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Aging and Job Security

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Aging and Job Security

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Abstract
We model a firm's choice as to the age composition of dismissed workers for different assumptions about the level of firing costs. We find that when the cost of firing is independent of age, a higher level of firing costs will induce firms to fire their younger workers while lower costs induce them to fire the older ones. A corresponding effect is not found in the age dimension of the hiring decision. It follows that job protection favours more senior workers even when the cost of firing is independent of age and seniority.

Keywords: Age-structure, tenure, firing decisions, real options.
JEL: E32, J23, J24, J54

State-mandated redundancy payments were introduced in many European countries from the late 1950s to the early 1970s. These restrictions have been assigned some of the blame for the poor employment performance of many European countries. There are also implicit costs of firing workers in the absence of such laws. For example, firing decisions may disrupt production, necessitate the training of replacement workers, have negative morale effects or shorten the expected future tenure of remaining workers. The economic effect of such implicit costs may be not much different from those stemming from more explicit employment protection. A number of studies have attempted to estimate the extent to which the poor performance of European countries can be explained by formal employment-protection legislation. There is also a body of research that describes the effect of different macroeconomic variables, such as real interest rates and expected productivity growth rates, on the firing decision.[1] However, the use of the representative agent framework prevents this research from illuminating some interesting insights.

We depart from the existing literature in modelling the firing decision by allowing for worker heterogeneity in terms of age. Various studies have documented how employment protection seems to be related to youth unemployment (see, amongst others, Scarpetta (1996)). Blanchard (2006) emphasises the magnitude of the youth

[1] See, amongst others, Bentolila and Bertola (1990),

unemployment problem in Europe. Bertola et al. (2002) describe the pattern of unemployment for different age groups for the OECD countries and find that high unemployment for the younger age group is particularly pronounced in the more unionised countries as well as in those having more employment protection. In particular, France, Italy and Spain experienced especially large declines in youth employment. Our goal is to discover to what extent redundancy payments – the size of which does not depend on workers' age or expected tenure – nevertheless have a differential effect on the firing decision depending on workers' expected tenure. By allowing workers to differ in terms of their age we can answer questions such as how firms reach a decision as to whether to dismiss a young or a more mature worker and how labour-market institutions such as employment protection affect this choice.

We are not the first ones to address this question. Lazear and Freeman (1997) find that in a downturn the youngest and the oldest workers should be the first to be laid off because the young have not been given any firm-specific skills while the productivity of older workers has declined relative to their wages. Layard et al. (1991) find the wage-push factor to be stronger for young workers due to higher turnover making their unemployment rate higher. This is because high turnover makes the prospect of unemployment spells less threatening which then makes unions more aggressive. Employment-protection legislation is not a part of the story in either case.[2]

1. An option-valuation approach

The option-valuation approach to investment has been popular since the seminal papers of Black and Scholes (1973) and Merton (1973) on the pricing of stock options. These methods of valuing stocks can be easily applied to *real options*, which denote the option-like characteristics of investment opportunities. The decision to invest (or the decision to exercise real options) becomes important with the existence of uncertainty and sunk costs. McDonald and Siegel (1986) show that the required rate of return on investment in many large industrial projects can be more than doubled by moderate amounts of uncertainty when the investment project is at least in partly irreversible.[3]

In most cases it is assumed that the real options are infinitely lived – the real-life investment opportunities are infinitely lived and never valueless (e.g. McDonald and

[2] Other reasons for age discrimination in firing involve the direct and indirect effects of pension schemes. Most pension schemes are defined-benefit schemes, making the benefits increase more rapidly as the age of retirement approaches since they are based on final salary at retirement. This makes employers want to lay off workers with a long tenure. A possible offsetting effect can be found in Orszag et al. (1999). Here old workers have a higher effort level because they have more to lose in the event of a dismissal.

[3] For an introduction, see Dixit and Pindyck (1994).

Siegel; 1984, 1986). However, some research deals with the non-perpetual real options (e.g. Paddock, Siegel and Smith, 1988). But it has been claimed that it is often not possible to solve such non-perpetual options analytically, making numerical methods essential.[4] Generally speaking, it is hard to solve for free-boundary time-dependent real options. This is partly due to functions of time-dependent options having a complicated shape, which may require several analytical functions for simulation. We will show in the case of real options that *approximate* analytical solutions do exist.[5] The approximate solutions of non-perpetual real options should share the same composite components as perpetual real options. The partial differential equation of non-perpetual real options can then be transformed into a convection-diffusion problem,[6] which can be solved for analytically using standard techniques of partial differential equations.

2. Modelling the firing decision

There is only one sector in our economy that uses labour as an input to produce a homogenous good. Since our focus is on labour demand, real wages are assumed to be fixed and their determination is not described. The source of uncertainty is stochastic productivity.

Current profits, measured in units of output, are defined as follows,

$$\Pi(g_t, N_t) = g_t N_t^{\theta} - wN_t - C(I_t), \qquad 0<\theta<1, \tag{1}$$

where N denotes the number of employed workers, w is the real wage, g is a measure of productivity, I represents gross changes of employment due to hiring and hiring – positive I denotes hiring and negative I firing – and C denotes the total costs of hiring and firing,[7] consisting of a fixed and variable component:

[4] One of the reasons for the non-existence of analytical solutions is that such options are similar to *American* stock options that can be exercised at any time up to the expiration date. It is well-known that American stock options can only be solved for using analytical approximations or numerical methods such as finite-difference methods. American call options with lump-sum dividends are an exception though in that their terminal and boundary conditions differ (see Roll, 1977; Geske, 1979; Whaley, 1981). The possible analytical solutions to partial differential equations vary greatly when boundary- and terminal (and/or initial) conditions change. Changes in such conditions can result in the non-existence of analytical solutions. The method used here is most similar to Barone-Adesi and Whaley (1987).

[5] '*Approximate*' is in a sense that the solutions might not be complete, but still a good proxy for real solutions.

[6] In physics, *convection* is the movement of the substance by the movement of the medium. Combined with a diffusion problem, it will be like the diffusion of a moving wave.

[7] The convex functional form of total adjustment costs is similar to the one used in physical capital (dis-) investment such as Abel and Eberly (1994), Dixit and Pindyck (1994).

$$C(I_t) = \begin{cases} c_h + p_h I_t + \frac{1}{2}\gamma_h I_t^2 & \text{for } I_t > 0, \\ 0 & \text{for } I_t = 0, \\ c_f - p_f I_t + \frac{1}{2}\gamma_f I_t^2 & \text{for } I_t < 0. \end{cases} \tag{2}$$

When the firm hires (fires) workers, it pays a fixed cost c_h (c_f) and positive unit costs of hiring (firing) which may be rising in the number of workers hired (fired), $p_h I + 1/2\gamma_h I^2$ (or $-p_f I + 1/2\gamma_f I^2$), respectively. Note that $-p_f I > 0$ for firing since $I < 0$ in this case. The coefficients c_h and c_f denote the fixed costs whenever the firm decides to hire or fire. These are usually related to advertising, the screening process, and so on; the fixed costs for firing are related to legal consultations, disputes about firing, and the trade union's cooperativeness. The parameters p_h and p_f refer the unit costs of hiring and firing respectively. The parameters γ_h and especially γ_f are also linked to labour market institution such as the strength of labour unions and unions consultations. All parameters in equation (2) are positive.

The net employment changes for the firm are denoted by hiring/firing minus quits

$$\frac{dN_s}{ds} = I_s - \lambda N_s, \tag{3}$$

where λ denotes the constant quit rate per unit time. It is assumed that each worker has a working life of T years. To simplify the model, we assume that workers die immediately after they retire and that all workers have the same productivity independent of their age. Moreover, it is assumed that g follows a geometric Brownian motion

$$dg_s = \eta g_s ds + \sigma g dW_s; \tag{4}$$

where W_s is a standard Wiener process; $dW_s = \varepsilon_s \sqrt{ds}$ and ε_s is a serially uncorrelated, normally distributed random variable with mean zero and a standard deviation of unity. Here η is the drift parameter (the expected growth rate of labour productivity) and σ the variance parameter.

The firm maximises the following expected intertemporal profits, V, by choosing hiring/firing, I_t, over time,

$$V = \max_I E\left[\int_t^T \left[g_t N_t^\theta - wN_t - C(I_t)\right] e^{-\rho(s-t)} ds \middle| g = g_t, N = N_t\right], \text{ s.t. (3) and (4)}, \tag{5}$$

where $E[\cdot]$ denotes the expectation operator given the information set available to the firm at period t, and ρ is the constant interest rate. Applying Ito's Lemma and the principle of dynamic programming, the expected intertemporal value of the firm can be represented by solving the following Bellman equation:

$$\rho V = \max_{I} E\left[gN^{\theta} - wN - C(I_t) + V_N(I - \delta N) + \eta g V_g + \frac{1}{2}\sigma^2 g^2 V_{gg} + V_t \right], \qquad (6)$$

The first three terms in the square bracket represent the firm's immediate profits at *t* after deducting the total wage bill and costs related to hiring and firing; the fourth term shows changes in the value of the firm due to fluctuations in *N* due to quitting or hiring/firing; the fifth and sixth terms denote the impact of changes in *g* on *V* over time; and the final term shows the effect of autonomous changes over time.

The boundary conditions for hiring and firing for equation (6) can be summarised by the following equations (see appendix A)

$$v = H = p_h + \sqrt{2c_h\gamma_h} \text{ for hiring thresholds,} \qquad (7.1)$$

$$v = -F = -\left(p_f + \sqrt{2c_f\gamma_f}\right) \text{ for firing thresholds,} \qquad (7.2)$$

where *H* denotes the *effective* hiring-cost threshold, depending on the fixed costs, unit costs and adjustment speed costs parameters of hiring; *F* is the *effective* firing-cost threshold; and $v = V_N$ is the marginal inter-temporal value of profits with respect to workers. In the inaction area where $-F < v < H$ the firm does nothing and the number of employees only falls due to quits. The magnitudes of *H* and *F* are positive functions of fixed costs $(c_{h/f})$ and the adjustment costs $(\gamma_{h/f})$. At the hiring (firing) thresholds, the firm would hire (fire) the following number of workers,

$$I = \sqrt{2c_h/\gamma_h} \text{ for hiring;} \qquad (8.1)$$

$$I = -\sqrt{2c_f/\gamma_f} \text{ for firing.} \qquad (8.2)$$

As fixed costs increase, the firm chooses to hire/fire more employees so that total benefit of hiring/firing outweighs the total cost. With lower values of $\gamma_{h/f}$ the firm would also hire/fire more workers. The inaction area is smaller for lower $c_{h/f}$ and $\gamma_{h/f}$ since the firm needs to pay lower fixed and adjustment costs for the same numbers of hires/fires.[8]

[8] Note that with null fixed costs and adjustment costs, equations (7.1) and (7.2) are reduced to the same forms as in Bentolila and Bertola (1990). Their implicit assumption is that the intertemporal value of marginal employees, *v*, never deviates outside of the inaction area of

$$v = \pm p_{h/f}.$$

A very small value of γ guarantees that the firm can hire/fire a lot of employees (according to equations (3) and (8))

$$dN/dt = I - \delta N \Rightarrow \Delta I = \left(\pm\sqrt{2c_{h/f}/\gamma_{h/f}} + \delta N\right)\Delta t.$$

This makes *v* fall back into the inaction area of

$$v = \pm p_{h/f}$$

immediately for any given shocks for very small values of γ.

The marginal intertemporal values of employees for the boundary conditions of hiring and firing, v, in equations (7.1) and (7.2) are subject to the following partial differential equation as show, as shown in Appendix A,

$$(\rho+\lambda)v = Y - w + \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY} + v_t\,, \tag{9}$$

where $Y = \theta g N^{\theta-1}$. Y has a drift term $\eta_Y = \eta + \lambda(1-\theta)$ for marginal labour productivity.

$$dY = \eta_Y Y ds + \sigma Y dW\ . \tag{10}$$

Equation (9) is equivalent to the following intertemporal integral without hiring/firing-related costs

$$v(Y,t;T) = \int_t^T (Y_s - w)e^{-(\rho+\lambda)(s-t)}ds\,. \tag{11}$$

The partial differential equation of equation (9) relates the value of workers to the value of the stochastic variable Y at each point in time. The firm will hire a marginal worker if

$$v(Y,t;T^h) \geq H$$

and fire a marginal worker if

$$-v(Y,t;T^f) \geq F\ ,$$

where T^h denotes T for the worker that the firm hires and T^f for the worker that the firm fires. T^h is different from T^f since hiring and firing decisions cannot happen at the same time.

The partial term with respect to time, v_t, in equation (9) make the differential equation difficult to solve. A simple way to get around it without resorting to pure numerical methods would be to assume that the analytical solutions have the same components as the infinitely-lived case found in Bentolila and Bertola (1990). In this case, equation (9) becomes a second-order ordinary differential equation in Y and as a result the option values of hiring and firing workers become independent of time. It follows that the options for hires and/or fires do not approach zero when workers age. One of the objectives of this paper is to correct for this and show how interesting implications arise.

The problem now is to solve for v, which is the value of employing a marginal worker. The solution for v consists of the particular integral and the general function. A convenient particular solution, v^P, for (9) can be obtained by integrating (11) directly

$$v^P = aY - bw, \tag{11}$$

where $a = \left(1 - e^{-(\rho+\lambda-\eta_Y)(T-t)}\right)/(\rho+\lambda-\eta_Y)$, $b = \left(1 - e^{-(\rho+\lambda)(T-t)}\right)/(\rho+\lambda)$ and it is assumed that the denominator of the parameter a is positive. As T approaches infinity, the

particular solution becomes identical to the one in the perpetual setup. The smaller value of T yields smaller value of particular solutions, which echoes the intuition that old workers have a lower intertemporal value for the firm.

The firm takes into account the option value of hiring in the future. There is also the option to fire the worker once he is employed. The two option values are measured by the general (or homogenous) solutions to equation (9). Now only focusing on the homogenous part of (9) and letting v^G be the value of the marginal option, we get

$$(\rho+\lambda)v^G = \eta_Y Y v_Y^G + \frac{1}{2}\sigma^2 Y^2 v_{YY}^G + v_t^G . \tag{12}$$

The general solutions of (12) are equal to the value of the options to hire or fire the marginal worker. When Y approaches zero, the value of the option to hire, v_H^G, has to go to zero. Similarly, the firing option, v_F^G, is equal to zero when Y goes to infinity. Thus, the general solutions for the hiring and firing options have to satisfy the following boundary conditions respectively,

$$\lim_{Y\to 0} v^G(Y,t;T) = 0 \text{ for the hiring option,} \tag{13.1}$$

$$\lim_{Y\to \infty} v^G(Y,t;T) = 0 \text{ for the firing option.} \tag{13.2}$$

A special case of equation (12) is when workers live forever (T=∞). Thus, the term v_t^G in equation (12) disappears and the values of the hiring- and firing options are (see Appendix B)

$$v_{0H} = A_1 Y^{\beta_1} \text{ for hiring option,} \tag{14.1}$$

$$v_{0F} = A_2 Y^{\beta_2} \text{ for firing option.} \tag{14.2}$$

The unknown parameters of A_1 and A_2 are determined by the value-matching and smooth-pasting conditions and β is determined by equation (15).

$$\frac{1}{2}\sigma^2\beta(\beta-1)+\eta_Y\beta-(\rho+\lambda)=0 . \tag{15}$$

The general solutions to (12) are then given by the following equations (see Appendix C):

$$v_H^G(Y,t;T) = A_1 Y^{\beta_1} N(d_1), \tag{16.1}$$

$$v_F^G(Y,t;T) = A_2 Y^{\beta_2} N(-d_2), \tag{16.2}$$

where A_1 , A_2 are unknown parameters, $d_{1/2} = \dfrac{\ln Y \pm \sigma^2(T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2}-\dfrac{1}{2}\right)^2+\dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$,

and $N(d) = \left(1/\sqrt{2\pi}\right)\int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi$, $0 \le N(d) \le 1$, is the cumulative normal distribution

function. The general solutions, or the real options to hire/fire, have the components similar to the ones of financial options such as in Black and Scholes (1973) while keeping its perpetual parts components. Such functional forms of general solutions make it possible to use the general solutions to solve the optimal stopping problem of hiring and firing.

Looking at the hiring- and firing options we find two separate cases:

Case 1: $T \to \infty$

It is easy to show that as T approaches infinity (workers live forever), the cumulative distribution functions of $N(d_1)$ and $N(-d_2)$ become unity. This reduces the firing- and hiring options to the case of perpetual options.

Case 2: $T \to t$

If $\ln Y > 0$, then $N(d_1)=1$ and $N(-d_2)=0$ as T approaches t. If the marginal profitability is high enough, firms mainly focus on the hiring decision. The firing option approaches zero because this workers will retire very soon.

If $\ln Y < 0$, then $N(d_1)=0$ and $N(-d_2)=1$ as T approaches zero. A small $(T-t)$ means that Y needs to be very small to reach the firing threshold. If the marginal profitability is low, firms mainly consider the firing decision. Since the marginal worker's T is very small when workers get fired, the possibility of re-hiring workers back is almost zero. Therefore, the hiring option approaches zero. Note that the options of hiring and firing approach zero automatically if T approaches t since the marginal profits for hiring/firing, particular solutions, would be zero in this situation.

The decision as to hire or fire workers depends on his value as given by equations (7.1), (7.2), (11), (16.1), and (16.2). The definition of the firing- and hiring barriers; Y_F and Y_H, are then given by the value-matching and smooth-pasting conditions:

Value-matching conditions

$$aY_H - bw + v_F^G\left(Y_H, t; T^h, A_2\right) = H + v_H^G\left(Y_H, t; T^h, A_1\right), \tag{16}$$

$$-\left(aY_F - bw\right) + v_H^G\left(Y_F, t; T^h, A_1\right) = F + v_F^G\left(Y_F, t; T^f, A_2\right). \tag{17}$$

The left-hand side of (16) has the marginal benefit of hiring which includes the acquired firing option. The right-hand side has the marginal cost of hiring, which includes the sacrificed hiring option. Similarly for equation (17), the left-hand side has the marginal benefit and the right-hand side the marginal cost of firing. In our numerical solutions below apart from Figure 1 of the general case of hiring and firing, we will only include the sacrificed firing option as part of the cost of firing; we will

not include the acquired hiring option as a benefit of firing. The reason is that firing workers is not going to alter a firm's chances at filling a vacancy in the future if there are many (homogeneous) unemployed people to start with.

There are four unknown variables, Y_H, Y_F, A_1, and A_2, in equations (16) and (17). The smooth-pasting conditions follow to ensure the slopes before and after thresholds with respect to Y_H and Y_F are the same:

Smooth-pasting conditions

$$a + \frac{\partial v_F^G\left(Y_H, t; T^h, A_2\right)}{\partial Y_H} - \frac{\partial v_H^G\left(Y_H, t; T^h, A_1\right)}{\partial Y_H} = 0, \tag{18}$$

$$a + \frac{\partial v_F^G\left(Y_F, t; T^f, A_2\right)}{\partial Y_F} - \frac{\partial v_H^G\left(Y_F, t; T^h, A_1\right)}{\partial Y_H} = 0, \tag{19}$$

where details of derivations of various $\partial v^G / \partial Y$ are shown in Appendix D.

Equations (16), (17), (18) and (19) are a non-linear systematic equations with four unknown parameters [Y_H, Y_F, A_1, and A_2] and can be solved for numerically, once beta roots, β_1 and β_2, are obtained from equation (15).

3. Hiring- and firing thresholds

We will now calculate the firing thresholds on the basis of equations (16), (17), (18) and (19). We calculate the hiring and the firing thresholds for a fixed level of firing costs in the two-threshold case when both the hiring- and the firing thresholds are calculated simultaneously. Though, *H* and *F* depend on three different parameters – fixed costs, unit costs, and adjustment speed-related costs of hiring and firing – we can show the aggregate effect of those by plotting firing thresholds against effective firing costs. This is shown in Figure 1 below.

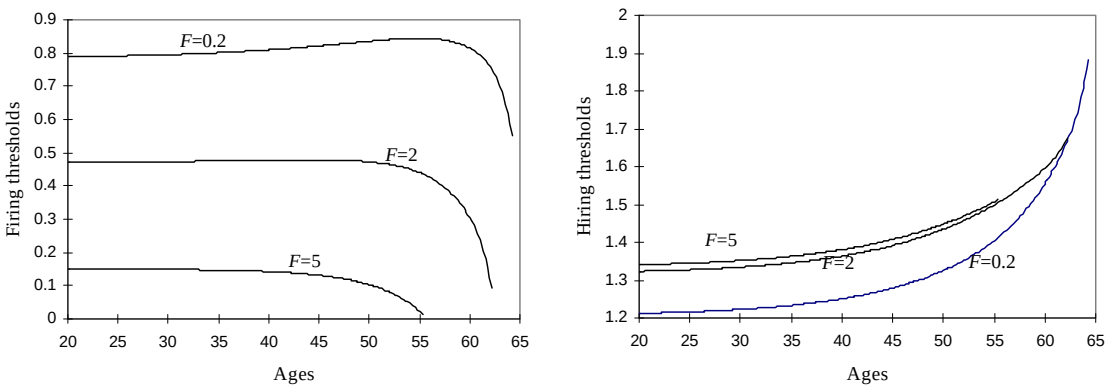

Figure 1.The effect of age on the hiring- and firing thresholds with different effective firing costs. Ages are equal to (65–$T_{h/f}$). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, H = 0.083, and t=0.

As the effective firing costs rise, the firm becomes more inclined to fire the younger among its workers. The reason is that part of the cost of firing workers is the sacrificed option of doing so in the future. This was shown in equation (17). This firing option is decreasing in both the level of the firing costs and in the worker's age. For low levels of effective firing costs, the marginal cost of firing the young workers is much higher than the cost of firing older ones for this reason. But at high firing costs, the difference is much smaller as the firing option is always very low – both for the young and the old workers. However, the marginal benefit of firing the young workers is always higher – that is for all levels of firing costs – because of their longer remaining tenure. It follows that the firm would choose to fire the young workers first if firing costs are high – the value of the firing option low – but at low firing costs it may choose to fire the older workers first since the marginal cost of doing so is much lower. Furthermore we find that firms always hire younger workers first no matter what level the firing costs are.

We can explain the effect of F on firing options in more details in the following simplified case. Consider the firing-only scenario with perpetual workers, the value-matching and smooth conditions are as follows

$$-\left(aY_F - bw\right) = F + A_2 Y_F^{\beta_2}, \tag{20}$$

$$a = \beta_2 A_2 Y_F^{\beta_2 - 1}, \tag{21}$$

where $A_2 Y_F^{\beta_2}$ represents the perpetual firing option as shown in (14.2). Equation (21) shows that the option to fire is a linear function of Y: $aY/\beta_2 = A_2 Y_F^{\beta_2}$. Thus, if the firing threshold Y falls due to a direct increase of F, the option to fire would also decrease accordingly. Though the fall of the Y thresholds due to increase in F leads to higher value of $Y_F^{\beta_2}$, the parameter A_2 makes sure that the whole firing option falls as F increases. The firing option denotes the waiting values from postponing exercising the option to fire workers. As the direct effective costs of firing F increase, the probability that the firm would fire the employees becomes smaller and thus the probability that Y would return the profitable value for the firm once the firing decision is made is smaller. Therefore, for a high value of F, the firm would not regret its decisions of firing, which implies that the value of waiting, that is the firing option, is less important to the firm. This relationship still holds numerically for workers who do not live forever and for the system of the general hiring/firing value-matching conditions of equations (16) and (17).

In this sense, the difference between the option of firing a young and an old worker becomes negligible for high F. This implies that with high firing costs, the firm tends to fire the young workers first in order to reduce its losses for a much longer period of time than when the older workers are fired. However with low firing costs, the firm values the option to be able to fire the workers at a later date when more information about the evolution of productivity is available. This option is worth more in the case of the young workers and hence the firm faces higher costs of firing the younger workers on this account. As a consequence, the firm tends to fire the older ones first when the cost of firing is low.

Note the difference between our setup and that of Lazear and Freeman (1997). They claim that it is optimal to fire the younger workers because they are less productive since the (firm-specific) skill accumulation has not been completed. We find that they should also, if there are significant costs of firing, be the first to go even if their productivity is no lower than that of older workers. Firing a young worker is more profitable than firing an older worker since his/her expected tenure is longer. Note also, that these results do not depend on firing costs rising over tenure. All that is needed is a high and fixed level of firing costs.

4. Macroeconomic implications

We have found that high firing costs provide more protection to the older workers than to the younger ones. It follows that the age structure of the population affects the tightness of employment-protection legislation; the ageing of the workforce has the same effect on the firing thresholds as an increase in the firing costs themselves. This has two implications.

First, when assessing the nature of a country's labour-market institutions one has to normalise for the age structure of the labour force. Two countries with similar legislation can nevertheless have different effective legislation in the sense that firms are more reluctant to fire workers in one of the countries. Second, changes in the age structure of the population over time may have important consequences. The maturing of the baby-boom generation in Europe can be one explanation why a given set of institutions started to generate different labour-market performance in the 1980s and 1990s from that of the 1950s, 1960s and 1970s. For example, the employment-protection legislation already in place in France, Italy and Spain may have been less restrictive in the 1960s and 1970s than in the 1980s and 1990s. We conclude that labour-market rigidity is a function of the age-structure of the population no less than of the nature of labour-market institutions.

There arises an interaction between the level of firing costs, on the one hand, and the age of workers, on the other hand, in determining the level of productivity at

which firms start firing each worker. With low firing costs, the firing threshold is monotonically rising in age making the more mature workers to be the first to lose their jobs in a downturn. But as the level of firing costs rises and, the sign of this relationship changes and the threshold becomes monotonically falling in age making the young workers the first to go if labour demand falls. We show the case of different levels of firing costs in Figure 2.[9]

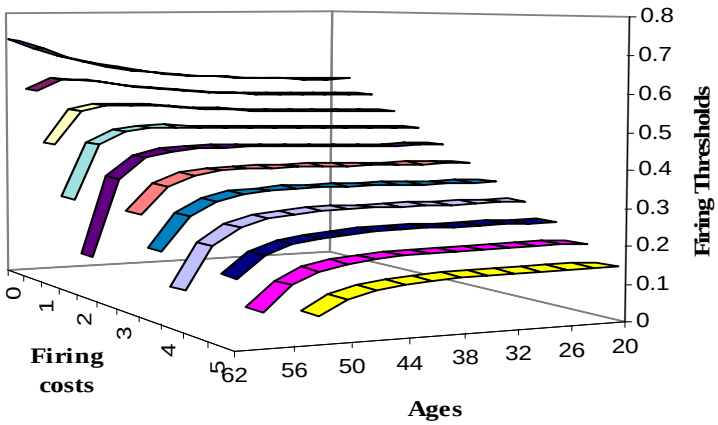

Figure 2.The effect of age on the firing threshold with different firing costs. Age is equal to (65–T_f). Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, and t=0.

Figure 3 below shows the firing thresholds as a function of age (expected future tenure) for both young (20 years) and old (60 years) workers who both will retire at age 65. The figure shows clearly that we fire old workers first with low F and then tend to fire the younger ones as F becomes high, as discussed in the previous section.

[9] Note that the numerical results in Figures 2 & 3 are obtained by running the value-matching/smooth-pasting conditions for *firing* thresholds only, with the assumption of null hiring options and no hiring decisions. As shown in Chen and Zoega (1999), this simplification doe not affect the qualitative results for firing thresholds when only discussing the effect of F.

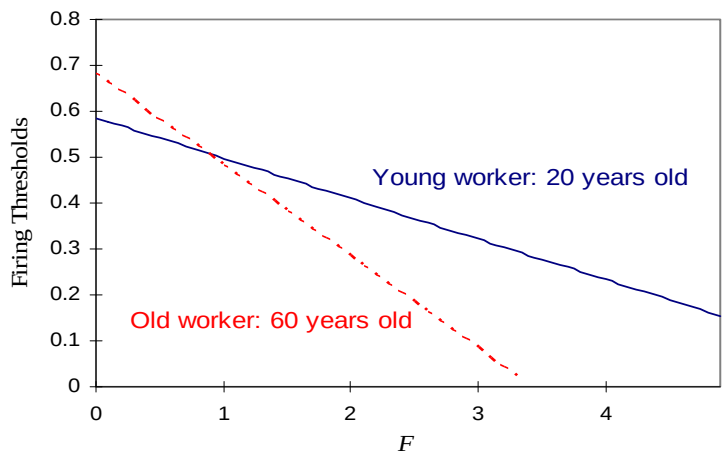

Figure 3.The effect of firing costs on the firing threshold for two age groups. Other parameters: σ=0.20, ρ=0.10, θ=0.7, η=0.02, λ=0.05, w=1, and t=0.

5. A quick look at the data

Following Nickell et al. (2005) we estimate an equation where the dependent variable is average unemployment in 1983-1988 and 1989-1994 for 19 OECD countries.[10] The explanatory variables include the unemployment-benefit replacement ratio, the maximum duration of benefits, union density, union coverage, union coordination, employer coordination, a measure of active labour-market policies, the average change in the inflation rate and a measure of employment protection.

The first two columns in the table below show the standard results when we do not allow for any interaction with the age structure of the population. The sign and significance of all variables is as expected. We note that employment protection (*epl*) has an insignificant coefficient. We then add the share of the labour force between the ages of 15 and 19 (*age*) as an interaction term with the *epl* in columns 3 and 4. The coefficients are not much affected apart from the constant term and the coefficient of *epl*. The effect of *epl* becomes stronger when we include the share of the labour force between the ages of 15 and 24. This is shown in the last two columns.

Table 2. Unemployment equations

	No interaction terms		Age (15-19) as interaction term		Age (15-24) as interaction term	
Variable	Coefficient	t-ratio	Coefficient	t-ratio	Coefficient	t-ratio
constant	6.29	1.5	-7.82	-0.8	-26.50	-2.4
dummy (89-94)	7.61	1.8	-6.54	-0.7	-24.29	-2.3
replace ratio	0.10	4.2	0.12	4.8	0.14	6.2
duration of benefits	0.70	2.1	0.58	1.7	0.71	2.4
union density	0.03	0.8	0.03	0.6	0.06	1.7

[10] We are grateful to Stephen Nickell for providing us with the data.

union coordination	-2.12	2.0	-2.21	-2.1	-2.74	-3.0
employer coordination	-3.77	4.2	-4.01	-4.5	-4.29	-5.4
labour market expenditures	-0.07	1.9	-0.10	-2.4	-0.11	-3.0
Δinflation	-1.01	1.5	-1.45	-2.1	-1.47	-2.4
union coverage	3.16	2.7	3.07	2.6	3.07	2.8
employment protection (epl)	0.49	0.7	-2.03	-0.3	-34.92	-1.9
age (15-19)	-0.42	1.4	0.74	1.0	1.02	2.6
epl*age			1.09	0.5	3.62	2.3
epl*age^2			-0.07	-0.7	-0.09	-2.5
observations: 38	R^2	DW				
period: 83-88	0.87	2.03				
period: 89-94	0.49	1.73				

In the last two regressions, the effect of *epl* is significant and a function of the age structure. This function is shown in the following figure when we use the age group 15-24. When the share of the labour force between 15 and 24 is less than 26%, the effect of the *epl* on unemployment is positive – greater *epl* gives higher unemployment – while the converse is true when the share is higher than 26%. We conclude that when the effect of *epl* on unemployment is allowed to depend on the age structure of the labour force, its effect becomes significant.

Finally, our results have implications for any empirical work done to test the employment effects of firing costs such as Lazear (1990), Scarpetta (1996), Elmeskov, Martin and Scarpetta (1998), Nickell (1998), DiTella and MacCulloch (1998) and Nickell, Nunziata and Ochel (2005). In another paper (Chen and Zoega, 1999) we have shown how the employment effects of employment protection depend on the nature of the stochastic process followed by productivity – trend growth, variance, degree of mean reversion – in addition to the rate of interest and workers' quit rates. Here we have shown that one also has to control for the age-distribution of the workforce when testing for the effect of firing costs on employment or unemployment.

6. Conclusions

This paper has shown that the effects of employment-protection legislation are likely to depend on the age structure of the population. Such legislation is most effective in deterring the dismissal of mature workers and, as a result, is more likely to lead firms to dismiss the younger ones. Our explanation is independent of the productivity- and wage profiles of workers and also independent of the type of pension schemes they have. The effect arises for the sole reason that the value of the firing option – that is a part of the marginal cost of firing – is decreasing in both the level of firing costs and in the age of the worker.

Appendix A:

The first-order conditions for gross employment changes of equation (6) in the text are denoted by

$$\pm p_{h/f} + \gamma_{h/f} I = v, \tag{A1}$$

where $v = V_N$. By substituting (A1) back into the Bellman equation (6) in the text, we obtain the following two differential equations for hiring and firing decisions:

$$\rho V = gN^{\theta} - wN - c_h + \frac{1}{2}\frac{(v - p_h)^2}{\gamma_h} - \lambda vN + \eta g V_g + \frac{1}{2}\sigma^2 g^2 V_{gg} + V_t; \tag{A2}$$

$$\rho V = gN^{\theta} - wN - c_f + \frac{1}{2}\frac{(v + p_f)^2}{\gamma_f} - \lambda vN + \eta g V_g + \frac{1}{2}\sigma^2 g^2 V_{gg} + V_t. \tag{A3}$$

Due to fixed costs of hiring and firing, the firm would only hire/fire employees whenever the total benefits of hiring/firing are greater than corresponding total adjustments costs. Therefore, for hiring decisions $(I \geq 0)$, the benefits of hiring I employees, Iv, must be greater than its total adjustment costs, $c_h + p_h I + \frac{1}{2}\gamma_h I^2$.

$$Iv - \left(c_h + p_h I + \frac{1}{2}\gamma_h I^2\right) \geq 0. \tag{A4}$$

In case of firing $(I \leq 0)$ in economic downturns, v is negative. Thus, the total benefits of firing $|I|$ employees is captured by Iv; while the total adjustment costs of firing are $c_f - p_f I + \gamma_f M^2/2$. The firm would fire employees as long as the following equation is satisfied:

$$Iv - \left(c_f - p_f I + \frac{1}{2}\gamma_f I^2\right) \geq 0. \tag{A5}$$

Equations (A4) and (A5) can be simplified by using (A1):

$$I \geq \sqrt{\frac{2c_h}{\gamma_h}} > 0 \text{ for hiring;} \tag{A6}$$

$$I \leq -\sqrt{\frac{2c_f}{\gamma_f}} < 0 \text{ for firing.} \tag{A7}$$

The boundaries of the inaction area are then represented by following equations:

$$v = p_h + \sqrt{2c_h\gamma_h} \text{ for hiring thresholds,} \tag{A8}$$

$$v = -p_f - \sqrt{2c_f\gamma_f} \text{ for firing thresholds.} \tag{A9}$$

Substituting (A8) and (A9) back into Bellman equations (A2) and (A3) gives the following differential equation for hiring and firing:

$$\rho V = gN^{\theta} - wN - \lambda vN + \eta g V_g + \frac{1}{2}\sigma^2 g^2 V_{gg} + V_t. \tag{A10}$$

We need to solve the boundaries of hiring and firing by (A8) and (A9). Thus, equation (A10) needs to be transformed into marginal intertemporal value of employees, v, by using the definitions $v = V_N$, $v_g = V_{Ng}$, $v_t = V_{Nt}$, $v_N = V_{NN}$ and $v_{gg} = V_{Ngg}$ and differentiating both sides of equation (A10) with respect to N:

$$(\rho+\lambda)v = \theta g N^{\theta-1} - w - \delta v_N N + \eta g v_g + \frac{1}{2}\sigma^2 g^2 v_{gg} + v_t . \qquad \text{(A11)}$$

Equation (A11) is equivalent to the following intertemporal *without hiring and firing*:

$$v(Y,t;T) \equiv v(Y_t,t;T) = \int_t^T (Y_s - w) e^{-(\rho+\lambda)(s-t)} ds , \qquad \text{(A12)}$$

where $Y = \theta g N^{\theta-1}$. To simplify equation (A11), we can use a new variable, $Y = \theta g N^{\theta-1}$, representing the marginal product of labour. By Ito's Lemma, we have

$$dY = \eta_Y Y ds + \sigma Y dW , \qquad \text{(A13)}$$

where $\eta_Y = \eta + \lambda(1-\theta)$. And the corresponding Bellman equation to (A12) without hiring and hiring is denoted by

$$(\rho+\lambda)v = Y - w + \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY} + v_t . \qquad \text{(A14)}$$

Appendix B:

As workers live forever (T=∞), equation (12) in the text is reduced to

$$(\rho+\lambda)v = \eta_Y Y v_Y + \frac{1}{2}\sigma^2 Y^2 v_{YY} . \qquad \text{(B1)}$$

(B1) is a homogenous equidimensional linear differential equation and is easily solvable. The solutions to (B1) are:

$$v_0 = A_1 Y^{\beta_1} + A_2 Y^{\beta_2} , \qquad \text{(B2)}$$

where A_1 and A_2 are coefficients and β_1 and β_2 are the roots of the following characteristic equation,

$$\frac{1}{2}\sigma^2 \beta(\beta-1) + \eta_Y \beta - (\rho+\lambda) = 0 , \qquad \text{(B3)}$$

and β_1 is positive and β_2 is negative,

$$\beta_1 = \frac{1}{2} - \frac{\eta_Y}{\sigma^2} + \sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}} > 0 , \qquad \text{(B4)}$$

$$\beta_2 = \frac{1}{2} - \frac{\eta_Y}{\sigma^2} - \sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}} < 0 . \qquad \text{(B5)}$$

The hiring and firing solutions for v_0 are

$$v_H = A_1 Y^{\beta_1} \text{ for hiring option,} \qquad \text{(B6)}$$

$$v_F = A_2 Y^{\beta_2} \text{ for firing option.} \qquad \text{(B7)}$$

These are equations (14.1) and (14.2) in the text respectively.

Appendix C:
Derivation of Equations (16.1) and (16.2)

We know that if workers are expected to have infinite lives, the hiring and firing options approach $A_1 Y^{\beta_1}$ and $A_2 Y^{\beta_2}$ respectively. Thus, the first guess for the solutions to equation (12) in the text would be

$$v^G(Y,t) = Y^\beta z(Y,t) . \qquad \text{(C1)}$$

Differentiating (C1) gives

$$v_Y^G = \beta Y^{\beta-1} z + Y^\beta z_Y ,$$
$$v_{YY}^G = \beta(\beta-1) Y^{\beta-2} z + 2\beta Y^{\beta-1} z_Y + Y^\beta z_{YY} ,$$
$$v_t^G = Y^\beta z_t .$$

Substituting into equation (12) in the text gives

$$\left[\frac{1}{2}\sigma^2\left[\beta(\beta-1)z + 2\beta Y z_Y + Y^2 z_{YY}\right] + \eta_Y(\beta z + Y z_Y) + z_t - (\rho+\lambda)z\right] Y^\beta = 0$$

or

$$\frac{1}{2}\sigma^2\left[\beta(\beta-1)z + 2\beta Y z_Y + Y^2 z_{YY}\right] + \eta_Y(\beta z + Y z_Y) + z_t - (\rho+\lambda)z = 0 .$$

Rearranging gives

$$\left[\frac{1}{2}\sigma^2\beta(\beta-1) + \eta_Y\beta - (\rho+\lambda)\right]z + \frac{1}{2}\sigma^2\left[Y^2 z_{YY} + 2\left(\beta + \frac{\eta_Y}{\sigma^2}\right)Y z_Y + \frac{2}{\sigma^2} z_t\right] = 0 . \quad \text{(C2)}$$

The first terms in the first bracket are equal to zero automatically due to the characteristic equation of equation (B3) in Appendix B. With the assumption that the solutions of options have the same components as the ones with infinite maturity, Y^β, the functions, $z(Y,t)$, then follow a Convection-Diffusion type partial differential equation:

$$Y^2 z_{YY} + 2\left(\beta + \frac{\eta_Y}{\sigma^2}\right)Y z_Y + \frac{2}{\sigma^2} z_t = 0 . \quad \text{(C3)}$$

It is time to get rid of the Y and Y^2 terms. Let

$$Y = e^y, \qquad -\infty < y < \infty , \ \frac{1}{2}\sigma^2 t = \frac{1}{2}\sigma^2 T - \tau,$$

where T is a constant. Then we have

$$z_y = Y z_Y , \ z_{yy} = Y^2 z_{YY} + Y z_Y , \text{ and } \frac{1}{2}\sigma^2 z_\tau = -z_t .$$

Substituting into (C3) gives

$$z_{yy} + 2\left(\beta + \frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)z_y - z_\tau = 0 . \quad \text{(C4)}$$

The boundary and conditions for options, equation (13.1) and (13.2) in the text become

$$z(-\infty,\tau) = 0 , \text{ for hiring options,} \quad \text{(C5.1)}$$
$$z(\infty,\tau) = 0 , \text{ for firing options.} \quad \text{(C5.2)}$$

Substituting the values of betas, β_1 and β_2, of equations (B4) and (B5) in Appendix B into (C4) gives

$$z_{yy} + 2\sqrt{\alpha} z_y - z_\tau = 0 , \text{ for hiring options,} \quad \text{(C6)}$$
$$z_{yy} - 2\sqrt{\alpha} z_y - z_\tau = 0 , \text{ for firing options,} \quad \text{(C7)}$$

where $\alpha = \left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}$.

Hiring options

We can simplify (C6) by setting

$$x = y + 2\sqrt{\alpha}\tau , \ \bar{\tau} = \tau .$$

Note that $\bar{\tau}$ is the same as τ. To rewrite (C6) in terms of $(x,\bar{\tau})$ we use the chain rule

$$z_\tau = z_x x_\tau + z_{\bar{\tau}} \bar{\tau}_\tau = 2\sqrt{\alpha} z_x + z_{\bar{\tau}},$$
$$z_y = z_x x_y = z_x, \text{ and } z_{yy} = z_{xx}.$$

Substituting into (C6) gives

$$z_{xx} = z_{\bar{\tau}}. \tag{C8}$$

A new variable that depends only on x and $\bar{\tau}$ is often used to solve the above partial differential equation:

$$\xi = \frac{x}{\sqrt{\bar{\tau}}}, \tag{C9}$$

so that $z(x,\bar{\tau}) = u(\xi)$. Differentiating shows that

$$z_{\bar{\tau}} = -\frac{1}{2\bar{\tau}} \xi u'(\xi),\ z_{xx} = \frac{1}{\bar{\tau}} u''(\xi).$$

Substituting into equation (C8) gives the following second-order ordinary differential equation:

$$u''(\xi) + \frac{1}{2}\xi u' = 0,\ \ -\infty < \xi < \infty, \tag{C10}$$

The boundary condition of (C5.1) becomes the following equation:

$$u(-\infty) = 0, \text{ for hiring options,} \tag{C11}$$

Separating the variables, (C10) becomes

$$u'(\xi) = B_1 e^{-\xi^2/4},$$

where B_1 is unknown constant. Integrating gives

$$u(\xi) = B_1 \int_{-\infty}^{\xi} e^{-s^2/4} ds + C_1, \tag{C12}$$

where C_1 is an unknown constant. Applying the boundary condition for hiring options (C11) gives

$$\lim_{\xi \to -\infty} u(\xi) = C_1 = 0.$$

Substituting into (C12) gives

$$u(\xi) = B_1 \int_{-\infty}^{\xi} e^{-s^2/4} ds.$$

It is convenient to make the change of variable $s = \sqrt{2}\varpi$, so that

$$u(\xi) = B_1 \sqrt{2} \int_{-\infty}^{\xi/\sqrt{2}} e^{-\varpi^2/2} d\varpi = A_1 \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\xi/\sqrt{2}} e^{-\varpi^2/2} d\varpi. \tag{C13}$$

where $A_1 = B_1 2\sqrt{\pi}$. Substituting (C13) into (C1) and using the facts of $Y = e^y$, $z(x,\bar{\tau}) = u(\xi)$, $\xi = \frac{x}{\sqrt{\bar{\tau}}}$, $\frac{1}{2}\sigma^2 t = \frac{1}{2}\sigma^2 T - \tau$, $x = y + 2\sqrt{\alpha}\tau$, and $\bar{\tau} = \tau$ gives the hiring options v_H^G,

$$v_H^G(Y,t) = A_1 Y^{\beta_1} N(d_1), \tag{C14}$$

where $d_1 = \dfrac{\ln Y + \sigma^2 (T-t)\sqrt{\left(\dfrac{\eta_Y}{\sigma^2} - \dfrac{1}{2}\right)^2 + \dfrac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$ and $N(d) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi,$.

Firing options

In a similar way, we can obtain the firing options. We can simplify (C7) by setting $x = y - 2\sqrt{\alpha\tau}$ and $\bar{\tau} = \tau$.

$$z_{xx} = z_{\bar{\tau}}. \tag{C15}$$

A new variable $\xi = \frac{x}{\sqrt{\bar{\tau}}}$ is used to solve the above partial differential equation so that $z(x,\bar{\tau}) = u(\xi)$. Differentiating and substituting into (C15) gives the following simple second order ordinary differential equation:

$$u''(\xi) + \frac{1}{2}\xi u' = 0, \quad -\infty < \xi < \infty.$$

Separating the variables, the above equation becomes

$$u'(\xi) = B_2 e^{-\xi^2/4},$$

where B_2 is unknown constant. Integrating gives

$$u(\xi) = B_2 \int_{-\infty}^{\xi} e^{-s^2/4} ds + A_2, \tag{C16}$$

where A_2 is an unknown constant. The boundary condition of (C5.2) becomes the following equation:

$$u(\infty) = 0, \quad \text{for firing options,} \tag{C17}$$

Applying the boundary condition for hiring options (C16) gives

$$\lim_{\xi\to\infty} u(\xi) = 2\sqrt{\pi}B_2 + A_2 = 0 \Rightarrow B_2 = -\frac{A_2}{2\sqrt{\pi}}.$$

Substituting the above relationship back into (C16) gives

$$u(\xi) = A_2\left(1 - \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{\xi} e^{-s^2/4} ds\right).$$

It is convenient to make the change of variable $s = \sqrt{2}\varpi$, so that

$$u(\xi) = A_2\left(1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\xi/\sqrt{2}} e^{-\varpi^2/4} d\varpi\right) = A_2 \frac{1}{\sqrt{2\pi}}\int_{\xi/\sqrt{2}}^{\infty} e^{-\varpi^2/2} d\varpi = A_2 \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{-\xi/\sqrt{2}} e^{-\varpi^2/2} d\varpi.$$

Thus, the firing options v_H^G becomes

$$v_F^G(Y,t) = A_2 Y^{\beta_2} N(-d_2), \tag{C18}$$

where $d_2 = \dfrac{\ln Y - \sigma^2(T-t)\sqrt{\left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho+\lambda)}{\sigma^2}}}{\sigma\sqrt{T-t}}$ and $N(d) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-\varpi^2/2} d\varpi..$

Appendix D:
Derivation of Equations for $\partial v^G / \partial Y$

Differentiating hiring and firing options – defined by (16.1) and (16.2) – with respect to Y gives

$$\frac{\partial v_H^G}{\partial Y} = A_1\beta_1 Y^{\beta_1 - 1} N(d_1) + A_1 Y^{\beta_1} N_Y(d_1), \tag{D1}$$

$$\frac{\partial v_F^G}{\partial Y} = A_2 \beta_2 Y^{\beta_2 - 1} N(-d_2) + A_2 Y^{\beta_2} N_Y(-d_2). \tag{D2}$$

Differentiation of the integral, $N(d)$, involves a parameter. Such differentiation can be obtained by using *Leibnitz's rule*. Suppose a function

$$\varphi(x) = \int_{a(x)}^{b(x)} f(x,s)ds, \tag{D3}$$

where f is such that the integration cannot be effected analytically. Using calculus gives

$$\varphi_x(x) = \int_{a(x)}^{b(x)} \frac{\partial f(x,s)}{\partial x} ds + f(x,b(x))b_x(x) - f(x,a(x))a_x(x). \tag{D4}$$

Applying (D4) to the differentiation of $N(d_1)$ and $N(-d_2)$ gives

$$N_Y(d_1) = \frac{e^{\frac{-\left[\ln Y + \sigma^2 (T-t)\sqrt{\alpha}\right]^2}{2\sigma^2 (T-t)}}}{\sigma Y \sqrt{2\pi (T-t)}}, \tag{D5}$$

$$N_Y(-d_2) = -\frac{e^{\frac{-\left[-\ln Y + \sigma^2 (T-t)\sqrt{\alpha}\right]^2}{2\sigma^2 (T-t)}}}{\sigma Y \sqrt{2\pi (T-t)}}. \tag{D6}$$

where $\alpha = \left(\frac{\eta_Y}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\rho + \lambda)}{\sigma^2}$.

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