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# Analysis and Synthesis of pHEMT Class-E Amplifiers With Shunt Inductor Including ON-State Active-Device Resistance Effects 

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#### Abstract

In this theoretical paper, the analysis of the effect that ON-state active-device resistance has on the performance of a Class-E tuned power amplifier using a shunt inductor topology is presented. The work is focused on the relatively unexplored area of design facilitation of Class-E tuned amplifiers where intrinsically low-output-capacitance monolithic microwave integrated circuit switching devices such as pseudomorphic high electron mobility transistors are used. In the paper, the switching voltage and current waveforms in the presence of ON -resistance are analyzed in order to provide insight into circuit properties such as RF output power, drain efficiency, and power-output capability. For a given amplifier specification, a design procedure is illustrated whereby it is possible to compute optimal circuit component values which account for prescribed switch resistance loss. Furthermore, insight into how ON-resistance affects transistor selection in terms of peak switch voltage and current requirements is described. Finally, a design example is given in order to validate the theoretical analysis against numerical simulation.


Index Terms-Class E, high efficiency, ON-channel resistance, pseudomorphic high electron mobility transistor (pHEMT), RF/microwave amplifiers.

## I. Introduction

THE CLASS-E power amplifier was firstly introduced by Sokal [1] in shunt-C/series-tuned configuration with an ideal RF Choke (infinite inductance). Other configurations such as series-C/parallel-tuned and series-L/parallel-tuned with transformer were briefly discussed in [2] by Raab. A revised topology which is simpler than those referenced above was named the "Class-E amplifier with shunt inductor" and was first presented in [3] by Kazimierczuk and later refined in [4]-[6].

The basic Class-E amplifier with shunt-inductor circuit topology shown in Fig. 1 consists of a transistor acting as a switch and a load network. The load network is formed by an inductor shunting the transistor under ac excitation, a series-tuned resonant circuit, and an RF resistive load. Essentially, this configuration is similar to the most common shunt-C/series-tuned configuration, [1], but importantly, for practical realization, it

[^0]
(a)

(b)

Fig. 1. Class-E power amplifier with shunt inductor. (a) Basic circuit. (b) Equivalent circuit.
uses a finite RF choke inductance as the storage element and dispenses with the need for a shunt capacitor. An additional attribute required for successful operation with a device such as a pseudomorphic high electron mobility transistor (pHEMT) is that this configuration requires zero-current switching (ZCS) and zero-current slope switching conditions for the optimum operation whereas the shunt-C/series-tuned Class-E amplifier requires zero-voltage switching (ZVS) and zero-voltage slope switching conditions.

The papers cited above [3]-[6] are orientated toward the design of amplifiers producing several watts of output power and operating at modest frequencies $\leq 1 \mathrm{MHz}$. The switching devices typically used exhibit low ON-resistance and large output capacitance. However, the losses produced by the latter can be neglected at low frequencies. In this paper, the emphasis is on RF applications where modest power $<1 \mathrm{~W}$ is required at low microwave frequencies $<3 \mathrm{GHz}$, and where ultimate physical realization is to be made using GaAs monolithic microwave
integrated circuit (MMIC) technology. The latter requirement suggests that pHEMT switching device technology be used. This type of device typically has low output-capacitance values in the range $0.2-0.5 \mathrm{pF}$, and ON -switch resistances of $2-3 \Omega$, [7], [8]. Therefore, we can neglect switch transistor output shunt capacitance effects, but we cannot ignore output switch series resistance.

Consequently, in this paper, we expand previously reported works by investigating the effect that ON -state transistor resistance has on the performance of a Class-E tuned amplifier with shunt inductor, whereas the analysis of the classic configuration, shunt-C/series-tuned, with an ideal RF choke and a finite dc feed inductance whereby the switching device ON-resistance is taken into account have been respectively discussed in [9]-[11].

## II. Circuit Analysis

An ideal Class-E power amplifiers offers efficiency approaching $100 \%$ since the nonzero switch current and voltage do not occur simultaneously, and therefore no power is dissipated within the switch. However, a real active device when used as a switch offers nonzero ON -state resistance and nonzero OFF-state output capacitance ( $C_{\mathrm{OUT}}$ ) both of which lead to power losses, since the switch voltage is now no longer zero during on-state and the output capacitance is discharged from the voltage $2 V_{\mathrm{CC}}\left(V_{\text {trans }}\right)$ to zero by the time the transistor switches on. At low frequencies the OFF-to-ON switching losses $\left(P_{\text {loss }}\right)$, expressed in (1), due to the discharge mechanism of the output capacitance can be neglected, but at higher frequencies, particularly if high device output capacitance is present this effect becomes comparable to the power losses due to the ON-resistance

$$
\begin{equation*}
P_{\text {loss }}=\frac{1}{2} C_{\text {OUT }} V_{\text {trans }}^{2} f \tag{1}
\end{equation*}
$$

where $f$ is the operating frequency. Fortunately for pHEMT devices [7], [8] operated at microwave frequencies below 3 GHz the effect of $C_{\text {OUT }}$ can be reasonably ignored since its value is low, $0.2-0.5 \mathrm{pF}$ and is responsible for, at most consuming $5 \%$ of RF output power at $V_{\mathrm{CC}}=3 \mathrm{~V}$ and $P_{\mathrm{OUT}}=27 \mathrm{dBm}$. However, once the ON-resistance of the transistor is introduced, the circuit component values derived in [3] are no longer valid. Consequently, a new theoretical analysis taking into account this ON-resistance is necessary.

The analysis presented here is made under following assumptions.

1) The components of the load network are ideal; no parasitic resistances, inductances and capacitances are included. In practice most of these parasitic elements can be absorbed into other load-network elements.
2) The switch duty ratio is $50 \%$, in [3], a $50 \%$ duty ratio was stated as one of the conditions for optimum amplifier operation.
3) The switch has zero output capacitance, zero saturation voltage, and infinite OFF-resistance, but nonzero ON-resistance.
4) The quality factor of series resonant circuit is infinite allowing only pure sinusoidal current flowing through the resistive load.

## A. Switch Current and Voltage Steady-State Waveforms

The basic current and voltage equations for Fig. 1(b) are expressed as

$$
\begin{align*}
i_{L}(t) & =i_{c}(t)+i_{o}(t)  \tag{2}\\
v_{L}(t) & =V_{\mathrm{CC}}-v_{c}(t)=L \frac{d i_{L}(t)}{d t} \tag{3}
\end{align*}
$$

The series-tuned resonant circuit here assumed to have infinite quality factor forces the RF output signal to be purely sinusoidal. $V_{o}$ and $\varphi$ in (4a) are yet to be determined

$$
\begin{align*}
v_{o}(t) & =V_{o} \sin (\omega t+\varphi)  \tag{4a}\\
i_{o}(t) & =\frac{V_{o}}{R} \sin (\omega t+\varphi) \tag{4b}
\end{align*}
$$

The amplifier operation is determined by the switch when it is ON and by the load network when it is OFF. When the switch turns OFF $(0 \leq t \leq \pi / \omega)$, it is open circuit and no current flows through it, consequently the current $i_{L}(t)$ is the same as $i_{o}(t)$ which is sinusoidal. Current $i_{L}(t)$ produces a voltage drop across inductor $L, v_{L}(t)$, which is a cosinusoidal waveform with a dc offset. The difference between the dc supply voltage, $V_{\mathrm{CC}}$, and $v_{L}(t)$ must be dropped within the switch, $v_{c}(t)$, as expressed as

$$
\begin{align*}
& i_{c_{-} \mathrm{OFF}}(t)=0  \tag{5}\\
& i_{L_{-} \mathrm{OFF}}(t)=i_{o}(t)=\frac{V_{o}}{R} \sin (\omega t+\varphi)  \tag{6}\\
& v_{c_{-} \mathrm{OFF}}(t)=V_{\mathrm{CC}}-\frac{V_{o} \omega L}{R} \cos (\omega t+\varphi) \tag{7}
\end{align*}
$$

where $i_{c_{-} \text {OFF }}(t), i_{L_{-} \text {OFF }}(t)$, and $v_{c_{-} \text {OFF }}(t)$ are, respectively, the currents $i_{c}(t), i_{L}(t)$, and the voltage $v_{c}(t)$ during OFF state.

When the switch is $\mathrm{ON}(\pi / \omega \leq t \leq 2 \pi / \omega)$, it is now represented by a nonzero ON-resistance $\left(R_{\mathrm{ON}}\right)$ and thus $\mathrm{vc}(\mathrm{t})$ is no longer zero as it was in the ideal case where $R_{\mathrm{ON}}=0 \Omega$. Kirchoffs current law (2) results in a first-order nonhomogeneous differential equation in terms of $v c(t)$ expressed in (8) and its solution is given by (9a) as follows:

$$
\begin{align*}
& \frac{L}{R_{\mathrm{ON}}} \frac{d v_{c_{-} \mathrm{ON}}}{d t}+v_{c_{-} \mathrm{ON}} \\
& \quad=V_{\mathrm{CC}}-\frac{\omega L V_{o}}{R} \cos (\omega t+\varphi)  \tag{8}\\
& v_{c_{-} \mathrm{ON}}(t) \\
& \quad=V_{\mathrm{CC}}+K e^{-\frac{R_{C N} t}{L} t}-\frac{V_{o}}{R} \frac{R_{\mathrm{ON}} \omega L^{2}}{R_{\mathrm{ON}}^{2}+(\omega L)^{2}} \\
& \quad \times\left\{\frac{R_{\mathrm{ON}}}{L} \cos (\omega t+\varphi)+\omega \sin (\omega t+\varphi)\right\} \tag{9a}
\end{align*}
$$

and further

$$
\begin{equation*}
i_{c_{-} \mathrm{ON}}(t)=\frac{v_{c_{-} \mathrm{ON}}(t)}{R_{\mathrm{ON}}} \tag{9b}
\end{equation*}
$$

The associated current flowing through inductor $L$ is given by

$$
\begin{equation*}
i_{L_{-} \mathrm{ON}}(t)=\frac{1}{L} \int_{\frac{\pi}{\omega}}^{t}\left\{V_{\mathrm{CC}}-v_{c_{-} \mathrm{ON}}(t)\right\} d t+i_{L_{-} \mathrm{ON}}\left(\frac{\pi}{\omega}\right) \tag{10}
\end{equation*}
$$

where $v_{c_{-} \mathrm{ON}}(t), i_{c_{-} \mathrm{ON}}(t)$, and $i_{L_{-} \mathrm{ON}}(t)$ are, respectively, the voltage $v_{c}(t)$ and the currents $i_{c}(t), i_{L}(t)$ during ON state.

The unknown constants $K$ and $i_{L_{-} \mathrm{ON}}(p i / \omega)$ in (9a) and (10), respectively, can be obtained from two boundary conditions which result from the continuity property of current $i_{L}(t)$, namely

$$
\begin{align*}
i_{L_{-} \mathrm{ON}}\left(\frac{\pi}{\omega}\right) & =i_{L_{-} \mathrm{OFF}}\left(\frac{\pi}{\omega}\right)=-\frac{V_{o}}{R} \sin \varphi  \tag{11}\\
i_{L_{-} \mathrm{OFF}}(0) & =i_{L_{-} \mathrm{ON}}\left(\frac{2 \pi}{\omega}\right)  \tag{12}\\
K & =\frac{2 V_{o}}{R} \frac{R_{\mathrm{ON}} e^{\pi / \beta}}{e^{-\pi / \beta}-1} \frac{\beta}{1+\beta^{2}}(\beta \sin \varphi+\cos \varphi) . \tag{13}
\end{align*}
$$

Equation (13) defines a new intermediate variable $\beta$ which is dependent on operating frequency, inductance $L$ and the series ON-resistance

$$
\begin{equation*}
\beta=\frac{\omega L}{R_{\mathrm{ON}}} \tag{14}
\end{equation*}
$$

The ZCS and zero-current slope switching conditions applicable in order to eliminate the power losses due to the ON-to-OFF switching transition of the transistor, respectively, are

$$
\begin{align*}
& i_{c}\left(\frac{2 \pi}{\omega}\right)=0  \tag{15}\\
& \left.\frac{d i_{c}}{d t}\right|_{t=\frac{2 \pi}{\omega}}=0 . \tag{16}
\end{align*}
$$

When applied to (9a) and (9b), these result in

$$
\begin{align*}
\varphi & =\tan ^{-1}\left\{\frac{\beta^{2}\left(e^{\pi / \beta}-1\right)-2}{\beta\left(e^{\pi / \beta}+1\right)}\right\}  \tag{17}\\
\frac{V_{o}}{R} & =\frac{V_{\mathrm{CC}}}{R_{\mathrm{ON}}} N \tag{18}
\end{align*}
$$

where

$$
\begin{equation*}
N=\frac{\left(1+\beta^{2}\right)\left(e^{\pi / \beta}-1\right)}{\beta\left(e^{\pi / \beta}+1\right)(\cos \varphi+\beta \sin \varphi)} \tag{19}
\end{equation*}
$$

It is important to note here that $\varphi$ can be expressed as a function of $\beta$. As a validity check, if we take the limit of $\varphi$ as $R_{\mathrm{ON}}$ approaches $0 \Omega(17)$ reduces to (20), which is the same as (10) in [3]

$$
\begin{equation*}
\lim _{R_{\mathrm{ON}} \rightarrow 0} \varphi=\tan ^{-1}\left(\frac{\pi}{2}\right) \tag{20}
\end{equation*}
$$

## B. DC and Fundamental-Frequency Components

The dc supply current $I_{\mathrm{dc}}$ is computed using the Fourier integral formula as follows:

$$
\begin{equation*}
I_{\mathrm{dc}}=\frac{\omega}{2 \pi} \int_{0}^{\frac{2 \pi}{\omega}} i_{c}(t) d t \tag{21}
\end{equation*}
$$

Since $I_{\mathrm{dc}}=V_{c c} G_{\mathrm{dc}}$, solving (21) leads to an expression for $G_{\mathrm{dc}}$ which is defined as the conductance that the amplifier presents to the power supply

$$
\begin{equation*}
G_{\mathrm{dc}}=\frac{1}{R_{\mathrm{ON}}}\left(\frac{1}{2}-\frac{\beta}{\pi} N \sin \varphi\right) \tag{22}
\end{equation*}
$$

Taking the limit of $G_{\mathrm{dc}}$ as $R_{\mathrm{ON}}$ approaches $0 \Omega$ reduces (22) to (23), which is the same as (50) in [3]

$$
\begin{equation*}
\lim _{R_{\mathrm{ON}} \rightarrow 0} G_{\mathrm{dc}}=\frac{1}{\pi \omega L} \tag{23}
\end{equation*}
$$

The highly nonlinear switching waveform $v_{c}(t)$ produces both fundamental and harmonic signals. The series-tuned resonant circuit $L s-C s$ filters out the harmonic signal contents and the voltage drop across it at the fundamental frequency is zero, leaving only $v_{1}(t)$ at the fundamental, see Fig. 1(b).

On this basis, using Fourier series, voltage $v_{1}(t)$ can be expressed as

$$
\begin{equation*}
v_{1}(t)=v_{x}(t)+v_{o}(t)=V_{x} \cos (\omega t+\varphi)+V_{o} \sin (\omega t+\varphi) \tag{24}
\end{equation*}
$$

where $V_{x}$ and $V_{o}$ can be obtained by means of Fourier integrals as follows:

$$
\begin{align*}
& V_{o}=\frac{\omega}{\pi} \int_{0}^{\frac{2 \pi}{\omega}} v_{c}(t) \sin (\omega t+\varphi) d t  \tag{25}\\
& V_{x}=\frac{\omega}{\pi} \int_{0}^{\frac{2 \pi}{\omega}} v_{c}(t) \cos (\omega t+\varphi) d t \tag{26}
\end{align*}
$$

Voltage $v_{c}(t)$ in (25) and (26) refers to (7) and (9a).
Voltage $v_{1}(t)$ is not purely sinusoidal because the integration described in (26) results in a nonzero $V_{x}$. In other words, an additional reactance $X$ is required to compensate the phase shift between $v_{o}(t)$ and $v_{1}(t)$. The corresponding results are

$$
\begin{align*}
V_{o} & =V_{\mathrm{CC}} P  \tag{27}\\
V_{x} & =-V_{\mathrm{CC}} H \tag{28}
\end{align*}
$$

where (29), shown at the bottom of the page, is true, and

$$
\begin{equation*}
H=\frac{\beta}{2} \frac{2+\beta^{2}}{1+\beta^{2}} N+\frac{2}{\pi} \frac{\beta}{1+\beta^{2}}(\beta \sin \varphi-\cos \varphi) \tag{30}
\end{equation*}
$$

It is important to note here that both $P$ and $H$ are a strong function of $\beta$. The amplitude of the RF output voltage $V_{o}$ is ex-

$$
\begin{equation*}
P=\beta \frac{\pi\left(1+\beta^{2}\right)\left(1-e^{\pi / \beta}\right)+2\left(1+e^{\pi / \beta}\right)\left\{2 \beta+\left(1+\beta^{2}\right) \sin 2 \varphi\right\}}{2 \pi\left(1+\beta^{2}\right)\left(1+e^{\pi / \beta}\right)(\cos \varphi+\beta \sin \varphi)} \tag{29}
\end{equation*}
$$



Fig. 2. Normalized $\varphi^{\prime}$ and $V_{o}^{\prime}$.
pressed by (27) and (29). Taking the limit of $P$ for $R_{\mathrm{ON}}$ approaches $0 \Omega$ reduces (29) to (31), which is (16) of [3]

$$
\begin{equation*}
\lim _{R_{\mathrm{ON}} \rightarrow 0} P=\frac{4}{\pi \sqrt{\pi^{2}+4}} \tag{31}
\end{equation*}
$$

The normalized output phase ( $\varphi^{\prime}$ ) and the normalized RF output voltage ( $V_{o}^{\prime}$ ) respectively defined as the ratio of (17) and (20) and the ratio of (29) and (31) are depicted in Fig. 2; for low $\beta$ (high $R_{\mathrm{ON}}$ ), the output phase and voltage are appreciably affected.

## C. Circuit Component Values

We now derive the circuit component values required for the synthesis of the circuit shown in Fig. 1(a) such that the effect of $R_{\text {ON }}$ can be accounted for. From (18) and (27)

$$
\begin{equation*}
R=\frac{P}{N} R_{\mathrm{ON}} \tag{32}
\end{equation*}
$$

Further

$$
\begin{equation*}
\lim _{R_{\mathrm{ON}} \rightarrow 0} R=\frac{8 \omega L}{\pi\left(\pi^{2}+4\right)} \tag{33}
\end{equation*}
$$

This relates the resistive load to the shunt inductor's value for the ideal case ( $R_{\mathrm{ON}}=0 \Omega$ ). The same result is obtained in [3], as (28).

Since the same current flows through load $R$ and the additional reactance $X$ (34) is valid

$$
\begin{equation*}
\psi=\tan ^{-1}\left(\frac{V_{x}}{V_{o}}\right)=\tan ^{-1}\left(\frac{X}{R}\right) . \tag{34}
\end{equation*}
$$

From (27), (28), and (34): if the ratio $-H / P$ is definite positive or negative then the reactance $X$ must be inductive or capacitive, respectively.
For $X$ capacitive

$$
\begin{align*}
X & =-\frac{1}{\omega C}  \tag{35}\\
C & =\frac{N}{H} \frac{1}{\omega R_{\mathrm{ON}}} . \tag{36}
\end{align*}
$$



Fig. 3. Drain efficiency $\eta$ and normalized $v_{c, \max }^{\prime}$ as a function of $\beta$.

Further

$$
\begin{equation*}
\lim _{R_{\mathrm{ON}} \rightarrow 0} C=\frac{2\left(\pi^{2}+4\right)}{\left(\pi^{2}+12\right) \omega^{2} L} . \tag{37}
\end{equation*}
$$

This relates the reactive capacitance with the shunt inductor's value for $R_{\mathrm{ON}}=0 \Omega$, the same result as obtained in [3], (28) and (29).
Provided that $R$ is known, the series-resonant capacitor and inductor values for a particular $Q_{L C}$ (quality factor of the seriestuned circuit) are forthcoming

$$
\begin{align*}
C s & =\frac{1}{\omega R Q_{L C}}  \tag{38}\\
L s & =\frac{1}{\omega^{2} C s} . \tag{39}
\end{align*}
$$

## D. Power and Drain Efficiency

The RF output power is defined as

$$
\begin{equation*}
P_{\mathrm{OUT}}=\frac{V_{o}^{2}}{2 R}=\frac{1}{2} \frac{V_{\mathrm{CC}}^{2}}{R_{\mathrm{ON}}} N P . \tag{40}
\end{equation*}
$$

Since the dc input power and drain efficiency are defined, respectively, as $P_{\mathrm{DC}}=V_{\mathrm{CC}}{ }^{2} \cdot G_{\mathrm{dc}}$ and $\eta=P_{\mathrm{OUT}} / P_{\mathrm{DC}}$, then upon substituting (22) and (40) we get

$$
\begin{equation*}
\eta=\frac{\pi N P}{\pi-2 N \beta \sin \varphi} \tag{41}
\end{equation*}
$$

The drain efficiency given by (41) is plotted versus $\beta$ in Fig. 3; as $\beta$ decreases from infinity to around 10 , the efficiency drops about $10 \%$ and below this value the efficiency-degradation rate increases rapidly.

## E. Peak Switch Voltage and Current

From (7) and (18), we can define the peak switch voltage as follows:

$$
\begin{equation*}
v_{c, \max }=V_{\mathrm{CC}}(1+N \beta) \tag{42}
\end{equation*}
$$



Fig. 4. Power-output capability, $P_{\max }$, and $M$ as a function of $\beta$.

Its normalized value which is the ratio of the peak switch voltage at any arbitrary $\beta$ values to that at infinite $\beta$, is expressed in (43) and is illustrated in Fig. 3

$$
\begin{equation*}
v_{c, \max }^{\prime}=\frac{2(1+N \beta)}{2+\sqrt{\pi^{2}+4}} \tag{43}
\end{equation*}
$$

It is observed that $v_{c, \text { max }}^{\prime}$ reduces as $R_{\mathrm{ON}}$ increases $(\beta$ decreases), which means that the breakdown failure of the transistor due to high peak switch voltage is minimized and therefore we can relax the requirement to select the appropriate transistor.

Based on (9) the analytical solution required to determine the peak switch current is cumbersome, however, it is possible to solve by means of a graphical approach and parametric study, which will be discussed later in Section III. It is observed that the peak switch current approximately occurs around

$$
\begin{equation*}
t=\frac{2(\pi-\varphi)}{\omega} \tag{44}
\end{equation*}
$$

Upon substituting (44) into (9a) and utilizing (9b), (13) and (18), the peak switch current can be approximated as follows:

$$
\begin{equation*}
i_{c, \max }=\frac{V_{\mathrm{CC}}}{R_{\mathrm{ON}}} M \tag{45}
\end{equation*}
$$

with

$$
\begin{equation*}
M=1+\frac{N \beta}{\beta^{2}+1}\left\{\beta \sin \varphi-\cos \varphi+\frac{2 e^{\frac{2 \varphi}{\beta}}}{1-e^{\frac{\pi}{\beta}}}(\beta \sin \varphi+\cos \varphi)\right\} \tag{46}
\end{equation*}
$$

and $M$ is illustrated in Fig. 4.

## F. Power-Output Capability

For $R_{\mathrm{ON}}=0 \Omega$, the power-output capability $\left(P_{\max }\right)$ of a Class-E amplifier with shunt inductor is 0.0981 (i.e., [3, (26)]), which is the same as that of shunt-C/series-tuned configuration [2, p. 732]. Now, we derive an expression for power-output capability of a Class-E amplifier with shunt inductor for any arbitrary values of $\beta$

$$
\begin{equation*}
P_{\max }=\frac{P_{\mathrm{OUT}}}{v_{c, \max } i_{c, \max }}=\frac{0.5 N P}{(1+N \beta) M} \tag{47}
\end{equation*}
$$



Fig. 5. RF output power versus shunt inductor.

Fig. 4 shows that $P_{\text {max }}$ is reduced when $\beta$ decreases. The maximum level of about 0.0981 is reached when $\beta$ is infinity ( or $R_{\mathrm{ON}}=0 \Omega$ ).

## III. Class-E Amplifier Design Procedure

In power amplifier design, the electrical specification is usually given in terms of dc supply voltage $\left(V_{\mathrm{CC}}\right)$, RF output power ( $P_{\text {OUT }}$ ), and operating frequency $(f)$. For a given transistor technology an active device with appropriate power handling capability can be selected and its ON-resistance $\left(R_{\mathrm{ON}}\right)$ can be accordingly determined. Next, using (40), the value of the shunt inductor $(L)$ is computed numerically. Here $N, P$, and the related phase $\varphi$ are respectively expressed by (19), (29) and (17). Once $L$ is obtained, the values of $R$ and $C$ can be computed by means of (32) and (36), here $H$ is expressed by (30). Finally the series-resonant components $C s$ and $L s$ are obtained from (38) and (39).

In order to further justify the theoretical analysis shown above a design example is now given. We require to design an amplifier whose center frequency $f=2.5 \mathrm{GHz}, V_{\mathrm{CC}}=5 \mathrm{~V}$ and $P_{\mathrm{OUT}}=$ 0.5 W . The switching transistor to be used is to be selected from a range of devices which exhibit ON-resistances, $R_{\mathrm{ON}}$, ranging from 0 to $4 \Omega$.

Based on the specified parameters and using (40), RF output power is plotted versus shunt inductor ( $L$ ), in Fig. 5. Fig. 5 shows that for any nonzero $R_{\mathrm{ON}}$ values, the output power $P_{\mathrm{OUT}}$ has a maximum peak. This implies that the existence of $R_{\text {ON }}$ apparently limits the maximum achievable output power for a given $V_{\mathrm{CC}}$. Importantly for a transistor with $R_{\mathrm{ON}} \gg 4 \Omega$, no value of $L$ can satisfy the output power specification required here i.e., $0.5 \mathrm{~W}(=27 \mathrm{dBm})$.

Now a specific design example is given for $R_{\mathrm{ON}}=2 \Omega$. From Fig. 5, in order to obtain the required output power $P_{\text {OUT }}=$ 0.5 W , the series inductor $(L)$ of 0.85 nH must be chosen. Next we can compute the parameters $\beta, \varphi, N, P$, and $H$, respectively, using (14), (17), (19), (29), and (30) as follows:

$$
\begin{align*}
\beta & =\frac{\omega L}{R_{\mathrm{ON}}}=6.664  \tag{48}\\
\varphi & =\tan ^{-1}\left\{\frac{\beta^{2}\left(e^{\pi / \beta}-1\right)-2}{\beta\left(e^{\pi / \beta}+1\right)}\right\} \\
& =0.96 \mathrm{rad} \text { or } 54.98^{\circ} \tag{49}
\end{align*}
$$

$$
\begin{align*}
N & =\frac{\left(1+\beta^{2}\right)\left(e^{\pi / \beta}-1\right)}{\beta\left(e^{\pi / \beta}+1\right)(\cos \varphi+\beta \sin \varphi)}=0.261  \tag{50}\\
P & =\beta \frac{\pi\left(1+\beta^{2}\right)\left(1-e^{\pi / \beta}\right)+2\left(1+e^{\pi / \beta}\right)\left\{2 \beta+\left(1+\beta^{2}\right) \sin 2 \varphi\right\}}{2 \pi\left(1+\beta^{2}\right)\left(1+e^{\pi / \beta}\right)(\cos \varphi+\beta \sin \varphi)} \\
& =0.306  \tag{51}\\
H & =\frac{\beta}{2} \frac{2+\beta^{2}}{1+\beta^{2}} N+\frac{2}{\pi} \frac{\beta}{1+\beta^{2}}(\beta \sin \varphi-\cos \varphi) \\
& =1.347 \tag{52}
\end{align*}
$$

The circuit element values $R, C, C s$, and $L s$ can then be evaluated using (32), (36), (38), and (39), respectively; $Q_{L C}$ is chosen, for example, 100

$$
\begin{align*}
R & =\frac{P}{N} R_{\mathrm{ON}}=2.34 \Omega  \tag{53}\\
C & =\frac{N}{H} \frac{1}{\omega R_{\mathrm{ON}}}=6.18 \mathrm{pF}  \tag{54}\\
C s & =\frac{1}{\omega R Q_{L C}}=0.27 \mathrm{pF}  \tag{55}\\
L s & =\frac{1}{\omega^{2} C s}=14.9 \mathrm{nH} \tag{56}
\end{align*}
$$

Further, $I_{\mathrm{dc}}, \eta, v_{c, \max }$, and $i_{c, \text { max }}$ are calculated as follows:

$$
\begin{align*}
I_{\mathrm{dc}} & =V_{\mathrm{CC}} G_{\mathrm{dc}}=114.4 \mathrm{~mA}  \tag{57}\\
\eta & =\frac{\pi N P}{\pi-2 N \beta \sin \varphi}=87.4 \%  \tag{58}\\
v_{c, \max } & =V_{\mathrm{CC}}(1+N \beta)=13.7 \mathrm{~V}  \tag{59}\\
i_{c, \max } & =\frac{V_{\mathrm{CC}}}{R_{\mathrm{ON}}} M=406 \mathrm{~mA} \tag{60}
\end{align*}
$$

with

$$
\begin{align*}
G_{\mathrm{dc}}= & \frac{1}{R_{\mathrm{ON}}}\left(\frac{1}{2}-\frac{\beta}{\pi} N \sin \varphi\right)=0.023 \Omega^{-1}  \tag{61}\\
M= & 1+\frac{N \beta}{\beta^{2}+1} \\
& \times\left\{\beta \sin \varphi-\cos \varphi+\frac{2 e^{\frac{2 \varphi}{\beta}}}{1-e^{\frac{\pi}{\beta}}}(\beta \sin \varphi+\cos \varphi)\right\} \\
= & 0.162 \tag{62}
\end{align*}
$$

The circuit component values for other $R_{\mathrm{ON}}$ values can be evaluated in the same way as just described and are presented in Table I.

The circuit in Fig. 1(b) was simulated using Agilent's Advanced Design System (ADS) harmonic balance (HB) and transient circuit simulation software. The former was intended to obtain the steady-state output parameter values to compute the output power and drain efficiency while the latter was used to determine the shape of the time-domain waveforms, in order to observe adherence to theoretical timing requirements. The active device used in the simulation is modeled as a switch which has ON-state resistance ranging from 0 to $4 \Omega$ and $10 \mathrm{k} \Omega$ OFF-state impedance.

Two sets of simulations were conducted; the first simulation assumes that circuit element values do not change as $R_{\mathrm{ON}}$ is varied (i.e., we use the ideal-case component values given in [3]); the second simulation uses the optimal component values

TABLE I
Circuit Component Values of Class-E Amplifiers With Shunt Inductor Where nonzero $R_{\text {ON }}$ IS Accounted for; and Comparison of Theoretical Analysis and Numerical Simulation Results

|  | $R_{O N}=0 \Omega$ | $R_{\text {ON }}=1 \Omega$ | $R_{\text {ON }}=2 \Omega$ | $R_{\text {ON }}=3 \Omega$ | $R_{O N}=4 \Omega$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| THEORETICAL ANALYSIS |  |  |  |  |  |
| $R(\Omega)$ | 2.92 | 2.65 | 2.34 | 1.97 | 1.45 |
| $C$ (pF) | 5.07 | 5.54 | 6.18 | 7.15 | 9.08 |
| $L$ ( nH ) | 1.01 | 0.94 | 0.85 | 0.74 | 0.6 |
| Cs (pF) | 0.22 | 0.24 | 0.27 | 0.32 | 0.44 |
| $L s(\mathrm{nH})$ | 18.6 | 16.9 | 14.9 | 12.5 | 9.23 |
| $\eta$ (\%) | 100 | 94.2 | 87.4 | 79 | 66.8 |
| $P_{\text {OUT }}(\mathrm{mW})$ | 500 | 500 | 500 | 500 | 500 |
| $I_{d c}(\mathrm{~mA})$ | 100 | 106.2 | 114.4 | 126.5 | 149.7 |
| ADS SIMULATION: Ideal-case component values [3] |  |  |  |  |  |
| $\eta$ (\%) | 100 | 94.8 | 90.1 | 85.8 | 81.9 |
| $P_{\text {OUT }}(\mathrm{mW})$ | 491 | 444 | 403 | 369 | 338 |
| $I_{d c}(\mathrm{~mA})$ | 98 | 94 | 90 | 86 | 83 |
| ADS SIMULATION: Optimal component values |  |  |  |  |  |
| $\eta$ (\%) | 100 | 94.1 | 87.4 | 79 | 66.7 |
| $P_{\text {OUT }}(\mathrm{mW})$ | 491 | 491 | 491 | 491 | 490 |
| $I_{d c}(\mathrm{~mA})$ | 98 | 104 | 112 | 124 | 147 |


(a)

(b)

Fig. 6. Steady-state waveforms. (a) Switch voltage. (b) Switch current.
as derived in this paper (following the design procedure explained above). From the second simulation, we can conclude that as $R_{\text {ON }}$ increases the efficiency decreases and the RF output power is not affected significantly (i.e., it remains close to the $0.5-\mathrm{W}$ specification) since the circuit draws more current. On the other hand, from the first simulation, we can observe that as $R_{\mathrm{ON}}$ increases the efficiency degradation is not as significant as that which is resulted from the second simulation, but the required RF output power $(0.5 \mathrm{~W})$ can not be maintained. This observation suggests that when a switching device with a known nonzero active-device ON -resistance is introduced into the circuit, the circuit component values need to be re-computed in order to give the output power required. As can be observed

TABLE II
Corrected Circuit Component Values $R^{\prime}, C^{\prime}$ and Simulation Results in the Presence of $C_{\text {Out }}$ for $R_{\text {ON }}=0 \Omega$ and $R_{\text {ON }}=2 \Omega$

| $C_{\text {OUT }}(\mathrm{pF})$ | $R^{\prime}(\mathbf{\Omega})$ | $C^{\prime}(\mathbf{p F})$ | $P_{\text {OUT }}(\mathrm{mW})$ |  | $I_{a c}(\mathbf{m A})$ |  | $\boldsymbol{\eta}$ (\%) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $R-C$ | $R^{\prime}-C^{\prime}$ | R-C | $R^{\prime}-C^{\prime}$ | $R-C$ | $R^{\prime}-C^{\prime}$ |
| $\mathbf{R}_{\text {ON }}=\mathbf{0} \boldsymbol{\Omega}$ |  |  |  |  |  |  |  |  |
| 0.2 | 3.2 | 4.9 | 572 | 491 | 116 | 101 | 98.6 | 97 |
| 0.5 | 3.6 | 4.6 | 722 | 507 | 146 | 104 | 98.6 | 97.4 |
| $\mathrm{R}_{\text {ON }}=\mathbf{2 \Omega}$ |  |  |  |  |  |  |  |  |
| 0.2 | 2.5 | 6 | 532 | 495 | 128 | 116 | 83.4 | 85.6 |
| 0.5 | 2.8 | 5.7 | 576 | 494 | 153 | 120 | 75.3 | 82.2 |



Fig. 7. Waveforms of the currents and voltages in the presence of $C_{\mathrm{OUT}}$ for $R_{\mathrm{ON}}=0 \Omega$; left: using $R$ and $C$, right: using $R^{\prime}$ and $C^{\prime}$. (a) $C_{\mathrm{OUT}}=0.2 \mathrm{pF}$. (b) $C_{\text {OUT }}=0.5 \mathrm{pF}$.
from the data presented in Table I, the second simulation results for $\eta, P_{\mathrm{OUT}}$ and $I_{\mathrm{dc}}$ are in excellent agreement with the analytical prediction equations derived in this paper.

Switch voltage and current waveforms in the steady state are presented in Fig. 6. Here, as $R_{\text {ON }}$ increases, the peak switch voltage during the OFF-state decreases (in line with the theoretical analysis, Fig. 3, $v_{c, \text { max }}^{\prime}$ ) but the peak switch current increases. As can be seen from Fig. 6(b), for $R_{\mathrm{ON}}=4 \Omega$, the peak
switch current increases to 1.5 times higher than the ideal case where $R_{\mathrm{ON}}=0 \Omega$. Therefore, for higher $R_{\mathrm{ON}}$ values care must be taken to choose an appropriate transistor which can sustain this larger peak current otherwise amplifier failure may occur.

## IV. Effect of Small Transistor Output Capacitance

In this section, we investigate the effect that small output capacitance of MMIC pHEMT device $\left(C_{\mathrm{OUT}}\right)$ has on the perfor-


Fig. 8. Waveforms of the currents and voltages in the presence of $C_{\mathrm{OUT}}$ for $R_{\mathrm{ON}}=2 \Omega$; left: using $R$ and $C$, right: using $R^{\prime}$ and $C^{\prime}$. (a) $C_{\mathrm{OUT}}=0.2 \mathrm{pF}$. (b) $C_{\text {OUT }}=0.5 \mathrm{pF}$.
mance of Class-E amplifier with shunt inductor. The presence of $C_{\text {OUT }}$ during OFF state changes the fundamental load-network impedance as required for optimum output power, and therefore an adjustment as described in (63) and (64) has to be made, see Fig. 1(b). Parameters $R^{\prime}, C^{\prime}$, and $L^{\prime}$ are the corrected $R, C$, and $L$, respectively. One of these parameters, for example $L^{\prime}$, may be fixed ( $L^{\prime}=L$ ) and consequently the other parameters $R^{\prime}$ and $C^{\prime}$ can be determined by solving

$$
\begin{align*}
\frac{(\omega C)^{2} R}{1+(\omega C R)^{2}}= & \frac{\left(\omega C^{\prime}\right)^{2} R^{\prime}}{1+\left(\omega C^{\prime} R^{\prime}\right)^{2}}  \tag{63}\\
\frac{\omega C}{1+(\omega C R)^{2}}-\frac{1}{\omega L}= & \frac{\omega C^{\prime}}{1+\left(\omega C^{\prime} R^{\prime}\right)^{2}} \\
& -\frac{1}{\omega L^{\prime}}+\omega C_{\text {OUT }} \tag{64}
\end{align*}
$$

ADS simulations are carried out to observe how deleterious the effect that small output capacitance of MMIC switching de-
vice such as pHEMT ranging from 0.2 to 0.5 pF has on the amplifiers performance in terms of output power and drain efficiency. Two cases are observed here i.e., for $R_{\mathrm{ON}}=0 \Omega$ and $R_{\mathrm{ON}}=2 \Omega$. Table II gives the revised component values of $R$ and $C$, i.e., $R^{\prime}$ and $C^{\prime}$ as well as the output parameters $P_{\mathrm{OUT}}$ and $\eta$ resulted from HB simulations for the case when $R_{\mathrm{ON}}=0 \Omega$ and $R_{\mathrm{ON}}=2 \Omega$. It is obvious from Table II that the specified output power (i.e., 0.5 W ) can be reached only when the revised circuit component values $R^{\prime}$ and $C^{\prime}$ are used.

Time-domain waveforms of the currents and voltages depicted in Figs. 7 and 8 for $R_{\mathrm{ON}}=0 \Omega$ and $R_{\mathrm{ON}}=2 \Omega$ respectively, are obtained from ADS transient simulations. By using uncorrected component values of $R$ and $C$, the switch voltage waveform, $v_{c}(t)$, is distorted in the presence of even small $C_{\text {OUT }}$ (see Figs. 7 and 8 left-hand side). On the other hand, when using $R^{\prime}$ and $C^{\prime}$ the switch voltage waveform can be maintained as in ideal case (zero $C_{\text {OUT }}$ ) however the ZCS and zero-current slope switching conditions can not be ful-
filled which leads to efficiency degradation (see switch current waveform, $i_{c}(t)$, in Figs. 7 and 8 right-hand side). More importantly, using the uncorrected component values $R, C$ when compared to $R^{\prime}, C^{\prime}$ results in higher switch peak voltage which may cause transistor's failure due to breakdown mechanisms. This problem becomes even more severe for higher $C_{\text {OUT }}$; comparing Fig. 7(a) with Fig. 7(b) and Fig. 8(a) with Fig. 8(b).

In Fig. 8, it is evident that a small rise of switch voltage occurs during ON state since nonzero ON resistance (i.e., $R_{\mathrm{ON}}=2 \Omega$ ) is included in the circuit. Further, it is also observed that the output parameters $P_{\text {OUT }}$ and $\eta$ computed from HB simulation results are in good agreement with those obtained from transient simulation results which are run until 150 periods.

## V. Conclusion

A new theoretical analysis in which the effects that activedevice ON-resistance has on the behavior of a Class-E power amplifier with shunt inductor has been presented. Introducing ON -state transistor resistance into the circuit changes its properties and therefore a revised design procedure is necessary in order to re-compute optimal circuit component values. A design example for a $0.5-\mathrm{W} 5-\mathrm{V} 2.5-\mathrm{GHz}$ Class-E amplifier simulated in ADS was given to validate the theoretical analysis presented in this paper. The results obtained show excellent agreement with the theoretical analysis. It was shown that by using a transistor which has an ON-state resistance up to $2 \Omega$ results in a dc-to-RF efficiency of above $85 \%$. Moreover the peak switch voltage and current decreases and increases respectively as $R_{\mathrm{ON}}$ increases; careful attention must be paid in the selection of an appropriate transistor which can sustain higher peak switch current. The power-output capability is also affected by the presence of $R_{\mathrm{ON}}$; as $R_{\mathrm{ON}}$ increases, the power-output capability decreases below 0.0981 . The work presented in this paper should facilitate the design of Class-E power amplifiers where intrinsically low-output-capacitance MMIC switching devices such as pHEMTs are to be used. Such amplifiers would find a broad application in mobile wireless and telemetry/sensor applications where high dc to RF efficiency is of importance.

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