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Publication date:
2010

Citation for published version (APA):
Kwiatkowski, A. (2010). Non-cooperative incentives to share knowledge in competitive environments. (Dundee Discussion Papers in Economics; No. 243). University of Dundee.

Dundee Discussion Papers in Economics

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Working Paper
No. 243
October 2010
ISSN:1473-236X

Non-cooperative incentives to share knowledge in competitive environments*

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Abstract

In this paper we study a model where non-cooperative agents may exchange knowledge in a competitive environment. As a potential factor that could induce the knowledge disclosure between humans we consider the timing of the moves of players. We develop a simple model of a multistage game in which there are only three players and competition takes place only within two stages. Players can share their private knowledge with their opponents and the knowledge is modelled as influencing their marginal cost of effort. We identify two main mechanisms that work towards knowledge disclosure. One of them is that before the actual competition starts, the stronger player of the first stage of a game may have desire to share his knowledge with the "observer", because this reduces the valuation of the prize of the weaker player of that stage and as a result his effort level and probability of winning in a fight. Another mechanism is that the "observer" may have sometimes desire to share knowledge with the weaker player of the first stage, because in this way, by increasing his probability of winning in that stage, he decreases the probability of winning of the stronger player. As a result, in the second stage the "observer" may have greater chances to meet the weaker player rather than the stronger one.

Keywords: knowledge sharing, strategic knowledge disclosure, multistage contest game, non-cooperative games

JEL classification: C72, D83

*I am deeply indebted to Carmen Beviá, Matthias Dahm, Bernardo Moreno and Enriqueta Aragones for their valuable comments and suggestions. I also want to thank Generalitat de Catalunya for financial support.

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1 Introduction

The problem of transmission of knowledge (or know-how)[1] between humans involved in some kind of interaction is a point of concern of many economists. This is an important issue for instance in personnel economics theory. The problem is of particular interest when the interaction admits a form of competition. This is so because knowledge sharing in competitive situations is very difficult, as it can be interpreted as a type of cooperative behavior, which is in opposition to competition. The conventional wisdom suggests that in such environments the disclosure enhances the likely performance of opponents in the actual contest, and thereby reduces one's own chances of winning it. As a result, non-cooperatively behaving humans acting as the agents of competition do not want to reveal any knowledge. The economic literature hasn't provided so far examples of mechanisms that could create any incentives to disclose knowledge in such situations. Therefore, in this paper we focus on the knowledge-sharing behavior of people, who act as agents of competitive interaction. Our main objective is to bridge the existing gap in the economic theory on knowledge sharing between humans. We want to shed some more light on this particular case of competitive interactions and address the question about possible situations where knowledge exchange between non-cooperatively behaving humans may emerge as an equilibrium solution.

Industrial-organization theory predicts that there are some circumstances in which competing agents have incentives to share their private knowledge with rivals[2]. The disclosure of know-how emerges there either by imposing cooperative behavior of firms in the stage preceding the actual competition where knowledge sharing decision is taken or, if

[1] In this paper knowledge is understood as facts or ideas of different quality acquired by study, investigation, observation, or experience, which are used by a player to perform efficiently, that is at lower levels of cost of effort. It can be therefore interpreted as know-how that a player possesses and potentially can exchange with other players.

[2] There knowledge (know-how) sharing between competitors takes the form of sharing of technology and the actual competition is to discover an innovation first, or in the market of a product. For instance, the know-how disclosure may be observed if we impose cooperative behavior of firms in the stage preceding the actual competition where knowledge sharing decision is taken. It may also emerge as a result of the increase in the present value of future profits of a firm that lost in a patent race. This increase happens because the firm may be able to benefit from the invention by imitating it, from using it after the patent expires and the invention still has a considerable commercial value, or from using it to obtain its own invention in a related field. The disclosure of know-how may be also observed when firms operate in different but complementary industries and in models that deal with licensing of the disclosed knowledge, or in models where R&D costs are already sunk.

cooperative behavior is not assumed, when sharing of knowledge with a rival has a potential positive effect for one's payoff. Unfortunately, the mechanisms that lead to knowledge sharing in this part of the economic literature are very often the result of some peculiar characteristics of industrial organization that simply cannot be applied in the interactions between human beings. In this paper we study a model where agents may exchange knowledge without resorting to any of the explanations quoted above: in our model agents always behave non-cooperatively, information is complete and there are no mechanisms that change the value of the prize for the winner. As a potential factor that could induce the disclosure of knowledge between humans we consider the timing of the moves of players. To concentrate only on incentives to share knowledge, we develop a simple model of a multistage game in which there are only three players and competition takes place only within two stages. In the first stage, two of three agents compete against each other and the winner goes to the second stage of the game. Here he competes with the third agent, who in the previous stage was not active (the "observer")[3]. The winner of the second stage gets the prize. Additionally, before the whole game starts, all agents decide to pass or not some of their private knowledge to each of their opponents, which affects their cost functions that they use in subsequent stages. Such a decision is taken again at the end of the first stage of a game by a survivor of that stage and the agent who was not active in that stage.

An interesting feature of our model is that players can share their private knowledge with their opponents. To win a contest a player exerts effort and the knowledge is modelled as influencing its marginal cost. This is done in such a way that more knowledge is associated with lower levels of this cost. Decision to pass knowledge to opponents is done twice: before the actual competition in the first stage of a game and at the end of that stage, when its winner is already determined. Players while making this decision need to consider all possible effects that may follow. The knowledge transfer doesn't generate any directs costs, but, as our analysis shows, there are other, strategic consequences of this

[3] Therefore our model reflects such competitive situations in which one of the game participants can be an "observer" of competition of his potential rivals. This may happen for instance if he is perceived to be "the leader" of another, parallel competition, which he wins with certainty or almost with certainty. It may be also someone, who is known to be a participant of the final stage before the game starts, because he is the winner of the parallel competition, which has already finished. Similar examples cover such cases in which the "observer" is somebody who goes directly to the final stage of a game by game rules, which happens in some sports. In some of them players with some level of maturity or skills, as assessed by their history, go to the finals without having to participate in semi-finals of a game.

decision, that may potentially lower the expected utility of a knowledge donor.

One of our main results is that knowledge sharing can occur even in purely non-cooperative environments. Namely, using our model we identified two main sources of the knowledge-disclosure incentives that may potentially appear in such cases. One such a mechanism is that before the actual competition starts, the stronger player of the first stage of a game may have desire to share his knowledge with the "observer", because this reduces the valuation of the prize of the weaker player of that stage. As a consequence, the effort exerted by the weaker player falls and so his probability of winning in a fight. We show that before the actual competition starts there is also another source of the knowledge-sharing incentives. Namely the "observer" may have sometimes desire to share knowledge with one of the two remaining players to enforce him in a fight. This happens when the difference in the marginal cost of effort between the weaker and the stronger player of the first stage of a game is high enough. In such a situation the "observer" may gain in expected terms by sharing knowledge to some extent with the weaker player of the first stage, because in this way, by increasing his probability of winning in that stage, he decreases the probability of winning of the stronger player. Through this mechanism, in the second stage the "observer" may have greater chances to meet the weaker player rather than the stronger one.

The lines of research that can be identified as related to our paper cover many areas of the economic theory. This paper is related to some works in the industrial organization literature, which concentrate on incentives that competing firms have to share their strategic knowledge on technology. In this area of research Poyago-Theotoky (1999) shows that cooperating (non-cooperating) firms choose maximal (minimal) disclosure levels (spillovers). De Fraja (1993) shows that if for whatever reason, a rival's success increases a firm's own profit, the latter could, by disclosing some of its knowledge, reduce the overall expected cost of the patent race, while reducing also the expected benefit. Katsoulacos and Ulph (1998) show that firms operating in different but complementary industries may choose to maximally reveal knowledge, even in the absence of cooperation. Bhattacharya, Glazer and Sappington (1990) derive conditions under which full sharing of knowledge can be motivated in R&D joint ventures. Atallah (2003) studies R&D joint ventures in terms of information sharing and the stability of cooperation. d'Aspremont, Bhattacharya and Gérard-Varet (2000) consider the problem of bargaining over the disclosure of interim research knowledge between two participants in an R&D race for an ultimate, patentable invention. Kovenocky, Morathz and Münster (2009) study incentives to share private information ahead of contests, such as markets with promotional competition, procurement contests, or R&D, when firms have independent values or common values of winning a contest. The issue of know-how sharing appears also in models that deal with licensing of the disclosed knowledge, as in Bhattacharya, Glazer and Sappington (1992), or in models where R&D costs are already sunk, as in Kultti and Takalo (1998). Our paper is also related to ones in other areas of economics, where the interaction between the agent who reveals the knowledge and others doesn't necessarily take the form of competition, but the issue of knowledge sharing is vital. This is so, for instance, in the literature on the principal-agent relationship, where a common problem is how much of knowledge the principal should pass to his employees. Revealing the knowledge makes an agent more productive, but also can cause the agent to become self-employed. This issue is investigated for instance in Kräkel (2002), Kräkel (2005), Pakes and Nitzan (1983), and in Barcena-Ruiz and Rubio (2000). Our model can be interpreted as a very simple two-stage version of an elimination contest. Therefore our work is also related to papers on such a type of competition. A paper that is the most closely related to ours in this field of the economic research is Amegashie and Runkel (2007), who study a two-stage elimination contest with sabotaging. Sabotaging appears there in the form of help to a weaker player to decrease winning chances of a stronger player (indirect sabotage). This is very similar to what we observe in our model, where any potential knowledge sharing can be interpreted as a form of such sabotaging help. However, there exist fundamental difference between the work of Amegashie and Runkel (2007) and ours. In their paper the relative performance of a player who received help is enhanced only in the stage of a game in which the sabotaging activity is performed. In our work, however, the effect of knowledge sharing (help) enhances the relative performance of a help receiver throughout the whole game in any potential fight and at any of its stages. Other works, in which elimination contests are studied, are for instance Amegashie (2004), Zhang (2008), Zhang and Wang (2009).

Although our model could find application in many particular situations, it is motivated mainly by competition between humans, such as in sports, in a workplace or even in some kind of TV shows (games). In many such cases we can view the competition as a multistage process in which only a winner of a stage proceeds with the game, and the loser drops out. As an example consider sports, where rivals are initially divided into groups within which they compete, and the group winners go to the next stage where they compete again. In this example knowledge sharing could potentially appear between players in different groups, such that a player in one group by disclosing some of his knowledge helps to a weaker player in other group to increase his chances to win and eliminate a stronger player(s). Note that this disclosed knowledge may be used by the weaker player not only to win the stage of a game just after the knowledge is disclosed to him but also during later stages of the competition. As a multistage game we can also view competition in a workplace, where some group of workers, for instance in one department, compete and the best among them is promoted to a post at a higher level. At this new level he competes again with another worker or workers. Here again, knowledge sharing could potentially appear between workers in different departments, such that a worker in one of them by disclosing some of his knowledge may help to a weaker member in other department to increase his chances to be promoted and eliminate in this way other, stronger rival(s). As in the previous example, the knowledge acquired by that weaker worker may be used by him just after it is disclosed but also later in subsequent stages of the competition.

This paper is organized as follows: In Section 2 we introduce our model. In Section 3 we perform the analysis of different incentives to share knowledge between competitors. Section 4 concludes. For the clarity of presentation all our proofs are delegated to Appendix.

2 The model

Let $N = \{1, 2, 3\}$ denote the set of risk-neutral individuals who compete against each other in a contest game. The contest is organized in the following way: It consists of three stages $S = \{0, 1, 2\}$. In stage 1 two of three agents compete against each other and the winner goes to the second part of the game – stage 2. Here he competes with the third agent, who in stage 1 was not active. The winner of stage 2 gets the prize. Before the whole game starts, in stage 0 all agents decide to pass or not some of their private knowledge to each of their opponents. Such a decision is taken again at the end of stage 1 by a survivor of stage 1 and the agent who was passive in that stage. Without loss of generality we assume that agents 1 and 2 participate in stage 1 of the game. The winner of this part goes to stage 2 and competes here with agent 3. Let N_s denote the subset of N of individuals who compete against each other in a stage s of the contest game. It follows that in each stage, 1 and 2, only two contestants $i, j \in N_s, i \neq j$ compete against each other, with $N_1 = \{1, 2\}$, and $N_2 = \{1, 3\}$ or $N_2 = \{2, 3\}$ depending on the result of the competition in stage 1. As all agents participate in stage 0, $N_0 = N = \{1, 2, 3\}$. To win the contest, in a stage $s = 1, 2$ a contestant $i \in N_s$ exerts an effort level $e_{is} \in R^+$, while his opponent – a contestant j

– exerts an effort level $e_{js} \in R^{+}$. It is assumed that all contestants have the same positive valuation V for the contested prize. The contestants differ in the respective "cost function" that captures the disutility of exerting effort e_{is}. It is assumed that for all $i \in N$ and for all $s = 1, 2$ this cost function is linear in e_{is} and multiplicative in α_{is}, such that:

$$c_{is}\left(e_{is}\right) = \alpha_{is} e_{is}, \tag{1}$$

where $\alpha_{is} > 0$ is a marginal cost parameter of an agent i in a stage s. By assumption, marginal cost of effort is finite and constant, and the knowledge level is related to it in such a way that a player with a higher knowledge level has lower marginal cost of effort. Formally, given a marginal cost of effort α_{is}, the level of the knowledge of an agent i, is defined as

$$\kappa_{is} = \frac{1}{\alpha_{is}}. \tag{2}$$

At the beginning of a game an agent i has initial marginal cost of effort $\alpha_{i0} > 0$, and consequently the knowledge level

$$\kappa_{i0} = \frac{1}{\alpha_{i0}}. \tag{3}$$

The knowledge levels of players in a stage s can be ordered in the following sense: if players i and j have knowledge levels κ_{is} and κ_{js}, respectively, with $\kappa_{is} > \kappa_{js}$, only a player j can gain from an exchange of knowledge. At best, his knowledge level can be raised to κ_{is}. Any knowledge transmission always benefits a knowledge receiver and never harms in terms of his knowledge level. Although other information structures are conceivable[4], the ordering we adopt seems natural in the cases that we want to consider in our model. In stage 0 of a game all agents, and at the end of stage 1 – a survivor of stage 1 and the agent who was passive in that stage, decide about their levels of knowledge disclosure to an opponent. We assume that while making this knowledge-sharing decision agents behave in a non-cooperative way. Let δ_{ij}^{s} be a parameter which reflects how much knowledge is disclosed by an agent i to an agent j in a stage s. Considering the way in which knowledge is transmitted in our model

$$\begin{cases} \delta_{ij}^{s} \in [0, \kappa_{is} - \kappa_{js}], \text{if } \kappa_{is} > \kappa_{js}, \\ \qquad \delta_{ij}^{s} = 0, \qquad\qquad \text{otherwise.} \end{cases} \tag{4}$$

[4] For instance that pieces of knowledge of different players are treated as complementary goods. In such a case, both players could benefit from exchange of knowledge and their knowledge levels could be potentially raised even to $\kappa_{i0}+ \kappa_{j0}$ for both of them.

Given δ_{ij}^{s} for all $i \in N, i \neq j$, in a stage $s+1$ the knowledge level of a player j becomes

$$\kappa_{j(s+1)} = \kappa_{js} + \max_{i \in N, i \neq j} \left\{ \delta_{ij}^{s} \right\}, \tag{5}$$

where the "max" operator reflects the fact that his different opponents may decide to pass him different amounts of their knowledge. In such a case we assume that the knowledge of a player j increases by the highest piece of knowledge that his opponents pass to him, $\max_{i \in N, i \neq j} \left\{ \delta_{ij}^{s} \right\}$. With the change in the knowledge level of a player j, his marginal cost of effort also changes accordingly and becomes:

$$\alpha_{j(s+1)} = \frac{1}{\kappa_{j(s+1)}}. \tag{6}$$

We assume that the contestants 1 and 2 are heterogenous in terms of their marginal cost parameter ex ante α_{i0} and are ordered, such that $\alpha_{10} < \alpha_{20}$, with normalization $\alpha_{10} = 1$. The marginal cost parameter ex ante of a contestant 3 is not restricted in this sense and may be below or above α_{10}. However, as we assumed earlier, it is always strictly positive.

The contestants perceive the outcome of the each stage of the contest game as probabilistic. However, they can influence the probability of winning by exerting effort, which means that the outcome depends on the vector of effort levels exerted by both individuals playing in a stage. In our model we will employ the following Contest Success Function (CSF) $p_i : R_+^2 \rightarrow [0, 1]$:

$$p_i(e_{is}, e_{js}) = \frac{e_{is}}{e_{is} + e_{js}}, \text{ for all } i \in N_s, s = 1, 2. \tag{7}$$

This function maps the vector of effort levels (e_{is}, e_{js}) into win probabilities for each contestant. It is a restricted version of a CSF axiomatized in Skaperdas (1996)[5]. If no contestant exerts positive effort it is assumed

[5] In Skaperdas (1996) a CSF has an exponential form $p_i(e_i, e_j) = \frac{\alpha e_i^r}{\alpha e_i^r + \alpha e_j^r} = \frac{e_i^r}{e_i^r + e_j^r}$, for all $i \in N$, with $\alpha > 0$ and $r > 0$. The parameter r measures the sensitivity of the outcome of the contest game with respect to differences in effort. To simplify our analysis and to focus only on the effects of knowledge exchange it is assumed that the CSF is linear with $r = 1$. Also with a general parameter $r > 0$ the existence of pure strategy equilibria cannot be guaranteed (see Baye, Kovenock and de Vries (1994) for details). With the restriction $r = 1$ all our equilibria are in pure strategies.

Apart from the exponential CSF, Skaperdas (1996) axiomatized also the logit CSF: $p_i(e_i, e_j) = \frac{e^{ke_i}}{e^{ke_i} + e^{ke_j}}$, for all $i \in N$, with $k > 0$. The parameter k, similar to the parameter r, measures the sensitivity of the outcome of the contest game with respect to differences in effort.

that none of the individuals receives the prize, i.e. $p_i(0,0) = 0$ for all $i \in N$[6].

In each stage $s = 1, 2$ a contestant $i \in N_s$ aims at maximizing his expected utility, which given the cost function (1) and the contest mechanism (7) takes the following (additive separable) form:

$$u_i\left(e_{is}, e_{js}\right) = p_i(e_{is}, e_{js})\Pi_{is} - c_{is}\left(e_{is}\right), \tag{8}$$

where

$$\Pi_{is} = \begin{cases} u_i\left(e_{i2}, e_{32}\right), \text{ if } s = 1, \text{ for all } i \in N_1, \\ V, \text{ if } s = 2 \text{ for all } i \in N_2 \end{cases}, \tag{9}$$

with (e_{i2}, e_{32}) denoting the effort levels of a player $i \in N_1$ and of player 3[7] in stage 2, and

$$\begin{aligned} u_i\left(e_{i2}, e_{32}\right) &= p_i(e_{i2}, e_{32})\Pi_{i2} - c_i\left(e_{i2}\right) \\ &= p_i(e_{i2}, e_{32})V - c_i\left(e_{i2}\right) \end{aligned}$$

denoting the expected payoff of a player i in that stage. The definition of Π_{is} reflects the fact that the payoffs in stage 2 are the players' valuations in stage 1.

It is assumed that in every stage of a game – 0, 1 and 2 – players make all their decisions simultaneously and behave in a non-cooperative way. Our game is formulated under complete information and the equilibrium concept that we use is Subgame Perfect Equilibrium. Players choose their strategies: effort levels and the knowledge disclose parameters δ_{ij}^s that maximize their expected utilities.

3 Analysis

We start by solving our model by backward induction for effort levels of players in stages 1 and 2.

Note first that as $N_1 = \{1, 2\}$, and $N_2 = \{1, 3\}$ or $N_2 = \{2, 3\}$ in either stage we have only two agents i and j, $i \neq j$ who compete against each other. So, first we solve our model in a general case, with agents i and j, $i, j \in N_s, i \neq j$ being the competitors. Plugging the CSF and the cost function as specified in equations (7) and (1) into the expected utility function of an agent i in eq. (8) and differentiating, produces the following first order condition:

$$\frac{e_{js}}{\left(e_{is} + e_{js}\right)^2}\Pi_{is} - \alpha_{is} = 0, \text{ for all } i \in N_s,$$

[6] Another convention in the contest-game literature is that $p_i(0,0) = \frac{1}{2}$ for all $i \in N$. The choice of either definition is not important in terms of the results that we obtain in this paper

[7] Note that in the definition we consider the fact that in stage 2 agent 3 always participates in a game, having as his opponent either agent 1 or agent 2.

which after some algebra produce the (sub)equilibrium effort candidate for $i \in N_s$:

$$e_{is} = \frac{(\Pi_{is})^2 \Pi_{js}\alpha_{js}}{(\Pi_{js}\alpha_{is} + \Pi_{is}\alpha_{js})^2}.$$

This effort candidate is strictly positive, given our assumptions on the parameters. The second order conditions can be expressed as

$$\frac{\partial^2 u_i (e_{is}, e_{js})}{\partial e_{is}^2} = -\frac{2e_{js}}{(e_{is} + e_{js})^3}\Pi_{is} < 0,$$

which proves concavity[8]. Thus the maximum exists and is interior and unique.

So it follows from our analysis, that in each stage $s = 1, 2$ there exists a unique interior (sub)equilibrium, in which players exert effort at positive levels. Those equilibrium effort levels are

$$e_{is}^* = \frac{(\Pi_{is})^2 \Pi_{js}\alpha_{js}}{(\Pi_{js}\alpha_{is} + \Pi_{is}\alpha_{js})^2}, \text{ for all } i \in N_s. \tag{10}$$

Plugging this result into eq. (8) (together with the CSF and the cost function as specified in equations (7) and (1)) produces the (sub)equilibrium expected payoffs in a stage $s = 1, 2$, that may be written as

$$u_i \left(e_{is}^*, e_{js}^*\right) = \frac{(\Pi_{is})^3 \alpha_{js}^2}{(\Pi_{js}\alpha_{is} + \Pi_{is}\alpha_{js})^2}, \text{for all } i \in N_s. \tag{11}$$

In turn, plugging the equilibrium effort levels into eq. (7) yields the (sub)equilibrium probabilities of success in a stage $s = 1, 2$ that may be expressed as

$$p_i(e_{is}^*, e_{js}^*) = \frac{\Pi_{is}\alpha_{js}}{\Pi_{js}\alpha_{is} + \Pi_{is}\alpha_{js}}, \text{ for all } i \in N_s. \tag{12}$$

3.1 Stage 2

Let's concentrate on stage 2 of the game. Depending on the result of the competition in stage 1, in stage 2 agent 3 either competes with agent 1 or with agent 2, that is $N_2 = \{1, 3\}$ or $N_2 = \{2, 3\}$.

In stage 2 by eq. (9) $\Pi_{i2} = V$ for all $i \in N_2$. Plugging this into eq. (10) and considering the act that now $s = 2$ produces the (sub)equilibrium effort levels of players in stage 2:

$$e_{i2} = \frac{\alpha_{j2}}{(\alpha_{i2} + \alpha_{j2})^2}V, \text{ for all } i \in N_2; j \in N_2, i \neq j.$$

[8] Here, we implicitly assume that Π_{is} which is defined in eq. (9) is strictly positive. As $V > 0$ by assumption, this requires that $\Pi_{i,1} > 0$ for all $i \in N_1$. As our subsequent analysis shows this is always satisfied in equilibrium.

Plugging again $\Pi_{i2} = V$ for all $i \in N_2$ into eq. (11) with $s = 2$ yields the (sub)equilibrium expected payoffs of players in stage 2, that may be expressed as

$$u_i\left(e_{i2}^*, e_{j2}^*\right) = \frac{\alpha_{j2}^2}{\left(\alpha_{i2} + \alpha_{j2}\right)^2} V, \text{ for all } i \in N_2; j \in N_2, i \neq j. \tag{13}$$

In this section we will be interested in effects of changes in values of the marginal cost parameters. To consider this fact we may rewrite eq. (13) as a function of the parameters α_{i2}:

$$u_i^*\left(\alpha_{i2}, \alpha_{j2}\right) = u_i\left(e_{i2}^*, e_{j2}^*\right), \text{ for all } i \in N_2, j \in N_2, i \neq j. \tag{14}$$

Recall that by definition of Π_{is} given in eq. (9) $\Pi_{i,1} = u_i\left(e_{i2}, e_{32}\right)$ for all $i \in N_1$, which using last expression implies that

$$\Pi_{i1} = \frac{\alpha_{32}^2}{\left(\alpha_{i2} + \alpha_{32}\right)^2} V, \text{ for all } i \in N_1, \tag{15}$$

which is always is strictly positive for all $i \in N_1$, given our assumptions on the parameters of the model.

Using again the fact that $\Pi_{i2} = V$ for all $i \in N_2$, and eq. (12), we may write the (sub)equilibrium probabilities of success of players in stage 2 as

$$p_i(e_{i2}^*, e_{j2}^*) = \frac{\alpha_{j2}}{\alpha_{i2} + \alpha_{j2}}, \text{ for all } i \in N_2; j \in N_2, i \neq j. \tag{16}$$

At the end of stage 1 – a survivor of stage 1 and the agent who was passive in that stage, decide about their levels of knowledge disclosure to an opponent. Our objective now is to analyze different incentives that govern this knowledge disclosure behavior of agents. To meet this objective, we need to study how the expected payoffs at the beginning of stage 2 react to changes in the knowledge levels of the players. However, by eq. (2) there exists one to one correspondence between the knowledge level and the marginal cost of effort of a player. Hence, to study knowledge disclosure and knowledge flows between the players it is enough to look at the changes in their levels of the marginal cost of effort and this is sufficient to derive any conclusions about their respective knowledge changes. We will use this fact to simplify our analysis.

We analyze the incentives of players of stage 2 of a game to disclose knowledge at the end of stage 1 in Proposition 1:

Proposition 1 *If all participants of stage* 2 *of a game have already been determined, then there exists no SPE in which they have incentives to share information between themselves, that is*

$$\delta_{ij}^{1*} = \delta_{ji}^{1*} = 0 \textit{ for all } i, j \in N_2, i \neq j.$$

It follows from Proposition 1 that when a set of participants of stage 2 of a game has already been determined, then those participants do not want to exchange their knowledge between themselves. The intuition behind this result is the following: Suppose that an agent i and an agent j are those who are to fight in stage 2 of a game and that the knowledge level of an agent i is not lower that the level of an agent j. In such a case an agent j is not capable of passing his knowledge to an agent i, so he doesn't do it, but an agent i has such possibility and may disclose some of his knowledge to an agent j. However, making an agent j stronger by disclosing some knowledge to him just before the fight in stage 2 only reduces the expected utility of an agent i. This is as a result of the increase in the effort level of an agent i and of lower probability of his success. This is not desired by an agent i, therefore he discloses no knowledge to an agent j.

This means that the knowledge levels determined by the players in stage 0 of a game do not change later – at the end of stage 1, and remain constant until the end of a game:

Conclusion 1 *The levels of the marginal cost of effort of players remain constant throughout stages 1 and 2 of a game, that is*

$$\alpha_{i2} = \alpha_{i1} \text{ for all } i \in N_2.$$

Note that Conclusion 1 allows us to express all our results for stage 1 and/or 2 of a game in terms of marginal cost parameters α_{i1}, $i \in N$ only. We will use this fact in our subsequent analysis and solve our model in terms of those marginal cost parameters only.

3.2 Stage 1

Let's concentrate on stage 1 of the game. In this stage agent 1 always competes with agent 2, that is $N_1 = \{1, 2\}$.

By eq. (9), the players' valuations in this stage, $\Pi_{i,1}$ for all $i \in N_1$, are their equilibrium expected payoffs in stage 2. Those are given in eq. (15). Plugging those valuations into eq. (10), eq. (11) and eq. (12) and considering Conclusion 1 produces for all $i \in N_1; j \in N_1, i \neq j$ respectively: the (sub)equilibrium effort levels of the players in stage 1:

$$e_{i1}^{*} = \frac{\alpha_{j1}\alpha_{31}^{2}\left(\alpha_{j1}+\alpha_{31}\right)^{2}}{\left(\alpha_{i1}\left(\alpha_{i1}+\alpha_{31}\right)^{2}+\alpha_{j1}\left(\alpha_{j1}+\alpha_{31}\right)^{2}\right)^{2}}V, \tag{17}$$

their (sub)equilibrium expected payoffs in stage 1:

$$u_{i}\left(e_{i1}^{*}, e_{j1}^{*}\right) = \frac{\left(\alpha_{j1}\alpha_{31}\right)^{2}\left(\alpha_{j1}+\alpha_{31}\right)^{4}}{\left(\alpha_{i1}+\alpha_{31}\right)^{2}\left(\alpha_{i1}\left(\alpha_{i1}+\alpha_{31}\right)^{2}+\alpha_{j1}\left(\alpha_{j1}+\alpha_{31}\right)^{2}\right)^{2}}V \tag{18}$$

and their (sub)equilibrium probabilities of success in stage 1:

$$p_i(e^*_{i1}, e^*_{j1}) = \frac{\alpha_{j1}\left(\alpha_{j1}+\alpha_{31}\right)^2}{\alpha_{i1}\left(\alpha_{i1}+\alpha_{31}\right)^2+\alpha_{j1}\left(\alpha_{j1}+\alpha_{31}\right)^2}. \tag{19}$$

3.3 Stage 0

Before the actual competition starts, in stage 0 all agents decide to pass or not some of their private knowledge to each of their opponents. In the following we are going to study different incentives that may lead to knowledge sharing between agents in that stage, assuming that they behave non-cooperatively. In this section we will be mainly interested in effects of changes in values of the marginal cost parameters. To consider this fact we may rewrite eq. (18) and (19) as functions of the parameters $\alpha_{i1}, i \in N$:

$$u^*_i\left(\alpha_{i1}, \alpha_{j1}, \alpha_{31}\right) \equiv u_i\left(e^*_{i1}, e^*_{j1}\right) \text{ for all } i \in N_1; j \in N_1, i \neq j, \tag{20}$$

and

$$p^*_i(\alpha_{i1}, \alpha_{j1}, \alpha_{31}) \equiv p_i(e^*_{i1}, e^*_{j1}), \text{ for all } i \in N_1; j \in N_1, i \neq j. \tag{21}$$

The last two equations express for an agent $i \in N_1$ the (sub)equilibrium expected payoff in a game at the beginning of stage 1 and the (sub)equilibrium probability of success in stage 1, given marginal cost parameters $\alpha_{i1}, i \in N$. Agent 3 by assumption doesn't participate actively in stage 1 of a game. Therefore his payoff at the beginning of this stage is given as the expected payoff of his payoffs in stage 2

$$u^*_3\left(\alpha_{11}, \alpha_{21}, \alpha_{31}\right) = p_1(e^*_{11}, e^*_{21})u_3\left(e^*_{32}, e^*_{12}\right) + p_2(e^*_{21}, e^*_{11})u_3\left(e^*_{32}, e^*_{22}\right),$$

which using equations: (13) and (19), and Conclusion 1 after some algebra yields

$$u^*_3\left(\boldsymbol{\alpha}\right) = \frac{\alpha_{11}\alpha_{21}\left(\alpha_{21}\left(\alpha_{11}+\alpha_{31}\right)^4+\alpha_{11}\left(\alpha_{21}+\alpha_{31}\right)^4\right)}{\varphi\left(\boldsymbol{\alpha}\right)}V, \tag{22}$$

where

$$\varphi\left(\boldsymbol{\alpha}\right) = \left(\alpha_{11}+\alpha_{31}\right)^2\left(\alpha_{21}+\alpha_{31}\right)^2\left(\alpha_{11}\left(\alpha_{11}+\alpha_{31}\right)^2+\alpha_{21}\left(\alpha_{21}+\alpha_{31}\right)^2\right)$$

and $\boldsymbol{\alpha} = (\alpha_{11}, \alpha_{21}, \alpha_{31})$.

Note that by eq. (6), eq. (5) and eq. (3), the value of α_{j1} depends on its initial value α_{j0}, which can be (only) reduced if any of agents i would decide to pass in stage 0 some of his knowledge to an agent j, that is when $\delta_{ij}^{0} > 0$ for some $i \in N, i \neq j$. Our ultimate objective is to analyze different incentives that govern such knowledge disclosure behavior of agents. To meet this objective, we need to study how the expected payoffs at the beginning of stage 1 react to changes in the knowledge levels of the players. As in the proof of Proposition 1, to simplify our analysis we will use the fact that there exists one to one correspondence between the knowledge level and the marginal cost of effort of a player. We will be studying how the expected payoffs at the beginning of stage 1 react to changes in the knowledge levels of the players by investigating the effects of the changes in their levels of the marginal cost of effort, rather than changes in their knowledge levels directly.

Our analysis of the knowledge sharing incentives in stage 0 of a game we begin by considering agents 1 and 2.

Proposition 2 *There exists no SPE in which in stage* 0 *of a game agent* 1 *and/or agent* 2 *share their knowledge between themselves, that is*

$$\delta_{12}^{0*} = \delta_{21}^{0*} = 0.$$

It follows from Proposition 2, that none of agents 1 and 2 has possibility or any incentives to pass any piece of his knowledge to his opponent in stage 0 of a game. Agent 2 simply can't do this, because his knowledge level is lower than the one of agent 1. Agent 1 in turn might do it, but sharing any knowledge with his opponent in this stage has only a detrimental effect – the reduction in his expected utility level. This is as a result of the decrease in his winning probability and of the increase in his effort level at the same time.

The result given in Proposition 2 has another important consequence. Namely, it implies that any knowledge flow towards agent 1 and/or agent 2 in stage 0 of a game may have its origin only in agent 3, and nor in agent 1 nor 2. We will use this observation in our subsequent analysis.

We are going to consider now sharing of knowledge in stage 0 of a game by agent 3 with agent 1. We begin this part of our analysis by stating the following lemma:

Lemma 1 *As long as* $\alpha_{11} < \alpha_{21}$*, there exists no SPE in which in stage* 0 *of a game agent* 3 *has incentives to share his knowledge with agent* 1*. That is in such a case*

$$\delta_{31}^{0*} = 0.$$

As we assumed, contestants 1 and 2 are ordered with respect to their ex ante marginal cost parameter α_{i0}, such that $\alpha_{10} < \alpha_{20}$. This fact together with the results given in Proposition 2 and Lemma 1 suggest that agent 1 would not receive in equilibrium in stage 0 of a game any piece of knowledge from any of his opponents – agent 2 and 3. This would imply that his equilibrium level of the marginal cost of effort in stage 1 is equal to its ex ante value, that is $\alpha_{11}^{*} = \alpha_{10} = 1$. However, for this to be really the case we need to check what happens in a potential equilibrium with the marginal cost parameter of agent 2. Still it might occur that in stage 0 of a game agent 3 has incentives to share his knowledge with agent 2, and moreover that his most preferred level of the marginal cost parameter of agent 2 is such that the condition $\alpha_{11} < \alpha_{21}$ wouldn't hold. In such a case Lemma 1 couldn't be applied. Specifically, we need to show now that the agent 3's most preferred level of the marginal cost parameter of agent 2 in stage 1, α'_{32}, is always greater than $\alpha_{11} = \alpha_{01} = 1$. This is done in Lemma 2:

Lemma 2 *If $\alpha_{11} = \alpha_{10} = 1$, then*

(i) there exists a level of the marginal cost parameter of agent 2 in stage 1, which is the most preferred by agent 3, α'_{32},

(ii) and this level is always greater than $\alpha_{10} = 1$.

Now, using Lemma 2 together with Proposition 2 and Lemma 1, we may state the following result about sharing knowledge by agent 3 with agent 1 in stage 0 of a game:

Proposition 3 *If $\alpha_{11} = \alpha_{10} = 1$, then there exists no SPE in which in stage 0 of a game agent 3 has incentives to share his knowledge with agent 1, that is*

$$\delta_{31}^{0*} = 0.$$

It follows from Proposition 3 that in stage 0 of a game agent 3 – the passive player in stage 1 of a game, never wants to make stronger agent 1 – the stronger active player of that stage. The intuition behind this result is the following: Agent 3 knows that his opponent in stage 2 of a game will be either agent 1 or agent 2 and that agent 1 is stronger than agent 2. Making agent 1 even stronger by disclosing knowledge to him has two effects for agent 3: first, it increases the winning probability of agent 1 in stage 1 and thus makes more probable that he will be an agent 3's competitor in stage 2, second it increases the winning probability of agent 1 in stage 2. Both effects are not desired by agent 3, therefore he discloses no knowledge to agent 1.

The result given in Proposition 3 together with the ones in Proposition 2 and Lemma 2 have an important consequence in term of the relation between the levels of a marginal cost parameter of agents 1 and 2 in stage 1 in a potential equilibrium:

Proposition 4 *If a knowledge sharing SPE exists, then always* $\alpha_{11}^* < \alpha_{21}^*$*, with* $\alpha_{11}^* = \alpha_{10} = 1$.

It follows that the relation ex ante between the levels of the marginal cost parameter of agents 1 and 2 – the active players of stage 1 of a game, will hold also in stage 1 in a potential equilibrium. Moreover, the level of the marginal cost parameter of agent 1 – the stronger active player of that stage, is always constant and doesn't change in equilibrium in stage 1.

Now, we are going to consider sharing of knowledge in stage 0 by agent 3 with agent 2. Note that by Lemma 2, there exists the level of the marginal cost parameter of agent 2 in stage 1, which is the most preferred by agent 3, α'_{32}. This implies that there exits also the corresponding most preferred knowledge level of agent 2, κ'_{32}. The fact that this knowledge level exists, however, doesn't automatically mean that in stage 0 there is always knowledge disclosure of agent 3 to agent 2. First, it may happen that this most preferred level is lower than the knowledge level ex ante of agent 2. Note that in our model, any knowledge transmission ($\delta_{ij}^s > 0$) is always beneficial to a knowledge receiver, as it increases his knowledge (and never reduces it). Therefore, in this case, in stage 0 of a game agent 3 would prefer not to disclose any piece of his knowledge to agent 2. Second, by eq. (4) there would be no knowledge transmission from agent 3 to agent 2, if the knowledge level ex ante of agent 3 is lower than the level of knowledge ex ante of agent 2. In other cases, there is the knowledge disclosure of agent 3 to agent 2. This occurs when the knowledge level ex ante of agent 3 and his most preferred level of the knowledge of agent 2 are both higher than the knowledge level ex ante of agent 2. This all is summarized in Proposition 5:

Proposition 5 *If there exists a knowledge sharing SPE, then in such an equilibrium in stage* 0 *of a game agent* 3 *may have incentives to share his knowledge with agent* 2 *and they are described by the following rule:*

$$\begin{cases} \delta_{32}^{0*} = 0, \mathit{if} (\kappa_{30} \leq \kappa_{20}) \cup (\kappa_{32}'^{*} \leq \kappa_{20}), \\ \delta_{32}^{0*} > 0, \qquad \qquad \mathit{otherwise.} \end{cases}$$

It follows from Proposition 5 that in a potential equilibrium agent 3 – the passive player of stage 1 of a game, may have sometimes incentives to share his knowledge in stage 0 of a game with agent 2 – the weaker active player of that stage. To understand the mechanism that leads to knowledge sharing in this case we need to consider possible effects that may play a role here. Note that passing a piece of knowledge in stage 0 by agent 3 to agent 2 improves the probability of winning of agent 2 in both stages, 1 and 2, of a game. In stage 2 of a game it tends to lower the expected payoff of agent 3, because he would have lower chances to win with agent 2 than before. However, the increase in probability of winning in stage 1 of a game doesn't necessarily have to be harmful for agent 3 in terms of his expected payoff. In this stage there are two potential rivals for agent 3, and of those two agent 3 prefers to compete in stage 2 with the weaker one – agent 2. In such a fight agent 3 has much higher chances to win and his expected payoff is higher as compared to a fight with agent 1 – the stronger player. So if agent 2 is very weak it may be beneficial for agent 3, by passing some knowledge to that player in stage 0 of a game, to increase his chances to win in stage 1. In this way agent 3 could have higher probability of competing in stage 2 with the weaker player than with the stronger one. As a result, the expected payoff of agent 3 could increase, because his chances to win with the weaker player are higher, as compared to the fight with the stronger player. Hence it may happen that as a result of passing knowledge in stage 0, the gains for agent 3 from increasing the winning probability of the weaker player in stage 1 of a game outweigh the losses from his higher probability of winning in stage 2. This creates incentives for agent 3 to share knowledge with agent 2.

Note that by Proposition 2 in stage 0 of a game agent 1 has no incentives to share knowledge with agent 2, which implies that the level of the marginal cost of effort of agent 2 can be affected only by agent 3, as defined in Proposition 5.

Now we are going to consider sharing of knowledge in stage 0 by agent 2 with agent 3.

Proposition 6 *There exists no SPE in which in stage* 0 *of a game agent* 2 *has incentives to share his knowledge with agent* 3*, that is*

$$\delta_{23}^{0*} = 0.$$

It follows from Proposition 6 that in stage 0 of a game agent 2 – the weaker player in stage 1 of a game, never wants to make stronger agent 3 – the passive player in that stage. The intuition behind this result is the following: Passing a piece of knowledge by agent 2 to agent 3 makes the latter one stronger than before, which results in lowering the valuations $\Pi_{i,1}$ in stage 1 of a game for both agents, 1 and 2 (see eq.(15)). This affects the performance of those players in stage 1 of a game, by reducing their effort levels. However, this detrimental effect for the performance of agent 2 in stage 1 of a game is much stronger than for agent 1, as a result of the difference in their marginal cost of effort. This effect is not desired by agent 2, therefore he discloses no knowledge to agent 3.

Now we are going to consider sharing of knowledge by agent 1 with agent 3. In the proof of the subsequent proposition we will show that there exists the level of the marginal cost parameter of agent 3 in stage 1, which is the most preferred by agent 1, α'_{13}. This implies that there exits also the corresponding most preferred knowledge level of agent 3, κ'_{13}. This fact, however, doesn't automatically mean that in stage 0 there is always knowledge disclosure of agent 1 to agent 3. We have already discussed a similar issue when we talked about the knowledge disclosure between agents 3 and 2. Applying the same reasoning as there, we define situations in which the knowledge disclosure occurs and when it doesn't. This all is summarized in Proposition 7:

Proposition 7 *If there exists a knowledge sharing SPE, then in such an equilibrium*

a) when $\kappa^*_{21} \geq \frac{1}{3}$, in stage 0 agent 1 has no incentives to share his knowledge with agent 3, that is

$$\delta^{0*}_{13} = 0.$$

b) when $\kappa^*_{21} < \frac{1}{3}$ agent 1 may have such incentives in stage 0 and they are described by the following rule:

$$\begin{cases} \delta^{0*}_{13} = 0, \, \mathit{if} (1 \leq \kappa_{30}) \cup (\kappa'^*_{13} \leq \kappa_{30}), \\ \delta^{0*}_{13} > 0, \qquad\qquad \mathit{otherwise}. \end{cases}$$

It follows from Proposition 7 that in a potential equilibrium agent 1 – the stronger active player of stage 1 of a game, may have sometimes incentives to share his knowledge with agent 3 – the passive player of that stage. To understand the mechanism that leads to knowledge sharing in this case we need to consider possible effects that may play a role here. Note that passing a piece of knowledge by agent 1 to agent 3 makes the latter one stronger than before, which results in lowering the valuations $\Pi_{i,1}$ in stage 1 of a game for both agents, 1 and 2 (see eq.(15)). This affects the performance of those players in stage 1 of a game, by reducing their effort levels. However, this detrimental effect for the performance of agent 2 in stage 1 of a game is much stronger than for agent 1, as a result of the difference in their marginal cost of effort. As Proposition 7 shows this difference must be big enough, as reflected by the restriction on the knowledge level of agent 2, which needs to be low enough in comparison to agent 1. As in our CSF relative and not absolute effort levels play a role in the determination of winning probabilities, as a consequence the probability of winning by agent 1 in stage 1 increases and by agent 2 falls. So it may happen that for agent 1 the gain in terms of the winning probability in stage 1 of a game outweighs the negative effect of the reduction in his valuation $\Pi_{1,1}$ in stage 1, which creates incentives for him to share knowledge with agent 3.

Note that by Proposition 6 agent 2 has no incentives to share knowledge with agent 3. It follows that the level of the marginal cost of effort of agent 3 can be affected only by agent 1, as defined in Proposition 7.

Now, we can summarize all our previous results in the form of the following proposition:

Proposition 8 *There exists a unique knowledge sharing SPE defined in the following way:*

1. *there is no knowledge disclosure in stage* 1 *of a game between the players in stage* 2*, if all participants of that stage have already been determined, that is*

$$\delta_{13}^{1*} = \delta_{31}^{1*} = \delta_{23}^{1*} = \delta_{32}^{1*} = 0,$$

2. *there is no knowledge disclosure in stage* 0 *of a game between the active players in stage* 1*, that is*

$$\delta_{12}^{0*} = \delta_{21}^{0*},$$

3. *there is no knowledge disclosure in stage* 0 *of a game from the weaker player in stage* 1 *to the passive player in that stage, that is*

$$\delta_{23}^{0*} = 0,$$

4. *there is no knowledge disclosure in stage* 0 *of a game from the passive player in stage* 1 *to the stronger player in that stage, that is*

$$\delta_{31}^{0*} = 0,$$

5. *when* $\kappa_{21}^{*} \geq \frac{1}{3}$*, then in stage* 0 *of a game there is no knowledge disclosure from the stronger player in stage* 1 *to the passive player in that stage, that is*

$$\delta_{13}^{0*} = 0,$$

6. *when* $\kappa_{21}^{*} < \frac{1}{3}$*, then in stage* 0 *of a game the knowledge disclosure may occur from the stronger player in stage* 1 *to the passive player in that stage, according to the following rule*

$$\begin{cases} \delta_{13}^{0*} = 0, if(1 \leq \kappa_{30}) \cup (\kappa_{13}'^{*} \leq \kappa_{30}), \\ \delta_{13}^{0*} > 0, \qquad\qquad otherwise, \end{cases}$$

7. *the knowledge disclosure may occur in stage* 0 *of a game from the passive player in stage* 1 *to the weaker player in that stage, according to the following rule*

$$\begin{cases} \delta_{32}^{0*} = 0, if(\kappa_{30} \leq \kappa_{20}) \cup (\kappa_{32}'^{*} \leq \kappa_{20}), \\ \delta_{32}^{0*} > 0, \qquad\qquad otherwise. \end{cases}$$

Proposition 8 is a summary of all our results obtained earlier. It shows that there exists a unique knowledge sharing SPE in which some agents have incentive to share their knowledge with their rivals. An important property of this equilibrium is that those incentives arise although agents behave non-cooperatively. Their common feature is that sharing of knowledge works here as an instrument to sabotage indirectly potential rivals. For instance, in stage 0 of a game agent 3 is willing to share knowledge with agent 2 – the weaker player in stage 1 of a game, because in this way he may try to eliminate from stage 2 agent 1 – the stronger rival. In a similar way, in stage 0 agent 1 is willing to share knowledge with agent 3, because in this way by reducing the relative performance of agent 2 can increase his own probability of winning in stage 1 of a game. A similar mechanism of sabotaging we can find in Amegashie and Runkel (2007). The authors study a two-stage elimination contest, where agents can perform sabotage activity only in the first stage. They show that indirect sabotage may appear in the form of help to a weaker player to decrease winning chances of a stronger player. This is very similar to what we observe in our model, where any potential knowledge sharing with one player, which can be interpreted as a form of helping him, acts as an instrument to eliminate another player. However, there exist fundamental difference between the work of Amegashie and Runkel (2007) and ours. In their paper the relative performance of a player who received help is enhanced only in the stage of a game in which the sabotaging activity is performed. In our work, however, the effect of knowledge sharing (help) enhances the relative performance of a help receiver throughout the whole game in any potential fight and at any of its stages.

3.4 Numerical example

To understand better different forces that govern the knowledge-sharing behavior of agents and to illustrate how they work in practice we simulated the equilibrium numerically. In this section we present the results of this analysis. By Proposition 4 in all our calculations we set the value of a parameter α_{11} equal to its ex ante value $\alpha_{10} = 1$. In terms of parameters α_{20} and α_{30} we considered different combinations of their values that are admissible in our model, that is ones that satisfy $\alpha_{20} > \alpha_{10} = 1$ and $\alpha_{30} > 0$[9]. Note that by Proposition 8 in equilibrium a parameter δ^1_{ij} is always zero for all pairs $\{i, j\}, i, j \in N_2, i \neq j$. Moreover, by the same proposition, a parameter δ^0_{ij} is zero for all pairs $\{i, j\}, i, j \in N, i \neq j$ except for $\{1, 3\}$ and $\{3, 2\}$, where it may be strictly positive. Therefore, in our analysis we concentrate on those two parameters only. We report our results on the equilibrium values of δ^0_{13} and δ^0_{32} in Tables 1 to 6. Tables 1 and 2 present the equilibrium values of the parameters in their nominal levels, Tables 3 and 4 present them as a percentage of the difference between the knowledge levels of an knowledge donor and a receiver, and finally Tables 5 and 6 – as a percentage of the knowledge level of a knowledge donor.

Our analysis reveals that there are no interior equilibria, where both δ^0_{13} and δ^0_{32} are strictly positive at the same time. It is also worth to notice that the knowledge sharing between agents 1 and 3 can be observed only at high levels of the parameter α_{30}, and that knowledge sharing between agents 3 and 2 appears even at its low levels. Comparing Tables 1 and 2, we see that the amount of knowledge shared between agents 3 and 2 in nominal terms is much bigger that between agents 1 and 3. This observation holds also in relative terms (Tables 3 to 6). If we go some more into detail by considering the parameter δ^0_{32} and Tables 1, 3 and 5 we will easily notice that knowledge sharing never occurs in equilibrium if $\alpha_{30} > \alpha_{20}$. In such a case the knowledge level of agent 3 is lower than the knowledge level of agent 2 and knowledge sharing is not possible by assumption. We may also notice that given α_{30} the amount of knowledge shared in equilibrium by agent 3 with agent 2 increases monotonically in α_{20}. It means that the lower is the knowledge level of agent 2, the more knowledge he receives from agent 3. This conclusion holds independently of whether we measure the level of knowledge sharing in nominal (Table

[9] In our numerical analysis we used α_{20} ={1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100} and α_{30} ={0.25, 0.5, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1000}. Note also that – as our previous considerations suggest – as long as a parameter $V > 0$, it doesn't affect potential maxima of the marginal cost parameters. Therefore our numerical analysis didn't require to define it explicitly.

1) or in relative terms (Tables 3 and 5). This is somehow different if we study the amount of knowledge shared by agent 3 given α_{20}. In such a case it decreases monotonically in α_{30} in nominal terms, but increases monotonically as a percentage of the difference between the knowledge levels of agent 2 and agent 3 (Table 3) or even increases first and decreases afterwards as a percentage of the knowledge level of agent 3 (Table 5). This means in nominal terms that the more knowledge agent 3 has, the more knowledge he is ready to disclose to agent 2. Moreover, although the amount of the knowledge shared in nominal terms decreases in α_{30}, at the same time – as the results in Table 3 suggest – this amount is more and more capable to cover the difference between the knowledge levels of agent 2 and agent 3. As a consequence, for some combinations of the parameters α_{20} and α_{30}, this difference is covered completely and disappears in equilibrium, so that both agents have then the same level of the marginal cost of effort. If we consider the parameter δ_{13}^{0} and Tables 2, 4 and 6, we may notice that given α_{30} the amount of the knowledge shared in equilibrium by agent 1 with agent 3 increases in α_{20} first and decreases afterwards, suggesting the existence of some maximum. Moreover, this knowledge sharing never occurs in equilibrium if $\alpha_{20} \leq 3$, as our analytical solution suggested. However, if we study the amount of knowledge shared by agent 1 with agent 3 given α_{20}, then this always monotonically increases in α_{30} with the property that this knowledge sharing never occurs in equilibrium if $\alpha_{30} \leq 30$. It means that agent 3 has to be very weak in comparison with agent 1, so that the latter one has incentives to disclose some of his knowledge to the former one. It means also that if knowledge sharing takes place ($\delta_{13}^{0} > 0$), then the more knowledge agent 3 possesses the less knowledge he receives from agent 1. All these conclusions hold independently of whether we measure the level of knowledge sharing in nominal (Table 2) or in relative terms (Tables 4 and 6).

4 Conclusions

In this paper we study a model where non-cooperatively behaving agents may exchange knowledge in a competitive environment. As a potential factor that could induce the disclosure of knowledge between humans we consider the timing of the moves of players. To concentrate only on incentives to share knowledge, we develop a simple model of a multistage game, in which there are only three players and competition takes place only within two stages. In the first stage, two of three agents compete against each other and the winner goes to the second stage of the game. Here he competes with the third agent, who in the previous stage was not active. An important feature of our model is that players can share their private knowledge with their opponents and the knowledge is modelled as influencing marginal cost of effort of players. This is done in such a way that more knowledge is associated with lower levels of marginal cost. Before the whole game starts, all agents decide to pass or not some of their private knowledge to each of their opponents, which affects their cost functions that they use in subsequent stages. Such a decision is taken again at the end of the first stage of a game by a survivor of that stage and the agent who was not active in that stage.

One of our main results is that knowledge sharing can occur even in purely non-cooperative environments. In particular, we managed to show that there is only one agent, who never has incentives to share his knowledge with opponents. This is the weaker participant of the first stage of a game. However, the stronger participant of that stage may have incentives to disclose his knowledge before the whole game starts to the "observer", if both the weaker agent and the "observer" are weak enough. In such a case the stronger agent benefits from decreasing the valuation of the prize of the weaker player. Our numerical example reveals that in such a case the "observer" has to be very weak ex ante in comparison with the stronger player, so that the latter one has incentives to disclose some of his knowledge to the former one. It also shows that the more knowledge the "observer" possesses the less knowledge he receives from the stronger player. In our analysis we also proved that the "observer" has incentives to share knowledge before the actual competition in the first stage with the weaker opponent of that stage. This happens when the difference in marginal cost of effort between the weaker and the stronger player of the first stage of a game is high enough. In such a situation the "observer" gains by sharing his knowledge to some extent with the weaker opponent, because in this way, by increasing the probability of winning of the weaker opponent in the fight in the first stage, reduces the probability of winning of the stronger one. Thus, through this mechanism, in the second stage the "observer" has greater chances to meet the weaker player rather than the stronger one. Our numerical example reveals that in such a case the lower is the knowledge level of the weaker player, the more knowledge he receives from the "observer". It also shows that the more knowledge the "observer" has the more knowledge he is ready to disclose to the weaker player. We show also that there is never exchange of knowledge between the active participants of the first stage of a game – the weaker and the stronger player, and also between participants of the final stage – a survivor of the first stage of a game and the "observer".

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Appendix

Proofs

Proof of Proposition 1. [10]To prove the proposition we will consider players i and j, $i, j \in N_2, i \neq j$, who participate in stage 2 of a game. W.l.o.g we assume that $\kappa_{j1} \leq \kappa_{i1}$ (or equivalently $\alpha_{i1} \leq \alpha_{j1}$). We will consider now an agent i and two cases:

a) $\kappa_{j1} < \kappa_{i1}$ (or equivalently $\alpha_{i1} < \alpha_{j1}$)

In such a case for an agent i it is always "technically" possible to disclose some of his knowledge to an agent j. We will show now that in spite of that fact, he never wants to do it. For that end we will use the (sub)equilibrium expected payoff of an agent i in stage 2 given in eq. (14). Proving that $\delta_{ij}^{1*} = 0$ requires to show that for any admissible values of parameters α_{i2}, α_{j2} and V the inequality

$$\frac{\partial u_i^* (\alpha_{i2}, \alpha_{j2})}{\partial \alpha_{j2}} > 0$$

is satisfied. This inequality reflects the fact that the decrease in the value of α_{j2} is never profitable for an agent i, as this results in the fall in his expected utility. Therefore, an agent i doesn't want to reduce α_{j2}. By eq. (6), eq. (5) and eq. (3), any knowledge transmission ($\delta_{ij}^s > 0$) reduces marginal cost of effort of a knowledge receiver. It follows that an agent i has no incentives to disclose any piece of his knowledge to an agent j and that $\delta_{ij}^1 = 0$.

By differentiating eq. (14) w.r.t. α_{j2} we obtain that

$$\frac{\partial u_i^* (\alpha_{i2}, \alpha_{j2})}{\partial \alpha_{j2}} = \frac{2\alpha_{i2}\alpha_{j2}}{(\alpha_{i2} + \alpha_{j2})^3} V.$$

It is obvious that this is always strictly positive, given our assumption on strictly positive values of the parameters. This proves that $\delta_{ij}^1 = 0$ for all $i, j \in N_2, i \neq j$, whenever $\kappa_{j1} < \kappa_{i1}$.

[10]All the proofs in this section were done with help of Mathematica (Wolfram Research).

b) $\kappa_{j1} = \kappa_{i1}$ (or equivalently $\alpha_{i1} = \alpha_{j1}$)

Note that in this case for an agent i it is always "technically" impossible to disclose some of his knowledge to an agent j. So by eq. (4) $\delta_{ij}^{1} = 0$, whenever $\kappa_{j1} = \kappa_{i1}$.

So it follows from our discussion that an agent i never has incentives to disclose some of his knowledge to an agent j, and $\delta_{ij}^{1} = 0$.

Consider now an agent j. Note that when $\kappa_{j1} \leq \kappa_{i1}$, then for an agent j it is always "technically" impossible to disclose some of his knowledge to an agent i. So by eq. (4) $\delta_{ji}^{1} = 0$. This completes the proof. ■

Proof of Proposition 2. To prove the first part of the proposition, that $\delta_{12}^{0*} = 0$, note first that by assumption $\alpha_{20} > \alpha_{10}$. This by eq. (3) means that $\kappa_{20} < \kappa_{10}$. Hence, for agent 1 it is always "technically" possible to disclose some of his knowledge to agent 2. We will show now that in spite of that fact, he never wants to do it. For that end we will use the (sub)equilibrium expected payoffs of agents 1 and 2 at the beginning of stage 1 given in eq. (20). Setting $i = 1$ and $j = 2$, this equation becomes

$$u_1^*(\alpha_{11}, \alpha_{21}, \alpha_{31}) = \frac{(\alpha_{21}\alpha_{31})^2 (\alpha_{21} + \alpha_{31})^4}{(\alpha_{11} + \alpha_{31})^2 \left(\alpha_{11} (\alpha_{11} + \alpha_{31})^2 + \alpha_{21} (\alpha_{21} + \alpha_{31})^2\right)^2} V. \tag{23}$$

Proving that $\delta_{12}^{1*} = 0$ requires to show that for any admissible values of parameters $\alpha_{11}, \alpha_{21}, \alpha_{31}$ and V the inequality

$$\frac{\partial u_1^*(\alpha_{11}, \alpha_{21}, \alpha_{31})}{\partial \alpha_{21}} > 0$$

is satisfied. This inequality reflects the fact that the decrease in the value of α_{21} is never profitable for agent 1, as this results in the fall in his expected utility. Therefore, agent 1 doesn't want to reduce α_{21}. By eq. (6), eq. (5) and eq. (3), any knowledge transmission ($\delta_{ij}^{s} > 0$) reduces marginal cost of effort of a knowledge receiver. It follows that agent 1 has no incentives to disclose any piece of his knowledge to agent 2 in stage 0 of a game and that $\delta_{12}^{1} = 0$.

By differentiating eq. (23) w.r.t. α_{21} we obtain that

$$\frac{\partial u_1^*(\alpha_{11}, \alpha_{21}, \alpha_{31})}{\partial \alpha_{21}} = \frac{2\alpha_{11}\alpha_{21}\alpha_{31}^2 (3\alpha_{21} + \alpha_{31}) (\alpha_{21} + \alpha_{31})^3}{\left(\alpha_{11} (\alpha_{11} + \alpha_{31})^2 + \alpha_{21} (\alpha_{21} + \alpha_{31})^2\right)^3} V.$$

It is obvious that this is always strictly positive, given our assumption on strictly positive values of the parameters. Note that this result holds for all admissible values of α_{21}, and in particular for $\alpha_{21} = \alpha_{20}$. This completes the first part of the proof.

Proving the second part of the proposition, that $\delta_{21}^{1}=0$, is straightforward. As we noted earlier $\kappa_{20}<\kappa_{10}$, so by eq. (4) $\delta_{21}^{1}=0$. This completes the proof. $\blacksquare$

Proof of Lemma 1. Using similar reasoning as in the proof of Proposition 2, to prove the lemma we need to show that for any admissible values of parameters $\alpha_{11},\alpha_{21},\alpha_{31}$ and V with $\alpha_{11}<\alpha_{21}$ the inequality

$$\frac{\partial u_3^*\left(\alpha_{11},\alpha_{21},\alpha_{31}\right)}{\partial\alpha_{11}}>0$$

holds. This inequality reflects the fact that the decrease in the value of α_{11} is never profitable for agent 3, as this results in the fall in his expected utility. Therefore, agent 3 doesn't want to reduce α_{11}. By eq. (6), eq. (5) and eq. (3), any knowledge transmission ($\delta_{ij}^{s}>0$) reduces marginal cost of effort of a knowledge receiver. It follows that in stage 0 agent 3 has no incentives to disclose any piece of his knowledge to agent 1 and that $\delta_{31}^{0}=0$.

Differentiating eq. (22) w.r.t. α_{11} yields

$$\frac{\partial u_3^*\left(\alpha_{11},\alpha_{21},\alpha_{31}\right)}{\partial\alpha_{11}}=\frac{\alpha_{21}\alpha_{31}N\left(\alpha_{11}\right)}{\left(\alpha_{11}+\alpha_{31}\right)^3\left(\alpha_{11}\left(\alpha_{11}+\alpha_{31}\right)^2+\alpha_{21}\left(\alpha_{21}+\alpha_{31}\right)^2\right)^2}V \tag{24}$$

where

$$\begin{aligned}N\left(\alpha_{11}\right)=&-3\left(2\alpha_{21}+\alpha_{31}\right)\alpha_{11}^5+\left(8\alpha_{21}^2-10\alpha_{21}\alpha_{31}-5\alpha_{31}^2\right)\alpha_{11}^4\\&-\left(\alpha_{31}\left(\alpha_{31}^2+2\alpha_{21}\alpha_{31}-21\alpha_{21}^2\right)\right)\alpha_{11}^3\\&+\left(\alpha_{31}^2\left(\alpha_{31}^2+2\alpha_{21}\alpha_{31}+19\alpha_{21}^2\right)\right)\alpha_{11}^2\\&+\left(\alpha_{21}\left(2\alpha_{21}^4+8\alpha_{21}^3\alpha_{31}+12\alpha_{21}^2\alpha_{31}^2+15\alpha_{21}\alpha_{31}^3+2\alpha_{31}^4\right)\right)\alpha_{11}\\&+\alpha_{21}^2\alpha_{31}^4.\end{aligned}$$

Using the assumptions about the values of the parameters we obtain after some algebra that the values produced by eq. (24) are always strictly positive, which completes the proof. $\blacksquare$

Proof of Lemma 2. To prove the part (i) of the lemma, we need to show that for $\alpha_{11}=1$ and any strictly positive value of parameters α_{21},α_{31} and V there exists α_{32}' that satisfies the following relation

$$\frac{\partial u_3^*\left(1,\alpha_{21}',\alpha_{31}\right)}{\partial\alpha_{21}}=0,$$

and that this value α_{32}' is the global maximum. Differentiating eq. (22) w.r.t. α_{21} produces

$$\frac{\partial u_3^*\left(\alpha_{11},\alpha_{21},\alpha_{31}\right)}{\partial\alpha_{21}}=\frac{\alpha_{11}\alpha_{31}O\left(\alpha_{21}\right)}{\left(\alpha_{21}+\alpha_{31}\right)^3\left(\alpha_{11}\left(\alpha_{11}+\alpha_{31}\right)^2+\alpha_{21}\left(\alpha_{21}+\alpha_{31}\right)^2\right)^2}V$$

where

$$O\left(\alpha_{21}\right)=-3\left(2\alpha_{11}+\alpha_{31}\right)\alpha_{21}^{5}+\left(8\alpha_{11}^{2}-10\alpha_{11}\alpha_{31}-5\alpha_{31}^{2}\right)\alpha_{21}^{4} \\ -\left(\alpha_{31}\left(\alpha_{31}^{2}+2\alpha_{11}\alpha_{31}-21\alpha_{11}^{2}\right)\right)\alpha_{21}^{3} \\ +\left(\alpha_{31}^{2}\left(\alpha_{31}^{2}+2\alpha_{11}\alpha_{31}+19\alpha_{11}^{2}\right)\right)\alpha_{21}^{2} \\ +\left(\alpha_{11}\left(2\alpha_{11}^{4}+8\alpha_{11}^{3}\alpha_{31}+12\alpha_{11}^{2}\alpha_{31}^{2}+15\alpha_{11}\alpha_{31}^{3}+2\alpha_{31}^{4}\right)\right)\alpha_{21} \\ +\alpha_{11}^{2}\alpha_{31}^{4}. \tag{25}$$

After equating it to zero, setting $\alpha_{11}=1$ and solving for α_{21} we get a maximum candidate

$$\alpha_{32}'=\begin{cases} O\left(\alpha_{21},3\right), \text{if}\, \alpha_{31}<7.94211, \\ O\left(\alpha_{21},5\right), \text{if}\, \alpha_{31}\geq 7.94211, \end{cases} \tag{26}$$

where $O(\alpha_{21},n)$ denotes the n-th root of the polynomial $O(\alpha_{21})$ defined for $\alpha_{11}=1$:

$$O\left(\alpha_{21}\right)=-3\left(\alpha_{31}+2\right)\alpha_{21}^{5}-\left(5\alpha_{31}^{2}+10\alpha_{31}-8\right)\alpha_{21}^{4} \\ -\left(\alpha_{31}\left(\alpha_{31}^{2}+2\alpha_{31}-21\right)\right)\alpha_{21}^{3} \\ +\left(\alpha_{31}^{2}\left(\alpha_{31}^{2}+2\alpha_{31}+19\right)\right)\alpha_{21}^{2} \\ +\left(2\alpha_{31}^{4}+15\alpha_{31}^{3}+12\alpha_{31}^{2}+8\alpha_{31}+2\right)\alpha_{21}+\alpha_{31}^{4}.$$

For α_{32}' to be the global maximum we need to show that $\frac{\partial u_3^*(1,\alpha_{21},\alpha_{31})}{\partial\alpha_{21}}$ is a strictly quasi-concave function of α_{21}. As our problem is unidimensional, this requires to show that for any strictly positive values of parameters α_{21},α_{31} and V the inequalities

$$\frac{\partial u_3^*\left(1,\alpha_{21},\alpha_{31}\right)}{\partial\alpha_{21}}>0, \text{ if } \alpha_{21}<\alpha_{32}',$$

and

$$\frac{\partial u_3^*\left(1,\alpha_{21},\alpha_{31}\right)}{\partial\alpha_{21}}<0, \text{ if } \alpha_{21}>\alpha_{32}'$$

hold. After examination of the properties of $\frac{\partial u_3^*(1,\alpha_{21},\alpha_{31})}{\partial\alpha_{21}}$ it turns out that the two last inequalities are always satisfied, which implies that there exist the global maximum given by eq. (26).

It follows that if $\alpha_{11}=\alpha_{10}=1$, then there exists the level of the marginal cost parameter of agent 2 in stage 1, which is the most preferred by agent 3, α_{32}', given by the following relation

$$\alpha_{32}'=\begin{cases} O\left(\alpha_{21},3\right), \text{if}\, \alpha_{31}<7.94211, \\ O\left(\alpha_{21},5\right), \text{if}\, \alpha_{31}\geq 7.94211, \end{cases} \tag{27}$$

where $O(\alpha_{21}, n)$ denotes the n-th root of the polynomial

$$\begin{aligned} O\left(\alpha_{21}\right) = & -3\left(\alpha_{31}+2\right)\alpha_{21}^{5} - \left(5\alpha_{31}^{2}+10\alpha_{31}-8\right)\alpha_{21}^{4} \\ & -\left(\alpha_{31}\left(\alpha_{31}^{2}+2\alpha_{31}-21\right)\right)\alpha_{21}^{3} + \left(\alpha_{31}^{2}\left(\alpha_{31}^{2}+2\alpha_{31}+19\right)\right)\alpha_{21}^{2} \\ & +\left(2\alpha_{31}^{4}+15\alpha_{31}^{3}+12\alpha_{31}^{2}+8\alpha_{31}+2\right)\alpha_{21}+\alpha_{31}^{4}, \end{aligned}$$

This proves the part (i) of the lemma.

To prove its part (ii), we need to show that for any strictly positive value of a parameter α_{31} the relation

$$\alpha_{32}' > 1$$

is always true. In particular this requires to check whether the two following inequalities

$$O\left(\alpha_{21}, 3\right) > \alpha_{10} = 1, \text{ if } \alpha_{31} < 7.94211,$$

and

$$O\left(\alpha_{21}, 5\right) > \alpha_{10} = 1, \text{ if } \alpha_{31} \geq 7.94211,$$

hold for any strictly positive value of α_{31}. It turns out after examination that they are always satisfied, which completes the proof. ■

Proof of Proposition 3. By Proposition 2 neither agent 1 nor agent 2 will share their knowledge between themselves in stage 0 of a game, so in that stage the only potential source of knowledge flow towards agent 1 or 2 that could change their marginal cost parameters is agent 3. By 2, if $\alpha_{11} = \alpha_{10} = 1$, then the level of the marginal cost parameter of agent 2 in stage 1, which is the most preferred by agent 3, α_{32}', is always greater than $\alpha_{10} = 1$, hence $\alpha_{10} < \alpha_{32}'$. In such a case it always holds that $\alpha_{11} < \alpha_{21}$ in a potential equilibrium, so by Lemma 1 in stage 0 of a game agent 3 has no incentives to share his knowledge with agent 1, that is $\delta_{31}^{0} = 0$. This completes the proof. ■

Proof of Proposition 4. We know that $\alpha_{10} = 1 < \alpha_{20}$ by assumption. By Proposition 2 in stage 0 of a game neither agent 1 nor agent 2 will share their knowledge between themselves, so in that stage the only potential source of knowledge flow towards agent 1 or 2 that could change their marginal cost parameters is agent 3. Moreover, by Proposition 3 in stage 0 of a game agent 3 has no incentive to share his knowledge with agent 1. It follows that in a potential equilibrium in that stage none of the opponents will share knowledge with agent 1, which implies that his marginal cost of effort in stage 1 remains at his ex ante level $\alpha_{10} = 1$, and $\alpha_{11}^{*} = \alpha_{10} = 1$. Moreover, by Lemma 2 if $\alpha_{11} = \alpha_{10} = 1$, then the level of the marginal cost parameter of agent 2 in stage 1, which is the most preferred by agent 3, α_{32}', is always greater than $\alpha_{10} = 1$. So it follows from our reasoning that $\alpha_{21}^{*}=\alpha_{32}'>\alpha_{10}=\alpha_{11}^{*}$, which completes the proof. ■

Proof of Proposition 5. By Proposition 4, in a potential equilibrium always $\alpha_{11}^{*}=\alpha_{10}=1$. Then by Lemma 2, there exists the level of the marginal cost parameter of agent 2 in stage 1 which is the most preferred by agent 3, α_{32}', defined in eq. (27), with the corresponding knowledge level

$$\kappa_{32}'=\frac{1}{\alpha_{32}'}. \tag{28}$$

It follows that in stage 0 of a game agent 3 may have sometimes incentives to pass a piece of his knowledge to agent 2. By sharing his knowledge agent 3 may try to modify the level of the marginal cost of effort of agent 2 in stage 1, so that it maximizes his own expected utility. In order to derive specific values of a parameter δ_{32}^{0}, we need to consider four cases:

a) $\kappa_{30}\leq\kappa_{20}$

In such a situation, agent 3 has a lower (or equal) level of knowledge ex ante than agent 2, which using eq. (4) implies that $\delta_{32}^{0}=0$.

b) $\kappa_{32}'\leq\kappa_{20}$

In such a situation, the level of the knowledge of agent 2 in stage 1, which is the most preferred by agent 3, is below (or equal to) the knowledge level ex ante of agent 2. In our model, any knowledge transmission ($\delta_{ij}^{s}>0$) is always beneficial to a knowledge receiver, as it increases his knowledge (and never reduces it). Therefore in stage 0 of a game agent 3 prefers not to disclose any piece of his knowledge to agent 2, which implies that $\delta_{32}^{0}=0$.

c) $(\kappa_{30}>\kappa_{20})\cap(\kappa_{30}>\kappa_{32}'>\kappa_{20})$

$\kappa_{30}>\kappa_{20}$, so in such a case for agent 3 it is "technically" possible to pass some of his knowledge to agent 2. Moreover $\kappa_{30}>\kappa_{32}'>\kappa_{20}$, so the level of the knowledge of agent 2 in stage 1, which is the most preferred by agent 3, is above the knowledge level ex ante of agent 2 and below the knowledge level ex ante of agent 3. This creates in stage 0 of a game incentives for agent 3 to disclose some of his knowledge to agent 2. Therefore, agent 3 has both: incentives and possibility to disclose some of his knowledge to agent 2. The knowledge is disclosed in such a way that the resulting level of the knowledge of agent 2 in stage 1 is equal to $\kappa_{12}=\kappa_{32}'$. This requires that agent 3 passes to agent 2 a piece of knowledge equal to $\delta_{32}^{0}=\kappa_{32}'-\kappa_{20}$.

d) $(\kappa_{30}>\kappa_{20})\cap(\kappa_{32}'\geq\kappa_{30})$

As in the previous case $\kappa_{30}>\kappa_{20}$, so in this case for agent 3 it is still "technically" possible to pass some of his knowledge to agent 2. Moreover $\kappa_{32}'\geq\kappa_{30}$, so the level of the knowledge of agent 2 in stage 1, which is the most preferred by agent 3, is above or equal to the knowledge level ex ante of agent 3 (and by the previous inequality also above the knowledge level ex ante of agent 2). This creates in stage 0 of a game incentives for agent 3 to disclose some of his knowledge to agent 2. Therefore, as in the previous case, agent 3 has both: incentives and possibility to disclose some of his knowledge to agent 2. Recall, however, that by the mechanism of the knowledge transmission that we assume in this paper, a resulting knowledge level of a knowledge receiver cannot be higher than a knowledge level of a knowledge donor. It follows that in our current case the knowledge is disclosed in stage 0 of a game in such a way that the resulting level of the knowledge of agent 2 in stage 1 is the same as the level ex ante of the knowledge of agent 3, that is $\kappa_{12} = \kappa_{30}$. This requires that agent 3 passes to agent 2 a piece of knowledge equal to $\delta_{32}^{0} = \kappa_{30} - \kappa_{20}$.

Summarizing all the results obtained in the four cases we obtain:

$$\delta_{32}^{0} = \begin{cases} 0, & \text{if } (\kappa_{30} \le \kappa_{20}) \cup (\kappa'_{32} \le \kappa_{20}), \\ \kappa'_{32} - \kappa_{20}, & \text{if } (\kappa_{30} > \kappa_{20}) \cap (\kappa_{30} > \kappa'_{32} > \kappa_{20}), \\ \kappa_{30} - \kappa_{20}, & \text{if } (\kappa_{30} > \kappa_{20}) \cap (\kappa'_{32} \ge \kappa_{30}), \end{cases}$$

If we focus only on the instances in which the knowledge disclosure occurs and in which it doesn't, we can simplify the last expression and rewrite it as:

$$\begin{cases} \delta_{32}^{0*} = 0, \text{if } (\kappa_{30} \le \kappa_{20}) \cup (\kappa'^{*}_{32} \le \kappa_{20}), \\ \delta_{32}^{0*} > 0, \quad \text{otherwise.} \end{cases}$$

This completes the proof. ■

Proof of Proposition 6. By differentiating eq. (20) w.r.t. α_{31} we obtain that for all $i \in N_1, i \neq j$

$$\frac{\partial u_i^* (\alpha_{i1}, \alpha_{j1}, \alpha_{31})}{\partial \alpha_{31}} = \frac{2\alpha_{i1}\alpha_{j1}^2\alpha_{31} (\alpha_{j1} + \alpha_{31})^3 M (\alpha_{31})}{(\alpha_{i1} + \alpha_{31})^3 \left(\alpha_{i1} (\alpha_{i1} + \alpha_{31})^2 + \alpha_{j1} (\alpha_{j1} + \alpha_{31})^2\right)^3} V, \tag{29}$$

where

$$M (\alpha_{31}) = (3\alpha_{i1} - \alpha_{j1}) \alpha_{31}^3 + 3 \left(2\alpha_{i1}^2 - \alpha_{i1}\alpha_{j1} + \alpha_{j1}^2\right) \alpha_{31}^2 \\ +3 \left(\alpha_{i1}^3 + \alpha_{j1}^3\right) \alpha_{31} + \alpha_{j1} \left(\alpha_{i1}^3 + \alpha_{j1}^3\right).$$

Setting $i = 2$ and $j = 1$, eq. (29) becomes

$$\frac{\partial u_2^* (\alpha_{21}, \alpha_{11}, \alpha_{31})}{\partial \alpha_{31}} = \frac{2\alpha_{11}^2\alpha_{21}\alpha_{31} (\alpha_{11} + \alpha_{31})^3 M (\alpha_{31})}{(\alpha_{21} + \alpha_{31})^3 \left(\alpha_{11} (\alpha_{11} + \alpha_{31})^2 + \alpha_{21} (\alpha_{21} + \alpha_{31})^2\right)^3} V, \tag{30}$$

with

$$M (\alpha_{31}) = (3\alpha_{21} - \alpha_{11}) \alpha_{31}^3 + 3 \left(\alpha_{11}^2 - \alpha_{11}\alpha_{21} + 2\alpha_{21}^2\right) \alpha_{31}^2 \\ +3 \left(\alpha_{11}^3 + \alpha_{21}^3\right) \alpha_{31} + \alpha_{11} \left(\alpha_{11}^3 + \alpha_{21}^3\right).$$

Using similar reasoning as in our previous proofs, to prove the proposition it is enough to show that for any admissible values of parameters $\alpha_{11}, \alpha_{21}, \alpha_{31}$ and V the inequality

$$\frac{\partial u_2^* (\alpha_{21}, \alpha_{11}, \alpha_{31})}{\partial \alpha_{31}} > 0 \tag{31}$$

is satisfied. In such a case the decrease in the value of α_{31} is never profitable for agent 2, as this results in the fall in his expected utility. Therefore, agent 2 doesn't want to reduce α_{31}. By eq. (6), eq. (5) and eq. (3), any knowledge transmission ($\delta_{ij}^s > 0$) reduces marginal cost of effort of a knowledge receiver. It follows that agent 2 has no incentives to disclose any piece of his knowledge to agent 3 in stage 0 of a game and that $\delta_{23}^0 = 0$.

Note, that by our assumption on the parameters, the denominator of eq. (30) is always strictly positive, and the sign of the nominator depends only on the sign of values produced by $M(\alpha_{31})$. Therefore we will concentrate only on studying the polynomial $M(\alpha_{31})$. By Proposition 4 in equilibrium $\alpha_{11} < \alpha_{21}$. Using this fact it is easy to notice that two first coefficients of the polynomial satisfy respectively

$$3\alpha_{21} - \alpha_{11} > 0$$

and

$$3\left(\alpha_{11}^2 - \alpha_{11}\alpha_{21} + 2\alpha_{21}^2\right) = 3\left(\alpha_{21}(\alpha_{11} + \alpha_{21}) + (\alpha_{21} - \alpha_{11})^2\right) > 0,$$

and the last two are always strictly positive. Hence all coefficients of the polynomial $M(\alpha_{31})$ are strictly positive. Therefore, given our assumption on strictly positive values of the parameters, $M(\alpha_{31})$ is always strictly positive, which implies that the inequality (31) is always satisfied. This completes the proof. ■

Proof of Proposition 7. Setting $i = 1$, $j = 2$ and considering the fact that by Proposition 4 $\alpha_{11} = \alpha_{10} = 1$, eq. (29) becomes

$$\frac{\partial u_1^* (1, \alpha_{21}, \alpha_{31})}{\partial \alpha_{31}} = \frac{2\alpha_{21}^2 \alpha_{31} (\alpha_{21} + \alpha_{31})^3 M(\alpha_{31})}{(1 + \alpha_{31})^3 \left((1 + \alpha_{31})^2 + \alpha_{21}(\alpha_{21} + \alpha_{31})^2\right)^3} V, \tag{32}$$

with

$$\begin{aligned} M(\alpha_{31}) = &(3 - \alpha_{21})\alpha_{31}^3 + 3\left(2 - \alpha_{21} + \alpha_{21}^2\right)\alpha_{31}^2 \\ &+3\left(1 + \alpha_{21}^3\right)\alpha_{31} + \alpha_{21}\left(1 + \alpha_{21}^3\right). \end{aligned}$$

We will prove the proposition considering two cases: of $\alpha_{21} \leq 3$ and of $\alpha_{21} > 3$.

Consider first the case of $\alpha_{21} \leq 3$. Applying similar reasoning as in our previous proofs, we will show now that for $\alpha_{21} \leq 3$ and any strictly positive values of parameters α_{31} and V the inequality

$$\frac{\partial u_1^* (1, \alpha_{21}, \alpha_{31})}{\partial \alpha_{31}} > 0, \tag{33}$$

is always satisfied. In such a case the decrease in the value of α_{31} is never profitable for agent 1, as this results in the fall in his expected utility. Therefore, agent 1 doesn't want to reduce α_{31}. By eq. (6), eq. (5) and eq. (3), any knowledge transmission ($\delta_{ij}^s > 0$) reduces marginal cost of effort of a knowledge receiver. It follows that if $\alpha_{21} \leq 3$, then agent 1 has no incentives to disclose any piece of his knowledge to agent 3 in stage 0 of a game and that $\delta_{13}^1 = 0$.

Note, that by our assumption on the parameters, the denominator of eq. (32) is always strictly positive, and the sign of the nominator depends only on the sign of values produced by $M(\alpha_{31})$. Therefore we concentrate only on studying the polynomial $M(\alpha_{31})$. It is easy to notice that if $\alpha_{21} \leq 3$, then its two first coefficients satisfy respectively

$$3 - \alpha_{21} \geq 0$$

and

$$3\left(2 - \alpha_{21} + \alpha_{21}^2\right) = 3\left(1 + \alpha_{21} + (1 - \alpha_{21})^2\right) > 0,$$

and the last two are always strictly positive. Hence all coefficients of the polynomial $M(\alpha_{31})$ are strictly positive or equal to zero, with the last term being always strictly positive. Therefore if $\alpha_{21} \leq 3$, then $M(\alpha_{31})$ is always strictly positive, given our assumption on strictly positive values of the parameters. This implies that the inequality (33) is always satisfied.

Consider now the second case of $\alpha_{21} > 3$. We will demonstrate now that for $\alpha_{21} > 3$ and any strictly positive values of parameters α_{31} and V there exists α_{13}' that satisfies the following relation

$$\frac{\partial u_1^* (1, \alpha_{21}, \alpha_{31}')}{\partial \alpha_{31}} = 0,$$

and that this value α_{13}' is the global maximum. We will also prove that this global maximum is always higher than $\alpha_{10} = 1$. After equating the expression (32) to zero and solving for α_{31} we get a maximum candidate

$$\alpha_{13}' = M(\alpha_{31,}1). \tag{34}$$

It can be verified that the value produced by this root is always strictly positive. For α_{13}' to be the global maximum we need to show that

$\frac{\partial u_1^*(1,\alpha_{21},\alpha_{31}')}{\partial \alpha_{31}}$ is a strictly quasi-concave function of α_{31}. As our problem is unidimensional, this requires to show that for $\alpha_{21} > 3$ and any strictly positive values of parameters α_{31} and V the inequalities

$$\frac{\partial u_1^*(1, \alpha_{21}, \alpha_{31})}{\partial \alpha_{31}} > 0, \text{ if } \alpha_{31} < \alpha_{13}',$$

and

$$\frac{\partial u_1^*(1, \alpha_{21}, \alpha_{31})}{\partial \alpha_{31}} < 0, \text{ if } \alpha_{31} > \alpha_{13}'$$

hold. After examination of properties of $\frac{\partial u_1^*(1,\alpha_{21},\alpha_{31}')}{\partial \alpha_{31}}$ it turns out that the two last inequalities are always satisfied, which implies that there exist the global maximum given by eq. (34). To prove that this global maximum is always higher than $\alpha_{10} = 1$, we need to verify whether the following relation

$$\alpha_{13}' > \alpha_{10} = 1$$

holds for all $\alpha_{21} > 3$. It turns out after examination that this is in fact true.

Now, using results on α_{13}' we will derive the values of the parameter δ_{13}^1 for $\alpha_{21} > 3$. As our previous analysis for $\alpha_{21} > 3$ reveals, there exists the most preferred by agent 1 level of the marginal cost parameter of agent 3 in stage 1, α_{13}', defined in eq. (34) with the corresponding knowledge level

$$\kappa_{13}' = \frac{1}{\alpha_{13}'}. \tag{35}$$

It follows that agent 1 may have sometimes incentives in stage 0 to pass a piece of his knowledge to agent 3. Specifically, by sharing knowledge agent 1 may try to modify the level of the marginal cost of effort of agent 3 in stage 1, so that it maximizes his expected utility. However, as the analysis in the previous paragraph shows, in stage 0 agent 1 never has incentives to reduce the level of the marginal cost of effort of agent 3 to the level which is below or equal to his own level of the marginal cost of effort. In other words, agent 1 never has incentives in stage 0 to increase the knowledge level of agent 3 in stage 1 to the level which is above or equal to his own knowledge level ex ante. In order to derive specific values of a parameter δ_{13}^0, we need to consider here three cases:

a) $(1 = \kappa_{10} \leq \kappa_{30})$

In such a situation, agent 1 has a lower (or equal) level of knowledge ex ante than agent 3, which using eq. (4) implies that $\delta_{13}^0 = 0$.

b) $(\kappa_{13}' \leq \kappa_{30})$

In such a situation, the level of the knowledge of agent 3 in stage 1, which is the most preferred by agent 1, is below (or equal to) the knowledge level ex ante of agent 3. In our model, any knowledge transmission

($\delta_{ij}^{s} > 0$) is always beneficial to a knowledge receiver, as it increases his knowledge (and never reduces it). Therefore in stage 0 agent 1 prefers not to disclose any piece of his knowledge to agent 3, which implies that $\delta_{13}^{0} = 0$.

c) $(1 = \kappa_{10} > \kappa_{30}) \cap (1 = \kappa_{10} > \kappa_{13}' > \kappa_{30})$

$\kappa_{10} > \kappa_{30}$, so in such a case for agent 1 it is "technically" possible to pass some of his knowledge to agent 3. Moreover $\kappa_{10} > \kappa_{13}' > \kappa_{30}$, so the level of the knowledge of agent 3 in stage 1, which is the most preferred by agent 1, is above the knowledge level ex ante of agent 3 and below the knowledge level ex ante of agent 1. This creates in stage 0 incentives for agent 1 to disclose some of his knowledge to agent 3. Therefore, agent 1 has both: incentives and possibility to disclose some of his knowledge to agent 3. The knowledge is disclosed in such a way that the resulting level of the knowledge of agent 3 in stage 1 is equal to $\kappa_{13} = \kappa_{13}'$. This requires that agent 1 passes to agent 3 a piece of knowledge equal to $\delta_{13}^{0} = \kappa_{13}' - \kappa_{30}$.

Summarizing all the results obtained in the three cases for $\alpha_{21} > 3$ we obtain:

$$\delta_{13}^{0} = \begin{cases} 0, & \text{if} \quad (1 \leq \kappa_{30}) \cup (\kappa_{13}' \leq \kappa_{30}), \\ \kappa_{13}' - \kappa_{30}, & \text{if} \, (1 > \kappa_{30}) \cap (1 > \kappa_{13}' > \kappa_{30}). \end{cases}$$

If we summarize all our results about δ_{13}^{0} for $\alpha_{21} \leq 3$ ($\kappa_{21} \geq \frac{1}{3}$) and $\alpha_{21} > 3$ ($\kappa_{21} < \frac{1}{3}$) we get

$$\delta_{13}^{0} = \begin{cases} 0, & \text{if} \left(\kappa_{21} \geq \frac{1}{3}\right) \cup \left(\left(\kappa_{21} < \frac{1}{3}\right) \cap \left((1 \leq \kappa_{30}) \cup (\kappa_{13}' \leq \kappa_{30})\right)\right), \\ \kappa_{13}' - \kappa_{30}, & \text{if} \quad \left(\kappa_{21} < \frac{1}{3}\right) \cap \left((1 > \kappa_{30}) \cap (1 > \kappa_{13}' > \kappa_{30})\right). \end{cases}$$

If we focus only on the instances in which the knowledge disclosure occurs and in which it doesn't, we can simplify the last expression and rewrite it as:

$$\delta_{13}^{0} = 0,$$

if $\kappa_{21} \geq \frac{1}{3}$, and

$$\begin{cases} \delta_{13}^{0} = 0, \text{if} \, (1 \leq \kappa_{30}) \cup (\kappa_{13}' \leq \kappa_{30}), \\ \delta_{13}^{0} > 0, \qquad \text{otherwise}, \end{cases}$$

if $\kappa_{21} < \frac{1}{3}$. This completes the proof. ■

Proof of Proposition 8. By Propositions 1, 2, 3, 5, 6 and 7 all best reply functions exist and are uniquely defined over their domains, which implies the existence of a unique equilibrium.

By Proposition 1

$$\delta_{13}^{1*} = \delta_{31}^{1*} = \delta_{23}^{1*} = \delta_{32}^{1*} = 0,$$

which is exactly the first expression in the statement of the proposition.

By Proposition 2

$$\delta_{12}^{0*} = \delta_{21}^{0*} = 0,$$

which is the second expression in the statement of the proposition.

By Proposition 6

$$\delta_{23}^{0*} = 0,$$

which is the third expression in the statement of the proposition.

By Proposition 3

$$\delta_{31}^{0*} = 0,$$

which is the fourth expression in the statement of the proposition.

By Proposition 7, if $\kappa_{21}^{*} \geq \frac{1}{3}$,

$$\delta_{13}^{0*} = 0,$$

and if $\kappa_{21}^{*} < \frac{1}{3}$

$$\begin{cases} \delta_{13}^{0*} = 0, \text{if} \left(1 \leq \kappa_{30}\right) \cup \left(\kappa_{13}'^{*} \leq \kappa_{30}\right), \\ \delta_{13}^{0*} > 0, \text{if} \qquad\qquad \text{otherwise}, \end{cases}$$

where $\kappa_{13}'^{*}$ is defined in eq. (35). These are the fifth and sixth expression of the proposition.

The last expression of the proposition follows from Proposition 5. Using it we obtain that

$$\begin{cases} \delta_{32}^{0*} = 0, \text{if} \left(\kappa_{30} \leq \kappa_{20}\right) \cup \left(\kappa_{32}'^{*} \leq \kappa_{20}\right), \\ \delta_{32}^{0*} > 0, \text{if} \qquad\qquad \text{otherwise}, \end{cases}$$

where $\kappa_{32}'^{*}$ is defined in eq. (28). This completes the proof. ■

Tables

δ_{32}^{0}		α_{02}																					
		1.5	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**6**	**7**	**8**	**9**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
α_{03}	**0.25**	0.0000	0.1496	0.2496	0.3162	0.3639	0.3996	0.4274	0.4496	0.4829	0.5067	0.5246	0.5385	0.5496	0.5996	0.6162	0.6246	0.6296	0.6329	0.6353	0.6371	0.6385	0.6396
	0.5	0.0000	0.1127	0.2127	0.2794	0.3270	0.3627	0.3905	0.4127	0.4461	0.4699	0.4877	0.5016	0.5127	0.5627	0.5794	0.5877	0.5927	0.5960	0.5984	0.6002	0.6016	0.6027
	1	0.0000	0.0521	0.1521	0.2188	0.2664	0.3021	0.3299	0.3521	0.3854	0.4093	0.4271	0.4410	0.4521	0.5021	0.5188	0.5271	0.5321	0.5355	0.5378	0.5396	0.5410	0.5421
	1.25	0.0000	0.0267	0.1267	0.1934	0.2410	0.2767	0.3045	0.3267	0.3600	0.3838	0.4017	0.4156	0.4267	0.4767	0.4934	0.5017	0.5067	0.5100	0.5124	0.5142	0.5156	0.5167
	1.5	0.0000	0.0038	0.1038	0.1705	0.2181	0.2538	0.2816	0.3038	0.3371	0.3610	0.3788	0.3927	0.4038	0.4538	0.4705	0.4788	0.4838	0.4872	0.4895	0.4913	0.4927	0.4938
	1.75	0.0000	0.0000	0.0830	0.1497	0.1973	0.2330	0.2608	0.2830	0.3164	0.3402	0.3580	0.3719	0.3830	0.4330	0.4497	0.4580	0.4630	0.4664	0.4688	0.4705	0.4719	0.4730
	2	0.0000	0.0000	0.0641	0.1307	0.1784	0.2141	0.2419	0.2641	0.2974	0.3212	0.3391	0.3530	0.3641	0.4141	0.4307	0.4391	0.4441	0.4474	0.4498	0.4516	0.4530	0.4541
	2.25	0.0000	0.0000	0.0444	0.1111	0.1587	0.1944	0.2222	0.2444	0.2778	0.3016	0.3194	0.3333	0.3444	0.3944	0.4111	0.4194	0.4244	0.4278	0.4302	0.4319	0.4333	0.4344
	2.5	0.0000	0.0000	0.0000	0.0667	0.1143	0.1500	0.1778	0.2000	0.2333	0.2571	0.2750	0.2889	0.3000	0.3500	0.3667	0.3750	0.3800	0.3833	0.3857	0.3875	0.3889	0.3900
	3	0.0000	0.0000	0.0000	0.0000	0.0476	0.0833	0.1111	0.1333	0.1667	0.1905	0.2083	0.2222	0.2333	0.2833	0.3000	0.3083	0.3133	0.3167	0.3190	0.3208	0.3222	0.3233
	4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0278	0.0500	0.0833	0.1071	0.1250	0.1389	0.1500	0.2000	0.2167	0.2250	0.2300	0.2333	0.2357	0.2375	0.2389	0.2400
	5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0333	0.0571	0.0750	0.0889	0.1000	0.1500	0.1667	0.1750	0.1800	0.1833	0.1857	0.1875	0.1889	0.1900
	6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0238	0.0417	0.0556	0.0667	0.1167	0.1333	0.1417	0.1467	0.1500	0.1524	0.1542	0.1556	0.1567
	7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0179	0.0317	0.0429	0.0929	0.1095	0.1179	0.1229	0.1262	0.1286	0.1304	0.1317	0.1329
	8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0139	0.0250	0.0750	0.0917	0.1000	0.1050	0.1083	0.1107	0.1125	0.1139	0.1150
	9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0111	0.0611	0.0778	0.0861	0.0911	0.0944	0.0968	0.0986	0.1000	0.1011
	10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0500	0.0667	0.0750	0.0800	0.0833	0.0857	0.0875	0.0889	0.0900
	20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0167	0.0250	0.0300	0.0333	0.0357	0.0375	0.0389	0.0400
	30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0083	0.0133	0.0167	0.0190	0.0208	0.0222	0.0233
	40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0050	0.0083	0.0107	0.0125	0.0139	0.0150
	50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0033	0.0057	0.0075	0.0089	0.0100
	60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0024	0.0042	0.0056	0.0067
	70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0018	0.0032	0.0043
	80	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0014	0.0025
	90	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0011
	100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	>100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 1. Equilibrium knowledge sharing parameter δ_{32}^{0} in its nominal levels.

δ_{13}^{0}		α_{02}																					
		1.5	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**6**	**7**	**8**	**9**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
α_{03}	**<=20**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	30	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	40	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0012	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	50	0.0000	0.0000	0.0000	0.0000	0.0000	0.0016	0.0048	0.0062	0.0063	0.0051	0.0035	0.0020	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	60	0.0000	0.0000	0.0000	0.0000	0.0000	0.0049	0.0082	0.0095	0.0096	0.0084	0.0069	0.0053	0.0038	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	70	0.0000	0.0000	0.0000	0.0000	0.0003	0.0073	0.0106	0.0119	0.0120	0.0108	0.0092	0.0077	0.0062	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	80	0.0000	0.0000	0.0000	0.0000	0.0021	0.0091	0.0123	0.0137	0.0138	0.0126	0.0110	0.0095	0.0080	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	90	0.0000	0.0000	0.0000	0.0000	0.0035	0.0105	0.0137	0.0151	0.0152	0.0140	0.0124	0.0108	0.0094	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	100	0.0000	0.0000	0.0000	0.0000	0.0046	0.0116	0.0148	0.0162	0.0163	0.0151	0.0135	0.0120	0.0105	0.0017	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	200	0.0000	0.0000	0.0000	0.0000	0.0096	0.0166	0.0198	0.0212	0.0213	0.0201	0.0185	0.0170	0.0155	0.0067	0.0031	0.0012	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	300	0.0000	0.0000	0.0000	0.0000	0.0113	0.0183	0.0215	0.0229	0.0230	0.0218	0.0202	0.0186	0.0172	0.0084	0.0048	0.0029	0.0017	0.0009	0.0003	0.0000	0.0000	0.0000
	400	0.0000	0.0000	0.0000	0.0000	0.0121	0.0191	0.0223	0.0237	0.0238	0.0226	0.0210	0.0195	0.0180	0.0092	0.0056	0.0037	0.0025	0.0017	0.0011	0.0007	0.0003	0.0001
	500	0.0000	0.0000	0.0000	0.0000	0.0126	0.0196	0.0228	0.0242	0.0243	0.0231	0.0215	0.0200	0.0185	0.0097	0.0061	0.0042	0.0030	0.0022	0.0016	0.0012	0.0008	0.0006
	600	0.0000	0.0000	0.0000	0.0000	0.0130	0.0199	0.0232	0.0245	0.0246	0.0234	0.0219	0.0203	0.0188	0.0101	0.0064	0.0045	0.0033	0.0025	0.0019	0.0015	0.0012	0.0009
	700	0.0000	0.0000	0.0000	0.0000	0.0132	0.0202	0.0234	0.0248	0.0249	0.0237	0.0221	0.0205	0.0191	0.0103	0.0067	0.0048	0.0036	0.0028	0.0022	0.0017	0.0014	0.0011
	800	0.0000	0.0000	0.0000	0.0000	0.0134	0.0203	0.0236	0.0249	0.0251	0.0238	0.0223	0.0207	0.0192	0.0105	0.0069	0.0049	0.0038	0.0029	0.0024	0.0019	0.0016	0.0013
	900	0.0000	0.0000	0.0000	0.0000	0.0135	0.0205	0.0237	0.0251	0.0252	0.0240	0.0224	0.0208	0.0194	0.0106	0.0070	0.0051	0.0039	0.0031	0.0025	0.0021	0.0017	0.0014
	1000	0.0000	0.0000	0.0000	0.0000	0.0136	0.0206	0.0238	0.0252	0.0253	0.0241	0.0225	0.0210	0.0195	0.0107	0.0071	0.0052	0.0040	0.0032	0.0026	0.0022	0.0018	0.0016

Table 2. Equilibrium knowledge sharing parameter δ_{13}^{0} in its nominal levels.

$\frac{\delta_{32}^{0}}{\kappa_{03}-\kappa_{02}}\cdot 100\%$		α_{02}																					
		1.5	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**6**	**7**	**8**	**9**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
α_{03}	**0.25**	0.00%	4.27%	6.93%	8.62%	9.80%	10.66%	11.31%	11.83%	12.60%	13.14%	13.54%	13.85%	14.09%	15.18%	15.54%	15.71%	15.82%	15.89%	15.94%	15.98%	16.01%	16.03%
	0.5	0.00%	7.51%	13.30%	16.76%	19.08%	20.73%	21.97%	22.93%	24.33%	25.30%	26.01%	26.56%	26.99%	28.86%	29.46%	29.76%	29.93%	30.05%	30.14%	30.20%	30.25%	30.29%
	1	0.00%	10.42%	25.35%	32.82%	37.30%	40.28%	42.41%	44.01%	46.25%	47.75%	48.81%	49.61%	50.23%	52.85%	53.67%	54.06%	54.30%	54.45%	54.56%	54.65%	54.71%	54.76%
	1.25	0.00%	8.90%	31.68%	41.44%	46.86%	50.31%	52.70%	54.45%	56.85%	58.41%	59.51%	60.33%	60.96%	63.56%	64.35%	64.74%	64.96%	65.11%	65.22%	65.30%	65.36%	65.41%
	1.5	0.00%	2.29%	38.93%	51.14%	57.25%	60.92%	63.36%	65.10%	67.43%	68.91%	69.94%	70.69%	71.26%	73.59%	74.29%	74.62%	74.82%	74.95%	75.04%	75.11%	75.16%	75.20%
	1.75	0.00%	0.00%	48.44%	62.88%	69.07%	72.50%	74.69%	76.20%	78.16%	79.38%	80.20%	80.80%	81.25%	83.05%	83.57%	83.83%	83.97%	84.07%	84.14%	84.19%	84.23%	84.26%
	2	0.00%	0.00%	64.08%	78.45%	83.24%	85.63%	87.07%	88.03%	89.22%	89.94%	90.42%	90.76%	91.02%	92.02%	92.30%	92.44%	92.52%	92.57%	92.60%	92.63%	92.65%	92.67%
	2.25	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	2.5	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	3	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	6	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	40	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	50	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	60	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%	100.00%
	70	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%	100.00%
	80	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%	100.00%
	90	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%
	100	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	>100	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 3. Equilibrium knowledge sharing parameter δ_{32}^{0} expressed as a percentage of the difference between the knowledge levels of agent 2 and agent 3.

$\frac{\delta_{13}^0}{\kappa_{01}-\kappa_{03}}\cdot 100\%$		α_{02}																					
		1.5	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**6**	**7**	**8**	**9**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
α_{03}	**<=20**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	40	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.12%	0.13%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	50	0.00%	0.00%	0.00%	0.00%	0.00%	0.16%	0.49%	0.63%	0.64%	0.52%	0.36%	0.20%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	60	0.00%	0.00%	0.00%	0.00%	0.00%	0.50%	0.83%	0.97%	0.98%	0.86%	0.70%	0.54%	0.39%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	70	0.00%	0.00%	0.00%	0.00%	0.03%	0.74%	1.07%	1.21%	1.22%	1.10%	0.94%	0.78%	0.63%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	80	0.00%	0.00%	0.00%	0.00%	0.21%	0.92%	1.25%	1.39%	1.40%	1.28%	1.12%	0.96%	0.81%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	90	0.00%	0.00%	0.00%	0.00%	0.35%	1.06%	1.39%	1.53%	1.54%	1.41%	1.26%	1.10%	0.95%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	100	0.00%	0.00%	0.00%	0.00%	0.47%	1.17%	1.50%	1.64%	1.65%	1.52%	1.37%	1.21%	1.06%	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	200	0.00%	0.00%	0.00%	0.00%	0.97%	1.67%	1.99%	2.13%	2.14%	2.02%	1.86%	1.70%	1.56%	0.68%	0.31%	0.12%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	300	0.00%	0.00%	0.00%	0.00%	1.13%	1.83%	2.16%	2.29%	2.31%	2.18%	2.03%	1.87%	1.72%	0.84%	0.48%	0.29%	0.17%	0.09%	0.03%	0.00%	0.00%	0.00%
	400	0.00%	0.00%	0.00%	0.00%	1.21%	1.91%	2.24%	2.38%	2.39%	2.26%	2.11%	1.95%	1.80%	0.93%	0.56%	0.37%	0.25%	0.17%	0.11%	0.07%	0.03%	0.01%
	500	0.00%	0.00%	0.00%	0.00%	1.26%	1.96%	2.29%	2.42%	2.44%	2.31%	2.16%	2.00%	1.85%	0.97%	0.61%	0.42%	0.30%	0.22%	0.16%	0.12%	0.08%	0.06%
	600	0.00%	0.00%	0.00%	0.00%	1.30%	2.00%	2.32%	2.46%	2.47%	2.35%	2.19%	2.03%	1.88%	1.01%	0.65%	0.45%	0.33%	0.25%	0.20%	0.15%	0.12%	0.09%
	700	0.00%	0.00%	0.00%	0.00%	1.32%	2.02%	2.35%	2.48%	2.49%	2.37%	2.21%	2.06%	1.91%	1.03%	0.67%	0.48%	0.36%	0.28%	0.22%	0.17%	0.14%	0.11%
	800	0.00%	0.00%	0.00%	0.00%	1.34%	2.04%	2.36%	2.50%	2.51%	2.39%	2.23%	2.07%	1.93%	1.05%	0.69%	0.50%	0.38%	0.30%	0.24%	0.19%	0.16%	0.13%
	900	0.00%	0.00%	0.00%	0.00%	1.35%	2.05%	2.38%	2.51%	2.52%	2.40%	2.24%	2.09%	1.94%	1.06%	0.70%	0.51%	0.39%	0.31%	0.25%	0.21%	0.17%	0.14%
	1000	0.00%	0.00%	0.00%	0.00%	1.36%	2.06%	2.39%	2.52%	2.53%	2.41%	2.26%	2.10%	1.95%	1.07%	0.71%	0.52%	0.40%	0.32%	0.26%	0.22%	0.18%	0.16%

Table 4. Equilibrium knowledge sharing parameter δ_{13}^0 expressed as a percentage of the difference between the knowledge levels of agent 1 and agent 3.

$\frac{\delta_{32}^{0}}{\kappa_{03}} \cdot 100\%$		α_{02}																					
		1.5	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**6**	**7**	**8**	**9**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
α_{03}	**0.25**	0.00%	3.74%	6.24%	7.91%	9.10%	9.99%	10.68%	11.24%	12.07%	12.67%	13.11%	13.46%	13.74%	14.99%	15.41%	15.61%	15.74%	15.82%	15.88%	15.93%	15.96%	15.99%
	0.5	0.00%	5.64%	10.64%	13.97%	16.35%	18.14%	19.53%	20.64%	22.30%	23.49%	24.39%	25.08%	25.64%	28.14%	28.97%	29.39%	29.64%	29.80%	29.92%	30.01%	30.08%	30.14%
	1	0.00%	5.21%	15.21%	21.88%	26.64%	30.21%	32.99%	35.21%	38.54%	40.93%	42.71%	44.10%	45.21%	50.21%	51.88%	52.71%	53.21%	53.55%	53.78%	53.96%	54.10%	54.21%
	1.25	0.00%	3.34%	15.84%	24.17%	30.12%	34.59%	38.06%	40.84%	45.00%	47.98%	50.21%	51.95%	53.34%	59.59%	61.67%	62.71%	63.34%	63.75%	64.05%	64.28%	64.45%	64.59%
	1.5	0.00%	0.57%	15.57%	25.57%	32.72%	38.07%	42.24%	45.57%	50.57%	54.14%	56.82%	58.91%	60.57%	68.07%	70.57%	71.82%	72.57%	73.07%	73.43%	73.70%	73.91%	74.07%
	1.75	0.00%	0.00%	14.53%	26.20%	34.53%	40.78%	45.64%	49.53%	55.37%	59.53%	62.66%	65.09%	67.03%	75.78%	78.70%	80.16%	81.03%	81.62%	82.03%	82.35%	82.59%	82.78%
	2	0.00%	0.00%	12.82%	26.15%	35.67%	42.82%	48.37%	52.82%	59.48%	64.25%	67.82%	70.59%	72.82%	82.82%	86.15%	87.82%	88.82%	89.48%	89.96%	90.32%	90.59%	90.82%
	2.25	0.00%	0.00%	10.00%	25.00%	35.71%	43.75%	50.00%	55.00%	62.50%	67.86%	71.88%	75.00%	77.50%	88.75%	92.50%	94.38%	95.50%	96.25%	96.79%	97.19%	97.50%	97.75%
	2.5	0.00%	0.00%	0.00%	16.67%	28.57%	37.50%	44.44%	50.00%	58.33%	64.29%	68.75%	72.22%	75.00%	87.50%	91.67%	93.75%	95.00%	95.83%	96.43%	96.88%	97.22%	97.50%
	3	0.00%	0.00%	0.00%	0.00%	14.29%	25.00%	33.33%	40.00%	50.00%	57.14%	62.50%	66.67%	70.00%	85.00%	90.00%	92.50%	94.00%	95.00%	95.71%	96.25%	96.67%	97.00%
	4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	11.11%	20.00%	33.33%	42.86%	50.00%	55.56%	60.00%	80.00%	86.67%	90.00%	92.00%	93.33%	94.29%	95.00%	95.56%	96.00%
	5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.67%	28.57%	37.50%	44.44%	50.00%	75.00%	83.33%	87.50%	90.00%	91.67%	92.86%	93.75%	94.44%	95.00%
	6	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14.29%	25.00%	33.33%	40.00%	70.00%	80.00%	85.00%	88.00%	90.00%	91.43%	92.50%	93.33%	94.00%
	7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.50%	22.22%	30.00%	65.00%	76.67%	82.50%	86.00%	88.33%	90.00%	91.25%	92.22%	93.00%
	8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	11.11%	20.00%	60.00%	73.33%	80.00%	84.00%	86.67%	88.57%	90.00%	91.11%	92.00%
	9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%	55.00%	70.00%	77.50%	82.00%	85.00%	87.14%	88.75%	90.00%	91.00%
	10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	50.00%	66.67%	75.00%	80.00%	83.33%	85.71%	87.50%	88.89%	90.00%
	20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	33.33%	50.00%	60.00%	66.67%	71.43%	75.00%	77.78%	80.00%
	30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	25.00%	40.00%	50.00%	57.14%	62.50%	66.67%	70.00%
	40	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	20.00%	33.33%	42.86%	50.00%	55.56%	60.00%
	50	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	16.67%	28.57%	37.50%	44.44%	50.00%
	60	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	14.29%	25.00%	33.33%	40.00%
	70	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.50%	22.22%	30.00%
	80	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	11.11%	20.00%
	90	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.00%
	100	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	>100	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

Table 5. Equilibrium knowledge sharing parameter δ_{32}^{0} expressed as a percentage of the knowledge level of the knowledge donor - agent 3.

$\frac{\delta_{13}^0}{\kappa_{01}} \cdot 100\%$		α_{02}																					
		1.5	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**6**	**7**	**8**	**9**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
α_{03}	**<=20**	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	40	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.12%	0.13%	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	50	0.00%	0.00%	0.00%	0.00%	0.00%	0.16%	0.48%	0.62%	0.63%	0.51%	0.35%	0.20%	0.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	60	0.00%	0.00%	0.00%	0.00%	0.00%	0.49%	0.82%	0.95%	0.96%	0.84%	0.69%	0.53%	0.38%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	70	0.00%	0.00%	0.00%	0.00%	0.03%	0.73%	1.06%	1.19%	1.20%	1.08%	0.92%	0.77%	0.62%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	80	0.00%	0.00%	0.00%	0.00%	0.21%	0.91%	1.23%	1.37%	1.38%	1.26%	1.10%	0.95%	0.80%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	90	0.00%	0.00%	0.00%	0.00%	0.35%	1.05%	1.37%	1.51%	1.52%	1.40%	1.24%	1.08%	0.94%	0.06%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	100	0.00%	0.00%	0.00%	0.00%	0.46%	1.16%	1.48%	1.62%	1.63%	1.51%	1.35%	1.20%	1.05%	0.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	200	0.00%	0.00%	0.00%	0.00%	0.96%	1.66%	1.98%	2.12%	2.13%	2.01%	1.85%	1.70%	1.55%	0.67%	0.31%	0.12%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	300	0.00%	0.00%	0.00%	0.00%	1.13%	1.83%	2.15%	2.29%	2.30%	2.18%	2.02%	1.86%	1.72%	0.84%	0.48%	0.29%	0.17%	0.09%	0.03%	0.00%	0.00%	0.00%
	400	0.00%	0.00%	0.00%	0.00%	1.21%	1.91%	2.23%	2.37%	2.38%	2.26%	2.10%	1.95%	1.80%	0.92%	0.56%	0.37%	0.25%	0.17%	0.11%	0.07%	0.03%	0.01%
	500	0.00%	0.00%	0.00%	0.00%	1.26%	1.96%	2.28%	2.42%	2.43%	2.31%	2.15%	2.00%	1.85%	0.97%	0.61%	0.42%	0.30%	0.22%	0.16%	0.12%	0.08%	0.06%
	600	0.00%	0.00%	0.00%	0.00%	1.30%	1.99%	2.32%	2.45%	2.46%	2.34%	2.19%	2.03%	1.88%	1.01%	0.64%	0.45%	0.33%	0.25%	0.19%	0.15%	0.12%	0.09%
	700	0.00%	0.00%	0.00%	0.00%	1.32%	2.02%	2.34%	2.48%	2.49%	2.37%	2.21%	2.05%	1.91%	1.03%	0.67%	0.48%	0.36%	0.28%	0.22%	0.17%	0.14%	0.11%
	800	0.00%	0.00%	0.00%	0.00%	1.34%	2.03%	2.36%	2.49%	2.51%	2.38%	2.23%	2.07%	1.92%	1.05%	0.69%	0.49%	0.38%	0.29%	0.24%	0.19%	0.16%	0.13%
	900	0.00%	0.00%	0.00%	0.00%	1.35%	2.05%	2.37%	2.51%	2.52%	2.40%	2.24%	2.08%	1.94%	1.06%	0.70%	0.51%	0.39%	0.31%	0.25%	0.21%	0.17%	0.14%
	1000	0.00%	0.00%	0.00%	0.00%	1.36%	2.06%	2.38%	2.52%	2.53%	2.41%	2.25%	2.10%	1.95%	1.07%	0.71%	0.52%	0.40%	0.32%	0.26%	0.22%	0.18%	0.16%

Table 6. Equilibrium knowledge sharing parameter δ_{13}^0 expressed as a percentage of the knowledge level of the knowledge donor - agent 1.