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Transmission and Generation Investment in Electricity Markets: The Effects of Market Splitting and Network Fee Regimes

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TRANSMISSION AND GENERATION INVESTMENT IN ELECTRICITY MARKETS: THE EFFECTS OF MARKET SPLITTING AND NETWORK FEE REGIMES

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ABSTRACT. In this paper we propose a three–level computational equilibrium model that allows to analyze the impact of the regulatory environment on transmission line expansion (by the regulator) and investment in generation capacity (by private firms) in liberalized electricity markets. The basic model analyzes investment decisions of the transmission operator (TO) and private firms in expectation of an energy only market and cost-based redispatch. In different specifications we consider the cases of one versus two price zones (market splitting) and analyze different approaches to recover network cost, in particular lump sum, capacity based, and energy based fees. In order to compare the outcomes of our multi–stage market model with the first best benchmark, we also solve the corresponding integrated planer problem. In two simple test networks we illustrate that energy only markets can lead to suboptimal locational decisions for generation capacity and thus, imply excessive network expansion. Market splitting heals those problems only partially. Those results obtain for both, capacity and energy based network tariffs, although investment slightly differs across those regimes.

Keywords: Electricity markets, Network Expansion, Generation Expansion, Investment Incentives, Computational Equilibrium Models, Transmission Management

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1. Introduction

Following the British privatization in the 1980s, various countries around the world liberalized their electricity sectors. Today, in most industrialized countries only network facilities remain regulated while private firms decide on investment in generation capacities, and trade electricity on markets. This structure challenges the planning of transmission and generation capacity expansion. While an entirely regulated electricity sector allows for simultaneous transmission and generation expansion planning, in a liberalized market, investment decisions in transmission and generation capacities are taken by different agents. Investment in generation capacities is typically made by firms and private investors, based on their expectations concerning the future regulatory environment and taking into account available network facilities. Network expansion, however, is decided on by regulated firms (or even the regulator), in anticipation of capacity investments by private firms. Traditional optimization approaches, while they reveal the optimal expansion plan for transmission and generation, do not offer valuable information on how to achieve those goals in a market environment. In a liberalized market, incentives induced by the interplay of market environment and regulation determine whether firms indeed make the appropriate investment choices. Liberalized electricity markets thus call for new tools to inform the various agents involved — regulators, electricity firms, investors, and other stakeholders.

In this paper we propose a model that allows to analyze investment in transmission expansion (by the transmission operator) and generation expansion (by firms) in liberalized electricity markets. We model environments with energy only markets and a regulated transmission operator who uses redispatch to deal with transmission constraints. In a multistage analysis we study transmission expansion decisions by the regulated transmission operator in anticipation of capacity expansion by private firms. In different specifications of our model we analyze the effects of market splitting (one versus multiple price zones) as well as different approaches to recover network cost, in particular lump sum, capacity based, and generation based fees. In order to compare the outcomes to the First Best we also solve the integrated planer problem. In this paper we restrict ourselves to solving stylized test cases to illustrate the applicability of our framework. Our results demonstrate that investment choices in a market environment can substantially differ from First Best solution. In our numerical examples, the absence of proper locational investment incentives for firms clearly affects investment choices of generators, which, in turn, leads to excessive line investment. Our results demonstrate that our model allows to compare different network management regimes and assess their effects on long run investment decisions. Our approach is, thus, an important extension of various studies that have mainly considered the short run properties of different transmission management regimes (see the literature review). As we show, transmission management has also important implications in the long run when generation and transmission expansion are taken into account.

Let us emphasize that our approach allows to assess the impact of different regulatory regimes on investment under conditions that obtain in various industrialized countries around the world. Especially in Europe, spot market trading does not fully account for transmission constraints, but capacities are shut down and called for by the transmission operator in case the spot market solution is not feasible. Under cost based redispatch (as it is used in Austria, Switzerland, or Germany) firms called into operation are just compensated for their variable production cost. Consequently, redispatch operations cannot contribute to the recovery of investment costs.[1] Several countries thus try to induce adequate locational investment incentives via market splitting, i.e the establishment of predefined zones where different spot market prices obtain in case of transmission constraints. As it is well known, various countries outside Europe go even further and introduced variants of nodal pricing (Australia, New Zealand, Russia or the United States, see Joskow, 2008), where electricity prices reflect transmission constraints and induce adequate locational investment incentives for generation capacities run by private firms. Since in our framework the first best solution coincides with the outcome under nodal pricing, our approach also allows to assess the long run benefits from a change to a nodal pricing system.

Let us briefly illustrate the relevance of our model for important policy considerations by example. In 2011, after the Fukushima nuclear accident, the German government decided to shut down all nuclear power plants by 2020. At the same time, renewable production was heavily promoted. Those developments caused an increasing concentration of generation capacities in the northern part of the country, since cheap wind power and coal plants have locational advantages there. In the south, increasing photovoltaic capacities require a backup to compensate for fluctuations in production. Alternative solutions to these problems are e.g. investment in gas plants located in the south or transmission line expansion to provide the south with power from the north. While global optimization suggests that gas plants solve arising problems at lower cost, the current German regulation does not provide sufficient investment incentives. As a consequence a substantial expansion of transmission capacities is planned, implying investment cost of over 30 bn. Euro. Power

[1]Market based redispatch (used, e.g., in Belgium, Finnland, France, or Sweden) yields rents for firms that are called for at the redispatch stage and thus, induces incentives to build plants at locations with systematic underprovision.

market models as the one suggested in this paper enable us to (i) identify those problems and their causes, and (ii) to analyse and compare different regulatory regimes that might be capable to induce adequate investment incentives. In the case of the German market sketched above, interesting scenarios include market splitting (so that producers in the south are rewarded for their locational choice) and changes in the redispatch system (to a system that rewards the right locational choices). Both measures would make production in the south more profitable and could contribute to increased investment incentives in the relevant locations.

We finally review the related literature. Prior to the liberalization of electricity sectors around the world, vertically integrated monopolists (either regulated or directly state owned) were responsible for generation and transmission expansion. To achieve their goals they needed insights on the cost minimal system configuration. As a consequence, traditionally, most of the contributions thus proposed frameworks and techniques to determine overall optimal expansion for generation and transmission facilities. Compare for example Gallego et al. (1997), Binato et al. (2001), Alguacil et al. (2003), or de Oliveira et al. (2005).

In liberalized electricity markets, however, we observe vertical unbundling of transmission and generation facilities. In addition to insights on the global optimum we thus need research on how the market environment affects decisions of the different stakeholders. By now a large literature has developed which explicitly analyzes incentives of private, potentially strategic firms to invest in generation facilities, completely disregarding limited transmission networks, however. Examples are Gabszewicz Poddar (1997), Joskow and Tirole (2007), Fabra et al. (2011), Fabra and Frutos (2011), Murphy and Smeers (2005), Zöttl (2011), or Grimm and Zoettl (2013).

Another recent strand of literature explicitly models both, generation and transmission investment, typically by making use of bi-level programs. Sauma and Oren (2006, 2009) are among the first ones to explicitly model investment incentives of generators and transmission network expansion in a such a way. In their contribution they quantify the impact of whether transmission investment anticipates resulting investment of strategic generation companies or not. Rho et al. (2007) propose a simulation framework to analyze investment of competitive generation companies and competitive merchant transmission companies. Rho et al. (2008) generalize this framework to also include a transmission system operator as a further agent.

Ryan et al. (2010) and in an extension Jin and Ryan (2011) analyze expansion of electricity generation and transmission capacities together with the expansion of a fuel transportation network. For electricity markets, Jenabi et al. (2013) propose a clear cut bi-level framework which considers optimal network expansion by the transmission company, anticipating investment of competitive of competitive generation companies. The resulting problem is solved based on recently developed complementarity theory techniques. Most interestingly, also based on a bi-level approach, O'Neill et al. (2013) propose an auction mechanism to implement optimal investment incentives by transmission companies. Those approaches, however, do not explicitly take into account the specific structure of the transmission management regime, but implicitly assume optimal management of the transmission network which is implemented by a regime of LMPs. While this nicely models incentives in markets where indeed a system of LMPs is implemented (as for example in the US or Canada), it limits insights with respect to other markets, which might rely on a system of market splitting/coupling and redispatch/countertrading which is not captured by the above modelling approaches. It is the purpose of our article to explicitly analyze the impact of specific design features of load management regimes as market splitting and redispatch on the generation and transmission investment incentives.

Let us note at this point that in recent years an extensive literature has developed that analyzes the impact of the specific rules of the load management regime on short run market outcomes, i.e. for fixed generation and transmission facilities. Prominent articles include Hogan (1999), Ehrenmann and Smeers (2005), Neuhoff et al. (2005), or Green (2007) who explicitly compare the short run implications of zonal systems with redispatch to the system of nodal pricing. Several articles explicitly analyze the incentives of different agents that are able to exercise market power under different load management regimes. Compare, for example, Oren (1997), Wei and Smeers (1999), Borenstein et al. (2000), Metzler et al (2003), Gilbert et al. (2004), or Hu and Ralph (2007). Recently Holmberg and Lazarczyk (2012), Oggoni and Smeers (2013), Oggoni et al. (2012) and Perigne and Söder (2013) explicitly compared different load management regimes based on market coupling/splitting with redispatch or countertrading. All those articles consider only the short run perspective, however, while it is our purpose to consider the long run effects on investment incentives.

This paper is organized as follows. Section 2 will present our theoretical framework. In section 3 we introduce the integrated planer approach, while section 4 presents the tri–level model with a cost-based redispatch system and market splitting. The tri–level model is reformulated using new decomposition ideas in section 5. The last part of our paper, in section 6, presents numerical results for test networks that illustrate applicability of our model. Section 7 concludes.

2. The Framework

2.1. **Overview.** In this section we present the basic framework for our tri-level power market model and the corresponding integrated planning approach that serves as a first best benchmark. The integrated planning approach, which we present in Section 3, replicates a stylized nodal price system with transmission and generation investment, the formulation of this benchmark can already be found in Jenabi et al. (2013). The tri-level model analyzes investment decisions (in network expansion) by the transmission operator (TO) and in generation capacity (by firms), in expectation of an energy only market and cost-based redispatch. The model allows to analyze different regulatory environments. In different specifications red we consider the cases of one versus two price zones (market splitting) and analyze different approaches to recover network cost, in particular capacity based versus generation based fees. For the sake of completeness, we present all sets, parameters and variables used in our model in the appendix.

2.2. **Network.** Consider a network $G = (N, L^{ex})$ with a set N of nodes and a set L^{ex} of edges. By $\mathfrak{L}$ we will denote different line types that are characterized by their thermal capacity $\bar{z}^{flow}$ and their susceptance b. Given different line types $\mathfrak{l} \in \mathfrak{L}$, the network operator decides on an optimal network expansion plan, i.e., on the construction of candidate power lines $l \in L^{new} = \bigcup_{\mathfrak{l} \in \mathfrak{L}} L^{new}_{\mathfrak{l}}$ and on the deconstruction of existing lines $l \in L^{ex} = \bigcup_{\mathfrak{l} \in \mathfrak{L}} L^{ex}_{\mathfrak{l}}$.

In our tri-level power market model, if we account for multiple price zones, we will consider a partition $Z = \left\{Z_1, \ldots, Z_{|Z|}\right\}$ of the node set N. Its elements $Z_1, \ldots, Z_{|Z|}$ refer to different price zones in the electricity market. By $L^{inter}_{\mathfrak{l}}$ we will denote lines connecting nodes that belong to different zones (inter-zone links). The following network example illustrates this concept:

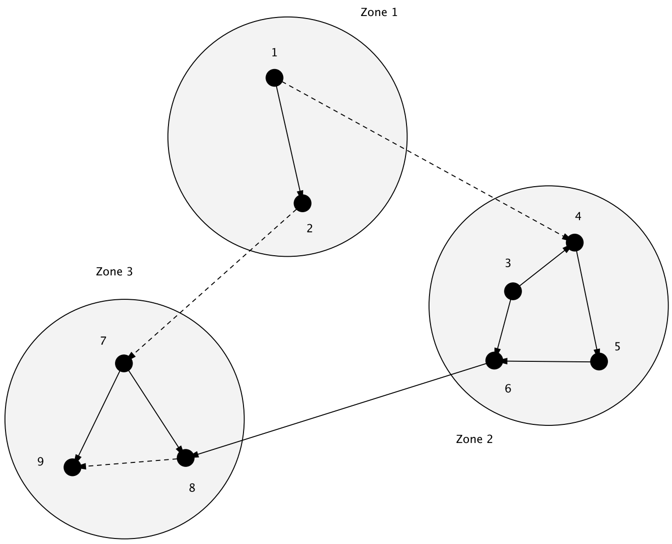
Example 2.1. Figure 1 depicts an electricity network with nine nodes, three price zones $Z = \{(1,2),(3,4,5,6),(7,8,9)\}$ (grey areas) and eleven existing transmission lines of two different line types $\mathfrak{L} = \{1,2\}$. Line type $\mathfrak{l} = 1$ is characterized by a thermal capacity of 1000 and a susceptance of 1 (solid arrows). Analogous, line type $\mathfrak{l} = 2$ has a thermal capacity of 500 and a susceptance of 0.5 (dashed arrows). This transmission network yields the following subsets of network links

$$L^{ex}_1 = \{(1,2),(3,4),(3,6),(4,5),(5,6),(6,8),(7,8),(7,9)\}$$

$$L^{ex}_2 = \{(1,4),(2,7),(8,9)\}$$

with $L^{inter}_1 = \{(6,8)\}$, $L^{inter}_2 = \{(1,4),(2,7)\}$, $L^{new}_1 = \emptyset$, and $L^{new}_2 = \emptyset$.

FIGURE 1. Nine–Node Network Economy with Market Coupling



When making line investment decisions, the transmission operator faces physical network constraints known as First and Second Kirchhoff's Laws. Note that we will use a linear approximation of real power flows known as the lossless direct current optimal power flow (DC) approximation (see also Kirschen and Strbac, 2004).

2.3. **Demand.** We denote demand nodes in the network by $D \subset N$. Consumers are located exclusively at those demand nodes, i.e., by assumption demand is zero at any other node $n \in N \setminus D$ in the network. We model elastic demand at a given demand node $d \in D$, in any given period $t \in T$, by continuous, strictly monotonic decreasing functions

$$Z_{d,t}^{price}(z_{d,t}^{dem}) : [0, \bar{z}_{d,t}^{dem}] \rightarrow [0, \infty[\,,$$

with $z_{d,t}^{dem}$ denoting demand in period $t \in T$ at demand node $d \in D$ and $Z_{d,t}^{price}$ being the resulting market price. $\bar{z}_{d,t}^{dem}$ denotes the saturation point. We will further assume that the integral of each demand function up to $z_{d,t}^{dem}$, to which we will refer to as the gross consumer surplus at node d in period t, is always convex, which is for instance the case for linear non-increasing demand functions.

2.4. **Investment.** Let G_n^{all} denote the set of technologies that firms can potentially dispose of in at a given network node $n \in N$. We will use the notations G_n^{ex} for already existing generation technologies at node $n \in N$. G_n^{new} will be used for candidate generation technologies that can be build at node $n \in N$, with $G_n^{all} = G_n^{new} \cup G_n^{ex}$. To account for the characteristics of different production technologies, we allow for technology specific efficiency parameters e_g^{gen}. Unit investment cost of building a new generation capacity is denoted by $i_g^{gen} \in \mathbb{R}^+$.

2.5. **Production/Supply.** We assume that all firms act in a competitive environment without any type of maket power and act as price takers. Variable production cost is denoted by $v_g^{gen} \in \mathbb{R}^+$. We assume that the variable costs v_g^{gen} are pairwise disjoint.

2.6. **Network Fees.** In the power market model the transmission operator has to collect network fees in order to cover expenses arising from i) line investment and ii) redispatch. We denote the revenue from fee collection by R. We consider three different types of network fee regimes (see also Section 4.2).

- **Lump Sum:** We denote by λ a lump sum fee paid by the consumers:

$$R_\lambda = \lambda$$

- **Transmission Based Fee:** We denote by ρ a per unit fee charged for each unit of electricity actually transmitted through the network. The corresponding revenue R_ρ is given by

$$R_\rho = \sum_{d \in D} \sum_{t \in T} \rho \cdot x_{d,t}^{dem}$$

- **Capacity Based Fee:** We denote by κ a per unit fee charged for each unit of (generation) capacity connected to the network. Revenues R_κ generated under this regime is

$$R_\kappa = \sum_{n \in N} \sum_{g \in G_n^{new}} \kappa \cdot x_g^{gncp}$$

3. The Integrated Planning Approach as a First Best Benchmark

As the first best benchmark we consider the integrated planning approach where an Integrated Generation and Transmission Company (IGTC) decides simultaneously on transmission and generation capacity expansion and chooses welfare maximizing production at the spot markets.[2] The IGTC maximizes total social welfare which is defined as the difference of gross consumer surplus (aggregate over all demand scenarios) and (i) generation investment cost, (ii) line investment cost as well as (iii) variable production cost (see (3.1)). Note that our approach yields the same investment

[2]As already discussed above several interesting approaches in the literature analyze integrated planning solutions, which serve a a benchmark in our setting. The formulation chosen herein is closely related to the clear cut formulation of the integrated planner solution in Jenabi et al. (2013).

and production outcomes as an idealized nodal pricing system.[3] Figure 2 visualizes the general structure of the model that is described in detail in (3.1) to (3.15).

FIGURE 2. Integrated Planning Approach

MAX **SOCIAL WELFARE**

S.T. A) GENERATION INVESTMENT CONSTRAINTS
B) PRODUCTION CONSTRAINTS
C) MARKET CLEARING CONDITION
D) POWER FLOW CONSTRAINTS
E) LINE INVESTMENT CONSTRAINTS

IDEALIZED MARKET (IGTC)

Considering different economic and physical constraints, the IGTC faces the following intertemporal optimization problem:

$$\max \sum_{d \in D} \sum_{t \in T} \left(\int_0^{z_{d,t}^{dem}} Z_{d,t}^{price}(a_{d,t})\, \mathrm{d}a_{d,t} \right) - \sum_{\mathfrak{l} \in \mathfrak{L}} \left(\sum_{l \in L_{\mathfrak{l}}^{new}} i_l^{line} \cdot z_l^{line} + \sum_{l \in L_{\mathfrak{l}}^{ex}} i_l^{line} \cdot z_l^{line} \right) - \sum_{n \in N} \left(\sum_{g \in G_n^{new}} i_g^{gen} \cdot z_g^{gncp} + \sum_{g \in G_n^{all}} \sum_{t \in T} \nu_g^{gen} \cdot z_{g,t}^{gen} \right) \tag{3.1}$$

[3]It is well known that the integrated planner approach yields the same outcome as a nodal price system in the short run (see, for instance, Hogan (2002)). Analogously, our results in section 5 imply that the solution of the long run integrated planner approach (the First Best) is equivalent to the outcome of a three-level nodal pricing model which also accounts for investments in transmission and generation capacity. In such a model, a regulated transmission operator decides on transmission expansion at the first stage. At the second stage, competitive firms decide on generation expansion investment and spot market bids. At the third stage, the transmission operator decides on the welfare maximizing feasible allocation and implements nodal prices.

subject to:

First Kirchhoff's Law (FKL):

$$z^{dem}_{n,t} = \sum_{g\in G^{all}_n} z^{gen}_{g,t} - \sum_{\mathfrak{l}\in\mathfrak{L}} \left(\sum_{n'\in N|(n,n')\in L_\mathfrak{l}} z^{flow}_{(n,n'),t} - \sum_{n'\in N|(n',n)\in L_\mathfrak{l}} z^{flow}_{(n',n),t} \right) \qquad \forall n \in N, t \in T \tag{3.2}$$

Second Kirchhoff's Law (SKL):

$$z^{flow}_{l,t} - b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \cdot z^{line}_l \qquad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{ex}_\mathfrak{l}, t \in T \tag{3.3}$$

$$z^{flow}_{l,t} - b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \left(1 - z^{line}_l \right) \qquad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{new}_\mathfrak{l}, t \in T \tag{3.4}$$

$$-z^{flow}_{l,t} + b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \cdot z^{line}_l \qquad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{ex}_\mathfrak{l}, t \in T \tag{3.5}$$

$$-z^{flow}_{l,t} + b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \left(1 - z^{line}_l \right) \qquad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{new}_\mathfrak{l}, t \in T \tag{3.6}$$

Voltage Phase Angle of Reference Node (VPA):

$$z^{angle}_{1,t} = 0 \qquad \forall t \in T \tag{3.7}$$

Transmission Flow Limits (TFL):

$$-\overline{z}^{flow}_l \left(1 - z^{line}_l \right) \leq z^{flow}_{l,t} \leq \left(1 - z^{line}_l \right) \overline{z}^{flow}_l \qquad \forall \mathfrak{l} \in \mathfrak{L}, l \in L^{ex}_\mathfrak{l}, t \in T \tag{3.8}$$

$$-z^{line}_l \cdot \overline{z}^{flow}_l \leq z^{flow}_{l,t} \leq z^{line}_{l^{new}_\mathfrak{l}} \cdot \overline{z}^{flow}_l \qquad \forall \mathfrak{l} \in \mathfrak{L}, l \in L^{new}_\mathfrak{l}, t \in T \tag{3.9}$$

Generation Capacity Limits (GCL):

$$z^{gen}_{g,t} \leq e^{gen}_g \cdot \overline{z}^{gncp}_g \qquad \forall n \in N, g \in G^{ex}_n, t \in T \tag{3.10}$$

$$z^{gen}_{g,t} \leq e^{gen}_g \cdot z^{gncp}_g \qquad \forall n \in N, g \in G^{new}_n, t \in T \tag{3.11}$$

Variable Restrictions (VR):

$$z^{gncp}_g \geq 0 \qquad \forall n \in N, g \in G^{new}_n \tag{3.12}$$

$$z^{gen}_{g,t} \geq 0 \qquad \forall n \in N, g \in G^{all}_n, t \in T \tag{3.13}$$

$$z^{dem}_{d,t} \geq 0 \qquad \forall d \in D, t \in T \tag{3.14}$$

$$z^{line}_l \in \{0,1\} \qquad \forall \mathfrak{l} \in \mathfrak{L}, l \in L^{ex}_\mathfrak{l} \cup L^{new}_\mathfrak{l} \tag{3.15}$$

Let us briefly explain the different constraints the IGTC is facing:

- First Kirchhoff's Law (see (3.2)) ensures power balance or flow conservation at any node $n \in N$ in the electricity network. To ensure flow balance, total power flow out of node $n \in N$ in any given period $t \in T$ must equal the sum of power flows in node $n \in N$.
- Constraints (3.3) to (3.6) correspond to Second Kirchhoff's Law and determine, together with the constraint group (3.7) the network's phase angles (and network flows).

- Constraint (3.7) ensures that the phase angle of node 1, the reference node of our network, equals zero in all periods $t \in T$.
- The transmission flow limit constraints in (3.8) and (3.9) guarantee that on a given transmission line total power flow always lies between its lower and upper bound. Here, line investment is explicitly taken into account.
- The next group of constraints, the generation capacity limits (constraints (3.10) to (3.11)), ensure that electricity production does not exceed installed generation capacity. These constraints explicitly account for the efficiency of the considered technologies.
- The last constraint group, (3.12) to (3.14) and (3.15), guarantees certain model variables being nonnegative and binary, respectively.

Note that by summing up (3.2) for all network nodes $n \in N$ yields the market clearing condition:

$$\sum_{d \in D} z_{d,t}^{dem} = \sum_{n \in N} \sum_{g \in G_n^{all}} z_{g,t}^{gen} \; \forall t \in T \tag{3.16}$$

4. The Tri-Level Power Market Model

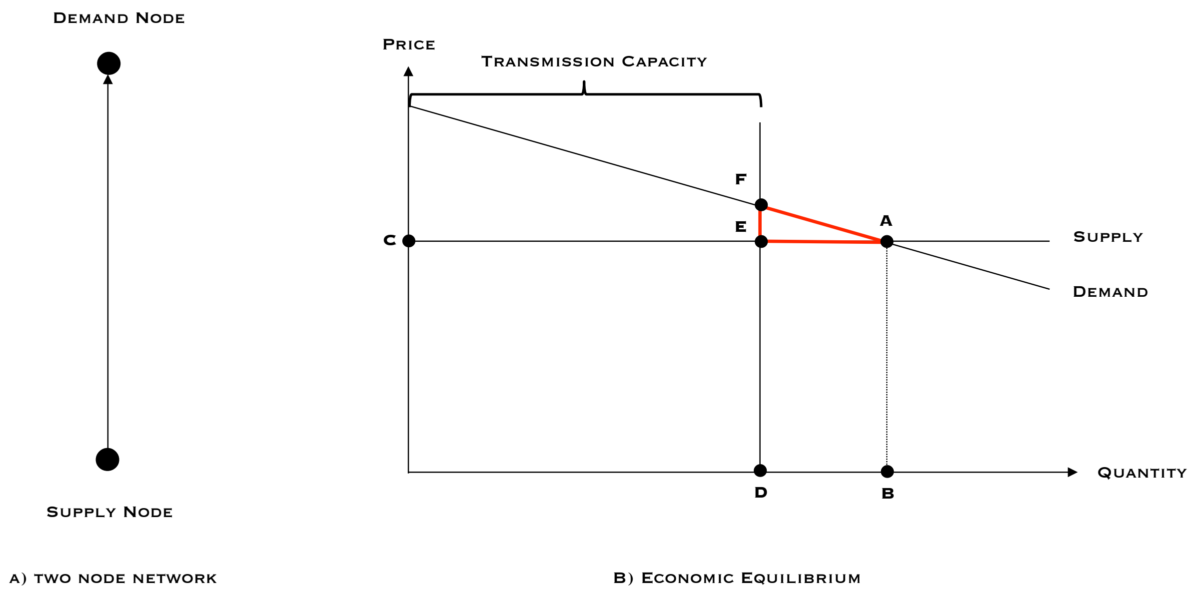
In most European countries, spot market trading does not fully account for network constraints (if at all). Therefore, congestion management mechanisms and the way network fees are collected play a crucial role for investment incentives, both, in network and generation capacity expansion. While it is most likely, that energy-only markets with redispatch do not result in an optimal incentive structure, the questions remain open (i) how far away we are from the first best and (ii) whether alternative mechanisms have the potential to improve the situation substantially.

In our power market model we consider an energy only market with cost-based redispatch, which is implemented in several European countries. Electricity trading and dispatch is organized as follows in such a system: Firms trade electricity day ahead (and possibly intra-day) at a power exchange that does not account for any transmission constraints. After closing of the market, the transmission operator (TO) checks feasibility of the resulting transmission flows. If the allocation is feasible, nothing is changed and electricity is generated and consumed as traded. If transmission is infeasible, the TO redispatches plants in the cheapest possible way that ensures feasibility of transmission flows. To that aim the TO obliges producers to (partially) shut down production and others on the other side of the congested link are called to step in instead. Plants that are shut down have to pay their marginal cost (that they save due to the shutdown) to the TO, while plants that are called to step in receive their variable production cost. Production cost of the called plants are necessarily higher than production cost of the plants shut down, since otherwise they would have been successful at the spot market. The resulting cost is collected by the TO through transmission fees. While in the early days of liberalized electricity markets redispatch was a rare event, nowadays the phenomenon becomes more and more important. See ENTSO–E (2012) for the case of Germany, where the decision to shut down nuclear power plants and increase the installation of renewables implies a much more uncertain supply structure.

Example 4.1. Before we formally introduce our power market model with cost-based redispatch, we briefly illustrate the main effects of cost-based redispatch in a stylized market with a simple two-node network. In the two-node network demand is located at the northern node and production is only feasible at the southern node (see also Figure 3). We further assume that demand is linear and that there is only one generator with constant marginal production cost. As illustrated by Figure 3, the demand function intersects the supply function (given by the constant marginal cost of production) in the point A. This determines the equilibrium market price C and the equilibrium quantity B. In the case of unlimited transmission capacities, the quantity B will be produced and consumed. However, if the market faces physical transmission constraints this is not possible. As shown in Figure 3, D is the maximum amount of electricity that can be transmitted from the production node to the consumption node. In this simplified scenario 'redispatch' implies that the transmission operator adjusts both demand and supply in order to meet the transmission capacity constraint at D. To this aim the TO obliges consumers (namely those with the lowest willingnesses to pay) to step back from their contracts. As a compensation, the TO pays these consumers the amount equal to the area of the polygon $ABDF$. Additionally, the transmission operator asks firms (typically those with the highest marginal cost, here marginal cost are the same) to shut down their production to an extent that resolves the congestion problem. The respective firms pay an amout equal to the area of the rectangle $ABDE$ to the transmission operator, which corresponds exactly to the variable costs that do not arise due to the redispatch. Obviously, the redispatch mechanism has no impact on the firms' profits if they are asked to decrease production. However, for the TO redispatch implies additional cost equal to the area of the triangle AEF that occur due to the reallocation procedure. These additional costs must be collected from the market participants when collecting network fees.

4.1. **Formal Representation as a Tri-Level Programming Problem.** Let us briefly sketch the structure of our model before we introduce the details at all levels. We consider a hierarchical tri-level problem that corresponds to a multistage game, where the transmission operator (TO) decides

FIGURE 3. Cost-Based Redispatch



about investment in network expansion anticipating a perfectly competitive electricity (energy only) market with cost-based redispatch. The timing of this game is illustrated in Figure 4. Note that investment choices are taken once (sequentially by the TO and competitive firms), followed by multiple periods of spot market trading and redispatch (in case of transmission constraints). We translate this game into a tri-level program as follows: At the first level, the TO decides to invest (or not) in line expansion, anticipating the outcomes at all subsequent levels. The TO's objective is to maximize welfare. At the second level, we model private firms' investment decisions (in generation capacity) as well as trading at a sequence of T spot markets with fluctuating demand. Other than the TO (who is driven by welfare concerns), firms take investment and supply decisions as to maximize profits. We assume competitive spot markets (no market power) and spot market rules yield no price signals that point to transmission constraints (energy only market). Redispatch at the T subsequent spot markets is modelled at the third level and taken into account by firms when they decide on investment and supply schemes at level two. Redispatch occurs whenever traded quantities turn out to be infeasible given transmission constraints. The TO redispatches plants as to minimize redispatch costs, which he/she recovers through network fees. Note that consideration of redispatch in a separate third stage is possible since, once network and generation capacities have been chosen, redispatch at time t cannot have any impact on supply decisions at any later point in time. Figure 5 illustrates the structure of our tri-level power market model. We point out that all levels of our power market model are interconnected: the competitive firms' investment and optimal spot market

FIGURE 4. Timing of the Corresponding Multistage Game

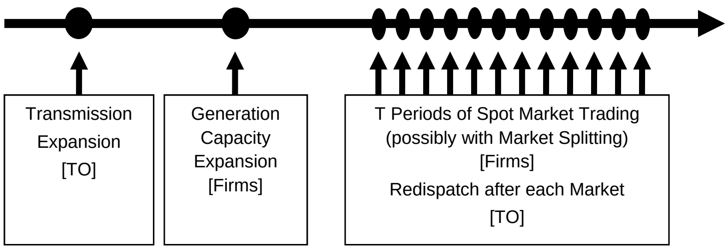
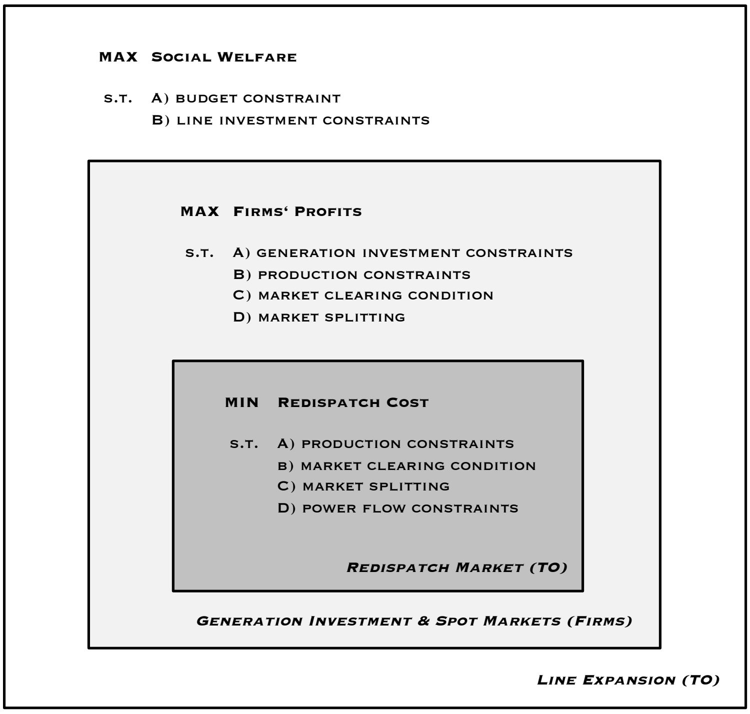


FIGURE 5. Structure of the Power Market Model



behaviour as well as the redispatch market are part of the transmission operator's constraints at the first level.

Let us finally emphasize that our power market model specification allows to account for multiple price zones $Z_1, \ldots, Z_{|Z|}$, which enables us to investigate the effect of market splitting on the extent of (cost of) redispatch. Under market splitting, spot market trading does take into account parts of the possible network constraints between (but not within) zones.

4.2. **First Level Problem: Optimal Line Expansion.** At the first level, the transmission operator (TO) decides on a line expansion plan as to maximize total welfare[4]. The TO is restricted by a budget constraint requiring that expenses for network expansion and redispatch are covered by revenues

[4]All variables that can be chosen by the transmission operator at the first level will be denoted by w.

from network fees. In particular, total expenses E equal (i) redispatch cost arising at the third level problem and (ii) line investment cost at level one,

$$E = \underbrace{\sum_{\mathfrak{l}\in\mathfrak{L}} \left(\sum_{l\in L_{\mathfrak{l}}^{new}} i_l^{line} \cdot z_l^{line} + \sum_{l\in L_{\mathfrak{l}}^{ex}} i_l^{line} \cdot z_l^{line} \right)}_{\text{Line Investment Cost}} + \underbrace{\sum_{d\in D}\sum_{t\in T} \left(\int_{z_{d,t}^{dem}}^{x_{d,t}^{dem}} Z_{d,t}^{price}(a_{d,t})\,\mathrm{d}a_{d,t} \right) + \sum_{n\in N}\sum_{g\in G_n^{all}}\sum_{t\in T} v_g^{gen} \cdot y_{g,t}^{gen}}_{\text{Redispatch Cost}},$$

where the first term represents (network) investment cost and the second and third term redispatch cost. Redispatch cost is composed of remuneration of consumers that cannot be supplied (second term), as well as transfers to (or, from) the plants that are redispatched (third term). $y_{g,t}^{gen}$ and $y_{d,t}^{dem}$ denote redispatch quantities, v_g^{gen} denote variable cost of technology g as introduced earlier.

At the first level the TO maximizes social welfare, which comprises of consumer surplus from all markets minus investment and generation cost. The budget constraint ($R = E$) allows to consider all three network fee regimes referred to in Section 2.6. We obtain the following line investment optimization problem:

$$\max \sum_{d\in D}\sum_{t\in T} \left(\int_0^{z_{d,t}^{dem}} Z_{d,t}^{price}(a_{d,t})\,\mathrm{d}a_{d,t} \right) - \sum_{\mathfrak{l}\in\mathfrak{L}} \left(\sum_{l\in L_{\mathfrak{l}}^{new}} i_l^{line} \cdot w_l^{line} + \sum_{l\in L_{\mathfrak{l}}^{ex}} i_l^{line} \cdot w_l^{line} \right) - \sum_{n\in N} \left(\sum_{g\in G_n^{new}} i_g^{gen} \cdot x_g^{gncp} + \sum_{g\in G_n^{all}}\sum_{t\in T} v_g^{gen} \cdot z_{g,t}^{gen} \right) \tag{4.1}$$

subject to:

$$R - E = 0 \qquad \text{(Budget Constraint, BC)} \tag{4.2}$$

$$w_l^{line} \in \{0,1\} \qquad \forall \mathfrak{l} \in \mathfrak{L}, l \in L_{\mathfrak{l}}^{ex} \cup L_{\mathfrak{l}}^{new} \tag{4.3}$$

4.3. **Second Level Problem: Generation Investment and Spot Market Behavior.** At the second level we model firms' behavior concerning (i) generation investment and (ii) spot market supply.[5] The generation and supply market is assumed to be perfectly competitive, i.e., no firm can directly affect prices by strategic investment or supply decisions. It has been shown in the literature[6] that a perfectly competitive environment yields welfare maximizing investment and production decisions

[5] All variables that are determined on the second stage will be denoted by x.
[6] See, e.g., Grimm and Zoettl (2013) for the context modelled here.

in our context. This simplifies our second level problem to a constrained welfare maximization problem.

$$\max \sum_{d\in D}\sum_{t\in T}\left(\int_0^{x_{d,t}^{dem}} Z_{d,t}^{price}(a_{d,t})\,\mathrm{d}a_{d,t}\right) - \sum_{n\in N}\left(\sum_{g\in G_n^{new}} i_g^{gen}\cdot x_g^{gncp} + \sum_{g\in G_n^{all}}\sum_{t\in T} v_g^{gen}\cdot x_{g,t}^{gen}\right) - R \tag{4.4}$$

subject to:

Generation Capacity Limits (GCL):

$$x_{g,t}^{gen} \le e_g^{gen}\cdot \overline{x}_g^{gncp} \qquad \forall n\in N, g\in G_n^{ex}, t\in T \tag{4.5}$$

$$x_{g,t}^{gen} \le e_g^{gen}\cdot x_g^{gncp} \qquad \forall n\in N, g\in G_n^{new}, t\in T \tag{4.6}$$

Zonal First Kirchhoff's Law (ZFKL):

$$\sum_{d|d\in Z_k} x_{d,t}^{dem} = \sum_{n|n\in Z_k}\sum_{g\in G_n^{all}} x_{g,t}^{gen} - \sum_{\mathfrak{l}\in\mathfrak{L}}\left(\sum_{(n,n')\in L_{\mathfrak{l}}^{inter}|n\in Z_k} x_{(n,n'),t}^{flow} - \sum_{(n',n)\in L_{\mathfrak{l}}^{inter}|n'\in Z_k} x_{(n',n),t}^{flow}\right) \qquad \forall t\in T, Z_k\in Z \tag{4.7}$$

Market Splitting Flow Restriction (MCF):

$$-w_l^{line}\cdot\overline{z}_l^{line} \le x_{l,t}^{flow} \le w_l^{line}\cdot\overline{z}_l^{line} \qquad \forall \mathfrak{l}\in\mathfrak{L}, l\in L_{\mathfrak{l}}^{inter}\cap L_{\mathfrak{l}}^{new}, t\in T \tag{4.8}$$

$$-\left(1-w_l^{line}\right)\cdot\overline{z}_l^{line} \le x_{l,t}^{flow} \le \left(1-w_l^{line}\right)\cdot\overline{z}_l^{line} \qquad \forall \mathfrak{l}\in\mathfrak{L}, l_{\mathfrak{l}}\in L_{\mathfrak{l}}^{inter}\cap L_{\mathfrak{l}}^{ex}, t\in T \tag{4.9}$$

$$x_g^{gncp} \ge 0 \qquad \forall n\in N, g\in G_n^{new} \tag{4.10}$$

$$x_{g,t}^{gen} \ge 0 \qquad \forall n\in N, g\in G_n^{all}, t\in T \tag{4.11}$$

$$x_{d,t}^{dem} \ge 0 \qquad \forall d\in D, t\in T \tag{4.12}$$

When making investment and supply decisions, firms only consider physical constraints for which they receive price signals. If the market is not divided into zones, firms receive no signals concerning network capacities and thus, will not account for them. If the market is divided into two or more zones, firms consider those physical constraints which are expressed in price differences due to market splitting: Between any pair of zones (Z_i, Z_j), electricity flow cannot exceed the maximum thermal capacity of the respective inter-zone network links – and congestion implies price differences across zones. Other physical restrictions like the phase angle determination do not play any role under market splitting.

To summarize, at level two we consider welfare maximizing generation investment and supply decisions where supply is constrained by generation capacities installed, and transmission constraints across zones.

Note that in (4.4), R again denotes the respective network fee payments (as introduced in Section 2.6) that affect demand and/or generation decisions on the spot market.

We finally point to the following observations regarding the Zonal First Kirchhoff's Law. Summing up (4.7) for all zones we obtain the well known market clearing condition:

$$\sum_{d\in D} x_{d,t}^{dem} = \sum_{n\in N} \sum_{g\in G_n^{all}} x_{g,t}^{gen} \; \forall t \in T \tag{4.13}$$

Thus, in a model with redispatch and only one zone ($|Z| = 1$), ZFKL coincides with a standard market clearing condition. In the case where every zone consists of exactly one network node ($|Z| = n$), ZFKL coincides with first Kirchhoff's law ensuring power balance of every network node. Intermediate cases require the market to clear within each zone, accounting for possible transmission constraints across zones through differences in the respective market clearing prices.

4.4. **Third Level Problem: Optimal Redispatch.** At the third level, the transmission operator simultaneously decides on redispatch for all T spot markets.[7] Reallocation of spot market outcomes is realized in a way that ensures compatibility with network constraints at the lowest possible cost.[8] The corresponding cost minimization problem (4.14) comprises of remunerations of consumers and firms that are involved in redispatch. The redispatch decision has to take into account all physical flow restrictions of installed network capacity as well as generation capacity limits, see (4.15) to (4.27).

$$\min \sum_{d\in D}\sum_{t\in T}\left(\int_{z_{d,t}^{dem}}^{x_{d,t}^{dem}} Z_{d,t}^{price}(a_{d,t})\,\mathrm{d}a_{d,t}\right) + \sum_{n\in N}\sum_{t\in T}\left(\sum_{g\in G_n^{all}}\left(v_g^{gen}\cdot y_{g,t}^{gen}\right)\right) \tag{4.14}$$

[7]Recall that we can consider redispatch separately since there are no temporal interdependencies, i.e. redispatch at time t has no effect on future supply decisions of any firm.

[8]Redispatch quantities will be denoted by y while quantities that result after redispatch will be denoted by z. Note that the following relationship between different variable types always holds: $z = y + x$.

subject to:

First Kirchhoff's Law (FKL):

$$z^{dem}_{n,t} = \left(\sum_{g \in G^{all}_n} z^{gen}_{g,t} \right) - \sum_{\mathfrak{l} \in \mathfrak{L}} \left(\sum_{n' \in N | (n,n') \in L_{\mathfrak{l}}} z^{flow}_{(n,n'),t} - \sum_{n' \in N | (n',n) \in L_{\mathfrak{l}}} z^{flow}_{(n',n),t} \right) \quad \forall n \in N, t \in T \tag{4.15}$$

Second Kirchhoff's Law (SKL):

$$z^{flow}_{l,t} - b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \cdot w^{line}_l \quad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{ex}_{\mathfrak{l}}, t \in T \tag{4.16}$$

$$z^{flow}_{l,t} - b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \left(1 - w^{line}_l \right) \quad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{new}_{\mathfrak{l}}, t \in T \tag{4.17}$$

$$-z^{flow}_{l,t} + b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \cdot w^{line}_l \quad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{ex}_{\mathfrak{l}}, t \in T \tag{4.18}$$

$$-z^{flow}_{l,t} + b_l \left(z^{angle}_{n,t} - z^{angle}_{n',t} \right) \leq \mathbf{M} \left(1 - w^{line}_l \right) \quad \forall \mathfrak{l} \in \mathfrak{L}, l = (n,n') \in L^{new}_{\mathfrak{l}}, t \in T \tag{4.19}$$

Voltage Phase Angle of Reference Node (VPA):

$$z^{angle}_{1,t} = 0 \quad \forall t \in T \tag{4.20}$$

Transmission Flow Limits (TFL):

$$-\left(1 - z^{line}_l \right) \overline{z}^{flow}_l \leq z^{flow}_{l,t} \leq \left(1 - z^{line}_l \right) \overline{z}^{flow}_l \quad \forall \mathfrak{l} \in \mathfrak{L}, l \in L^{ex}_{\mathfrak{l}}, t \in T \tag{4.21}$$

$$-z^{line}_l \cdot \overline{z}^{flow}_l \leq z^{flow}_{l,t} \leq z^{line}_l \cdot \overline{z}^{flow}_l \quad \forall \mathfrak{l} \in \mathfrak{L}, l \in L^{new}_{\mathfrak{l}}, t \in T \tag{4.22}$$

Generation Capacity Limits (GCL):

$$z^{gen}_{g,t} \leq e^{gen}_g \cdot \overline{x}^{gncp}_g \quad \forall n \in N, g \in G^{ex}_n, t \in T \tag{4.23}$$

$$z^{gen}_{g,t} \leq e^{gen}_g \cdot x^{gncp}_g \quad \forall n \in N, g \in G^{new}_n, t \in T \tag{4.24}$$

Spot and Redispatch Quantities (SRQ):

$$z^{dem}_{d,t} = x^{dem}_{d,t} + y^{dem}_{d,t} \quad \forall d \in D, t \in T \tag{4.25}$$

$$z^{gen}_{g,t} = x^{gen}_{g,t} + y^{gen}_{g,t} \quad \forall n \in N, g \in G^{all}_n, t \in T \tag{4.26}$$

$$z^{flow}_{l,t} = x^{flow}_{l,t} + y^{flow}_{l,t} \quad \forall \mathfrak{l} \in \mathfrak{L}, l \in L^{inter}_{\mathfrak{l}}, t \in T \tag{4.27}$$

$$z^{dem}_{d,t} \geq 0 \quad \forall d \in D, t \in T \tag{4.28}$$

$$z^{gen}_{g,t} \geq 0 \quad \forall n \in N, g \in G^{all}_n, t \in T \tag{4.29}$$

We conclude this section with the following relation between our integrated planning approach and the tri-level power market model:

Lemma 4.2. *Denote the solution of the integrated planer approach by f^{IP} and by f^{MS} the solution of the tri-level power market program with market splitting. Then*

$$f^{IP} \geq f^{MS}.$$

Proof. Consider a solution of the tri-level program with market splitting f^{MS}. Obviously, this solution is a feasible solution of the integrated planer approach as well. Thus, f^{MS} is a lower bound for the solution of the integrated planer approach f^{IP}. □

5. Decomposition of the Tri-Level Program and Solution Strategy

The presented tri-level programming approach is a special case of a general multi-level programming problem that takes the following form (see also Dempe, 2002)

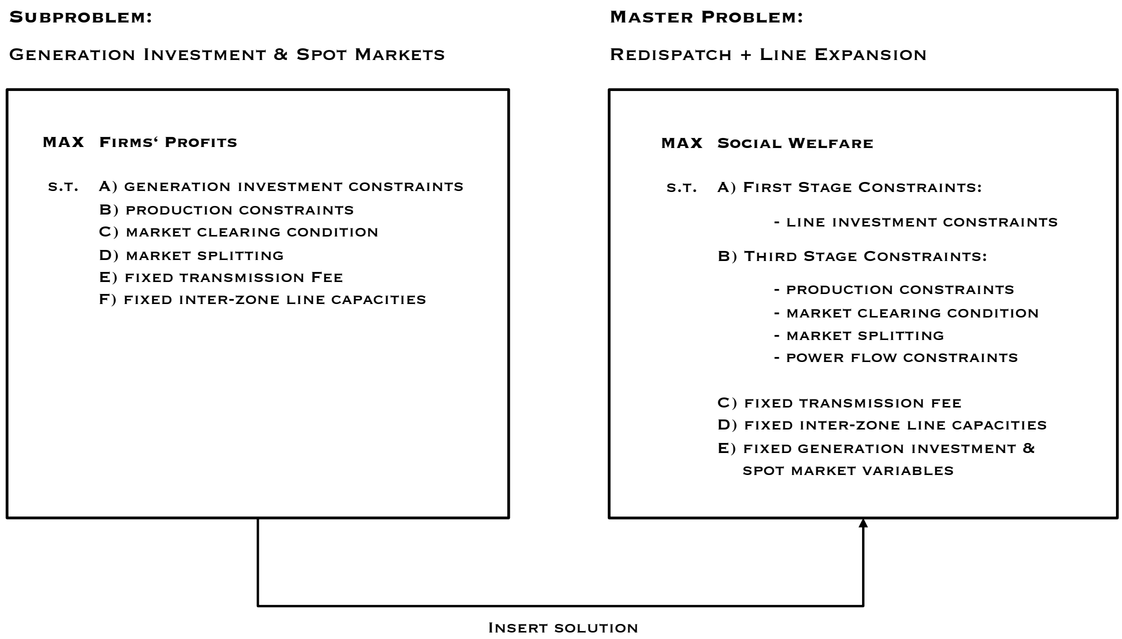
$$\begin{aligned}
&\min_{x_1} f_1(x_1, x_2, \ldots, x_p)\\
&\text{s.t. } g_1(x_1, x_2, \ldots, x_p) \geq 0\\
&\quad x_2, \ldots, x_p \text{ solve}\\
&\qquad \min_{x_2} f_2(x_1, x_2, \ldots, x_p)\\
&\qquad \text{s.t. } g_2(x_1, x_2, \ldots, x_p) \geq 0\\
&\qquad\quad \vdots\\
&\qquad\quad x_p \text{ solves}\\
&\qquad\qquad \min_{x_p} f_p(x_1, x_2, \ldots, x_p)\\
&\qquad\qquad \text{s.t. } g_p(x_1, x_2, \ldots, x_p) \geq 0
\end{aligned} \tag{5.1}$$

with f_i being a real-valued function and g_i describing vectors of real-valued functions. In the case $p = 1$, we face a single-level problem (as our integrated planner model). It is a key feature of every multi-level optimization problem that the first level problem cannot be solved until the (conditional) solutions to the lower level problems are known and adequately incorporated into the solution process. Of course, the lower level problem(s) can in general not be solved until the optimal solution of the first level is known. These interdependencies make general multilevel problems very hard to solve. In Jeroslow (1985) it is shown that a simple linear bi-level problem is already NP-hard. Most algorithmic approaches for bi-level problems make use of some version of Karush-Kuhn-Tucker (KKT) conditions and solve the resulting mathematical program with equilibrium constraints (MPEC).[9] However, such problem reformulations explicitly rely on nonconvex and nondifferentiable optimization problems. An obvious drawback is that in these problem formulations the Mangasarian-Fromovitz constraint qualification (MFCQ) is violated. Therefore, various local optimality or stationarity conditions such as Clarke, Mordukhovich, Strong or Bouligand stationarity have been developed (see Scheel and Scholtes, 2000). For more infomation on the topic of multilevel programming and MPECs we refer the interested reader to the monographs Cottle et al. (1992), Dempe (2002) and Luo et al. (1996). In this section we will present a new decomposition approach that allows us to find a global optimal solution to the presented tri-level problem without explicitly using KKT systems. Recall that we analyze three different network fee regimes. Our basic decomposition approach for solving the tri-level programming problem will be slightly modified depending on the respective network fee regime we consider. Before we present our formal decomposition approach, we point out that the tri-level problem has always a finite global optimal solution, since the feasible region is bounded.

5.1. **Basic Decomposition Idea.** Our decomposition approach builds on a detailed two step analysis of the connection between the three problem levels. In a first step, we fix all non-second level variables in the second-level problem and use them as parameters in the second level. This allows us to iteratively solve independent single-level sub problems (i.e., the second level problem for given scenarios with respect to line investment) and corresponding bi-level master problems consisting of the original first and third level problems. In a second step, we will show that instead of solving a master problem of bi-level type, we can solve an easier single-level master problem. Combining the results of step 1 and step 2, we will iteratively solve single-level sub and master problems arriving at a global optimal solution to the initial tri-level problem. Figure 6 depicts the decomposition of the tri-level problem into two independent single-level (sub and master) problems.

[9]For applied studies we refer the reader for instance to Hobbs (2001), Jenabi et al. (2013) or Jin and Ryan (2011).

FIGURE 6. Decomposed Tri-Level Problem: Sub and Master Problem



5.1.1. *Step 1: From the Original Tri-level Problem to Single-Level Sub and Bi-level Master Problem.* As a starting point, note that the second-level problem (generation investment and production decisions) includes only first and second level variables. The reason is basically that for cost based redispatch firms never receive additional rents from the third stage. Consequently, although the second-level problem is connected to the third (redispatch) level via the line investment problem at the first level, there is no direct interconnection.[10] To be more precise, the stages' interconnection is purely driven by inter-zonal line investment variables and the transmission fee variable from the first level problem. We can thus fix those variables and solve the second-level market sub problem for every possible realization of those variables. We then replace the respective variables in the first-level *and* the third-level problem by this solution. Doing so reduces the problem to a bi-level master problem with a reduced number of variables. Note that the approach to substitute in the optimal second-level values requires uniqueness of the second-level solution. In Grimm et al. (2014) we prove that under pairwise disjoint variable cost of production the second stage solution (optimal generation investments and spot market production) is unique if competitive firms interact on a given network structure.

[10]Such a connection would result if the redispatch mechanism would allow for profits that depend on decisions taken at the second stage.

5.1.2. *Step 2: From a Bi-level to a Single-Level Master Problem.* We will now show how the bi-level master problem consisting of the line investment and the redispatch stage can be solved efficiently. To develop our algorithmic approach we make use of the following lemma proving that the third stage objective f_3 is an affine linear transformation of the first level objective f_1. To be more precise, we have $f_1 = -f_3 + b$ with

$$b = -\sum_{\mathfrak{l}\in\mathfrak{L}}\left(\sum_{l\in L_{\mathfrak{l}}^{new}} i_l^{line}\cdot w_l^{line} + \sum_{l\in L_{\mathfrak{l}}^{ex}} i_l^{line}\cdot w_l^{line}\right) + \sum_{d\in D}\sum_{t\in T}\left(\int_0^{x_{d,t}^{dem}} Z_{d,t}^{price}(a_{d,t})\,\mathrm{d}a_{d,t}\right)$$
$$-\sum_{n\in N}\sum_{g\in G_n^{all}}\left(\sum_{t\in T}\left(v_g^{gen}\cdot x_{g,t}^{gen}\right) + \sum_{g\in G_n^{new}}\left(i_g^{gen}\cdot x_g^{gncp}\right)\right)$$

depending only on spot market variables x and line investment variables w. This insight reveals that the first and the third stage problem have identical objective functions and thus, identical optimization directions.

Lemma 5.1. *There exists a* $b\in\mathbb{R}$ *such that*

$$f_1 = -f_3 + b.$$

We will now exploit the fact that in order to solve a general mixed-integer nonlinear bi-level problem with identical objective functions, it is possible to solve an easier single-level problem which is equivalent. This single-level problem includes both, the constraints of the upper level program and the lower level problem.

Lemma 5.2. *Consider a mixed-integer nonlinear bi-level problem of the general form given in (5.1) but with identical objective functions. Denote by* $\mathfrak{S}^{BLP}$ *the solution set of the bi-level problem. Let* $\mathfrak{S}^{SLP}$ *denote the set of optimal solutions to the single-level problem (SLP) whose constraint system equals the union of the upper and lower level constraints:*

$$\begin{aligned} &min_{x_1,x_2}\ f(x_1,x_2)\\ &s.t.\ g_1(x_1,x_2)\geq 0\\ &\quad\ \ g_2(x_1,x_2)\geq 0\end{aligned}$$

Then, we have:

$$\mathfrak{S}^{SLP} = \mathfrak{S}^{BLP}$$

Proof. Consider an optimal solution $(x_1^{BLP}, x_2^{BLP}) \in \mathfrak{S}^{BLP}$ to the bi-level optimization problem. Obviously, (x_1^{BLP}, x_2^{BLP}) is also feasible for the single-level problem. Let $(x_1^{SLP}, x_2^{SLP}) \in \mathfrak{S}^{SLP}$ be an optimal solution to the single-level problem. Then, we get:

$$f(x_1^{BLP}, x_2^{BLP}) \geq f(x_1^{SLP}, x_2^{SLP}) \tag{5.2}$$

Now, consider an optimal solution $(x_1^{SLP}, x_2^{SLP}) \in \mathfrak{S}^{SLP}$ to the single-level optimization problem. Then, there exists $\bar{x}_2^{SLP}$ such that

$$f(x_1^{SLP}, x_2^{SLP}) \geq f(x_1^{SLP}, \bar{x}_2^{SLP}) \geq f(x_1^{BLP}, x_2^{BLP}) \tag{5.3}$$

where $(x_1^{SLP}, \bar{x}_2^{SLP})$ is a feasible solution to the bi-level problem and $(x_1^{BLP}, x_2^{BLP}) \in \mathfrak{S}^{BLP}$ is again an optimal solution to the bi-level problem. Combining (5.2) and (5.3) we obtain:

$$f(x_1^{BLP}, x_2^{BLP}) = f(x_1^{SLP}, x_2^{SLP}) \tag{5.4}$$

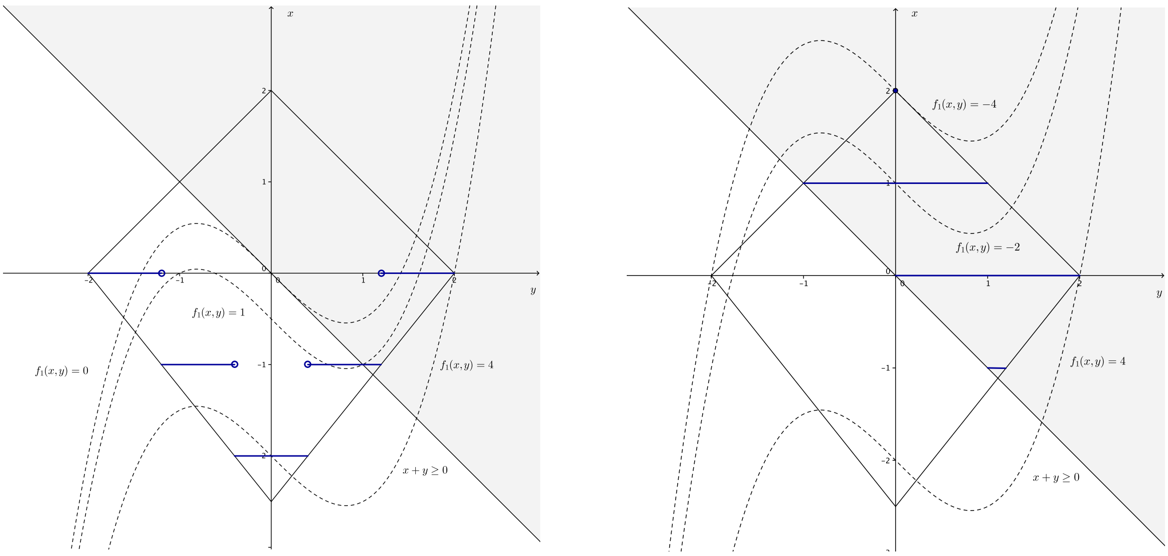
□

Let us note that the above result can easily be generalized to p-level programming problems. For the sake of completeness let us also state the following theorem which is an immediate consequence:

Theorem 5.3. *Assume a general p-level programming problem in the form 5.1 with f_i denoting the i^{th} problem stage's objective function, for all $i \in \{1, \ldots, p\}$. If there exist affine linear transformations $\Gamma_i = a_i \cdot f_i + b_i$ with $\Gamma(f_i) = f_1$ and $a_i > 0$ for all $i \in \{2, \ldots, p\}$, then the multilevel problem can be solved as a single-level program.*

To illustrate the intuition of these results, we will apply the above theorem to an adapted mixed-integer bi-level problem example taken from the monograph on bi-level programming in Dempe (2002):

FIGURE 7. A Mixed-Integer Nonlinear Bi-level Programming Problem



A) ORIGINAL BI-LEVEL PROBLEM

B) CORRESPONDING SINGLE-LEVEL PROBLEM

Example 5.4. Let us assume the following mixed-integer nonlinear bi-level problem based on an example in Dempe (2002) (p. 256):

$$\max f_1 = y^3 - 2y - 2x$$

$$\text{s.t. } x + y \geq 0$$

$$y \in [-5, 5]$$

$$x \text{ solves}$$

$$\min f_2 = x$$

$$\text{s.t. } x + y \leq 2$$

$$x - y \leq 2$$

$$-4x + 5y \leq 10$$

$$-4x - 5y \leq 10$$

$$x \in \mathbb{Z}$$

Figure 7a) depicts both the feasible set of the lower level problem (blue lines) and different first level isoobjective lines (dashed curves). Obviousely, the bi-level's optimal solution is $(x^* = 0, y^* = 2)$ with an objective value of $f_1 = 4$. However, by the above theorem we can also solve a single-level problem given the following relationship between the upper and lower level objective:

$$(-2) \cdot f_2 + y^3 - 2y = x = y^3 - 2y - 2x = f_1$$

Figure 7b) depicts the feasible set of the corresponding single-level problem (blue lines). It shows that its optimal value coincides with the optimal bi-level objective value.

With these results our two level master problem can be solved as a single-level problem. Combining step 1 and step 2, we arrive at both single-level sub and single level master problems. Figure 6 depicts the result of our decomposition.

5.2. **Linear vs. Binary Search Strategies.** In this subsection we will discuss how all the relevant sub and master problems can be evaluated in an efficient way using an inner and an outer loop. Accounting for all possible network expansion plan and transmission fee pairs, in an inner loop we will consider fixed inter-zone network capacities and iteratively solve sub and master problems for all the relevant transmission fees. In an outer loop we will consider the different network expansion plans. In the following two subsections we will first describe a linear search strategy and then a binary search procedure.

5.2.1. *Linear Search.*

Inner Loop: Transmission Fees. Solving the decomposed tri-level problem for all possible values of the transmission fee is not possible, given that the three different transmission fees are all real valued. Therefore, we will restrict λ, ρ and κ to the case having at most $m < \infty$ fractional digits. A natural choice seems $m = 2$, in which case we calculate a fee that is accurate to the cent. Given upper bounds $\bar{\lambda}$, $\bar{\rho}$ and $\bar{\kappa}$ and the common lower bound zero, we need an exponential number of inner loop iterations in the input length to find a solution that is optimal for a given network expansion plan.

Outer Loop: Inter-Zone Network Expansion Plans. Not having to check all possible inter-zone network configurations, we will group construction choices that yield identical transmission capacities between zone pairs (Z_k, Z_l), with $Z_k \neq Z_l$. By CC we will denote the set of all the relevant network configurations or construction choices that we will evaluate. Note that a reduction in sub problems is

TABLE 1. Transmission Line Data

Start Zone-End Zone	Susceptance	Max. Flow
1-2	0.25	500
	0.5	500
	1	500
2-3	1	1000
3-4	0.5	500
	1	500
4-1	0.5	1000
	1	1000

only possible given that physical line characteristics other than thermal capacities do not play a role on the spot market. The next example illustrates that grouping the corresponding line construction decisions and fixing thermal capacities in the second level problem can indeed significantly reduce the number of sub problems and therfore the number of iterations:

Example 5.5. Let a four zone network be given. Assume that there are no existing lines connecting the different zones. Table 1 gives the candidate transmission lines that can be build to connect the four zones. Solving the problem for all construction choices yields $2^8 = 256$ different sub problems (for a given transmission fee). Grouping line construction decisions we only have to solve $4 \cdot 2 \cdot 3 \cdot 3 = 72$ sub problems (for a given transmission fee). Thus, in the present example we will check less than 30 percent of all existing sub problems by introducing line construction groups.

Denoting by φ one of the three network fees λ, κ or ρ, Algorithm 1 depicts the general structure of this solution process.

Algorithm 1: Tri-Level Decomposition Approach for a Transmission Fee φ

Input : Parameter set for the tri-level power market problem
Output : Optimal solution w^*, x^*, y^* and z^* to the tri-level problem

```
Set f = 0
forall the network configurations c ∈ CC do
    forall the network fees φ ∈ [0, φ̄] do
        Solve the sub problem
        Substitute its optimal values x*_{c,φ} into the master problem and solve
        if f(w*_{c,φ}, x*_{c,φ}, y*_{c,φ}, z*_{c,φ}) > f then
            Update: f(w*_{c,φ}, x*_{c,φ}, y*_{c,φ}, z*_{c,φ}) → f
Return f
```

Remark 5.6. Obviously, the above algorithm can easily be generalized to the case where a mix of the three different network fee regimes is used. In this case, the number of inner loops increases to the cross product of the different fees.

Further Algorithmic Simplifications in the Case of a Lump Sum. In the case of a lump sum we can further simplify our presented search strategy. Obviousely, electricity demand is totally independent of income, i.e., the income elasticity $\eta_{d,t}$ ist zero for all $d \in D$ and $t \in T$. This total inelasticity means that spot market outcomes are completely independent of the lump sum. As a consequence, our inner loop will only have one iteration. With this in mind we can solve the sub problem with a normalized lump sum of zero, drop the budget constraint in the master problem and solve this relaxed single-level master problem for fixed spot market and generation investment values. Then, we calculate ex post the transmission operator's total cost E. Given that in any optimal solution to the original problem total cost must equal total revenues R_λ, the a priori unknown lump sum λ will be equal to the calculated cost. Algorithm 2 depicts the structure of this simplyfied search strategy.

Algorithm 2: Tri-Level Decomposition Approach for a Lump Sum

Input : Parameter set for the tri-level power market problem
Output : Optimal solution w^*, x^*, y^* and z^* to the tri-level problem with a lump sum

Set $f = 0$
forall the *network configurations* $c \in CC$ **do**
    Solve the sub problem
    Substitute its optimal values x_c^* into the master problem and solve
    if $f(w_c^*, x_c^*, y_c^*, z_c^*) > f$ **then**
        Update: $f(w_c^*, x_c^*, y_c^*, z_c^*) \to f$
Return f

5.2.2. *Binary Search.* In this subsection we will discuss a binary search strategy with a linear number of inner loop iterations to solve the tri-level energy market model. Given the above simplifications in the case of a lump sum, φ will be restricted to capacity fees κ and energy fees ρ. Let us now consider the transmission operator's budget constraint. We will explicitly define the profit function $M(w, x, y, z, \varphi)$ with an optimal solution (and any other feasible solution) to the tri-level problem $(w^*, x^*, y^*, z^*, \varphi^*)$ realizing a nonprofit situation, i.e., a situation with $M(w^*, x^*, y^*, z^*, \varphi^*) = 0$. We will further introduce the following optimal value function:

$$V(\varphi) = M(w^*(\varphi), x^*(\varphi), y^*(\varphi), z^*(\varphi), \varphi)$$

$V(\varphi)$ describes the transmission operators profits for varying transmission fee and explicitly takes the optimal variable values $w^*(\varphi), x^*(\varphi), y^*(\varphi)$ and $z^*(\varphi)$ into account. To use a binary search solution algorithm, we will have to prove that $V(\varphi)$ is a monotonic increasing function within an interval $\left[\underset{\sim}{\varphi}, \widetilde{\varphi}\right]$. Such a monotonicity seems quite plausible: Increasing the network fee, revenues of the network operator should also increase. On the other hand, higher network fees should lead to lower generation investment and lower production. This effect should yield a less congested network and therefore less need for redispatch. In a consequence, network expenses including redispatch cost should decrease. Indeed, our numerical test instances strongly suggest that the profit function should be monotonic increasing. Eventhough Grimm et al. (2014) prove the monotonicity of the transmission operator's revenues, a complete analytical proof for the operator's profits does not seem straightforward. Possible or plausible values for $\underset{\sim}{\varphi}$ and $\widetilde{\varphi}$ are discussed in the next section. We will summarize these considerations in the following claim:

Claim 5.7. *$V(\varphi)$ is a monotonic increasing function in the transmission fee $\varphi \in \left[0, \widetilde{\varphi}\right]$.*

Algorithm 3 depicts the structure of the binary search.

6. Numerical Results

In this section we will discuss two prominent test networks. The first example is taken from Jenabi et al. (2013) and consists of three nodes. Applying our power market model to this simple network will illustrate the scope of our approach. Our second test network is a six node example by Chao and Peck (1998) that has widely been used in the energy market literature. For both scenarios (consisting of the network structure and assumed demand and cost parameters) we will calculate i) the welfare optimum (which coincides with the market outcome of a nodal price system in our framework), as well as the market outcomes in the case of ii) a redispatch model without market splitting and iii) a redispatch model with market splitting (i.e. price zones). For the two redispatch models (ii and iii) we will consider the three different network fee regimes, a lump sum fee, an energy based fee, as well as a capacity based fee. Note that we transformed all parameters into equivalent annual hourly values.

6.1. **Implementation.** We implemented the redispatch model as well as the integrated planning model in ZIMPL and used SCIP's file writer to generate corresponding mps files. With these input files we solved the problems using the CPLEX interface in our Java code. All experiments were

Algorithm 3: Binary Search: Tri-Level Decomposition Approach

Input : Parameter set for the tri-level power market problem
Output : Optimal solution w^*, x^*, y^* and z^* to the tri-level problem

Set $f = 0$
forall the *network configurations* $c \in CC$ **do**
    Set $\mathtt{l} = 0$ and $\mathtt{u} = \tilde{\varphi}$
    Solve the sub problem for $\varphi = \mathtt{l}$ and $\varphi = \mathtt{u}$
    Substitute its optimal values $x^*_{c,\varphi}$ into the master problem and solve
    while $\mathtt{l} \leq \mathtt{u}$ **do**
        Calculate $m = 0.5(\mathtt{l} + \mathtt{u})$ and set $\varphi = m$
        Solve the sub problem for $\varphi = \mathtt{m}$
        Substitute its optimal values $x^*_{c,\varphi}$ into the master problem and solve
        if $|V(m)| \leq 0.009$ **then**
            STOP
        else
            if $V(m) \leq 0$ **then**
                Update: $\mathtt{l} \leftarrow m$
            else
                Update: $\mathtt{u} \leftarrow m$
    if $f(w^*_{c,\varphi}, x^*_{c,\varphi}, y^*_{c,\varphi}, z^*_{c,\varphi}) > f$ **then**
        Update: $f(w^*_{c,\varphi}, x^*_{c,\varphi}, y^*_{c,\varphi}, z^*_{c,\varphi}) \rightarrow f$
Return f

performed on a 12 core computer equipped with two AMD Opteron(tm) 2435 Processors, with 2x6 MB cache and 64 GB DDR2-RAM, running Linux (in 64 bit mode).

Given the importance of appropriate transmission fee bounds, we will take the following strategy. In all transmission fee regimes we will use zero as a lower bound excluding negative transmission fees. Then, we will calculate monopoly solutions and derive the respective monopoly markups. Based on the implicit assumption that the TO will not charge fees that are larger than monopoly markups, we use these markups to derive upper bounds. Choosing the respective upper bound, we will consider the different network fee regimes separately: In the case of a per unit fee charged for each electricity unit actually produced, we will use the average monopoly markup across all time periods and market zones as an upper bound. In the case of a per unit fee charged for each unit of capacity connected to the network, we will use the cumulative monopoly markup across all time periods where total generation capacity is binding in a given zone. To obtain the upper bound we then take averages over all market zones.

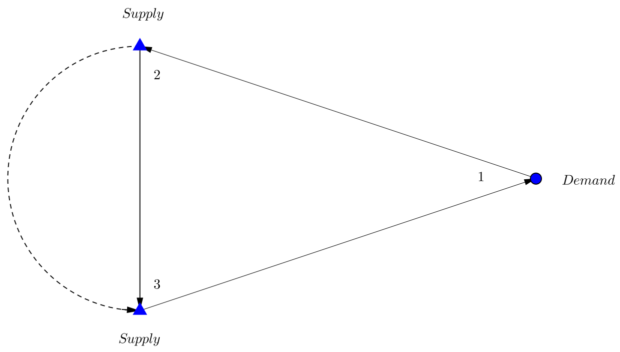


FIGURE 8. 3 Node Test Network

As can easily be seen, in the capacity fee regime the calculated upper bound will in general be much larger than in the energy fee model. This immediately results in relatively long linear search solution times for the capacity fee models. Indeed, this was confirmed by our numerical examples. In our six node example with 2 price zones, given upper capacity fee bounds between 1000 and 1500 the maximum running time amounted up to days. All other model variants including the energy fee models were solved in a few minutes or hours using a linear search procedure. In contrast, our binary search algorithm solved all problems in only a few minutes. This indicates that our binary search seems quite promising also for larger test instances.

6.2. **A Three Node Test Example.** Our first test example consists of three nodes and four time periods. We allow for both, investment in the transmission network and in generation units. Before we present our main results, we briefly review the input data that is directly taken from Jenabi et al. (2013).

Network Parameters. Similar to Jenabi et al. (2013), we assume a complete graph with three nodes as depicted in figure 8. We number the three existing links (solid lines) counterclockwise, i.e., line 1 connects network node 1 and 2 and so forth. Each line has a susceptance of 8. Lines 1 and line 3 have thermal capacity of 2 MW and line 2 has a capacity of 0.25 MW. To explicitly model line investment, we allow for investment in a new transmission line connecting node 2 and node 3 (dashed line). This candidate transmission line has a capacity of 0.25 MW, susceptance of 10, and investment cost of 4 $/MW.

Generation Parameters. Nodes 2 and node 3 are generation nodes. We assume that at the beginning of the planning horizon there are no existing generators but explicitly model generation investment. Technology 1 can be built at node 2 at investment costs of 160 $/MW and produces at constant variable cost equal to 10 \$/MWh. Technology 2 can be built at node 3 at investment cost of 120 \$/MW and produces at constant variable cost of production of 11 \$/MWh. On average, 80% of capacity is available.

Demand Parameters. Consumers are located at node 1. We will assume the following basic linear demand function:

$$Z^{price}(x^{dem}) = 550 - 500x^{dem}$$

To give the four periods $t = 1, 2, 3, 4$ some simple interpretation, we will refer to them as the four seasons spring, summer, autumn and winter. For this reason, we will multiply the intercept of the above demand function by $1, 0.5, 1$ and 1.5 in period 1 to 4, respectively[11].

Numerical Results. In our three node test framework the integrated planner approach yields a social welfare of \$ 1083. This would also be the welfare level under a nodal pricing model. A redispatch model with no market splitting (1 price zone) implies a welfare loss as compared to the first best solution in all three network fee regimes. In the case of a capacity based fee, an electricity based fee and a lump sum fee the realized welfare levels are \$ 1050, \$ 1044 and \$ 1040, respectively. This welfare loss is mainly driven by a distortion of generation investment: As Figure 9 illustrates, the optimal solution implies positive investment in both technologies, where investment in technology 1 is much larger, however. The tri-level power market model, on the contrary, yields generation investment only in technology 1. Investment in technology 1 is higher than in the welfare optimum for all network fee regimes (lowest for capacity based fees, highest for lump sum fees). The candidate line connecting the two nodes 2 and 3 is build in all three scenarios, and also in the welfare optimum. Introducing a second price zone does not change the presented results. This can be explained by the fact that no matter how the two price zones are designed, there is always at least one non–congested line that connects the two zones. This suggests that a meaningful analysis of market splitting requires larger networks.

6.3. **Test Example by Chao and Peck (1998).** In order to illustrate the possibilities to analyze effects of market splitting within our model we consider an adapted 52 period test example based on Chao and Peck (1998). The network we use has been extensively analyzed in the literature to illustrate the abilities of different energy market models. We extend the original test example by

[11]Such an interpretation implicitly assumes a simplified temperature course like in Europe where a lot of energy is consumed for heating in winter time. However, by switching the weights for the different seasons we could also simulate other temperature courses where most energy is for instance consumed in summer time for running air-conditioning systems.

FIGURE 9. Generation Investments

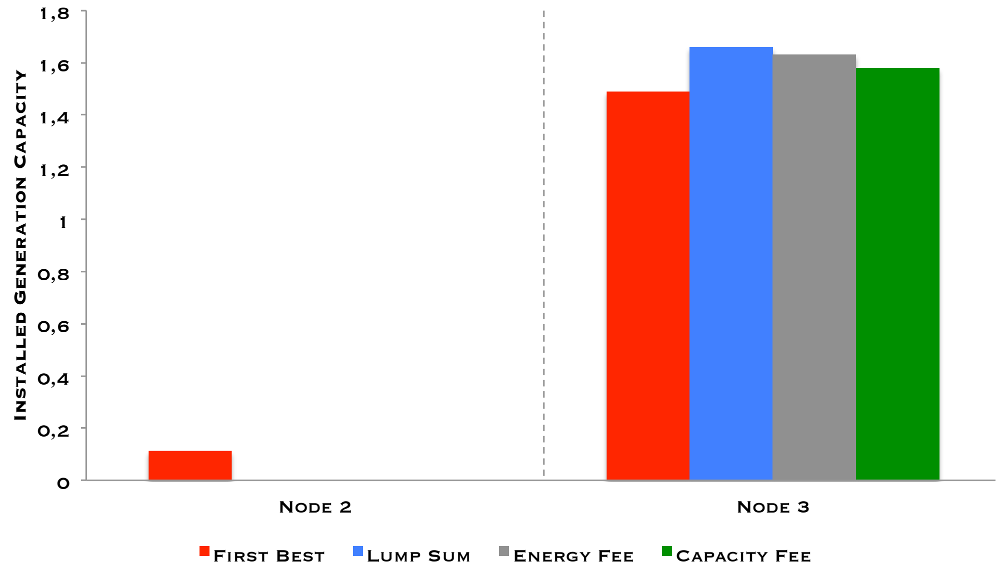


FIGURE 10. Electricity Prices

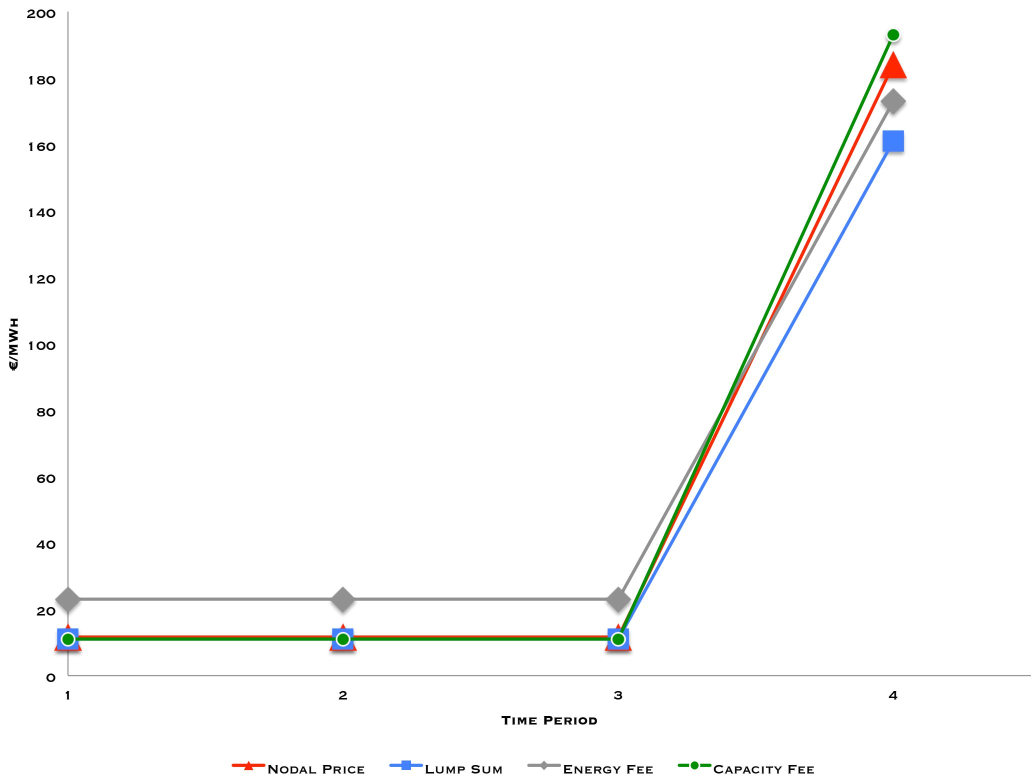
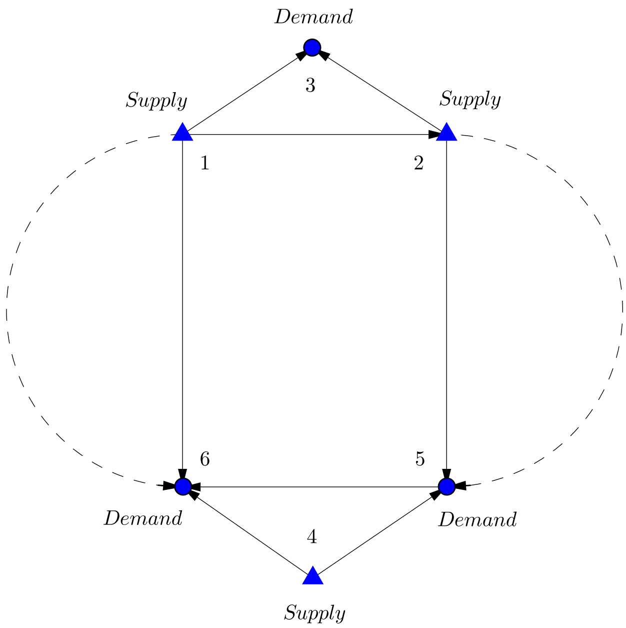


FIGURE 11. Six–Node Test Network



Chao and Peck (1998) by several aspects important for our approach such as fluctuating demand, generation investment, network investment, as well as a cost-based redispatch mechanism.

Network Parameters. As Figure 11 illustrates, the network consists of six nodes interconnected by eight existing transmission lines (solid lines). Northern nodes (nodes 1 to 3) and southern nodes (nodes 4 to 6) are interconnected by lines with unlimited capacities. The "northern zone" and the "southern zone" are interconnected by only two lines (1—6 and 2—5) that have limited capacities (200 MW and 250 MW, respectively). We consider a situation where two different candidate transmission lines (dashed lines) can be build to alleviate congestion problems. Detailed transmission line data including investment cost can be found in table 2.

Generation Parameters. We assume that there are no existing generators initially but explicitly model generation investment. Generation units can be build at nodes 1, 2, and 4 (see the blue triangles in Figure 11). Variable cost of technology 1, 2, and 4 are 10 $/MWh, 15 $/MWh and 42.5 $/MWh, respectively. Investment cost are 700 $/MW, 600 $/MW and 200 $/MW, repectively. Average availability of capacity is 80%.

Demand Parameters. Demand is located at nodes 3, 5 and 6 (see the blue dots in Figure 11). The basic demand parameters used in our numerical example are provided in Table 3. Assuming these three basic demand functions taken from Chao and Peck (1998) we use real world data to induce

TABLE 2. Transmission Line Data

Start-End	Existing/Candidate Line	Susceptance	Max. Flow (MW)	Inv. Cost ($/MW)
1-2	Existing	1	∞	-
1-3	Existing	1	∞	-
1-6	Existing	0.5	200	-
	Candidate	0.5	200	230000
2-3	Existing	1	∞	-
2-5	Existing	0.5	250	-
	Candidate	0.5	200	230000
4-5	Existing	1	∞	-
4-6	Existing	1	∞	-
5-6	Existing	1	∞	-

TABLE 3. Basic Demand Parameters

Network Node	Intercept	Slope
3	37.5	0.05
5	75	0.1
6	80	0.1

demand fluctuation across the 52 periods at all nodes.[12] In particular, we use factors derived from the German 2011 demand realizations for shifting the above demand functions. Demand levels across nodes in a given period are always shifted by the same factor, which accounts for the fact that typically the level of demand in a given period is correlated across demand nodes.

Numerical Results. Figure 12 illustrates welfare in the scenarios with one and two price zones, respectively, as compared the the welfare optimum. Obviously, market splitting increases welfare but does not achieve the maximum possible level. Whereas welfare levels under different network tariff regimes are rather similar with two price zones we find substantial welfare differences for different network regimes without market splitting.

Table 4 and Figure 13 illustrate investment decisions in transmission and generation capacities in our different scenarios. In the welfare optimum the social planer refrains from transmission line expansion and generation capacities are installed at all nodes. Obviously, from a welfare perspective, investment in generation capacity at node 4 is preferable to any line investment, although variable

[12]Alternatively, one could choose random draws from a distribution that reflects the nature of demand fluctuation in electricity markets.

FIGURE 12. Welfare Under Different Regimes

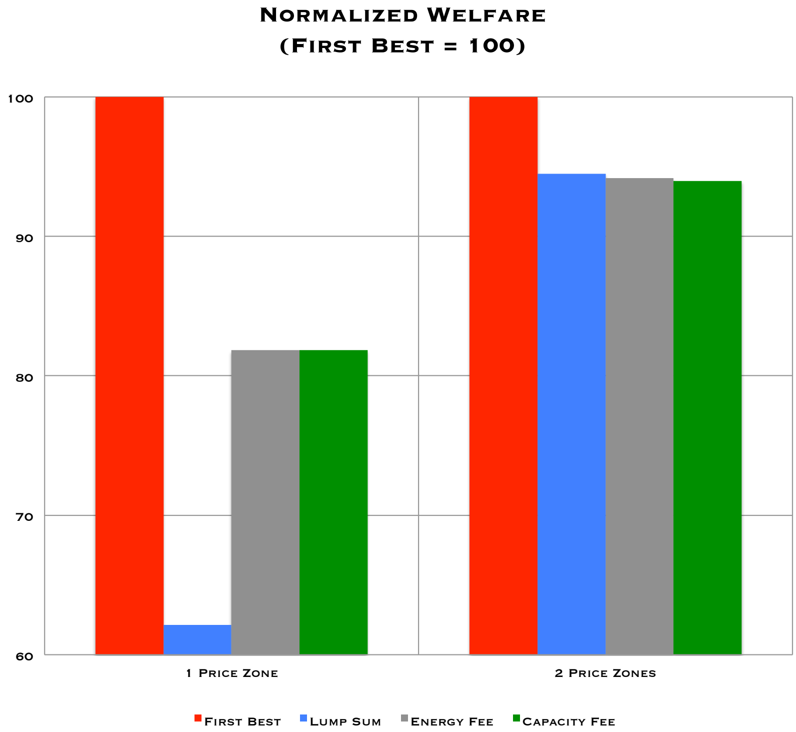


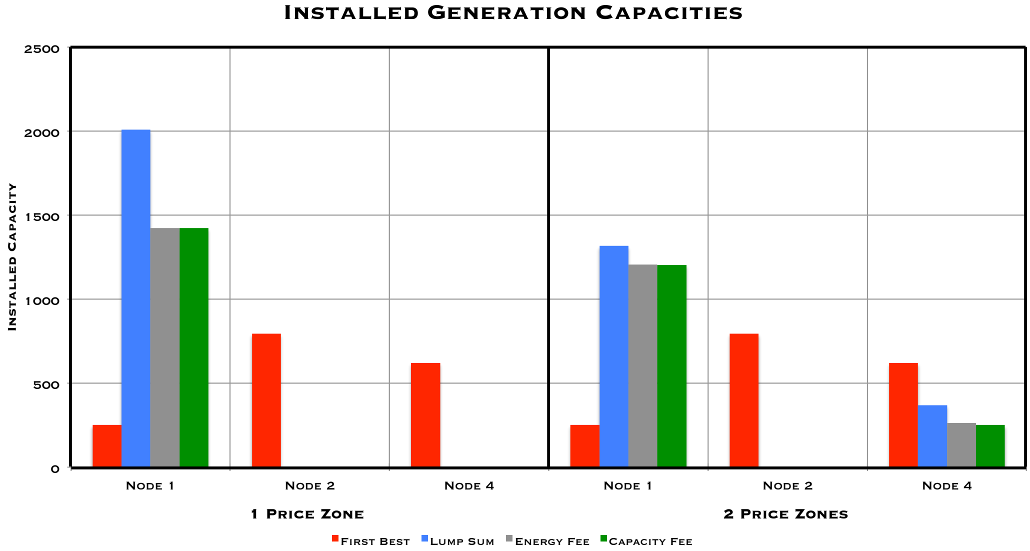
TABLE 4. Transmission Line Investment

Model Variant	**Network Fee Regime**	**Candidate Line 1**	**Candidate Line 2**
Nodal Price System	n.a.	NO	NO
1 Price Zone	Lump Sum	YES	NO
	Energy Fee	YES	YES
	Capacity Fee	YES	YES
2 Price Zones	Lump Sum	YES	NO
	Energy Fee	YES	NO
	Capacity Fee	YES	NO

production cost of the available technology is substantially higher than variable cost of technologies available at northern nodes. Furthermore, due to the fact that demand satisfied by northern generators is relatively low in the absence of line investment, generation capacity is build up at node 2, where investment cost is lower than at node 1, but unit production cost is higher.

Turing to the case of a power market without market splitting, we find that both candidate lines are built and that generation capacity is exclusively installed at node 1. The intuition is straightforward: Recall that we have no price zones and that transmission constraints are not accounted for at the

FIGURE 13. Generation Investment Under Different Regimes

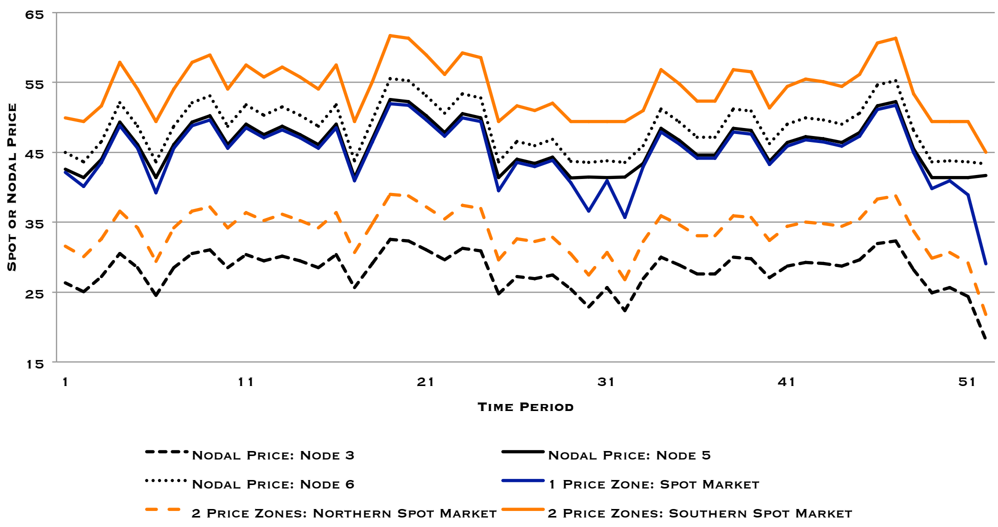


spot market. Thus, the generator with the lowest unit production cost will trade at the spot market, other generators will predominantly be called for at the redispatch stage. The fact that redispatch reimbursement covers only marginal cost implies immediately, that those generators cannot profitably operate. Consequently, all capacity investment occurs at the node where the sum of production and investment cost is lowest given that the whole market has to be supplied — which is at node 1. Anticipating this decision, the social planner prefers to install both candidate lines (except for the case of a lump sum network fee).

Finally, consider the scenario with two price zones, north and south. Obviously, market splitting allows a generator in the south to earn rents whenever it is impossible to satisfy demand in the south by production from northern generators due to transmission constraints. Thus, firms have an incentive to install capacity at node 4, which in turn decreases the incentives of the social planer to install additional transmission capacity. Consequently, anticipation of generation investment at node 4 leads to less transmission investment by the TO — only one line is built. Thus, market splitting moves the whole system closer to the welfare optimal scenario.

Prices in Figure 14 support the above intuition for the case of the energy based fee. In the welfare optimal solution, prices at the consumption nodes are low in the north (node 3) and high in the south (nodes 5 and 6). With one price zones, the spot market price is relatively high, however, generators with high unit production cost have no chance to contract their supply at the spot market.

FIGURE 14. Spot Market and Nodal Prices



Two price zones imply also a north south spread which allows the southern generators to recover their investment cost.

Overall, the 6–node test examples nicely illustrates the capability of our framework to identify the effects of different regulatory regimes and to potentially quantify them.

7. Conclusion and Outlook

This article analyzes the long run impact of different transmission management regimes on investment incentives of generating firms in a market environment with a regulated transmission operator. We propose a tri–level programming approach to model an electricity market with redispatch both, with and without market splitting. As a First Best benchmark we also solve the corresponding integrated planer problem where an Integrated Generation and Transmission Company (as a central planer) controls both the, grid and generation units. In order to solve our tri–level problem numerically, we present a decomposition approach that relies on a detailed analysis of interconnection between the three problem levels. We apply our approach to two simple test problems in order to demonstrate the capabilities to analyze transmission and capacity expansion in a market environment with the proposed methodology.

Our results clearly show that in a market environment investment choices by the transmission operator and private firms can substantially differ from socially optimal choices. Obviously, investment in generation units is driven by the incentives for private investors induced by the particular market environment. Absence of proper incentives affects locational decisions of generators and this, in turn, can have substantial effects on optimal line investment. In two examples, we demonstrate that welfare optimal line investment is crucially affected by the investment in generation capacity anticipated by the transmission operator. We thus demonstrate that our model allows to compare different network management regimes and assess their effects on long run investment decisions. Our approach is, thus, an important extension of various studies that up to now have mainly considered the short run properties of different transmission management regimes. As we show, transmission management has also important implications in the long run when generation and transmission expansion are taken into account.

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Appendix A. Notations and Symbols

In this subsection we will present all the sets, parameters and variables used in our models.

Table 5. Sets

Symbol:	**Description:**
	General Sets
N	Set of all network nodes
$D \subset N$	Set of demand nodes
T	Set of time periods
Z	Set of zones for market splitting
	Generators
G_n^{all}	Set of all generation technologies at node $n \in N$
$G_n^{ex} \subset G_n^{all}$	Set of existing generation technologies at node $n \in N$
$G_n^{new} \subset G_n^{all}$	Set of candidate generation technologies at node $n \in N$
	Transmission Lines
$\mathfrak{L}$	Set of all line types
$L_{\mathfrak{l}}^{ex} \subset N \times N$	Set of existing transmission lines of type $\mathfrak{l} \in \mathfrak{L}$
$L_{\mathfrak{l}}^{new} \subset N \times N$	Set of candidate transmission lines of type $\mathfrak{l} \in \mathfrak{L}$
$L_{\mathfrak{l}}^{inter} \subset N \times N$	Set of inter-zone transmission lines of type $\mathfrak{l} \in \mathfrak{L}$

TABLE 6. Parameters

Symbol:	**Description:**	**Unit:**
	General Parameters	
M	Big M parameter	1
	Demand Parameters	
$d_{d,t}^{inter}$	Intercept of demand function at node $d \in D$ in time period $t \in T$	€
$d_{d,t}^{slope}$	Slope of demand function at node $d \in D$ in time period $t \in T$	MW/€
$\bar{z}_{d,t}^{dem}$	Satiation point of demand function $d \in D$ in time period $t \in T$	MW
	Generator Parameters	
i_g^{gen}	Investment cost of candidate generation technology $g \in G_n^{new}$	€ /MW
v_g^{gen}	Variable cost of generation technology $g \in G_n^{all}$	€ /MW
$\overline{x}_g^{gncp}$	Maximum power generation capacity of generator $g \in G_n^{ex}$	MW
e_g^{gen}	Generation efficiency parameter of $g \in G_n^{all}$ (equivalent availability)	1
	Transmission Line Parameters	
i_l^{line}	Investment cost for candidate transmission line $l \in L_{\mathfrak{l}}^{new}$	€
i_l^{line}	Deletion cost for existing transmission line $l \in L_{\mathfrak{l}}^{ex}$	€
b_l	Susceptance of transmission line $l \in L_{\mathfrak{l}}^{all}$	S
$\bar{z}_l^{flow}$	Maximum power flow on line $l \in L_{\mathfrak{l}}^{all}$	MW

TABLE 7. Variables

Symbol:	**Description:**	**Unit:**
	First Stage Variables: Line Expansion	
w_l^{line}	Decision variable with value 1, if line $l \in L_\mathfrak{l}^{new}$ is build, and 0 otherwise	1
w_l^{line}	Decision variable with value 1, if line $l \in L_\mathfrak{l}^{ex}$ is deleted, and 0 otherwise	1
E	Transmission operator's expenses	€
R	Transmission operator's revenues	€
λ	Lump sum fee	€
ρ	Per unit fee charged for each unit of electricity consumed/produced	€ /MW
κ	Per unit fee charged for each unit of capacity installed	€ /MW
	Second Stage Variables: Spot Market	
x_g^{gncp}	Amount of new generation capacity installed of generator $g \in G_n^{new}$	MW
$x_{g,t}^{gen}$	Power generation of generator $g \in G_n^{all}$ in time period $t \in T$	MW
$x_{d,t}^{dem}$	Demand/Power sold at node $d \in D$ in period $t \in T$	MW
$x_{l,t}^{flow}$	(Net-)Power flow on inter-zone line $l \in L_\mathfrak{l}^{inter}$ in time period $t \in T$	MW
	Third Stage Variables: Redispatch Market	
$y_{g,t}^{gen}$	Redispatched electricity generation of generator $g \in G_n^{all}$ in time period $t \in T$	MW
$y_{d,t}^{dem}$	Redispatched electricity demand at node $d \in D$ in period $t \in T$	MW
$y_{l,t}^{flow}$	Redispatched Power flow on inter-zone line $l \in L_\mathfrak{l}^{inter}$ in time period $t \in T$	MW
$z_{g,t}^{gen}$	Final electricity generation of generator $g \in G_n^{all}$ in time period $t \in T$	MW
$z_{d,t}^{dem}$	Final electricity demand at node $d \in D$ in period $t \in T$	MW
$z_{n,t}^{angle}$	Nodal angle in node $n \in N$ at time period $t \in T$	rad
$z_{l,t}^{flow}$	Final (Net-)Power flow on line $l \in L_\mathfrak{l}^{all}$ in time period $t \in T$	MW