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Hiring and Firing: A Tale of Two Thresholds

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Hiring and Firing: A Tale of Two Thresholds*

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Abstract

The negative effect of quits on the willingness of firms to provide on-the-job training is well-documented in the theoretical literature (Becker, 1964). In this paper we explore the strength of this effect by solving a firm's dynamic optimisation problem where there is uncertainty about future productivity and non-zero firing costs. We find that the degree to which quit rates affect *hiring and training* depend on the ratio of firing to hiring costs. As this ratio rises, the negative effect of quits becomes less important, eventually reversing itself. We also describe how quit rates affect the *firing* decision. We conclude by highlighting some testable implications of our analysis.

Keywords: quitting, hiring, firing, human capital, real options.
JEL: E32, J 23, J 24, J54

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I. Introduction

According to human-capital theory, firms investing in specific human capital prefer hiring and employing workers who – for a given level of innate productivity – are unlikely to quit for non-wage reasons. Investing in the training of these more ‘loyal’ workers yields higher expected returns. This quitting effect is often used as an explanation of male-female wage differentials: firms invest less in women because they are more likely to quit for non-wage reasons. An objective of our paper is to show how the strength of this result depends on the ratio of firing to hiring costs when there is uncertainty about the level of future labour productivity.

In the presence of uncertainty about future productivity, employment protection or other firing costs result in a reduced incentive for firms to hire and train new workers, and introduce a distortion into the economy. Under these circumstances, quits may act as a second distortion that alleviates some of the adverse consequences of the first in the spirit of the theory of the ‘second best’ (Lancaster-Lipsey, 1956). When firms are prevented from firing their employees at will, higher quit propensities are no longer seen as reducing the attractiveness of possible hires to the same extent. Employers who – in a world with no demand uncertainty and no firing costs – would avoid hiring more mobile workers will become more inclined to hire them as uncertainty and employment protection costs increase. However, it still holds that the higher are the costs of training, the more firms value worker loyalty as in Becker (1964:29). We demonstrate below that the attractiveness of a mobile worker is increasing in both the ratio of firing-to-hiring costs and in the degree of uncertainty about future productivity.

Our analysis also has some interesting implications for the firing decision. With employment protection and uncertainty about future productivity, the firing decision is also an inter-temporal investment decision. The Becker analysis would then tell us that – *ceteris paribus* – when productivity is not expected to recover, firing a loyal worker would yield higher expected returns, as the firm is reducing its future losses for a longer expected period of time. But our analysis suggests that this too is an investment under uncertainty. This is because productivity may recover in the future, thereby making it optimal to employ the worker. Thus the firm may wait longer and suffer a larger drop in productivity before firing the loyal worker – especially if the cost of hiring a worker is high. Our results in this regard mirror our analysis of the

hiring decision. When the ratio of firing-to-hiring costs is low (high), and there is uncertainty about future productivity, firms are more inclined to fire a mobile (a loyal) worker.

This combination of uncertainty and a high level of firing costs relative to hiring and training costs is thus likely to reduce the bias of employers' hiring and firing decision against mobile individuals. Such individuals may now find it easier to find jobs and be less likely to lose them. In Booth and Zoega (1999) we derived a dynamic efficiency-wage model with uncertainty about the level of future productivity, and in which firms set wages to deter quits. There were no layoffs in that model, and hence the level of firing costs did not form part of the argument. In this present paper we extend that analysis, in order to show how the choice between workers who differ only in terms of their quit rate may depend on the level of firing costs. The higher are firing costs, the weaker is the quitting effect.

What are employers' views of labour turnover? Survey evidence on this is hard to come by, but analysis of the 1991 Employers' Manpower and Skills Practices Survey of British establishments suggests that firms may not dislike some turnover of their workforce. Using these data, Martin (1999) reports that over two out of three respondents felt that labor turnover is at the right level. Only one in four thought it was too high, while 4% of respondents thought their turnover was too low. Martin estimates the upper and the lower bounds to optimal turnover rates as 20% and 1.5% respectively. The fact that the lower bound is non-zero suggests that there are benefits from turnover. The upper bound is lower where training and hiring costs are greater. However, little variation is found in the lower bound across establishments. These findings are consistent with our model, since we provide a rationale for firms to prefer some turnover if the ratio of firing to hiring costs is very high.¹

Empirical evidence shows that individuals do differ in their quit rates. Studies using individual-level surveys suggest that heterogeneity in quit rates can be captured in part by differences in observable characteristics. Both British and US studies show that women (especially those with young children) tend to quit more than men (Viscusi, 1980; Blau and Kahn, 1981; Meitzen 1986, Royalty, 1998; Booth

¹ Roughly half of all establishments had turnover rates of less than or equal to 10%, while the remainder had turnover rates in excess of this. Of those workplaces with the lower turnover rates, 9% thought turnover was too high, while 84% thought it was 'about right'. Of the high turnover workplaces, 39%

Francesconi and Garcia-Serrano, 1999).² Royalty (1998) using a sample of US workers aged 22-30 in 1987, finds that less-educated women differ significantly in their turnover rates from more highly educated women (who behave not much differently from men), and that marital status and the presence of a child also have a significant impact. Jaeger and Huff Stevens (1999) and Neumark, Polsky and Hansen (1999) find that women, blacks and younger workers are more likely to have shorter job tenure than men, a finding that is however consistent with both higher quit rates and with higher layoff rates for those groups. Booth *et al* (1999) use the work history data from the 1993 wave of the British Household Panel Survey to show that women are more likely to leave *voluntarily* (either to another job or to non-employment) than men *ceteris paribus*, and that younger workers are also more likely to quit.

In the model developed below, we assume worker heterogeneity in quitting propensities. These affect their probability of being hired both directly – by affecting the value of their hiring option – and indirectly through wages. We suppose that wages are determined following the efficiency wage approach extensively used in the literature (see for Solow, 1979; Calvo, 1979; Shapiro and Stiglitz, 1984; Phelps, 1994). This provides a rationale for real-wage rigidity, so that changes in labour demand primarily affect the level of employment and not so much the level of real wages.

The rest of the paper is set out as follows. In the next section, we outline the background to the model developed in the paper, and summarise the principal assumptions underlying the analysis. In subsequent sections, we develop the model and discuss its testable implications.

II. Background and Assumptions

A. Assumptions

The representative firm's problem is to consider its demand for labour – that is, which type of worker to hire and the productivity threshold at which to hire each, where workers differ only in their propensity to quit. This decision is made by the firm in the

thought turnover too high and almost 60% thought they were about right.

presence of exogenously given firing costs and technologically given hiring costs that vary across two kind of jobs in accordance with the production technology. We make the following assumptions:

- (i) We consider a representative firm that provides *specific* training to workers. For simplicity, we set the reservation wage of workers who are willing to undergo training to zero. All training is provided by the firm, whose production technology involves only trained workers. There are two jobs: a manual job (for example a typist) requiring little training, and a more complex job (for example a computer programmer) involving more training.
- (ii) Workers have stochastic preferences, in the sense that we assume that the individual quit rate is exogenously given by b . Workers are heterogeneous with respect to their quitting probability, but homogeneous in all other respects.
- (iii) The level of severance pay is exogenous to our model,³ and it does not vary with the type of job.
- (iv) The firm chooses which worker-type to employ in the high- or low-training-cost jobs, and which worker-type to lay off first. This is analogous to the standard ‘right-to-manage’ assumption used in the trade union literature, whereby the firm retains its prerogative of which workers to employ or layoff when workers are of equal productivity. To our knowledge there are no rules in Europe stipulating affirmative-action hiring or layoff policies, so we believe the right-to-manage assumption is a plausible one in this context.
- (v) There is uncertainty about the level of future productivity. This uncertainty takes two forms. First, we consider a stochastic process exhibiting persistence, namely the continuous time equivalent of a random walk – geometric Brownian motion. Second, we consider a stochastic process with mean reversion.

² Royalty (1998) uses National Longitudinal Survey of Youth data for 1987 (for a sample aged 22-30 at that date). In her first set of specifications she does not distinguish between quits and layoffs, a distinction that is vital for our purposes. In her second set of results, she drops job spells ending in layoffs from her estimating sub-sample. She finds that less-educated women differ significantly in their turnover rates from more highly educated women (who behave not much differently from men).

³ We use the terms redundancy pay and severance pay interchangeably in this paper to mean a cost faced by the firm when it has to make a worker redundant due to low productivity and not due to the

B. Theoretical Background

There is a long-standing literature on the importance of quits for firms' training decisions and the possibility of market failure, in particular training externalities. This literature goes back to the work of Pigou (1912), who suggested that the amount of training provided by firms will be less than socially optimal owing to a quitting externality. It also includes the seminal contribution of Becker (1964). With respect to specific training, Becker notes (page 29) that worker turnover imposes capital losses on firms. "Firms can discourage such quits by sharing hiring costs and the return with employees, but they would have less need to discourage them and would be more willing to pay for hiring costs if insurance were provided." Recent contributions to this literature include Stevens (1994, 1996), Acemoglu and Pischke (1998) and Chang and Wang (1996), amongst others, who claim that imperfect competition in the labour market may open the way for market failures in the case of general training, Becker's thesis notwithstanding. The role of uncertainty about future output demand and productivity has been mostly ignored with a few exceptions (see for example Hashimoto 1981; Chang and Wang 1996; Booth and Zoega, 1999).

A unifying theme of the literature so far has been the contention that quits discourage training (both general and specific) by firms. We confine ourselves to the case of specific training and show how the strength of this result depends on the ratio of firing to hiring costs. Moreover, we show how it can potentially be reversed if the level of firing costs is very high in relation to the level of hiring costs and there is uncertainty about future productivity. Our analysis incorporates the insights of Malliaris and Brock (1982), Dixit and Pindyck (1994) and Bentolila and Bertola (1990) by explicitly allowing for the option values associated with hiring and firing to affect the firm's employment decisions.⁴ In our model, these option values vary across workers, reflecting their heterogeneous quitting propensities. Unlike the Bentolila and Bertola analysis, we relax the exogenous wage assumption. We let firms set wages to deter shirking among workers and then describe the microfoundations of the shirking

worker voluntarily quitting.

⁴ Paxson and Sicherman (1996) use a similar sort of framework to model an individual's mobility and hours adjustment, when desired hours are stochastic owing to individual preferences, and follow a geometric Brownian motion.

decision. In particular, we show how a higher quit rate leads – *ceteris paribus* – to higher wages.

C. A Measure of the Quitting Effect

We assume that both training and firing costs are fixed per worker. We will calculate a productivity threshold at which it becomes optimal to hire and train the marginal worker and relate its value to the worker’s quit rate. In Figure 1 we plot three such hypothetical thresholds. The steepness of the threshold is a measure of the strength of the relationship between quits and training – the steeper is the threshold, the stronger is this effect. Thus a worker with a quit rate of b^* would only be hired if productivity reaches the levels marked on the figure – a level which is higher, the steeper is the threshold. In the extreme case, a negative slope implies a preference for workers with a high quit propensity. We will show using a model how the slope depends on the level of firing costs, hiring costs and the magnitude of uncertainty.

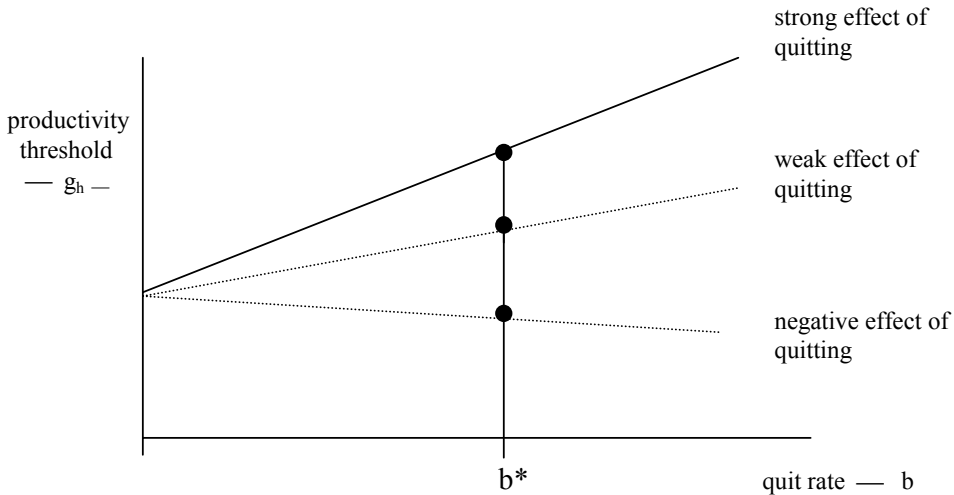


Figure 1. Measuring the Strength of the Quitting Effect

III. The Model

A. The Stochastic Environment and the Firm’s Decision Problem

We model the hiring- and firing decisions as inter-temporal investment decisions under conditions of uncertainty about future productivity. This amounts to an optimal stopping problem with respect to filling (or “emptying”) the two positions – the job

requiring little training (the typist) and the job requiring more training (the computer programmer) – and the firm has to make a binary choice in each case. One alternative is to hire (fire) a worker based on his or her quitting probability, while the other involves continuation with the current workforce – further waiting.

Current profits are defined as follows in the absence of hiring and firing,

$$\Pi(g_t, w, N_t) = g_t (N_t (1 - l(w, u; b)))^\theta - wN_t, \quad \theta \leq 1, \quad (1)$$

where N denotes the number of employed workers, l is the level of shirking (the fraction of the work day spent in an unproductive manner), w the real wage measured in units of output, g is a measure of productivity and also the source of uncertainty in the model and b denotes the (exogenously given) quit rate. Productivity follows the (potentially) mean-reverting process

$$dg = \mu(\bar{g} - g)gdt + \sigma g dz, \quad (2)$$

where μ is the speed of reversion, \bar{g} is average productivity, σ the variance parameter, and z a Wiener process. Productivity follows a geometric Brownian motion if $\mu = 0$ and a mean-reverting process if $\mu > 0$.

In order to calculate the value of g at which it becomes optimal to hire each type of worker given the size of the current workforce, one first has to calculate the value of employing this marginal worker. This can then be compared to the hiring costs. Use of Itô's Lemma gives the following Bellman equation for the value of the stock of workers; $V(N, g)$, in the continuation region where the value of future hires or fires is not taken into account,

$$\rho V = \max_w \left[g(N(1 - l(w, u; b)))^\theta - wN \right] - \lambda N V_N + \mu(\bar{g} - g)g V_g + \frac{1}{2} \sigma^2 g^2 V_{gg}, \quad (3)$$

and ρ is the real rate of interest. This is an asset-pricing equation for the value of the firm when wages are set optimally at each moment in time. The first term in square brackets on the right-hand side shows current profits, equal to the difference between output and the total wage bill when wages are at their optimal level. The third term represents the loss caused by current quits. The last two terms show the increase in the value of the firm caused by changes in the level of productivity.

The representative firm sets wages at each moment in time in order to maximise profits. This decision is based on the effect wages have on on-the-job effort or shirking l . Thus reducing wages may cause profits to fall if workers respond by

reducing their effort. We describe the microeconomic foundations of this function in Section IV below. Here it needs to be said that the optimal wage does not depend on the level of employment in the representative firm but instead on wages, the rate of unemployment – which is exogenous to each firm – quitting, discount rates etc. Thus we can continue the analysis of hiring and firing by taking the level of wages as given and determined by the equation below – the first-order condition with respect to wages:

$$-g\theta[N(1-l(w,u,b))]^{\theta-1}l_w(w,u,b)=1 \quad (4)$$

The left-hand side has the marginal of raising wages benefit – in terms of increased output due to less shirking – while the right-hand side has the marginal cost – in terms of higher wage costs. We now solve equation (4) for the optimal wage w^* and put it into equation (3) assuming that the effect of changes in g on w^* are sufficiently small to be ignored for the time being.⁵

In the appendix, we show the calculations used to derive the expected present value of the marginal employed worker $v^P(N, g)$.

$$v^P(N, g) = \theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{e^{-(\theta\lambda+\rho)t}}{1 + \left(\frac{\bar{g}-g}{g}\right) e^{-\bar{g}t\mu}} dt - \frac{w^*}{\rho + \lambda}. \quad (5)$$

The general solutions for the hiring and firing options have the following forms respectively (see the appendix and Dixit and Pindyck (1994), p.162),

$$v_H^G(N, g) = A_1 \left(g N^{\theta-1} (1-l)^\theta \right)^{\beta_1} H(g; \beta_1), \quad (6)$$

$$v_F^G(N, g) = A_2 \left(g N^{\theta-1} (1-l)^\theta \right)^{\beta_2} H(g; \beta_2), \quad (7)$$

where $H(\cdot)$ denotes the confluent hypergeometric function and β_1 and β_2 are the positive and negative roots of the characteristic equation (A15) respectively. To satisfy the boundary conditions that $v_H^G(N, 0) = 0$ and $v_F^G(N, \infty) = 0$ we use the positive solution for $v_H^G(N, g)$ and the negative solution for $v_F^G(N, g)$. The value of the marginal employed worker is therefore equal to the sum of $v^P(N, g)$ and $v_F^G(N, g)$ in the continuation region. This presupposes that a part of the value of

⁵ We show below that this requires l_w – the derivative of shirking with respect to wages – to be large.

employing a worker is the value of the option to fire her in bad times. The value of the marginal unemployed worker is then equal to the value of the option to hire that worker, $v_H^G(N, g)$.

B. The Hiring and the Firing Thresholds Defined

The definitions of the hiring and firing barriers; g_H and g_F , are given by the standard value-matching and smooth-pasting conditions. The firm would find it optimal to exercise the option to hire or fire the marginal worker once g hit one of the two barriers. The value-matching conditions follow

$$\begin{aligned} \theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{e^{-(\theta\lambda+\rho)t}}{1 + \left(\frac{\bar{g} - g_H}{g_H} \right) e^{-\bar{g}\mu t}} dt - \frac{w^*}{\rho + \lambda} + A_2 (g_H N^{\theta-1} (1-l)^\theta)^{\beta_2} H(g_H; \beta_2) \\ = T + A_1 (g_H N^{\theta-1} (1-l)^\theta)^{\beta_1} H(g_H; \beta_1), \end{aligned} \quad (8)$$

$$\begin{aligned} -\theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{e^{-(\theta\lambda+\rho)t}}{1 + \left(\frac{\bar{g} - g_F}{g_F} \right) e^{-\bar{g}\mu t}} dt + \frac{w^*}{\rho + \lambda} + A_1 (g_F N^{\theta-1} (1-l)^\theta)^{\beta_1} H(g_F; \beta_1) \\ = F + A_2 (g_F N^{\theta-1} (1-l)^\theta)^{\beta_2} H(g_F; \beta_2), \end{aligned} \quad (9)$$

where T and F denote training and firing costs respectively.

The left-hand sides of (8) and (9) show the marginal benefit from hiring/firing a worker and the right-hand sides the marginal costs. The marginal benefit of hiring a worker is equal to the sum of the present discounted value of his or her productivity net of wages, on the one hand, and the value of the option to fire the worker, on the other hand. The marginal cost of hiring is then the sum of the direct hiring costs and the sacrificed option to hire the worker in the future. The interpretation of the firing decision is similar. The smooth-pasting conditions follow.

$$\begin{aligned} \theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{\bar{g} \cdot e^{-(\theta\lambda+\rho+\bar{g}\mu)t}}{[g_H + (\bar{g} - g_H) e^{-\bar{g}\mu t}]^2} dt = A_1 (N^{\theta-1} (1-l)^\theta)^{\beta_1} [\beta_1 g_H^{\beta_1-1} H(g_H; \beta_1) \\ + g_H^{\beta_1} H'(g_H; \beta_1)] - A_2 (N^{\theta-1} (1-l)^\theta)^{\beta_2} [\beta_2 g_H^{\beta_2-1} H(g_H; \beta_2) + g_H^{\beta_2} H'(g_H; \beta_2)] \end{aligned} \quad (10)$$

$$\begin{aligned}
& -\theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{\bar{g} \cdot e^{-(\theta\lambda + \rho + \bar{g}\mu)t}}{[g_F + (\bar{g} - g_F)e^{-\bar{g}\mu}]^2} dt = -A_1 \left(N^{\theta-1} (1-l)^\theta \right)^{\beta_1} [\beta_1 g_F^{\beta_1-1} H(g_F; \beta_1) \\
& + g_F^{\beta_1} H'(g_F; \beta_1)] + A_2 \left(N^{\theta-1} (1-l)^\theta \right)^{\beta_2} [\beta_2 g_F^{\beta_2-1} H(g_F; \beta_2) + g_F^{\beta_2} H'(g_F; \beta_2)] \quad (11)
\end{aligned}$$

Equations (8), (9), (10) and (11) are non-linear systematic equations with four unknown parameters, g_H , g_F , A_1 and A_2 , and can be solved for numerically once the solutions for β_1 and β_2 are found from (A15).

Layard, Nickell and Jackman (1991, p. 420) report that the cost of firing ranges from 0.48 months salary in Denmark to 5.24 months salary in France to 15.86 months in Italy. We calculate the two thresholds for firing costs equal to 0.25 months salary, 1 months salary, 2 months salary, 4 months salary and 12 months salary respectively. These are in the range of those observed in Denmark (0.48 months), Belgium (1.24 months), France (5.24 months) and Norway (12 months), but below those for Spain (13.56 months) and Italy (15.86 months).

The critical productivity thresholds at which it would become optimal to hire (and to fire) workers when we ignore the effect of quitting on wages are shown in Figures 2-5. They show the effect of the expected quit rate on the hiring and firing thresholds for both the case of productivity following a mean-reverting process and a random walk when we ignore the effect of changes in quitting on the optimal wage and level of shirking. We then consider the indirect effect of quitting through wages in a later section. The hiring threshold is always rising in the quit rate for the lower values of the firing cost, but for higher values it is falling in the quit rate.

C. Low Hiring costs – the Case of a Typist

Figure 2a shows the hiring threshold as a function of the quit rate for the case of low hiring costs and no mean-reversion. It can be seen that for low levels of firing costs, quits reduce the willingness to train – the threshold is upward sloping. If the cost of firing is close to 2/12 (2 monthly wages), the threshold is approximately horizontal. For higher levels of firing costs, the thresholds becomes – surprisingly – negative sloping. Thus for a given level of hiring costs and a given level of uncertainty, the slope of the threshold depends on the level of firing costs. The magnitude of the (adverse) effect of quits on hiring and training is a negative function of the level of

firing costs – the higher is the level of firing costs, the smaller is the positive slope of the threshold.

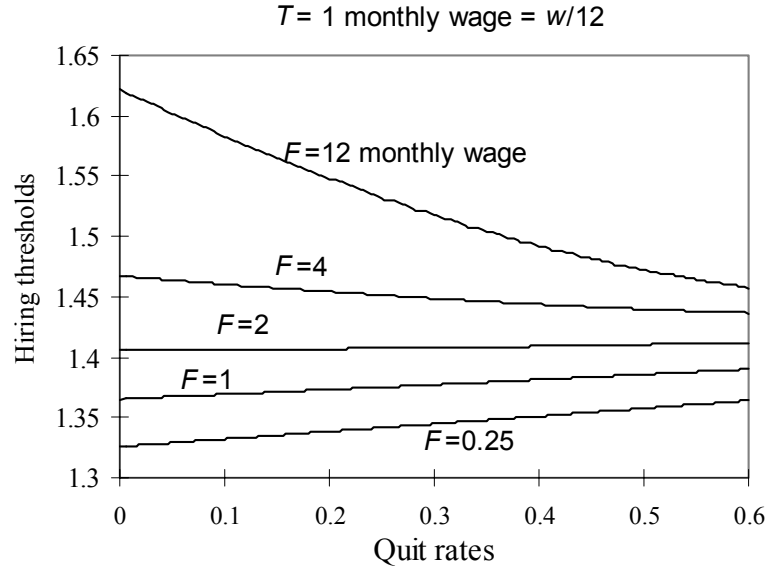


Figure 2a. The effect of quit rates on the hiring threshold of g when $T=1 \times w/12$ (one month wage) and $\mu=0.0$; $F = 0.25, 1, 2, 4, 12$ monthly wages respectively. Other parameters: $\sigma=0.25$, $\rho=0.05$, $\bar{g} = 1$, $\theta=1$, $w=1$ (annual wage), $[1 - I(w^*; u, b)]^p = 0.9$ and $N=1$.

In terms of equations (8) and (9), higher firing costs reduce the value of the firing option. An increase in the quit rate reduces the value of both options. It follows that starting with a higher firing cost the effect of higher quit rates is found in a lower value of the hiring option – hence a lower cost of hiring – while with hiring costs exceeding firing costs the effect is found to a greater extent in the firing option – hence in the marginal benefit of hiring. This explains our results. We conclude that while a higher quit rate reduces the expected discounted value of employing a typist, it also makes hiring less risky in the presence of firing costs – less risky implying a lower value of the hiring option.

The question arises how the time- or age dependance of quit rates would affect our results if we relax our assumption about constant and exogenous quit rates b . For example, would the firm prefer workers with rising quit rates to those with falling quit rates over their lifetimes? Without complicating the model above we can state the answer to this question. Because productivity follows a Brownian motion, changes in the productivity over any finite interval of time are normally distributed with a

variance that increases linearly with the time interval. For this reason, the firm would ideally prefer a worker who is not likely to quit initially but also not likely to stay for a very long period.⁶

We now do the calculations for the case of mean-reverting productivity, reflecting the situation of a transitory productivity shock. In Figure 3a we show that the threshold is now more likely to be upward sloping than before as the hiring decision is less risky.

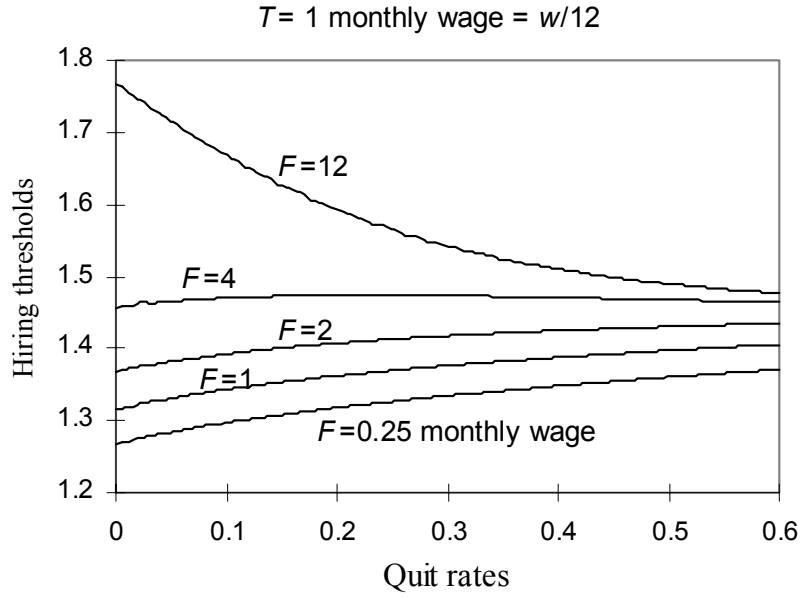


Figure 3a. The effect of quit rates on the hiring threshold of g when $T=1 \times w/12$ (one month wage) and $\mu=0.4$; $F = 0.25, 1, 2, 4, 12$ monthly wages respectively. Other parameters the same as in Fig 2a.

D. High Hiring costs – the Case of a Computer Programmer

How do the hiring- and firing thresholds for the computer programmer differ from those for the typist in light of the higher hiring costs involved? Will the firm also choose a mobile worker as the first to write a programme if firing costs are substantial?

⁶ We are grateful to a referee for this point.

To see this, we now raise the value of the hiring costs and solve the four equations again. The results are shown in Figure 4a for the geometric Brownian motion case. They show that, *ceteris paribus*, it would take higher productivity for the firm to invest in the training of a programmer. The figure also shows the threshold is almost always upward sloping and steep. The threshold only has a downward-sloping segment for the very highest levels of firing costs. Workers with very high quit rates are no longer an attractive choice – it does not make sense to train someone to write computer programmes who is very likely to quit in the near future. We conclude that the adverse effect of quitting on training is much stronger for a given level of firing costs in this case. In terms of equations (8) and (9) the quit rate reduces primarily the firing option – hence the marginal benefit of hiring. The hiring option is worth less than in the case of the typist due to the higher hiring costs.

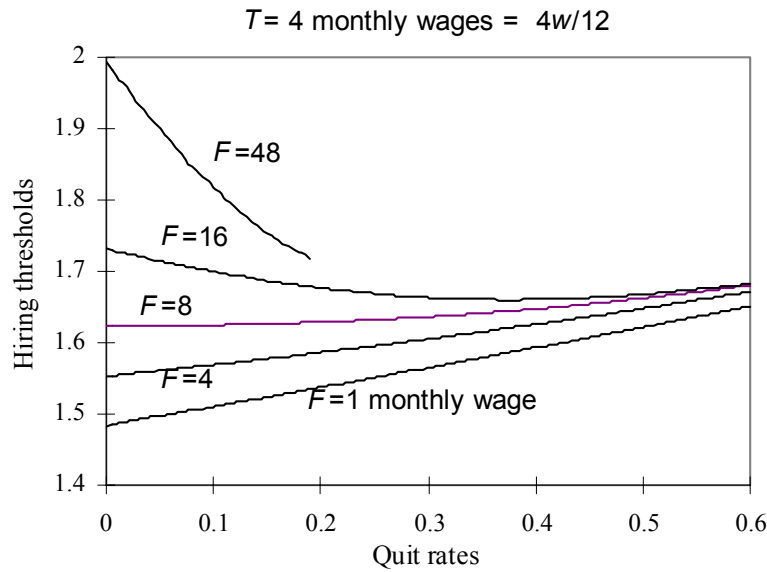


Figure 4a. The effect of quit rates on the hiring threshold of g when $T=4 \times w/12$ (four month wages) and $\mu=0.0$; $F = 1, 4, 8, 16, 48$ monthly wages respectively. Other parameters the same as in Fig 2a.

In Figure 5a we again see that loyalty becomes more desirable when productivity is mean reverting.

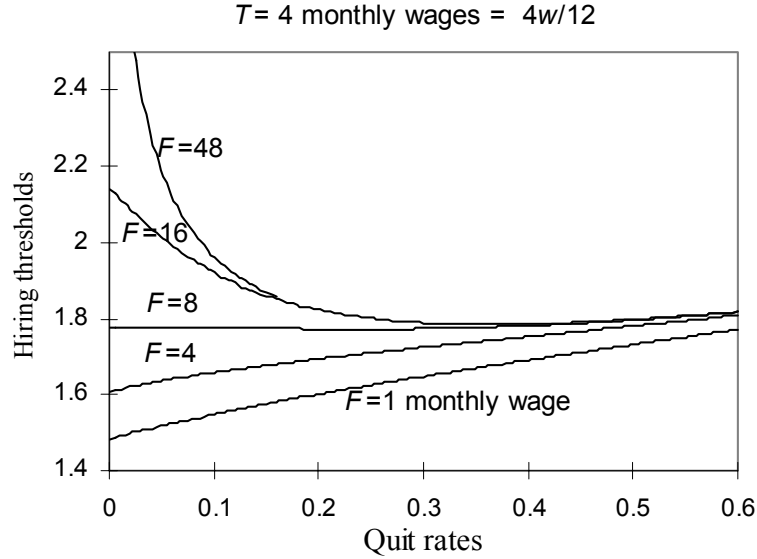


Figure 5a. The effect of quit rates on the hiring threshold of g when $T=4 \times w/12$ (four month wages) and $\mu=0.4$; $F = 1, 4, 8, 16, 48$ monthly wages respectively. Other parameters the same as in Fig 2a.

E. The Firing Decision

What about the possibility of firing the workers in the future? The same intertemporal considerations have to be taken into account when deciding on their dismissal in the future since firing them is also an investment. Firms incur fixed costs of firing each worker and in return reduce future losses. Now hiring costs will take the place of firing costs in the hiring decision and vice versa.

In Figures 2b-5b we find the optimal ranking for the firing decision. In all cases but those when the firing costs are trivial, the firm chooses to fire the loyal workers first. By firing the loyal workers, the firm gets a higher return on its firing investment as they would have stayed longer with the firm if not pushed out. This is the Becker argument put on its head – the firing decision is also an intertemporal investment decision and quits matter just as much as in the case of the hiring decision

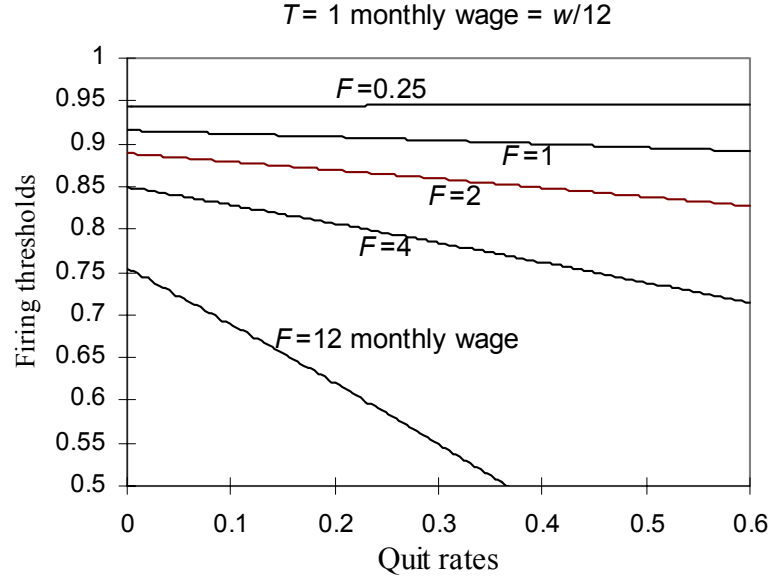


Figure 2b. The effect of quit rates on the firing threshold of g when $T=1 \times w/12$ (one month wage) and $\mu=0.0$; $F = 0.25, 1, 2, 4, 12$ monthly wages respectively. For other parameters see Fig. 2a.

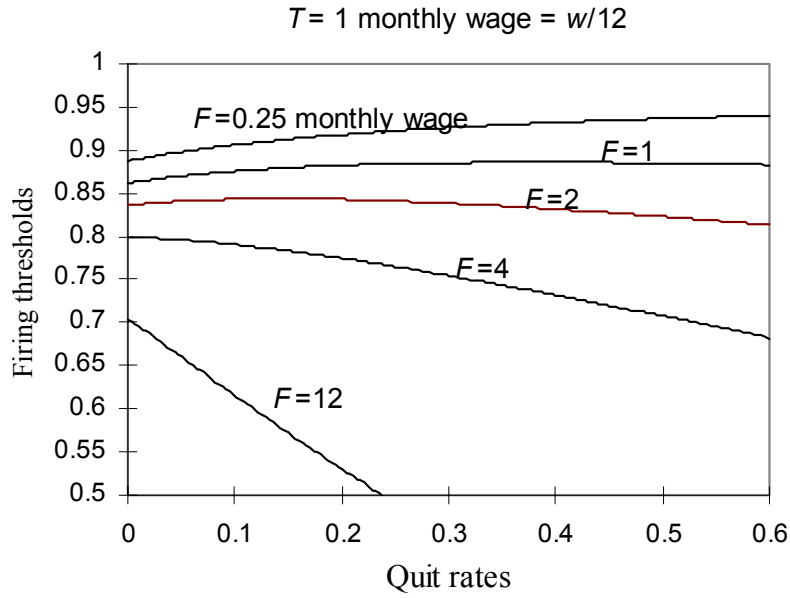


Figure 3b. The effect of quit rates on the firing threshold of g when $T=1 \times w/12$ (one month wage) and $\mu=0.4$; $F = 0.25, 1, 2, 4, 12$ monthly wages respectively. Other parameters the same as in Fig 2a.

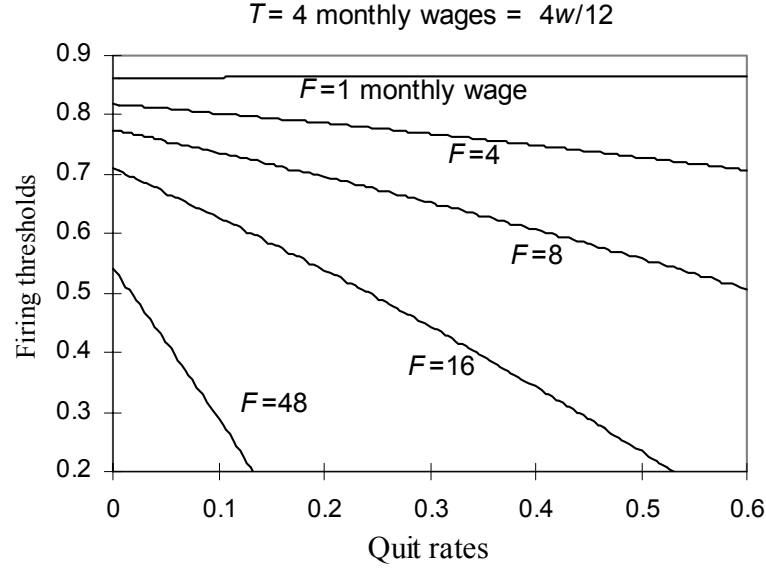


Figure 4b. The effect of quit rates on the firing threshold of g when $T=4 \times w/12$ (four month wages) and $\mu=0.0$; $F = 1, 4, 8, 16, 48$ monthly wages respectively. Other parameters the same as in Fig 2a.

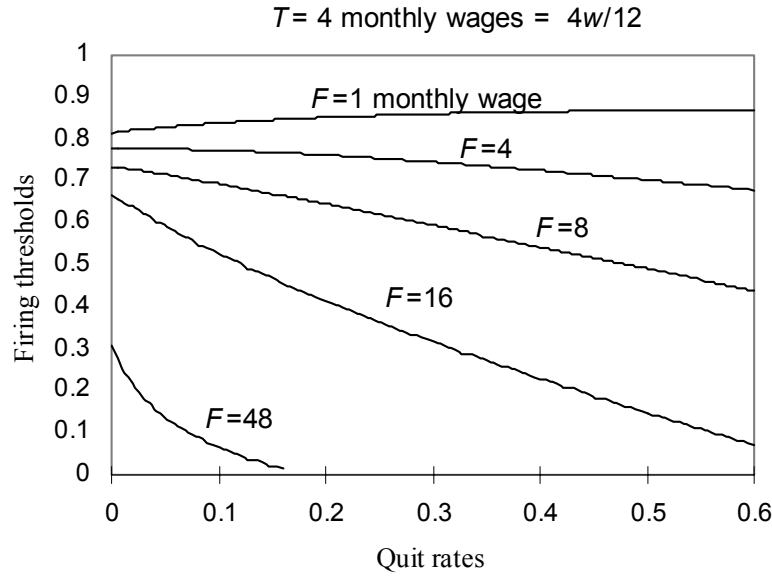


Figure 5b. The effect of quit rates on the firing threshold of g when $T=4 \times w/12$ (four month wages) and $\mu=0.4$; $F = 1, 4, 8, 16, 48$ monthly wages respectively. Other parameters the same as in Fig 2a.

An apparent contradiction arises. Should not considerations of risk enter the calculation again? Is it not least risky to fire a mobile worker as he would have left anyway? This turns out to depend again on the level of hiring costs relative to the

level of firing costs. We have seen that these affect the relative size of the hiring- and the firing options. Since both are falling in the quit rate, our results depend on which has the larger value at zero quit rates. If hiring costs were increased, an increase in the quit rate would primarily work to reduce the value of the firing option which is part of the marginal cost of firing. This would make the firm fire the mobile workers first.

F. A Summary of our Results

We now summarise the results with two propositions:

The negative effect of quits on training is increasing in magnitude in the ratio of hiring- to firing costs. Conversely, the higher is the ratio of firing- to hiring costs, the smaller is the negative effect of quits on training.

The chances of a loyal worker – that is one with a low quit rate – losing his job are decreasing in the ratio of hiring to firing costs. The chances of a worker with a high quit rate are increasing in this ratio.

It follows that as we raise the level of firing costs, loyal workers lose some of their advantages in terms of both getting a job as well as retaining it. Raising the level of firing costs would thus benefit mobile workers at the expense of the loyal ones.

Our numerical solutions show that it is the ratio of hiring to firing costs that matters for both decisions, not their absolute level.⁷ Moreover, the results do not depend on the particular functional form chosen for the production function apart from the requirement that it exhibit diminishing returns to labour. The value chosen for the parameter θ in the production function does not affect our results qualitatively.⁸

IV. Wage Determination

We now turn to the microfoundations of the shirking function l . This is important because differences in the quit rate between workers may possibly affect wages and hence the slope of the thresholds in sections III-c to III-e. We choose a well-known model of the shirking decision that gives real-wage rigidity in the face of demand fluctuations. This is an amended version of the efficiency wage model of Shapiro and

⁷ These are available from the authors upon request.

⁸ Numerical solutions for different values of θ – which measures the returns to labour – are available from the authors upon request.

Stiglitz (1984).⁹

An employed worker gets utility from both consumption C^1 and shirking, l .¹⁰ These are his two control variables that he adjusts to maximise expected discounted utility subject to the dynamic budget constraints below

$$\frac{dA^1}{dt} = w + \rho A^1 - C^1 \quad (12)$$

where A^1 denotes the real wealth – in the form of public bonds – of an employed worker and ρ is the real rate of interest. By raising his level of current consumption, the worker accumulates less wealth and hence consumes less in the future. By increasing the level of shirking, the worker gains instantaneous utility but increases the risk that she will be dismissed. The probability of dismissal is equal to $m \cdot l$ where m is the employer's monitoring intensity.

The instantaneous utility function – or felicity function – is the following:

$$U(C^1, l), \quad U_{C^1} > 0, \quad U_l > 0, \quad U_{C^1 C^1} < 0, \quad U_{ll} < 0 \quad (13)$$

Letting V denote discounted lifetime utility, the first-order conditions from the maximisation of this lifetime utility level by an employed worker are the following:

$$U_{C^1}(C^1, l) = V_A^1 \quad (14)$$

$$U_l(C^1, l) = m[V^1 - V^0] \quad (15)$$

The worker equates the marginal utility of private consumption to his marginal utility of wealth. The worker, similarly, equates the marginal benefit and the marginal cost of shirking. The marginal benefit is equal to the instantaneous marginal utility from shirking. The marginal cost is equal to the expected fall in lifetime utility due to a rise in the probability of detection and dismissal.

Using the solution values for C^1 and l and the underlying Bellman equation gives the familiar equation (16) where b denotes the quit rate, μ is the rate of pure time preference, and $ml+b$ is the probability of moving from the employed to the unemployed state:

$$\mu V^1 = U(C^1, l) + V_A^1 \frac{dA^1}{dt} + (ml+b)[V^0 - V^1] \quad (16)$$

⁹ We make shirking be a continuous variable while Shapiro and Stiglitz assume that it can only take two values, 0 and 1. We also allow for wealth accumulation.

¹⁰ We can think of the variable l as measuring the proportion of time spent at work which is not used for productive activities.

This is an asset equation which describes the value of being employed V^1 as a function of the instantaneous utility of being employed, the value of being unemployed and the transition probability between the two states. The left-hand side has the required return and the right-hand side the sum of instantaneous utility (dividend) and the expected gain from both wealth accumulation and a change in employment status (expected capital gain).

The maximisation problem to be solved by an unemployed worker is slightly simpler. The worker now only gets utility from consumption, C^0 being his only control variable,

$$U(C^0), \quad U_{C^0} > 0, \quad U_{C^0 C^0} < 0 \quad (17)$$

and faces the dynamic budget constraint

$$\frac{dA^0}{dt} = \rho A^0 - C^0 \quad (18)$$

where A^0 is denotes the holdings of public bonds by an unemployed worker. The first-order condition is given in equation (19):

$$U_{C^0} = V_A^0 \quad (19)$$

As in the case of the employed worker, optimal consumption is at the level where the marginal utility of private consumption is set equal to his marginal utility of wealth. Using the solution value for C^0 and the underlying Bellman equation gives the asset-pricing equation

$$\mu V^0 = U(C^0) + V_A^0 \frac{dA^0}{dt} + a[V^1 - V^0] \quad (20)$$

where a is the transition probability out of unemployment. The left-hand side is again the required return and the right-hand side the sum of the instantaneous utility and the expected gain from both wealth accumulation and a change in employment status.

Since the only form of nonhuman wealth is public bonds and all public bonds must be held by someone, equations (12) and (18) are related in the following way,

$$u[\rho A^0 - C^0] + (1-u)[w + \rho A^1 - C^1] = 0 \quad (21)$$

where u is the rate of unemployment and the first term denotes wealth decumulation by the unemployed – the selling of bonds – while the second term denotes the wealth

accumulation by the employed – the buying of bonds.

Equations (15), (16) and (20) can be simplified and imply equation (22) :

$$U_l(C^1, l)m^{-1}[\mu + ml + a + b] = U(C^1, l) + V_A^1[w + \rho A^1 - C^1] - U(C^0) - V_A^0[\rho A^0 - C^0] \quad (22)$$

We are interested in the effect of quits on wages. To learn from equation (22) about the level of wages, we need to go back to equation (4) that describes the firm's wage-setting decision. From that equation it follows that

$$\frac{dw}{db} = - \frac{(\theta-1)(1-l)^{-1}l_w l_b - l_{wb}}{(\theta-1)(1-l)^{-1}(l_w^2) - l_{ww}} \quad (23a)$$

and

$$\frac{dw}{dg} = \frac{l_w}{g(\theta-1)(1-l)^{-1}l_w^2 - gl_{ww}} \quad (23b)$$

The total differential of equation (22) can be used to find the effect of changes in quitting and wages on shirking and hence evaluate the sign of (23). The partial derivatives found in equation (23) follow:

$$l_b = \frac{\partial l}{\partial b} = - \frac{U_l m^{-1}}{U_{ll} m^{-1}(\mu + ml + a + b)} > 0 \quad (24)$$

$$l_w = \frac{\partial l}{\partial w} = \frac{V_A^1}{U_{ll} m^{-1}[\mu + ml + a + b]} < 0 \quad (25)$$

$$l_{wb} = \frac{\partial l_w}{\partial b} = - \frac{V_A^1}{U_{ll} m^{-1}(\mu + ml + a + b)^2} > 0 \quad (26)$$

$$l_{ww} = \frac{\partial l_w}{\partial w} = 0 \quad (27)$$

From equations (23)-(27) we conclude that wages are an increasing function of productivity – $dw/dg > 0$ – but that this effect is diminishing in the size of l_w . A high value of this term justifies our assumption of fixed wages when deriving the numerical solutions in sections III-b to III-e.

The effect of quitting on wages – dw/db – is ambiguous. This can be anticipated from equation (4) and becomes clear when we substitute equations (24) to (27) into equation (23a). There are two effects. First, a higher quit rate gives more shirking, hence lower output and hence a higher marginal product of labour which raises the marginal benefit of higher wages in terms of increased output. On this count the wage should be a positive function of shirking. Second, the wage turns out to be more potent in reducing shirking the higher is the quit rate – $l_{wb} > 0$ – and this acts to make the optimal wage be falling in the quit rate. However, we are interested in circumstances in which the indirect effect of quits on hiring and firing going through wages acts to reverse our two propositions below. We now assume that the optimal wage is a positive function of b :

$$\frac{dw}{db} = f(b) > 0 \quad (23')$$

This gives

$$w = F(b), \quad F'(b) > 0 \quad (28)$$

where the functional form depends on the form of the utility and hence the value function V_A^1 .

The question now arises if the positive effect of quitting on optimal wages is sufficient to make our propositions in the previous section invalid. In particular, does the induced effect of quitting on wages prevent firms from ever preferring a worker with a high propensity to quit. We now approximate equation (28) by the following equation:

$$w = \alpha_0 + \alpha_1 \log(1+b) \quad (29)$$

where the quit rates are written as percentages. We then solve for the hiring- and the firing threshold while setting α_0 equal to one.

The dependence of wage on the quit rate b in equation (29) affects both parts of our propositions in Section III – that is both the hiring- and the firing decision. We start with the effect on the hiring decision.

A. Effect of endogenous wages on the hiring decision

A positive effect of quitting on wages can make it optimal for firms never to prefer mobile workers. We therefore redid the numerical solutions for the case of a high

firing costs and low hiring costs and having wages set according to equation (29). We found that for $\alpha_1 < 0.03$ our results did not change while for $\alpha_1 \geq 0.03$ the firm would never prefer mobile workers. However, note that this depends on the value of σ – our measure of uncertainty. The greater is uncertainty, the higher is this threshold value of α_1 . The critical wage function is plotted in Figure 6.

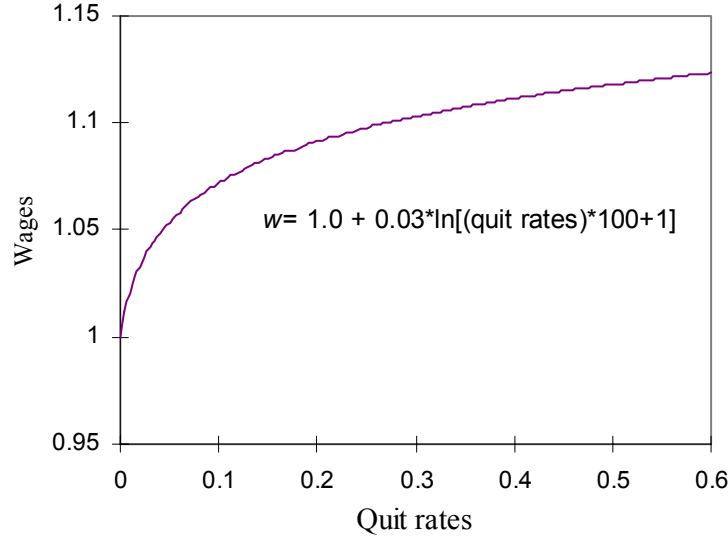


Figure 6. Wages and quit rates with wage function: $w=1+0.03 \times \log[b+1]$.

We can see that in the case of $F = 1$ and $T = 1/12$, firms will never hire a mobile worker.¹¹ The thresholds are shown in Figure 7.

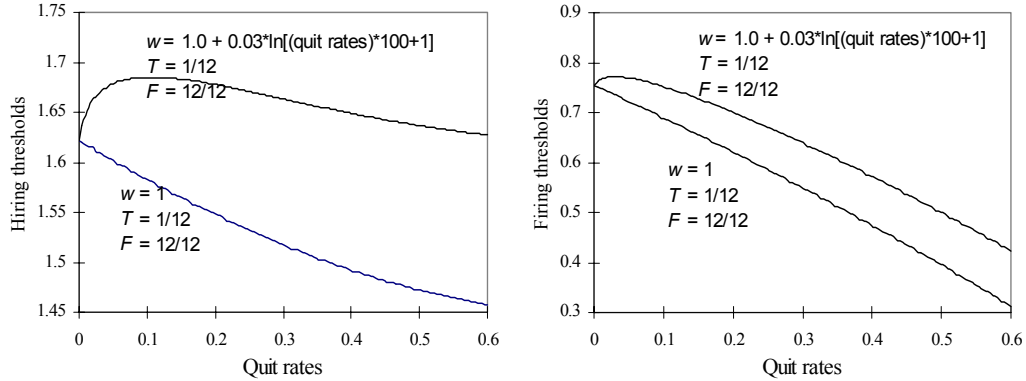


Figure 7. The effect of quit rates on the hiring and firing thresholds of g with $w = 1 + 0.03 \times \ln(b \times 100 + 1)$; $T = 1/12$, $F = 1$. Other parameters are the same as in Figure 2a.

¹¹ The explanation here somehow is irrelevant to value-matching conditions. An increase in wages will lead to a fall in effective firing costs and to a rise in effective hiring costs.

In the left-hand side panel we find that the hiring threshold is downward sloping for fixed wages $w = 1$ – firms prefer quitters to those who are more likely to stay – while the most loyal individual (zero quit rate) is always preferred for our wage function (29) with $\alpha_0 = 0$ and $\alpha_1 = 0.03$. However, the ranking does not change in the case of the firing decision although the firing threshold becomes less steep – the loyal workers are the first to be fired.

B. Effect of endogenous wages on the firing decision

We also need to look at the case of a high ratio of hiring- to firing costs. In this case firms preferred to fire mobile workers as shown in Figure 2b above. However, the wage function (29) now reinforces the earlier effect – hence the second part of our propositions above – instead of mitigating it as was the case with the hiring decision. When a higher quit rate raises the level of wages, the mobile workers become a primary target for dismissals. First, because they receive higher wages. Second, because firing them is less risky since they are more likely to leave on their own accord.

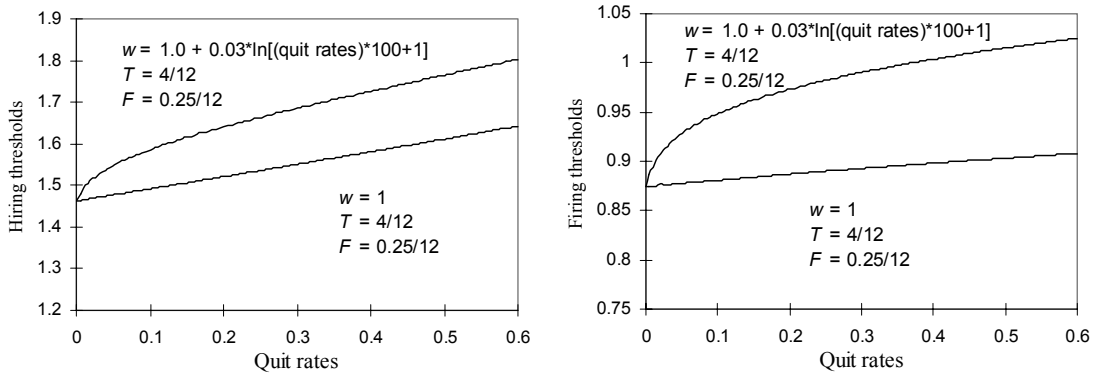


Figure 8. The effect of quit rates on the hiring and firing thresholds of g with $w = 1 + 0.03 \times \ln[(\text{quit rates}) \times 100 + 1]$; $T = 4/12$ (four months wages), $F = 0.25/12$ (one week wages). Other parameters are the same as Figure 4a.

We conclude that while endogenising wages may moderate the implications of our analysis for the hiring decision – in particular firms may always prefer loyal workers – this is not so in the case of the firing decision. When the ratio of firing- to hiring costs is low, firms may prefer firing mobile workers first despite, or as we have shown in part because of, their higher wages.

V. Some Testable Implications

A number of interesting implications emerge from our analysis as relates to the hiring- and the firing decision. Most studies show that women quit more than men (see above), especially those with young children. The same applies to teenage workers. The following two implications follow:

- With high hiring and training costs – that is relative to firing costs – and uncertainty about future productivity, firms would prefer prime-age workers to young workers and male workers to females. Thus when labour productivity improves, prime-age males are the first to benefit.
- With high hiring and training costs – again relative to firing costs – firms would prefer laying off young workers over prime-aged workers and women over men.

It follows that both mean and median expected tenure for young workers should be lower than for prime-aged workers, and also lower for women than for men. This is borne out by the data (see *inter alia* Shimer (1999), Jaeger and Huff Stevens (1999) and Neumak, Polsky and Hansen (1999) for the US, and Booth, Francesconi and Garcia Serrano, 1999 for Britain).¹²

In contrast, firms with high firing costs – relative to their hiring costs – show a greater tendency to hire women and young workers and to lay off their prime-aged men in recessions. The preference for laying off loyal and stable prime-aged workers may seem surprising but comes quite naturally out of our model – it is the Pigou-Becker effect put on its head, since quitting reduces the expected return from firing workers. This gives a third empirical prediction of our analysis:

¹² Although the US is not characterised by high statutory employment protection costs as in Europe, US firms may still face significant layoff costs through, for example, the costs of sabotage that may be imposed on firms by disgruntled workers who have been fired. It is hard to measure the extent of sabotage costs, since the ex-ante expectation of sabotage induces firms to employ policies to deter it (for example avoiding of advance notification, insistence of instant clearing out of desks, offering a generous final payment in lieu of advance notification, etc. See Addison and Chilton (1997) and Kuhn

- When firing costs are raised, the negative effect of quitting on the hiring and training of new workers is at least partially alleviated. Firms become more willing to hire workers with higher quit rates.

The positive effect of employment-protection legislation on on-the-job training has been advanced in the literature (see for example Booth and Chatterji, 1997; Saint Paul, 1996) but for a different reason – to compensate for lost shared training investments. Here we find another reason: Firms may prefer workers with a higher quit rate because the option to hire them is worth less with stringent employment protection – it takes a higher level of productivity for a firm to give a loyal worker the job security that comes along with employment protection.

Our model also has implications for worker composition at the industry and occupational level. Women, displaced older men approaching retirement, and young workers, should be disproportionately represented in industries characterised by uncertain future prospects and relatively low hiring and training costs. These typically involve low skill/low pay jobs or what has been called secondary-sector jobs in the theory of dual labour markets (see Saint-Paul, 1996) among others. More stable industries, and those characterised by high hiring- and training costs, on the other hand, will have a more permanent workforce of loyal – presumably prime-aged – workers. In Table 1, we summarise some of the testable implications of our theory. These predictions might be tested against cross-country data (in order to get variations in employment protection costs) disaggregated by sector, and with measures of sector-specific demand volatility.

(1992) for theoretical rationales for advance notification in a different context.

Table 1: Testable Predictions

Variable	Highly volatile occupations, low ratio of hiring to firing costs [1]	High ratio of hiring to firing costs, both high volatility and low volatility [2]
Employment composition	Relatively more youth, women, people near retirement age than in [2].	Proportionately more prime-aged men than in [1].
Employment inflows and outflows	Very careful vetting of hirees: Relatively more youth, women, people near retirement age hired. Proportionately more voluntary outflows than in [2]	Very careful vetting of hirees - few quitters are hired, so inflows are proportionately more prime-aged men. Voluntary outflows lower than in [1]. Women, men approaching retirement and young disproportionately casualties of layoffs.
Duration of Unemployment	Longer for the prime-aged men.	Longer for women, youth and older people cet. par.
Average job tenure	High for prime-aged men, lower for women, youth and older workers.	Workforce comprises proportionately more 'loyal' workers who have longer tenure, so average job tenure here longer than [1].
Average unemployment	Higher for prime-aged men.	Lower for prime-aged men.

VI. Conclusions

We have described the determinants of the strength of the adverse effect of quitting on hiring and training of new workers when there is uncertainty about the level of future productivity. We have found that the effect is stronger, the larger is the ratio of hiring to firing costs. While the dependence on the level of hiring costs comes as no surprise, the value added of our paper is the role of firing costs: A fall in firing costs enhances the quitting effect just as an increase in the level of hiring costs does. The quitting effect emphasised by authors from Pigou (1912) to Acemoglu and Pischke (1998) is thus endogenously determined and affected by labour-market institutions.

We also demonstrated that the firing decision is a mirror image of the hiring decision where hiring- and firing costs change places. So when productivity is not expected to recover, firms prefer laying off low-quit groups (i.e. prime-age workers), if the ratio of firing- to hiring costs is large. This may seem counterintuitive at first but the argument is identical to that of why high quits discourage training – by laying off a loyal worker the firm is reducing future losses by more since he would not have left on his own accord. But when hiring costs become more significant, this becomes a risky strategy as it becomes costly to replace workers if productivity recovers. Thus with high hiring costs and uncertainty about the future, firms may opt for laying off their more mobile workers.

References

- Acemoglu**, Daron and Jörn-Steffen **Pischke** (1998). “Why Do Firms Train? Theory and Evidence,” *Quarterly Journal of Economics*, February Vol. CXIII, No.1, pp.79-119.
- Addison**, John T and John B **Chilton** (1997). “Non-disclosure as a Contract Remedy: Explaining the Advance-Notice Puzzle,” *Journal of Labor Economics* 15 (1), January: 143-164.
- Becker**, Gary (1964), *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*, National Bureau of Economic Research, New York.
- Bentolila**, Samuel and Giuseppe **Bertola** (1990), “Firing Costs and Labor Demand: How Bad is Eurosclerosis,” *Review of Economic Studies*, 57, 381-402.
- Bentolila**, Samuel and **Saint-Paul**, Gilles (1994). “A Model of Labor Demand with Linear Adjustment Costs,” *Labour Economics*, 1: .
- Blau**, Francine D. and Lawrence M. **Kahn** (1981), “Race and Sex Differences in Quits by Young Workers,” *Industrial and Labor Relations Review*, Vol. 34, 563-77.
- Booth**, Alison L and Monojit **Chatterji** (1989), “Redundancy Payments and Firm-specific Training”, *Economica*, 56, 505-521.
- Booth**, Alison L (1996), “An Analysis of Firing Costs and their Implications for Unemployment Policy,” Chapter 12 in D.J. Snower and G. de la Dehesa *Unemployment Policy* (1996), Cambridge University Press.
- Booth**, Alison L. and Gylfi **Zoega** (1999), “Do Quits cause Under-training?” *Oxford Economic Papers*, 51, February.

Booth Alison L., Marco **Francesconi** and Carlos **Garcia-Serrano** (1999), "Job Tenure and Job Mobility in Britain", *Industrial and Labor Relations Review*, 53(1), October, pp. 43-70.

Calvo, Guillermo (1979), "Quasi-Walrasian Models of Unemployment." *American Economic Review*, 69. 102-107.

Chang, Chun and Yijiang **Wang** (1996). "Human Capital Investment under Asymmetric Information; The Pigovian Conjecture Revisited." *Journal of Labor Economics* 14 (3), July: 505-519.

Dixit, Avinash K. and Robert S. **Pindyck** (1994), *Investment under Uncertainty*, Princeton: Princeton University Press.

European Commission (1999) 'Employment in Europe 1998', *Employment & social affairs*, Employment and European Social Fund, Luxembourg: Office for the Official Publications of the European Communities.

Jaeger David A and Ann **Huff Stevens** (1999) "Is Job Stability in the United States Falling? Reconciling Trends in the CPS and PSID", *Journal of Labor Economics* 17(4), October, S1-S28.

Kuhn, Peter (1992). "Mandatory Notice." *Journal of Labor Economics* 14 (3), April: 117-137.

Hashimoto, Masanori (1981). "Firm Specific Human Capital as a Shared Investment." *American Economic Review*, 71(3), June :475-482.

Grubb David and William **Wells** (1993). "Employment Regulations and Patterns of Work in EC countries", *OECD Economic Studies*, 21, 7-58.

Layard, Richard, S.J. **Nickell** and R **Jackman** (1991). *Unemployment*. Oxford: Oxford University Press.

Lazear, Edward P (1990) "Job Security Provisions and Employment." *The Quarterly Journal of Economics*, 699-726.

Lipsey, R. G. and **Kelvin Lancaster** (1956), "The General Theory of Second Best." *Review of Economic Studies*, December.

Malliari, A. G. and William A. **Brock** (1982), *Stochastic Methods in Economics and Finance*. New York: North-Holland.

Martin, Christopher (1999), "The Impact of Labor Turnover: Evidence from UK Microdata," mimeo, Brunel University, Middlesex.

Meitzen, Mark E. (1986), "Differences in Male and Female Job-quitting Behavior," *Journal of Labor Economics*, Vol.4, No.2, 151-67.

- Neumark** David, Daniel **Polsky** and Daniel **Hansen** “Has Job Stability Declined yet? New Evidence for the 1990s”, *Journal of Labor Economics* 17(4), October, S29-S64.
- Paxson**, Christina H, and Nachum **Sicherman** (1996). “The Dynamics of Dual Job Holding and Job Mobility.” *Journal of Labor Economics* 14 (3), July: 357-393.
- Phelps**, Edmund S. (1994), *Structural Slumps*, Harvard University Press.
- Pigou**, Arthur C. (1912), *Wealth and Welfare*, London: Macmillan.
- Royalty**, Anne Beeson (1998) “Job-to-Job and Job-to-Nonemployment Turnover by Gender and Education Level.” *Journal of Labor Economics* 16 (2), April: 392-443.
- Saint-Paul**, Gilles (1996), *Dual Labor Markets: A Macroeconomic Perspective*, The MIT Press, Cambridge, Mass.
- Solow**, Robert E. (1979), “Another Source of Wage Stickiness.” *Journal of Macroeconomics*, 1, 79-82.
- Shapiro**, R and Joseph Stiglitz (1984), “Equilibrium Unemployment as a Worker Discipline Device.” *American Economic Review*, 74, 433-444.
- Shimer**, Robert (1999), “Why is the U.S. Unemployment Rate So Much Lower?”, NBER Macroeconomics Annual, 11-61.
- Stevens**, Margaret (1994), “A Theoretical Model of On-the-job Training with Imperfect Competition,” *Oxford Economic Papers*, 46, 537-562.
- Stevens**, Margaret (1996), “Transferable Training and Poaching Externalities,” in A. L. Booth and D. Snower (eds.), *Acquiring Skills*, Cambridge University Press.
- Viscusi**, Kip. (1980), “Sex Differences in Worker Quitting,” *Review of Economics and Statistics*, Vol. 62, No. 3, 388-98.

Appendix

DERIVATION OF EQUATION (5)

The problem is to solve equation (3) for $V(N, g)$, which is the value of employing all current workers. The solution for $V(N, g)$ consists of a particular integral and a complementary function. The particular integral of this equation can be written as,

$$V^P(N, g) = (1-l)^\theta \int_0^\infty \left[g_t (Ne^{-\lambda t})^\theta - w(Ne^{-\lambda t}) \right] e^{-\rho t} dt. \quad (A.1)$$

Eq. (2) in the text is an explicitly solvable stochastic differential equation known as the stochastic Verhulst equation. [see Kloeden and Platen (1992), p125] The value of g at time t is

$$g_t = \frac{g_0 \exp[(\mu \bar{g} - \sigma^2 / 2)t + \sigma z]}{1 + \mu g_0 \int_0^t \exp[(\mu \bar{g} - \sigma^2 / 2)s + \sigma z] ds}. \quad (A2)$$

(A1) can be reduced to the following equation when σ approaches zero,

$$g_t = \frac{\bar{g}}{1 + [(\bar{g} - g_0)/g_0] \cdot e^{-\bar{g}\mu t}}. \quad (A3)$$

Substituting (A3) into (A1) and differentiating with respect to N give (5) in the text.

$$v^P(N, g) = \theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{e^{-(\theta\lambda + \rho)t}}{1 + \left(\frac{\bar{g} - g}{g} \right) e^{-\bar{g}\mu t}} dt - \frac{w^*}{\rho + \lambda}. \quad (A4)$$

Numerical calculations show that (A3) is a good proxy for (A2) when σ is small. Rewrite (2) as a discrete stochastic differential equation

$$g_{t+\Delta t} - g_t = \mu(\bar{g} - g_t)g_t \Delta t + \sigma \sqrt{\Delta t} g_t \varepsilon_t, \quad \varepsilon_t \sim N(0,1). \quad (A5)$$

The integral part of (A1) is

$$N_0^\theta \int_0^\infty g_t \exp[-(\rho + \theta\lambda)t] dt \equiv N_0^\theta \sum_{n=0}^{\max} g_{n\Delta t} \exp[-(\rho + \theta\lambda)n\Delta t] \Delta t. \quad (A6)$$

The results of Monte Carlo numerical calculations of (A6) are shown in Fig A-1. The error term is the percentage difference when g_t follows (A5) or (A3).

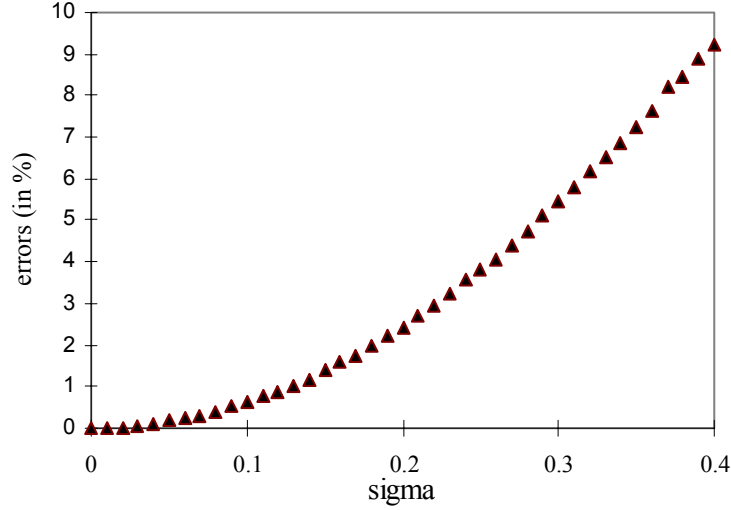


Figure A-1. Parameters: $\mu = 0.4$, $g_0 = 1.4$, $\bar{g} = 1$, $\Delta t = 0.01$, $\rho = 0.05$, $\theta = 1$, $\lambda = 0.15$, and total number of Monte Carlo calculations, max, is 100,000.

DERIVATION OF EQUATIONS (6)-(7)

Now only focusing on the homogenous part of equation (3), we define v^G as the value of the marginal worker and differentiate the homogenous part with respect to N . This gives,

$$(\rho + \lambda)v = \frac{1}{2}\sigma^2 g^2 v_{gg} + \mu(\bar{g} - g)gv_g - \lambda N v_N. \quad (A7)$$

Suppose the general solution to equation (A7) has the following functional form

$$v = A g^\beta N^\alpha h(g), \quad (A8)$$

where $A = B \cdot (1-l)^{\alpha\theta/(\theta-1)}$ and B is constant. Note that l should be a function of quits and thus will affect option values. However, since l is not a state variable, it would be convenient to treat $(1-l)$ as constant when deriving the option values. This gives the following relationships

$$v_N = A \alpha g^\beta N^{\alpha-1} h(g), \quad (A9)$$

$$\mu(\bar{g} - g)gv_g = [\beta\mu(\bar{g} - g)Ag^\beta h(g) + \mu(\bar{g} - g)gAg^\beta h'(g)]N^\alpha, \quad (A10)$$

$$\frac{1}{2}\sigma^2 g^2 v_{gg} = \left[\frac{1}{2}\sigma^2 g^2 \beta(\beta-1)Ag^\beta h(g) + \sigma^2 g \beta Ag^\beta h'(g) + \frac{1}{2}\sigma^2 g^2 Ag^\beta h''(g) \right] N^\alpha. \quad (A11)$$

Substituting (A9), (A10) and (A11) into (A7) gives

$$g^\beta N^\alpha h(g) \left[\frac{1}{2}\sigma^2 \beta(\beta-1) + \mu\bar{g}\beta - (\alpha\lambda + \rho + \lambda) \right] + g^{\beta+1} N^\alpha \left[\frac{1}{2}\sigma^2 gh''(g) + (\sigma^2 \beta + \mu(\bar{g} - g))h'(g) - \beta\mu h(g) \right] = 0. \quad (A12)$$

Equation (A12) must hold for any value of v , so that bracketed terms in both the first and second lines of the equation must equal zero. First we choose β to set the bracketed terms in the first line of the equation equal to zero:

$$\frac{1}{2}\sigma^2\beta(\beta-1)+\mu\bar{g}\beta-(\alpha\lambda+\rho+\lambda)=0. \quad (\text{A13})$$

The value of α is arbitrary and needs to be determined by economic restrictions. In equilibrium, the integral of (4) becomes

$$v^P(N, g) = \theta \bar{g} N^{\theta-1} (1-l)^\theta \int_0^\infty \frac{e^{-(\theta\lambda+\rho)t}}{1+e^{-\bar{g}\mu t}} dt. \quad (\text{A14})$$

The option value $v(N, g)$ should have the same composite in N and g . That is, $v(N, g)$ should depend on just the single composite variable, $gN^{\theta-1}$. Therefore, α should equal $\beta(\theta-1)$. Substituting into (A13) gives

$$\frac{1}{2}\sigma^2\beta(\beta-1)+(\mu\bar{g}-(\theta-1)\lambda)\beta-(\rho+\lambda)=0. \quad (\text{A15})$$

From the second line of equation (A12), we get

$$\frac{1}{2}\sigma^2 gh''(g) + (\sigma^2\beta + \mu(\bar{g}-g))h'(g) - \beta\mu h(g) = 0. \quad (\text{A16})$$

By making the substitution $x = 2\mu g/\sigma^2$, we can transform equation (A16) into a standard form. Let $h(g) = f(x)$. Then (A13) becomes

$$xf''(x) + (b-x)f'(x) - \beta f(x) = 0, \quad (\text{A17})$$

where $b = 2\beta + 2\mu\bar{g}/\sigma^2$. Equation (A17) is known as Kummer's equation. Its solution is the confluent hypergeometric function $H(x; \beta, b)$, which has the following series representation:

$$H(x; \beta, b) = 1 + \frac{\beta}{b}x + \frac{\beta(\beta+1)}{b(b+1)}\frac{x^2}{2!} + \frac{\beta(\beta+1)(\beta+2)}{b(b+1)(b+2)}\frac{x^3}{3!} + \dots \quad (\text{A18})$$

Thus, (A8) becomes

$$v^G(N, g) = A_1 (gN^{\theta-1})^{\beta_1} H(g; \beta_1) + A_2 (gN^{\theta-1})^{\beta_2} H(g; \beta_2), \quad (\text{A19})$$

where $H(g; \beta_1) \equiv H\left(\frac{2\mu}{\sigma^2}g; \beta_1, b_1\right)$ and $H(g; \beta_2) \equiv H\left(\frac{2\mu}{\sigma^2}g; \beta_2, b_2\right)$.

$$v^G(N, g) = A_1 (gN^{\theta-1})^{\beta_1} H(g; \beta_1) + A_2 (gN^{\theta-1})^{\beta_2} H(g; \beta_2),$$

As long as l is not a function of g and N , we can substitute $A = B \cdot (1-l)^{\alpha\theta/(\theta-1)} = B \cdot (1-l)^{\beta\theta}$ into the above equation, which gives

$$v^G(N, g) = B_1 (gN^{\theta-1}(1-l)^\theta)^{\beta_1} H(g; \beta_1) + B_2 (gN^{\theta-1}(1-l)^\theta)^{\beta_2} H(g; \beta_2), \quad (\text{A20})$$

where $H(g; \beta_1) \equiv H\left(\frac{2\mu}{\sigma^2}g; \beta_1, b_1\right)$ and $H(g; \beta_2) \equiv H\left(\frac{2\mu}{\sigma^2}g; \beta_2, b_2\right)$.