 = \frac{\ln g - \sigma^2(T-t) \sqrt{\left(\frac{\eta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}}{\sigma \sqrt{T-t}},$$

$$(A8) \quad N(-d_2) = \left(1/\sqrt{2\pi}\right) \int_{-\infty}^{-d_2} e^{-\varpi^2/2} d\varpi.$$

The d_2 function has the components of β_2 from perpetual real options to shirk. We can then prove that (A6) is one of possible solutions to (A3) by plugging (A6) back to (A3). Differentiation $V^R(g, t; T) = A_2(1/\sqrt{2\pi})g^{\beta_2} \int_{-\infty}^{-d_2} e^{-\varpi^2/2} d\varpi$ by using Leibnitz rule gives

$$(A9) \quad \eta g V_g^R = \eta \left[\beta_2 g^{\beta_2} \int_{-\infty}^{-d_2} e^{-\varpi^2/2} d\varpi - g^{\beta_2} \frac{1}{\sigma \sqrt{T-t}} e^{-d_2^2/2} \right] \frac{A_2}{\sqrt{2\pi}},$$

$$(A10) \quad V_t^R = \left[-\frac{\ln g}{2\sigma(T-t)\sqrt{T-t}} - \frac{\sigma}{2\sqrt{T-t}} \sqrt{\left(\frac{\eta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \right] \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} e^{-d_2^2/2},$$

$$(A11) \quad \begin{aligned} \frac{1}{2} \sigma^2 g^2 V_{gg}^R &= -\frac{\sigma}{2\sqrt{T-t}} \beta_2 \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} e^{-d_2^2/2} + \frac{\sigma^2}{2} \beta_2 (\beta_2 - 1) \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} \int_{-\infty}^{-d_2} e^{-\varpi^2/2} d\varpi \\ &+ \frac{1}{2(T-t)} \frac{A_2}{\sqrt{2\pi}} g^{\beta_2} \left(\frac{\left[\ln g - \sigma^2(T-t) \sqrt{\left(\frac{\eta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \right]}{\sigma \sqrt{T-t}} \right) e^{-d_2^2/2} \\ &- \frac{\sigma}{2\sqrt{T-t}} (\beta_2 - 1) \frac{A_2}{\sqrt{2\pi}} Y^{\beta_2} e^{-d_2^2/2}. \end{aligned}$$

Substituting (A6), (A9) – (A11) back to equation (A3) and collecting terms gives

$$(A12) \quad \begin{aligned} &\left[\frac{\sigma^2}{2} \beta_2 (\beta_2 - 1) + \eta \beta_2 - \rho \right] \frac{A_1}{\sqrt{2\pi}} Y^{\beta_2} \int_{-\infty}^{-d_2} e^{-\varpi^2/2} d\varpi \\ &+ \left[-\beta_2 + \frac{1}{2} - \frac{\eta}{\sigma^2} - \sqrt{\left(\frac{\eta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \right] \frac{\sigma}{\sqrt{T-t}} \frac{A_2}{\sqrt{2\pi}} Y^{\beta_2} e^{-d_2^2/2} = 0 \end{aligned}$$

The items in two brackets are equal to zero due to equation (9) in the text (or (A4)) and (A5), which concludes the proof that $V^R(g, t; T) = A_2 g^{\beta_2} N(-d_2(g, t; T))$ is the general solutions to equation (7) in the text. Combining the particular solutions and homogenous solutions [A(2) and (A6)] gives equation (8) in the text.