

# MULTI-ATTRIBUTE EXTENSION FUZZY OPTIMIZED DECISION-MAKING MODEL OF SCHEME DESIGN

*Ti-chun Wang, Ai-jun Yang, Shi-sheng Zhong*

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Due to the uncertain interval and the incomplete attribute information in the procedure of the complex mechanism scheme design, a computing method for the multi-attribute extension decision making of fuzzy number is proposed. The proposed model for the multi-attribute extension decision making of fuzzy number is developed based on the fuzzy system theory and the extension theory. By adopting the fuzzy-matter element, the multiple-attribute extension decision making matrix is standardized, and the dummy ideal region and the negative ideal region of fuzzy-matter element with integrated attribute information are proposed. Then, the extension distance of fuzzy information is presented, and the synthetic weights of indexes that include both subjective preference and objective information are given based on the extension distance. Based on the extension fuzzy optimal membership degree of design schemes, the optimum design scheme is achieved in the whole system. Finally, an example is provided to examine the model and algorithm.

**Keywords:** *extension theory, fuzzy-matter element, multi-attribute decision making, scheme design*

## Fuzzy optimizirani model za donošenje odluke o projektu sheme zasnovan na multi-atributnoj ekstenziji

Izvorni znanstveni članak

Zbog nesigurnog intervala i nepotpune informacije o karakteristikama u postupku projektiranja sheme složenog mehanizma, predlaže se računalna metoda za donošenje odluke uz multi-atributnu ekstenziju neodređenog (fuzzy) broja. Predloženi model za donošenje odluke uz multi-atributnu ekstenziju neodređenog broja razvijen je na osnovu teorije fuzzy sustava i teorije ekstenzije. Prihvatanjem elementa fuzzy-materije, matrica za donošenje odluke uz multi-atributnu ekstenziju je standardizirana i predlaže se zamjensko (dummy) idealno područje i negativno idealno područje elementa fuzzy-materije uz informacije o karakteristikama (atributima). Zatim se prezentira ekstenzijska udaljenost fuzzy informacije i daju sintetičke težine indeksa uključujući i subjektivne preferencije i objektivnu informaciju na temelju udaljenosti ekstenzije. Na temelju fuzzy optimalnog stupnja ekstenzije projektnih shema, optimalna projektna shema se postiže u cijelom sustavu. Konačno, daje se primjer u svrhu ispitivanja modela i algoritma.

**Ključne riječi:** *element fuzzy-materije, multi-atributno donošenje odluka, projekt sheme, teorija ekstenzije*

## 1 Introduction

Structural configuration has characteristics like multi-level, multi-attribute, creativity and so on, especially in complex mechanism scheme design of aerospace, power generation equipment and other industries. During the design process of such products, not only the performance requirements of the mechanism design, but also the overall planning system should be ensured. So we must anatomize the technical indicators, economic indicators and social indicators in the structural configuration of the complex mechanism scheme design. This is a decision-making problem about the technical and economic system which is multi-attribute and contains many non-deterministic factors [1 ÷ 3]. Traditional fuzzy theory studies into the problems of cognitive uncertainty, the problems have the characteristics of clear meaning and unclear extension [4, 5]. Extension theory studies the problem at contradiction in both qualitative and quantitative points. It uses extension methods for association analysis. For example, Tichun WANG [6], Meng-Hui Wang [7] and Yong-xiu He [8] have put forward the methods and models of the problems of extension data mining based on the extension theory. And YiFei Chen [9], Wang Tichun [10] and Zhao Yanwei [11] have discussed the extension configuration models of product Scheme design. Liu Yang [12], Jianzhou Gong [13] and Meng-Hui Wang [14] have studied the extension methods based on the matter-element. At present, there are some scholars who use extenics to study the issues of product design decision and design analysis, and obtain the corresponding results. Such as Yan-chao Yin [15], JIA Chun-rong [16] and Dongjun Liu [17] have given the extension fuzzy optimization and evaluation models based

on the extension theory. Thus, we combine the fuzzy mathematics with the extension theory and use this combination to solve the complex multi-attribute mechanism scheme design with non-deterministic factors. In the process, we gain a new decision-making method to solve such fuzzy problem. In recent years, many scholars have conducted research in this area. They applied it to engineering area and have all obtained good application results [18 ÷ 21]. However, in the existing literature, it mostly represents fuzzy decision-making based on the matter-element model and does research about decision-making based on the ideal scheme, without considering the deviation arising from the known information. Strictly speaking, such method cannot really solve fuzzy decision-making problems. In summary, this paper provides an improved multi-attribute extension fuzzy optimized decision-making model based on the fuzzy theory and the extension theory. In this paper, we will give the specific process with examples. Firstly, the basic definition of extension fuzzy decision-making is given in Section 2. Then, a multi-attribute extension fuzzy optimized decision-making model is described in Section 3. Then, an example about the multi-attribute extension fuzzy optimized decision-making model is provided in Section 4. Finally, the discussions and acknowledgments are given in Section 5 and Section 6, respectively.

## 2 Basic definition of extension fuzzy decision-making

In order to analyse and calculate things qualitatively and quantitatively, extenics describes a variety of things formally with matter-element. That means it describes things with triples  $R = (N, c, v)$  as the basic unit, the triples is made up of variable  $N$ , characteristic  $c$  and the

variable  $v$  which is changed with  $c$ . Where the variable  $N$  has  $n$  characteristics,  $\mathbf{R}$  is called  $n$ -dimensional matter-element. If the eigenvalue of matter-element is fuzzy, then the matter-element is called fuzzy matter-element, denoted as:

$$\tilde{\mathbf{R}}_n = (N, C, V(f(x))) = \begin{bmatrix} N & c_1 & v_1(f_1(x)) \\ & c_2 & v_2(f_2(x)) \\ & \dots & \dots \\ & c_n & v_n(f_n(x)) \end{bmatrix} \quad (1)$$

where  $\tilde{\mathbf{R}}_n$  is an  $n$ -dimensional fuzzy matter-element,  $N$  is the event,  $c_i$  is the NO.  $i$  decision-making characteristic of  $N$ ,  $v_i(f_i(x))$  is the fuzzy value of  $c_i$ , it can express both the interval value of  $c_i$  and the fuzzy membership degree of  $c_i$ .

**Definition 1.** If there are  $m$   $n$ -dimensional matter-elements, each of them has  $n$  characteristics as  $c_1, c_2, \dots, c_n$ , then the corresponding fuzzy values of the matter-element characteristics are  $v_1(f_{i1}(x)), v_2(f_{i2}(x)), \dots, v_n(f_{in}(x))$  ( $i = 1, 2, \dots, m$ ).

They are called  $n$ -dimensional fuzzy composite-elements, denoted as:

$$\tilde{\mathbf{R}}_{m \otimes n} = \left\{ \tilde{\mathbf{R}}_n(1), \tilde{\mathbf{R}}_n(2), \dots, \tilde{\mathbf{R}}_n(m) \right\}$$

$$= \begin{bmatrix} C_1 & C_2 & \dots & C_n \\ N_1 & v_1(f_{11}(x)) & v_2(f_{12}(x)) & \dots & v_n(f_{1n}(x)) \\ N_2 & v_1(f_{21}(x)) & v_2(f_{22}(x)) & \dots & v_n(f_{2n}(x)) \\ \dots & \dots & \dots & \dots & \dots \\ N_m & v_1(f_{m1}(x)) & v_2(f_{m2}(x)) & \dots & v_n(f_{mn}(x)) \end{bmatrix}_{m \times n} \quad (2)$$

**Definition 2.** Suppose the fuzzy composite-element corresponded to the decision-making scheme is  $\tilde{\mathbf{R}}_{m \otimes n} = \{ \tilde{\mathbf{R}}_n(1), \tilde{\mathbf{R}}_n(2), \dots, \tilde{\mathbf{R}}_n(m) \}$ , then all the decision-making characteristics of the fuzzy matter-element are positive:  $v_{0j}^+(f_j(x)) = \max(v_j(f_{ij}(x)) | 1 \leq i \leq m)$ , we call it the positive ideal region of fuzzy composite-element  $\tilde{\mathbf{R}}_{m \otimes n}$ , referred to the characteristic  $c_j$ . If  $v_j(f_{ij}(x))$  is an interval number, then  $v_j(f_{ij}(x)) = [v_j^L(f_{ij}(x)), v_j^R(f_{ij}(x))]$ ,  $v_j^L(f_{ij}(x)) \leq v_j^R(f_{ij}(x))$  and  $v_{0j}^+(f_j(x)) = [\max(v_j^L(f_{ij}(x)) | 1 \leq i \leq m), \max(v_j^R(f_{ij}(x)) | 1 \leq i \leq m)]$ .

$v_{0j}^-(f_j(x)) = \min(v_j(f_{ij}(x)) | 1 \leq i \leq m)$ , we call it the negative ideal region of fuzzy composite-element  $\tilde{\mathbf{R}}_{m \otimes n}$ , referred to matter-element characteristic  $c_j$ . If  $v_j(f_{ij}(x))$  is an interval number, then  $v_j(f_{ij}(x)) = [v_j^L(f_{ij}(x)), v_j^R(f_{ij}(x))]$ ,  $v_j^L(f_{ij}(x)) \leq v_j^R(f_{ij}(x))$ , and  $v_{0j}^-(f_j(x)) = [\max(v_j^L(f_{ij}(x)) | 1 \leq i \leq m), \min(v_j^R(f_{ij}(x)) | 1 \leq i \leq m)]$ .

So  $\tilde{\mathbf{R}}_{m \otimes n}^+$  is the positive ideal matter-element which corresponds to the fuzzy composite-element  $\tilde{\mathbf{R}}_{m \otimes n}$ :

$$\tilde{\mathbf{R}}_{m \otimes n}^+ = \begin{bmatrix} N & c_1 & v_{01}^+(f_1(x)) \\ & c_2 & v_{02}^+(f_2(x)) \\ & \dots & \dots \\ & c_n & v_{0n}^+(f_n(x)) \end{bmatrix} \quad (3)$$

$\tilde{\mathbf{R}}_{m \otimes n}^-$  is the negative ideal matter-element which corresponds to the fuzzy composite-element  $\tilde{\mathbf{R}}_{m \otimes n}$ :

$$\tilde{\mathbf{R}}_{m \otimes n}^- = \begin{bmatrix} N & c_1 & v_{01}^-(f_1(x)) \\ & c_2 & v_{02}^-(f_2(x)) \\ & \dots & \dots \\ & c_n & v_{0n}^-(f_n(x)) \end{bmatrix} \quad (4)$$

**Definition 3.** Suppose the ideal region of the fuzzy composite-element  $c_j$  is  $v_{0j}(f_j(x)) = [v_{0j}^L(f_j(x)), v_{0j}^R(f_j(x))]$  The matter-element characteristic quantity which corresponds to the design scheme  $i$  is  $v_j(f_{ij}(x))$ , then the dot extension distance of the fuzzy matter-element characteristic is  $K_{ij}$ , it can be expressed as:

$$K_{ij} = \rho(v_j(f_{ij}(x)), \langle v_{0j}^L(f_j(x)), v_{0j}^R(f_j(x)) \rangle)$$

$$= \left| v_j(f_{ij}(x)) - \frac{v_{0j}^L(f_j(x)) + v_{0j}^R(f_j(x))}{2} \right|$$

$$- \frac{1}{2} (v_{0j}^R(f_j(x)) - v_{0j}^L(f_j(x))) \quad (5)$$

When  $v_j(f_{ij}(x))$  is fuzzy and  $v_j(f_{ij}(x)) = [v_j^L(f_{ij}(x)), v_j^R(f_{ij}(x))]$  the dot extension distance  $K_{ij}$  can be expressed as:

$$K_{ij} = \rho(\langle v_j^L(f_{ij}(x)), v_j^R(f_{ij}(x)) \rangle, \langle v_{0j}^L(f_j(x)), v_{0j}^R(f_j(x)) \rangle)$$

$$= \frac{1}{2} (\rho(v_j^L(f_{ij}(x)), \langle v_{0j}^L(f_j(x)), v_{0j}^R(f_j(x)) \rangle) + \rho(v_j^R(f_{ij}(x)), \langle v_{0j}^L(f_j(x)), v_{0j}^R(f_j(x)) \rangle)) \quad (6)$$

Based on the extension theory, we know that the smaller  $K_{ij}$  is, the closer the matter-element characteristic  $c_j$  will get to the design goal.

**Definition 4.** If the membership degree of the positive ideal matter-element  $\tilde{\mathbf{R}}_{m \otimes n}^+$  is  $\psi_i$ , where the positive ideal matter-element  $\tilde{\mathbf{R}}_{m \otimes n}^+$  is attached to the fuzzy composite-element  $\tilde{\mathbf{R}}_{m \otimes n}$ , then the membership degree of the corresponding negative ideal matter-element is  $1 - \psi_i$ . In order to determine the extension optimal membership degree of the fuzzy matter-element  $\psi_i$ , the objective function is proposed as:

$$F(\psi) = \min \left\{ (\psi_i * d_i^+)^2 + ((1 - \psi_i) * d_i^-)^2 \right\} \wedge (0 \leq \psi_i \leq 1). \quad (7)$$

From the extension relatedness, we can get  $d_i^+ = \sqrt{\sum_{j=1}^n \left( w_j * \left( K_{ij} - \min_{1 \leq i \leq m} (K_{ij}^+) \right)^2 \right)}$ , that means the closer minimum extension distance gets to the positive region of the matter-element, the more superior it is;  $d_i^- = \sqrt{\sum_{j=1}^n \left( w_j * \left( K_{ij} - \min_{1 \leq i \leq m} (K_{ij}^-) \right)^2 \right)}$ , that means the closer minimum extension distance gets to the negative region of the matter-element, the more inferior it is; where  $K_{ij}^+$  is the extension distance of the positive fuzzy matter-element region and  $K_{ij}^-$  is the extension distance of the negative fuzzy matter-element region.  $w_j$  is the weight of matter-element characteristic  $c_j$  of the fuzzy matter-element  $\tilde{R}_n(i)$  with  $\sum_{j=1}^n w_j = 1$ .

Let  $\partial F(\psi) / \partial \psi_i = 0$ , solve the Eq. (7) based on the extremum principle, then the computational model of fuzzy matter-element's extension optimal membership degree can be expressed as:

$$\psi_i = 1 / \left( 1 + (d_i^+ / d_i^-)^2 \right), i = 1, 2, \dots, m. \quad (8)$$

### 3 Multi-attribute Extension Fuzzy Optimized Decision-Making Model

$$A_{m \times n} = \begin{bmatrix} [v_1^L(f_{11}(x)), v_1^R(f_{11}(x))] & [v_2^L(f_{12}(x)), v_2^R(f_{12}(x))] \\ [v_1^L(f_{21}(x)), v_1^R(f_{21}(x))] & [v_2^L(f_{22}(x)), v_2^R(f_{22}(x))] \\ \vdots & \vdots \\ [v_1^L(f_{m1}(x)), v_1^R(f_{m1}(x))] & [v_2^L(f_{m2}(x)), v_2^R(f_{m2}(x))] \\ \dots & \dots \\ \dots & \dots \\ \dots & \dots \\ \dots & \dots \\ [v_n^L(f_{1n}(x)), v_n^R(f_{1n}(x))] & \\ \dots & [v_n^L(f_{2n}(x)), v_n^R(f_{2n}(x))] \\ \dots & \vdots \\ \dots & [v_n^L(f_{mn}(x)), v_n^R(f_{mn}(x))] \end{bmatrix}. \quad (9)$$

Suppose there are  $m$  product design schemes called  $P$ , and it's fuzzy matter-elements are  $\tilde{R}_n(1), \tilde{R}_n(2), \dots, \tilde{R}_n(m)$ , it has  $n$  same decision-making indexes, that are fuzzy matter-element characteristics  $c_1, c_2, \dots, c_n$ , and the corresponding weights are  $w_1, w_2, \dots, w_n$  (model of weight distribution please see 2.3), the fuzzy value of the fuzzy matter-element with regard to matter-element characteristic  $c_j$  is  $V_{ij} = [v_j^L(f_{ij}(x)), v_j^R(f_{ij}(x))]$ , where  $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ , and the corresponding fuzzy

composite-element is  $\tilde{R}_{m \otimes n}$ , then the extension fuzzy decision-making matrix can be expressed as Eq. (9).

#### 3.1 Normative approach of fuzzy matter-element

Decision-making of complex mechanism design scheme contains varieties of indexes like technical indexes, economic indexes and social indexes. Among them there are both precise and imprecise indexes, cost-type indexes and efficiency indexes. Some of them are moderate-type indexes, but always with different dimensions. For easy analysis of the extension fuzzy decision-making, we have to normatively approach the extension fuzzy decision-making matrix  $A_{m \times n}$ .

If the matter-element characteristic  $c_j$  is a cost-type index, then

$$u_{ij} = \begin{cases} 1, & v_j(f_{ij}(x)) \leq v_{0j}^-(f_j(x)) \\ (v_{0j}^+(f_j(x)) - v_j(f_{ij}(x))) / (v_{0j}^+(f_j(x)) - v_{0j}^-(f_j(x))), & v_{0j}^-(f_j(x)) < v_j(f_{ij}(x)) < v_{0j}^+(f_j(x)) \\ 0, & v_j(f_{ij}(x)) \geq v_{0j}^+(f_j(x)) \end{cases} \quad (10)$$

If the matter-element characteristic  $c_j$  is an efficiency index, then

$$u_{ij} = \begin{cases} 0, & v_j(f_{ij}(x)) \leq v_{0j}^-(f_j(x)) \\ (v_j(f_{ij}(x)) - v_{0j}^-(f_j(x))) / (v_{0j}^+(f_j(x)) - v_{0j}^-(f_j(x))), & v_{0j}^-(f_j(x)) < v_j(f_{ij}(x)) < v_{0j}^+(f_j(x)) \\ 1, & v_j(f_{ij}(x)) \geq v_{0j}^+(f_j(x)) \end{cases} \quad (11)$$

If the matter-element characteristic  $c_j$  is a moderate-type index, then

$$u_{ij} = \begin{cases} 0, & v_j(f_{ij}(x)) \leq v_{0j}^-(f_j(x)) \vee v_j(f_{ij}(x)) \leq v_{0j}^-(f_j(x)) \\ \frac{\left| v_j(f_{ij}(x)) - \frac{v_{0j}^+(f_j(x)) + v_{0j}^-(f_j(x))}{2} \right|}{\left| \frac{v_{0j}^+(f_j(x)) + v_{0j}^-(f_j(x))}{2} - v_{0j}^-(f_j(x)) \right|}, & v_{0j}^-(f_j(x)) < v_j(f_{ij}(x)) < v_{0j}^+(f_j(x)) \\ \wedge v_j(f_{ij}(x)) \neq \frac{v_{0j}^+(f_j(x)) + v_{0j}^-(f_j(x))}{2} \\ 1, & v_j(f_{ij}(x)) = \frac{v_{0j}^+(f_j(x)) + v_{0j}^-(f_j(x))}{2} \end{cases} \quad (12)$$

Let  $v_j(f_{ij}(x)) = v_j^L(f_{ij}(x))$ , we can get the corresponding  $u_{ij}^L$ , and let  $v_j(f_{ij}(x)) = v_j^R(f_{ij}(x))$ , we can get  $u_{ij}^R$ , after the normative approach, the extension fuzzy decision-making matrix can be expressed as follows:

$$U_{m \times n} = \begin{bmatrix} [u_{11}^L, u_{11}^R] & [u_{12}^L, u_{12}^R] & \cdots & [u_{1n}^L, u_{1n}^R] \\ [u_{21}^L, u_{21}^R] & [u_{22}^L, u_{22}^R] & \cdots & [u_{2n}^L, u_{2n}^R] \\ \vdots & \vdots & \cdots & \vdots \\ [u_{m1}^L, u_{m1}^R] & [u_{m2}^L, u_{m2}^R] & \cdots & [u_{mn}^L, u_{mn}^R] \end{bmatrix} \quad (13)$$

**3.2 Analysis of extension fuzzy decision-making**

The word ‘fuzzy’ among ‘fuzzy multi-attribute decision-making analysis’ means there are extension uncertain factors in the decision-making information of the mechanism scheme design. Based on describing the uncertain relationship of range position by the extension distance, the above-mentioned uncertain relationship and the extension distance can expand and complement each other. From Definition 2, based on the positive ideal region  $u_{0j}^R$  and negative ideal region  $u_{0j}^L$  of the fuzzy composite-element which result from the extension fuzzy decision-making matrix  $U_{m \times n}$ , we can get the positive ideal matter-element  $\tilde{R}_{m \otimes n}^+$  and the negative ideal matter-element  $\tilde{R}_{m \otimes n}^-$ .

$$\tilde{R}_{m \otimes n}^+ = \begin{bmatrix} N & c_1 & u_{01}^+ \\ & c_2 & u_{02}^+ \\ & \cdots & \cdots \\ & c_n & u_{0n}^+ \end{bmatrix} \quad (14)$$

$$\tilde{R}_{m \otimes n}^- = \begin{bmatrix} N & c_1 & u_{01}^- \\ & c_2 & u_{02}^- \\ & \cdots & \cdots \\ & c_n & u_{0n}^- \end{bmatrix} \quad (15)$$

From Definition 3, based on the positive ideal matter-element  $\tilde{R}_{m \otimes n}^+$  and the negative ideal matter-element  $\tilde{R}_{m \otimes n}^-$ , we calculate the extension distance of the point value and the interval value of the extension fuzzy decision-making matrix  $U_{m \times n}$ , then we can get matrixes of the corresponding positive and negative extension distance  $K_{m \times n}^+$  and  $K_{m \times n}^-$ :

$$K_{m \times n}^+ = \begin{bmatrix} K_{11}^+(u) & K_{12}^+(u) & \cdots & K_{1n}^+(u) \\ K_{21}^+(u) & K_{22}^+(u) & \cdots & K_{2n}^+(u) \\ \vdots & \vdots & \cdots & \vdots \\ K_{m1}^+(u) & K_{m2}^+(u) & \cdots & K_{mn}^+(u) \end{bmatrix} \quad (16)$$

$$K_{m \times n}^- = \begin{bmatrix} K_{11}^-(u) & K_{12}^-(u) & \cdots & K_{1n}^-(u) \\ K_{21}^-(u) & K_{22}^-(u) & \cdots & K_{2n}^-(u) \\ \vdots & \vdots & \cdots & \vdots \\ K_{m1}^-(u) & K_{m2}^-(u) & \cdots & K_{mn}^-(u) \end{bmatrix} \quad (17)$$

From Definition 4, calculate the extension optimal membership degree of matrix  $K_{m \times n}^+$  and  $K_{m \times n}^-$ , then we can get the optimal membership degree of the fuzzy matter-element and the fuzzy ideal matter-element  $\tilde{R}_{m \otimes n}^+$ , that is:

$$\psi_i(K_{ij}(u)) = 1 / \left( 1 + \left( d_i^+(K_{ij}^+(u)) / d_i^-(K_{ij}^-(u)) \right)^2 \right) \quad (18)$$

**3.3 Model of attribute weight distribution based on extension distance**

In the multi-attribute decision, the incomplete information of weight distribution will lead to the nondeterminacy of choosing the best decision-making scheme, so how to deal with the subjective information and the objective information is very important. In this paper, we establish a model of attribute weight distribution based on the extension distance. Experts propose the fuzzy region of attribute weight based on the design constraints, and they consider design constraints as evaluation criteria of the multi-attribute weight distribution. So the model can take all the factors of design constraints into consideration and can fully embody the design requirements and design intent. It could realize the amalgamation of subjective information and the objective information well.

Suppose there are  $S$  design constraints of the attribute weight distribution called  $T$ , then invite  $Q$  experts to give scores of every design constraint using the ratio scale 1~9. Finally, we get the scale sequence of the design constraint  $T_i : Y_i = (y_i(1), y_i(2), \dots, y_i(Q))$ , where  $y_i(j) = [y_i^L(j), y_i^R(j)]$ ,  $i = 1, 2, \dots, S$ ,  $j = 1, 2, \dots, Q$ .

The ideal rate scale region of the design constraint is  $y_0(j) = (y_0^L(j), y_0^R(j)) = (\max_{1 \leq i \leq S} y_i^L(j), \max_{1 \leq i \leq S} y_i^R(j))$ , and the corresponding rate scale sequence is  $Y_0 = (y_0(1), y_0(2), \dots, y_0(Q))$ .

Then the extension distance of  $y_i(j)$  and  $y_0(j)$  is  $K_{ij}(T)$ , it can be expressed as:

$$\begin{aligned} K_{ij}(T) &= \rho(\langle y_i^L(j), y_i^R(j) \rangle, \langle y_0^L(j), y_0^R(j) \rangle) \\ &= \frac{1}{2} \left( \rho(y_i^L(j), y_0^L(j)) + \rho(y_i^R(j), y_0^R(j)) \right) \end{aligned} \quad (19)$$

the extension relational degree of design constraint  $T_i$  and  $Y_0$  is  $r_i$ , it can be expressed as:

$$r_i(T) = \frac{1}{Q} \sum_{j=1}^Q (9 - K_{ij}(T)) \quad (20)$$

the important degree of the design constraint  $T_i$  can be expressed as :

$$w_{si}(T) = r_i(T) / \sum_{i=1}^S r_i(T). \tag{21}$$

Suppose there are  $P$  decision-making characteristics of the scheme design, and invite  $Q$  experts to give scores of every decision-making characteristic using ratio scale 1~9. Finally, we get the scale sequence of the decision-making characteristic  $J_i: X_k = (X_k(1), X_k(2), L, X_k(Q))$ , where  $X_k(j) = [X_k^L(j), X_k^R(j)]$ ,  $k = 1, 2, L, P$ ,  $j = 1, 2, L, Q$ . Consider every design constraint sequence  $Y_i$  as ideal scale sequence  $Y_0$ , and based on the relational degree  $r_{ik}(J)$  of scale sequence  $X_k$  and  $Y_0$  which resulted from Eqs. (19) ÷ (21), then we can get the relationship matrix between design constraint and decision-making characteristics as follows:

$$B_{S \times P} = \begin{bmatrix} r_{11}(J) & r_{12}(J) & \cdots & r_{1P}(J) \\ r_{21}(J) & r_{22}(J) & \cdots & r_{2P}(J) \\ \vdots & \vdots & \ddots & \vdots \\ r_{S1}(J) & r_{S2}(J) & \cdots & r_{SP}(J) \end{bmatrix}. \tag{22}$$

Then the weight of decision-making characteristic  $k$  is  $w_k$ :

$$w_k = \left( \sum_{i=1}^S (w_{si}(T) * r_{ik}(J)) \right) / \sum_{k=1}^P \sum_{i=1}^S (w_{si}(T) * r_{ik}(J)). \tag{23}$$

### 3.4 Multi-attribute Extension Fuzzy Optimized Decision-Making Model and algorithm implement

From Eq. (18) and based on the closest principle of multi-attribute extension fuzzy decision-Making, we can get the extension fuzzy optimal membership degree  $\psi_i(K_{ij}(u))$  of the fuzzy matter-element  $\tilde{R}_n(i)$ , if

$$\psi_i = \max \{ \psi_1(K_{1j}(u)), \psi_2(K_{2j}(u)), \dots, \psi_m(K_{mj}(u)) \}. \tag{23}$$

Then, the decision scheme  $P_i$  is the best mechanism design scheme, so we call  $P_i$  the optimal design scheme based on the extension fuzzy optimal membership degree.

In summary, through the formal representation and the normative treatment of decision-making schemes, and by using fuzzy matter-element, we can get the corresponding fuzzy composite-element and its positive and negative ideal region. Then based on the above, the positive and negative ideal matter-element of the fuzzy composite-element can be set up. By combining with the matter-element characteristics weight distribution model of the fuzzy matter-element, we can also get the extension fuzzy distance based on the positive and negative ideal matter-element. Then we construct the extension optimal membership degree of the fuzzy matter-element. We can obtain the best decision-making scheme from the comparison of optimal membership

degree. The multi-attribute extension fuzzy optimized decision-making model is shown in Fig. 1.

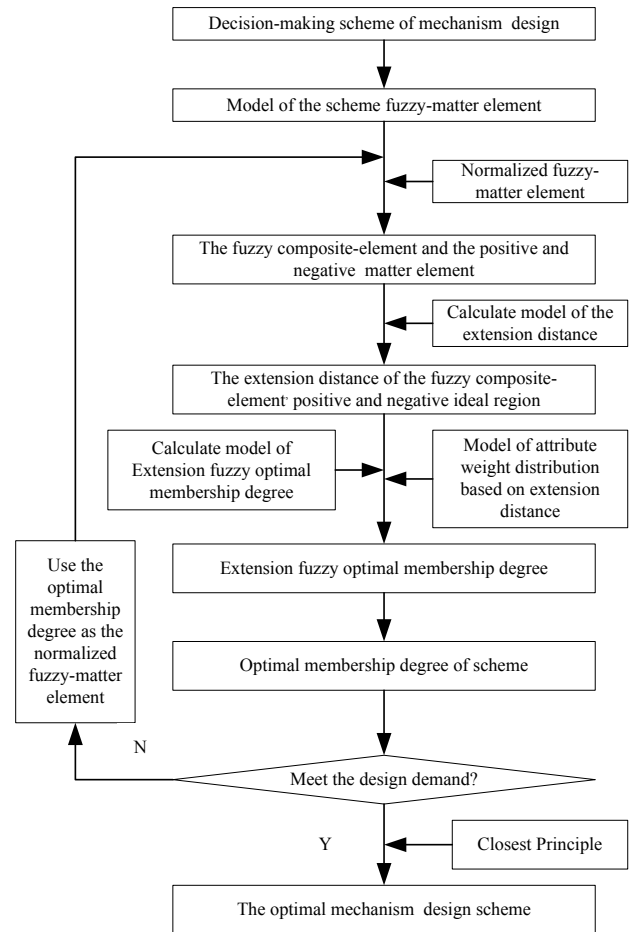


Figure1 Multi-attribute extension fuzzy optimized decision-making model of mechanism scheme design

The specific implementation steps of multi-attribute extension decision-making are as follows:

**Step 1.** Utilize the decision-making characteristics of the mechanism design scheme and the corresponding fuzzy eigenvalue to construct the fuzzy matter-element  $\tilde{R}_n(i)$ , which corresponds to the mechanism design scheme  $i$ ;

**Step 2.** Use Definition 1 to construct the fuzzy composite-element of the mechanism design scheme  $\tilde{R}_{m \otimes n}$ , and get the corresponding extension fuzzy decision-making matrix  $A_{m \times n}$ . Based on the exposition in 3.1, utilize Eqs. (10), (11) and (12) to normatively approach the matrix  $A_{m \times n}$ , then get the extension fuzzy matrix  $U_{m \times n}$ ;

**Step 3.** According to Definition 2, based on the extension fuzzy matrix  $U_{m \times n}$  utilize Eq. (14) and (15) to construct the positive and negative ideal region of every fuzzy matter-element characteristic, then generate the positive ideal matter-element  $\tilde{R}_{m \otimes n}^+$  and the negative ideal matter-element  $\tilde{R}_{m \otimes n}^-$ ;

**Step 4.** According to Definition 3, utilize Eq. (16) and (17) to calculate the extension distance of the ideal region of the positive and negative fuzzy matter-element,

then get the corresponding matrixes of the extension distance  $K_{m \times n}^+$  and  $K_{m \times n}^-$ ;

**Step 5.** Based on the exposition in 3.3, construct the corresponding design constraints and the decision-making characters, then based on the extension distance utilize Eq. (19) to (23) to get the multi-attribute weight  $w_k$  with the experts' fuzzy interval scale method.

**Step 6.** Combining with Definition 4, based on the results of Step 4 and Step 5 utilize Eq. (18) to get the extension optimal membership degree of the extension matter-element  $\tilde{R}_n(i)$  called  $\psi_i$ , and then construct the sequence of the fuzzy extension optimal membership degree  $\psi = (\psi_1, \psi_2, \dots, \psi_m)$ ;

**Step 7.** If the extension fuzzy decision-making is multipurpose and multi-level, then it is needed to determine whether the fuzzy extension optimal membership degree is a program-level one, if it is so, then implement Step 8, if else, implement Step 2 or Step 6, and use the unprogram-level one as the new fuzzy extension optimal membership degree.

**Step 8.** Based on the fuzzy extension optimal membership degree sequence  $\psi = (\psi_1, \psi_2, \dots, \psi_m)$  and utilizing the closest principle put the decision-making schemes in sequence, then the decision-making scheme which corresponds to  $\psi_i = \max(\psi_1, \psi_2, \dots, \psi_m)$  is the best one.

**4 Application example**

Use the large scaled hydraulic turbine scheme design as example to explain the above-mentioned model in detail. Because of the complex fluid motion and the incomplete design information of the mechanism scheme design, the large scaled hydraulic turbine design scheme is mostly based on the similarity theory and uses model experiment to get the performance parameter of the hydraulic turbine and the model runner. We analyse and calculate the characteristic parameter and performance parameter of the prototype machine. Then we get the primary scheme. The primary scheme has to meet the needs of the design primary scheme and the general plan of the hydropower stations construction. So it is a complex engineering decision-making problem of technical and economic system. Through the model experiment we get the characteristic parameter and performance parameter with lots of uncertain information. To deal with the uncertain information, we use the extension fuzzy decision-making method to choose the optimal hydraulic turbine type and runner model of the hydropower stations construction. The result can provide a reasonable qualitative foundation to the designer. The domain experts consider the runner force, runner efficiency, cavitation performance, operating characteristic, runner diameter and the manufacturing burden as the main influencing factors of the large scaled hydraulic turbine scheme. Then they get 3 schemes which all contain the water discharge of the hydropower stations construction.

**Step 1.** Based on the domain experts' opinion, determine the 6 matter-element characteristics of the

design scheme model. Among the 6 matter-element characteristics, take the cavitation index as the yardstick of the cavitation performance, and limit the yardstick of the operating characteristic in the high efficiency field of working condition. Then the corresponding matter-element model of the large scaled hydraulic turbine scheme design can be expressed as  $\tilde{R}_6(1)$ ,  $\tilde{R}_6(2)$  and  $\tilde{R}_6(3)$ ;

$$\tilde{R}_6(1) = \begin{bmatrix} \text{Turbine1} & \text{force (MW)} & [1,25; 1,34] \\ & \text{efficiency (\%)} & [84,10; 84,30] \\ & \text{cavitation performance} & [0,141; 0,143] \\ & \text{operating characteristic} & [0,8410; 0,8450] \\ & \text{diameter (m)} & 0,60 \\ & \text{burden (million)} & 1,1180 \end{bmatrix};$$

$$\tilde{R}_6(2) = \begin{bmatrix} \text{Turbine2} & \text{force (MW)} & [1,28; 1,36] \\ & \text{efficiency (\%)} & [81,03; 81,15] \\ & \text{cavitation performance} & [0,145; 0,147] \\ & \text{operating characteristic} & [0,9040; 0,9070] \\ & \text{diameter (m)} & 0,60 \\ & \text{burden (million)} & 1,3160 \end{bmatrix};$$

$$\tilde{R}_6(3) = \begin{bmatrix} \text{Turbine3} & \text{force (MW)} & [1,60; 1,72] \\ & \text{efficiency (\%)} & [85,90; 86,10] \\ & \text{cavitation performance} & [0,129; 0,131] \\ & \text{operating characteristic} & [0,8915; 0,8936] \\ & \text{diameter (m)} & 0,71 \\ & \text{burden (million)} & 1,2960 \end{bmatrix};$$

**Step 2** Construct the fuzzy composite-element of the large scaled hydraulic turbine design scheme:

$$\tilde{R}_{3 \otimes 6} = \begin{bmatrix} & \text{force (MW)} & \text{efficiency (\%)} & \text{cavitation performance} \\ \text{Turbine1} & [1,25; 1,34] & [84,10; 84,30] & [0,141; 0,143] \\ \text{Turbine2} & [1,28; 1,36] & [81,03; 81,15] & [0,145; 0,147] \\ \text{Turbine3} & [1,60; 1,72] & [85,90; 86,10] & [0,129; 0,131] \\ \text{operating characteristic} & & & \\ & \text{diameter (m)} & \text{burden (million)} & \\ [0,8410; 0,8450] & 0,60 & 1,1880 & \\ [0,9040; 0,9070] & 0,60 & 1,3160 & \\ [0,8915; 0,8936] & 0,71 & 1,2960 & \end{bmatrix};$$

From the analysis, we know the cavitation performance, runner diameter, and manufacturing burden are cost-type indexes, the runner force, runner efficiency and operating characteristic are efficiency indexes. Based on the exposition in 3.1 and utilizing Eq. (10) and (11) to normatively approach the cost-type indexes and efficiency indexes, we get the extension fuzzy decision-making matrix:

$$U_{3 \times 6} = \begin{bmatrix} [0,615; 0,689] & [0,977; 0,979] & [0,803; 0,831] \\ [0,640; 0,705] & [0,941; 0,943] & [0,746; 0,775] \\ [0,901; 1,000] & [0,998; 1,000] & [0,972; 1,000] \\ [0,838; 0,848] & [1,000; 1,000] & [1,000; 1,000] \\ [0,993; 1,000] & [1,000; 1,000] & [0,903; 0,903] \\ [0,962; 0,967] & [0,845; 0,845] & [0,917; 0,917] \end{bmatrix}$$

**Step 3.** Use the extension fuzzy decision-making matrix  $U_{3 \times 6}$  to construct the positive ideal matter-element  $\tilde{R}_{3 \otimes 6}^+$  and the negative ideal matter-element  $\tilde{R}_{3 \otimes 6}^-$ :

$$\tilde{R}_{3 \otimes 6}^+ = \begin{bmatrix} \text{Turbine} & \text{force} & [0,901; 1,000] \\ & \text{efficiency} & [0,998; 1,000] \\ \text{cavitation performance} & & [0,972; 1,000] \\ \text{operating characteristic} & & [0,993; 1,000] \\ & \text{diameter} & [1,000; 1,000] \\ & \text{burden} & [1,000; 1,000] \end{bmatrix}$$

$$\tilde{R}_{3 \otimes 6}^- = \begin{bmatrix} \text{Turbine} & \text{force} & [0,615; 0,689] \\ & \text{efficiency} & [0,941; 0,943] \\ \text{cavitation performance} & & [0,746; 0,775] \\ \text{operating characteristic} & & [0,838; 0,848] \\ & \text{diameter} & [0,845; 0,845] \\ & \text{burden} & [0,903; 0,903] \end{bmatrix}$$

**Step 4.** According to Definition 3, get the extension distance matrix of the positive and negative fuzzy ideal region  $K_{3 \times 6}^+$  and  $K_{3 \times 6}^-$  as follows:

$$K_{3 \times 6}^+ = \begin{bmatrix} 0,249 & 0,020 & 0,155 & 0,150 & 0,000 & 0,000 \\ 0,228 & 0,056 & 0,212 & 0,000 & 0,000 & 0,097 \\ 0,000 & 0,000 & 0,000 & 0,028 & 0,155 & 0,083 \end{bmatrix}$$

$$K_{3 \times 6}^- = \begin{bmatrix} 0,000 & 0,035 & 0,042 & 0,000 & 0,155 & 0,097 \\ 0,009 & 0,000 & 0,000 & 0,149 & 0,155 & 0,000 \\ 0,256 & 0,056 & 0,211 & 0,116 & 0,000 & 0,014 \end{bmatrix}$$

**Step 5** With the experts' opinion, construct the design constraints and the corresponding ratio scale of the fuzzy interval (in Tab. 1), construct the ratio scale of the fuzzy interval (in Tab. 2). With Eq. (19) to (23), calculate the weight sequence of the decision-making characteristics:  $w = \{0,178; 0,178; 0,138; 0,154; 0,179; 0,173\}$ .

**Step 6** Based on the above-mentioned results, utilize Eq. (10) to get the extension optimal membership degree of every fuzzy matter-element, then construct the sequence of the fuzzy extension optimal membership degree:  $\psi = (0,303; 0,161; 0,714)$ .

**Step 7** For the scheme is single stage and the fuzzy extension optimal membership degree is on program level, we can know  $\psi_{\max} = \psi_3$  by the closest principle, so the optimal scheme is the third one.

**Table 1** The interval ratio scale of every design constraint

design constraint	interval ratio scale
water head	([7,0-8,0], [7,5-8,0], [7,0-8,0], [8,0-8,5], [8,0-9,0], [7,5-8,0])
force	([9,0-9,0], [9,0-9,0], [9,0-9,0], [9,0-9,0], [9,0-9,0], [9,0-9,0])
efficiency	([7,5-8,0], [8,0-8,5], [8,0-8,5], [7,5-8,0], [7,5-8,5], [8,0-9,0])
cavitation corrosion	([6,0-7,0], [6,5-7,5], [6,0-6,5], [7,0-7,5], [7,5-8,0], [7,0-8,0])
low wear	([6,5-7,5], [7,5-8,0], [7,0-8,0], [7,0-7,5], [8,0-8,5], [7,5-8,0])
operational reliability	([7,5-8,0], [7,5-8,0], [8,0-9,0], [8,0-8,5], [7,5-8,0], [7,5-8,0])
long performance life	([6,5-7,5], [7,0-7,5], [8,0-8,5], [7,0-8,0], [7,5-8,0], [7,0-8,0])
low energy consumption	([6,5-7,5], [6,0-6,5], [6,0-7,0], [7,5-8,0], [7,0-7,5], [6,0-7,0])
low pollution	([7,0-7,5], [6,0-6,5], [6,0-7,0], [8,0-8,5], [7,0-7,5], [7,0-8,0])
little noise	([5,0-6,0], [5,0-6,0], [4,5-5,0], [5,0-5,5], [5,0-5,5], [4,5-5,0])
compact conformation	([6,0-7,0], [7,0-7,5], [7,5-8,0], [6,0-6,5], [6,0-6,5], [7,0-7,5])
light weight	([8,0-8,5], [8,0-8,5], [7,5-8,0], [8,5-9,0], [8,0-8,5], [8,5-9,0])

**Table 2** The interval ratio scale of every decision-making characteristic

Decision-making characteristics	interval ratio scale
runner force	([8,0-9,0], [7,5-8,0], [8,0-8,5], [8,0-9,0], [7,5-8,0], [8,0-8,5])
runner efficiency	([7,0-8,0], [7,0-8,0], [7,0-7,5], [8,0-8,5], [7,0-7,5], [8,0-9,0])
cavitation performance	([4,0-4,5], [5,0-5,5], [4,5-5,0], [5,0-6,0], [4,5-5,0], [4,0-4,5])
operating characteristic	([5,5-6,0], [5,5-6,0], [5,0-6,0], [5,5-6,0], [5,0-6,0], [5,0-5,5])
runner diameter	([7,0-7,5], [6,5-7,0], [7,5-8,0], [7,0-7,5], [7,0-7,5], [7,0-8,0])
manufacturing burden	([8,0-8,5], [8,5-9,0], [8,5-9,0], [8,0-9,0], [8,5-9,0], [8,0-9,0])

Multi-attribute extension fuzzy decision-making model is a favourable support for the large hydraulic turbine design scheme to be carried out successfully. The program designer could utilize the primary scheme and the technology of knowledge reuse to design the structural concept. It helps him to finish the overall scheme design of the large hydraulic turbine. The structural concept includes the module of rotary actuator, flush mounting, water distributor, layout system and so on. The program designer can also utilize the primary scheme and the extension design method to export more adaptive schemes. Space lacks for a detailed description of it. We will discuss and analyse it in another paper.

## 5 Conclusion

Aiming at the uncertain optimization problems of large complex product scheme design, the paper puts forward an improved multiple attribute extension fuzzy decision model. The model combines the fuzzy theory with the extension theory. It solves problems based on the fuzzy distance and the relational grade among the

decision-making schemes. The model utilizes the deviation caused by the known design information to deal with the decision-making problem with uncertain design information. At the same time, the model selects the optimal design scheme from an overall perspective by constructing the positive and negative ideal matter-element and sets up the fuzzy extension optimal membership degree among the design scheme and the positive and negative ideal matter-element. The optimal design scheme makes the evaluate results fairly objective. In addition, the paper provides a model of attribute weight distribution based on extension distance. It can consider all the influences of multi-attribute weight caused by the design requirement and design constraints taken into consideration. The model of attribute weight distribution can also roundly embody the design requirement and design constraints of the product. It also provides support for the large complex mechanism design scheme to be carried out successfully.

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### 6 References

- [1] Zhang, J. J.; Wu, D. S.; Olson, D. L. The method of grey related analysis to multiple attribute decision making problems with interval numbers. // *Mathematical and Computer Modelling*. 42, (2005), pp. 991-998.
- [2] Ayag, Z. An integrated approach to evaluating conceptual design alternatives in a new development environment. // *International Journal of Production Research*. 43, 4(2005), pp. 687-713
- [3] Fan, Z. P.; Ma, J.; Zhang, Q. An approach to multiple attribute decision making based on fuzzy preference information on alternatives. // *Fuzzy Sets and Systems*. 131, 1(2002), pp. 101-106.
- [4] Vanegas, L. V.; Labib, A. W. Fuzzy approaches to evaluation in engineering design. // *ASME Journal of Mechanical Design*. 127, 1(2005), pp. 24-33.
- [5] Ling-juan Li; Ling-tong Shen. An improved multilevel fuzzy comprehensive evaluation algorithm for security performance. // *The Journal of China Universities of Posts and Telecommunications*. 13, 4(2006), pp. 48-53.
- [6] Tichun Wang; Sai Zhao; Bingfa Chen. Association Rule Extension Mining and Reuse in Scheme Design of Large-scale Hydraulic Turbines. // *Information: An International Interdisciplinary Journal*. 15, 6(2012), pp. 2403-2409.
- [7] Meng-Hui Wang; Yi-Feng Tseng; Hung-Cheng Chen et al. A novel clustering algorithm based on the extension theory and genetic algorithm. // *Expert Systems with Applications*. 36, 4(2009), pp. 8269-8276.
- [8] Yong-xiu He; Ai-ying Dai; Jiang Zhu et al. Risk assessment of urban network planning in china based on the matter-element model and extension analysis. // *International Journal of Electrical Power & Energy Systems*. 33, 3(2011), pp. 775-782.
- [9] Yi-Fei, Chen; Dong-Sheng, Liu; Ji-Cheng, Wu et al. Extension based clustering method and approach to support adaptable design of the product. // *Proceedings of the 2007 International Manufacturing Science and Engineering Conference / Atlanta, Georgia, USA, 2007*, pp. 1-9.
- [10] Wang, Tichun; Huang, Xiang. Complex Mechanism Extension Scheme Design Model Based on Knowledge Reuse. // *Journal of Nanjing University of Aeronautics & Astronautics*. 44, 4(2012), pp. 548-552.
- [11] Zhao Yanwei; Zhang Guoxian. A new integrated design method based on fuzzy matter-element optimization. // *Journal of Materials Processing Technology*. 129, 1-3(2002), pp. 612-618.
- [12] Liu Yang. Assessment of City Environmental Quality in Western China Based on Matter Element Extension-a Case Study Of Chongqing. // *Energy Procedia*. 5, (2011), pp. 619-623.
- [13] Jianzhou Gong; Yansui Liu; Wenli Chen. Land suitability evaluation for development using a matter element model A case study in Zengcheng, Guangzhou, China. // *Land Use Policy*. 29, 2(2012), pp. 464-472.
- [14] Meng-Hui Wang; Yi-Feng Tseng. A novel analytic method of power quality using extension genetic algorithm and wavelet transform. // *Expert Systems with Applications*. 38, 10(2011), pp. 12491-12496.
- [15] Yan-chao Yin; Lin-fu Sun; Cheng Guo. A policy of conflict negotiation based on fuzzy matter element particle swarm optimization in distributed collaborative creative design. *Computer-Aided Design*. 40, 10-11(2008), pp. 1009-1014.
- [16] Jia Chun-rong; Zhang Jun. Based on Fuzzy Weight Matter Element to Evaluate the Water Quality of Jialing River in Nanchong, China. // *Procedia Environmental Sciences*. 11, (2011), pp. 631-636.
- [17] Dongjun Liu; Zhihong Zou. Water quality evaluation based on improved fuzzy matter-element method. // *Journal of Environmental Sciences*. 24, 7(2012), pp. 1210-1216.
- [18] Ma Xi-xia; Li Yan; Wan Jia-quan, et al. Model about comprehensive evaluation of water resources allocation based on multi-level fuzzy matter-element. // *Journal of Water Resources & Water Engineering*. 21, 6(2010), pp. 26-29 .
- [19] Liu Zhifeng; Wang Shuwang; Wan Juyong et al. Green Product Assessment Method Based on Fuzzy-Matter Element. // *China Mechanical Engineering*. 18, 2(2007), pp. 166-170.
- [20] Wang Guiping; Jia Yazhou; Zhou Guangwen. Evaluation Method and Application of CNC Machine Tool's Green Degree Based on Fuzzy-EAHP. // *Journal of Mechanical Engineering*. 46, 3(2010), pp. 141-147.
- [21] Zhang Genbao; Pang Jihong; Chen Guohua et al. A comprehensive evaluation method of fuzzy matter element for CNC equipment quality. // *Journal of Chongqing University*. 34, 1(2011), pp. 36-41.



**Authors' addresses*****Ti-chun Wang, Ph.D., Associate Professor***

Corresponding author

Institution:

1) College of Mechanical and Electrical Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

2) Jiangsu Key Laboratory of Precision and Micro-Manufacturing Technology, Nanjing 210016, China

3) Wuxi TJ Innova Auto-design Co., Ltd., Wuxi 214072, China

Postal address:

Nanjing University of Aeronautics and Astronautics, 344#, NO. 29 yudao street, Nanjing 210016, China

E-mail: wangtichun2010@nuaa.edu.cn

***Ai-jun Yang, postgraduate***

Institution:

College of Mechanical and Electrical Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China;

Postal address:

Nanjing University of Aeronautics and Astronautics, 344#, NO. 29 yudao street, Nanjing 210016, China

E-mail: wtc\_200\_cn@sina.com

***Shi-sheng Zhong, Ph.D., Professor***

Institution:

School of Mechatronics Engineering, Harbin Institute of Technology, Harbin 150001, China

Postal address:

Harbin Institute of Technology, NO. 92 xidazhi street, Harbin 150001, China

E-mail: zhongss@hit.edu.cn