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Evaluating the approximation of the affinity laws and improving the estimate of the efficiency for variable speed pumping

by

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15   **Abstract**

16   Affinity laws relate to the characteristics of pumps operating at different speeds, and in a

17   water distribution context are usually used to predict the pump curve of variable speed pumps

18   (VSPs). VSPs can adjust the pump curve so as to meet the network requirements more

19   efficiently with resultant savings of energy. The estimation of the effectiveness of a VSP is

20   based on hydraulic simulations, where the behaviour of VSPs is described using the affinity

21   laws. The affinity laws, however, contain approximations, as they do not take into account

22   factors that do not scale with velocity. In particular, the approximation inherent in the affinity

23   law that computes power and efficiency can produce a misleading result, especially for small

24   size pumps. The research reported in this paper estimates the error in efficiency for a wide

25   range of pump sizes and tests the use of the formula proposed by Sarbu and Borza (1998) as

26 an alternative to the affinity law. Results show that a better estimation can be achieved for the

27 efficiency of small and medium size pumps. Moreover the formula can be easily

28 implemented in hydraulic solvers.

29

30 **Keywords:** efficiency; variable speed pump; water distribution system; EPANET.

31

32 **Introduction**

33 In water distribution systems (WDSs), the component that consumes the most energy is the

34 pump (Carlson & Walburger, 2007), and a reduction in energy use in the system can only be

35 achieved by reducing pump inefficiencies and power demands. System designers wanting to

36 reduce power consumption and costs, therefore, often evaluate the possibility of replacing

37 fixed speed pumps with variable speed pumps (VSPs). VSPs have motors linked to a variable

38 speed drive so that the motor can be run at different speeds, resulting in a change of the

39 characteristic pump curve. Therefore, with VSPs, the pump operating point can be more

40 efficiently adapted to the system requirements than with standard fixed speed pumps

41 (Lingireddy and Wood 1998).

42 The estimate of the energy savings achievable with VSPs is usually accomplished by using

43 hydraulic solvers to simulate WDS and pumping system behavior. Although the energy

44 consumption has to be estimated by taking into account the efficiency of all VSP components

45 (pump, motor and variable speed drive) (Bernier and Bourret, 1999; Walski, 2001; Walski et

46 al. 2003; Walski, 2005; Marchi et al. 2012a, Marchi et al. 2012b) , in the initial assessment,

47 only the pump power is usually considered, because of the difficulty in finding motor and

48 variable speed drive data and because of the lower efficiency of pumps compared to the other

49 VSP components (motors and variable speed drives often have efficiencies as high as or

50 greater than 95%). Note that, despite this, motor and variable speed drive efficiencies affect

51 the wire to water efficiency, and cannot be neglected in the computation of the total power

52 and energy required. Moreover, the effectiveness of VSPs has to be assessed for each specific

53 WDS, taking into account water distribution system requirements, costs and savings

54 throughout the design life of the intervention.

55 This paper focuses on the efficiency of the pump in a VSP system. When the speed is

56 changed, the pump efficiency is usually estimated using the efficiency-flow curve at the

57 nominal pump speed and the affinity laws, which describe the relationship between the

58 variables involved in pump performance, such as flow, total head and power, and the pump

59 speed. However, affinity laws contain approximations that can affect the estimation of pump

60 efficiency. This paper highlights the magnitude of this approximation and proposes the use of

61 a formula presented by Sarbu and Borza (1998) as an alternative method for estimating the

62 efficiency of VSPs.

63

64 **Affinity laws for VSPs**

65 The affinity laws are commonly used to describe pump behaviour (flow, head and power)

66 when pump speed is changed. The laws reflect the fact that dimensionless characteristics

67 (dimensionless flow, C_Q, dimensionless head, C_H, and dimensionless power, C_P) are constant

68 for similar pumps. Dimensionless pump characteristics presented in Eq. 1 directly relate flow,

69 Q (m³/s), head, H (m), and power, P (W) to the speed, N (rpm) and to the impeller diameter,

70 D (m), of the pump.

71   a) $C_Q = \dfrac{Q}{N \cdot D^3}$ b) $C_H = \dfrac{gH}{N^2 \cdot D^2}$ c) $C_P = \dfrac{P}{\rho \cdot N^3 \cdot D^5}$ (Eq. 1)

72   where ρ is the liquid density (kg/m^3) and g is the acceleration of gravity (m/s^2).

73   The efficiency, η, is indirectly described by Eq. 1c as the power, P, is

74
$$P = \frac{\rho \cdot g \cdot Q \cdot H}{\eta}$$
(Eq. 2)

75   For variable speed pumps, only the pump speed (and not the impeller diameter) is modified,

76   thus the following equations, known as affinity laws, can be derived:

77   a) $\dfrac{Q_1}{Q_2} = \dfrac{N_1}{N_2}$ b) $\dfrac{H_1}{H_2} = \left(\dfrac{N_1}{N_2}\right)^2$ c) $\dfrac{P_1}{P_2} = \left(\dfrac{N_1}{N_2}\right)^3$ (Eq. 3)

78   where subscripts 1 and 2 refer to two different pump speeds.

79   The affinity laws state that the change in pump flow, head and power is a linear, squared and

80   cubic function of the change in pump speed, respectively. In particular, equations 1c and 3c

81   assume that a point on the pump curve maintains the same efficiency when the pump is run at

82   different speeds as shown in Figure 1. Hence the efficiency of the best efficiency point (BEP)

83   is constant. Figure 1 also shows that the difference between the use of Eq. 1c or 3c when the

84   pump speed is changed is that in a graph $\eta\text{-}C_Q$ the efficiency points of a pump operated at

85   different speeds lie on the same curve, while in a graph $\eta\text{-}Q$ the curve is scaled according to

86   Eq. 3a.

87

88   **Approximations in the affinity laws**

89   As mentioned, the affinity laws relate flow, head and power (and hence efficiency) to the

90   speed of the pump. However, they cannot take into account the factors that do not scale with

91   velocity and whose magnitude depends on the machine size, and must, therefore, rely on

92   approximations. For centrifugal pumps, Eqs. 3a and 3b are a good approximation of real

93   pump behavior for a wide range of speeds, and the impact of the factors affected by the

94   machine size can be easily neglected. In contrast, the approximation in the power and

95   efficiency relationships is larger, especially for smaller pumps.

96   Table 1 shows the differences between the measured best efficiency point (BEP) and the

97   efficiency predicted by the affinity laws for pumps with different power specifications. As

98   can be seen, for the large HPL 54-30-20 Sulzer pump (data from Ulanicki et al. 2008)

99   reducing the speed from $N_1 = 1525$ rpm to $N_2 = 1182$ rpm (relative speed=0.775) does not

100   decrease appreciably the efficiency (BEP at $N_1 = 83.6\%$, while BEP at $N_2 = 83.5\%$).

101   However, this is a large pump that reaches 556 kW of power at the higher flow. If a small

102   pump is considered, the decrease in efficiency is greater. For example, for the small pump

103   50x32-160H.T. (5.5 kW), the affinity laws predict a BEP equal to 56% instead of the 52%

104   measured for the BEP at 2000 rpm.

105

106   **Improving the efficiency estimation**

107   The decrease in efficiency when the pump speed is lowered has been reported by several

108   authors. Morton (1975) proposed a graphical method to identify the new curve, while Sarbu

109   and Borza (1998) related the new efficiency to the original efficiency and the speed. More

110   general approaches for tackling the problem of efficiency scaling previously considered that

111   the efficiency is a function of the Reynolds number ($Re = VD/v$, where V is the velocity of the

112   fluid (m/s), D is the diameter (m) and v is the kinematic viscosity (m^2/s)) and the interior

113   roughness of the pipe. As reported by Gűlich (2003), where a comprehensive review of the

114   past works can be found, one of the earliest methods to scale the efficiency of similar pumps

115   was formulated by noting that only a part of the energy losses, K, is a function of the

116   Reynolds number:

117
$$\frac{1-\eta_2}{1-\eta_1} = K + (1-K)\left(\frac{Re_1}{Re_2}\right)^m \qquad (4)$$

118   where η_1 and Re_1 are the efficiency and the Reynolds number at the speed N_1 and η_2 and Re_2

119   are the efficiency and the Reynolds number at the reduced speed N_2. By assuming that all

120   losses are independent of Re (K=0) and the value of m is equal to 0.1, Eq. 4 can be rewritten

121   as the Sarbu and Borza formula (Eq. 5). Note that the ratio of Reynolds numbers in Eq. 4

122   (Re_1/Re_2) is replaced by the ratio of the pump speeds (N_1/N_2) as the velocity of the fluid is

123   proportional to the pump speed, while diameter and viscosity do not change.

124
$$\eta_2 = 1 - (1-\eta_1) \cdot \left(\frac{N_1}{N_2}\right)^{0.1} \qquad (5)$$

125   According to Eq. 5, the efficiency decrease is greater for a large reduction of speed and

126   further decreases the efficiency at already low efficiency points on the pump curve. Although

127   the approach presented in Eq. 4 and 5 is limited by the fact that the exponent m depends on

128   the Reynolds number and the roughness (Osterwalder, 1978; Gűlich, 2003), the Sarbu and

129   Borza formula can provide reasonably good results if the pump speed is not reduced below

130   70% of the nominal speed.

131   Figure 2 shows the efficiency prediction using Eq. 5 for a wide range of efficiency points and

132   speeds. It can be seen that the predicted efficiency can be nearly equal to the original one if

133   the point has a high efficiency and the relative speed is above 0.7 as suggested by Sarbu and

134   Borza (1998). In contrast, if the efficiency is below 0.5 already, the formula predicts a lower

135   efficiency even for a small reduction in speed. The formula can also produce negative

136   efficiencies. However, these results are located in low efficiency regions of the pump ($\eta<0.3$)

137   and for small relative speeds ($N_1/N_2<0.40$) that are usually avoided in practice.

138   Figure 3 compares the data for the small pump 50x32-160H.T. (5.5 kW), the curves of which

139   were taken from the Thompsons Kelly and Lewis catalogue (1989), and the prediction using

140   the affinity laws and the Sarbu and Borza (1998) relation. A reduced speed of 2000 rpm (0.56

141   of the nominal speed equal to 3600 rpm) has been chosen to better visualize the differences.

142   As can be seen, the affinity laws can predict well the *Q-H* relation and the small differences

143   can be attributed to errors in retrieving the actual data. However, the prediction of efficiency

144   is appreciably inaccurate. The pump does not maintain the same efficiency. In particular, the

145   BEP at 2000 rpm is 52% instead of the 56% obtained with the affinity laws. Efficiency

146   results using Eq. 5 are unequivocally a better approximation than the affinity law.

147   Although the prediction of efficiency still contains some approximation, the use of the Sarbu

148   and Borza formula (Eq. 5) instead of the affinity law related to power, enables a more

149   accurate estimation of efficiency for a wide range of pump speeds. The formula overestimates

150   the efficiency only for small pumps. For medium-large pumps, it can predict well the BEP

151   (see Table 1, 125x80-250 Hydro-Titan 90 kW) and, only for very large pumps, does it

152   underestimate the BEP and therefore estimate a larger pump power.

153   Finally it should be noted that Eq. 5 can be easily implemented in hydraulic solvers, as it

154   requires the same input data as those used by the affinity laws: the efficiency at the nominal

155   speed, η_1, and the inverse of the pump relative speed. Appendix A shows how to modify the

156   popular hydraulic solver EPANET 2 source code (Rossman, 2000) to compute VSP

157   efficiency at the reduced speeds using the Sarbu and Borza relationship.

158

159

160

161 **Conclusion**

162 Assessing the cost effectiveness of variable speed pumps requires, amongst other factors (e.g.

163 estimating motor and variable speed drive efficiency), the estimate of the efficiency of the

164 pump at reduced speeds. This task is usually accomplished by using the affinity laws.

165 However, the affinity laws cannot take into account the effects of factors that do not scale

166 with velocity, the efficiency computed using the affinity laws unavoidably contains an

167 approximation. The use of the Sarbu and Borza (1998) relationship to estimate the pump

168 efficiency of VSPs is proposed in order to decrease the error in the efficiency estimation,

169 especially for small pumps. As this formula requires the same input data as that used by the

170 affinity laws, it can be easily implemented in hydraulic solvers: the modification of the

171 EPANET 2 solver has been presented.

172

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206 **Appendix A: How to compute VSP efficiency in EPANET 2 using Sarbu and Borza**

207 **equation**

208 This appendix shows how to modify EPANET 2 to compute VSP efficiency using Sarbu and

209 Borza (1998) equation. Only the function *getenergy()* needs to be modified: the few lines of

210 code added are shown in italics. The Sarbu and Borza formula only requires the knowledge of

211 the efficiency at the nominal speed, *i.e.* η_1 in Eq. 3 and *eN* in the following source code, and

212 of the relative speed, *i.e.* the inverse of N_1/N_2 in Eq. 3 and *patMult* in the source code. Note

213 that *eN* is initially a percentage because of the way the efficiency curve is inserted in

214 EPANET and has to be converted in a rate to be used in Sarbu and Borza formula.

215 Afterwards, *eN* has to be reconverted in a percentage to continue the energy computation.

216 Note that the variable *eN* is introduced for clarity, but can be omitted and substituted with the

217 variable *e* of EPANET 2.

218

219 void getenergy(int k, double *kw, double *eff)

220 {

221   int   i,j;

222   double dh, q, e;

223   *double q4eff, patMult; //corresponding flow at the nominal speed, relative speed factor*

224   *long p, temp; //variables used to compute the relative speed if it is inserted as a pattern*

225   *double eN; //efficiency at the nominal speed*

226   [...]

227   /* For pumps, find effic. at current flow */

228   if (Link[k].Type == PUMP)

229   {

230   j = PUMPINDEX(k);

231   e = Epump;

```
232        if ( (i = Pump[j].Ecurve) > 0)

233     { //compute the relative speed patMult for the specific time step

234        if ( Pump[j].Upat > 0 ) {

235           p = (Htime+Pstart)/Pstep;

236           temp = p % (long) Pattern[Pump[j].Upat].Length;

237           patMult = Pattern[Pump[j].Upat].F[temp];

238         }

239       else patMult = K[k];

240      q4eff = q/patMult;

241      eN = interp(Curve[i].Npts,Curve[i].X,Curve[i].Y,q4eff*Ucf[FLOW]);

242      eN = eN/100; // convert to use in Sarbu and Borza formula

243      e = 1- ((1-eN)*pow((1/patMult),0.1)));  // Sarbu and Borza formula

244      e = e*100; // convert in percentage to use with EPANET     }

245

246      e = MIN(e, 100.0);

247  [...]


248
```

249

Table 1. Comparison of data and predictions of efficiency.

Pump	Power (kW)	Data		Efficiency Prediction		Difference (%)	
		BEP at N_1	BEP at N_2	BEP Affinity laws	BEP Sarbu & Borza	BEP Affinity laws	BEP Sarbu & Borza
HPL 54-30-20 [i]	556	83.6	83.5	83.6	83.2	0.12	-0.36
Hydro-Tytan 125x80-250 [ii]	90	80.5	79.1	80.5	79.3	1.77	0.25
50x32-160H.T. [ii]	5.5	56.0	52.0	56.0	53.3	7.69	2.50

251 [i] Sulzer; data reported by Ulanicki et al. (2008). N_1 = 1525 rpm, N_2 = 1182 rpm;

252 [ii] Thompsons Kelly and Lewis (1989) N_1 = 3600 rpm, N_2 = 2000 rpm.
253