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Integrated inventory model for single vendor–single buyer with probabilistic demand

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Abstract: In this paper, we consider single vendor–single buyer integrated inventory model with probabilistic demand and equal delivery lot size. The model contributes to the current literature by relaxing the deterministic demand assumption which has been used for almost all integrated inventory models. The objective is to minimise expected total costs incurred by the vendor and the buyer. We develop effective iterative procedures for finding the optimal solution. Numerical examples are used to illustrate the benefit of integration. A sensitivity analysis is performed to explore the effect of key

parameters on delivery lot size, safety factor, production lot size factor and the expected total cost. The results of the numerical examples indicate our integrated model gives a significant cost savings over independent model.

Keywords: inventory; probabilistic demand; safety stock; supply chain.

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1 Introduction

The single vendor–single buyer integrated inventory problem received a lot of attention in recent years. This renewed interest is motivated by the growing focus on supply chain management where collaboration and integration have been considered as key factors in managing modern supply chain. Firms are realising that a more efficient management of inventories across the entire supply chain through better coordination and more cooperation are necessary for reducing costs and increasing service level. Such collaboration is facilitated by the advances in information technology providing faster and cheaper communication means.

The integrated inventory model in the supply chain has been of interest since more than three decades ago. Goyal (1976) is among the earliest publication addressing the problem of two organisations in a supply chain jointly optimising production and delivery

quantities. Then, this paper is followed by various contributions including, for example, Banerjee (1986), Goyal (1988), Hill (1997), Goyal and Nebebe (2000), Pujawan and Kingsman (2002) and Hoque and Goyal (2000). Most of those papers, however, assume that the demand is deterministic. Considering that demand is almost always uncertain in real life, assuming demand to be deterministic is a too restrictive assumption. In this paper, we relax the deterministic assumption and take into account the uncertainty factor.

More recently, Sarmah et al. (2006) and Ben-Daya et al. (2008) present a comprehensive literature review on vendor–buyer integrated model. They pointed out that there are opportunities in extending single vendor–single buyer inventory model. The extensions may lead to relaxing assumptions that may not be realistic such as the assumption of deterministic demand, perfect product quality and completely reliable production system.

In this paper, we consider integrated inventory problem in a two-tier supply chain that consists of a single vendor and single buyer. A vendor produces batches of product and delivers them to the buyer with an equal shipment size. Both parties have flexibility in determining the order quantity and production batch based on optimal delivery lot size. The first research dealing with this problem is Pujawan and Kingsman (2002). They compared an inventory model with lot streaming and without lot streaming for two different cases. Finally, the model has shown that synchronising the order times and agreeing on the delivery lot size, allowing the buyer to determine the order quantity and the supplier the production lot size independently, is virtually as good as jointly agreeing on the relevant lot sizes. Furthermore, Chan and Kingsman (2007) extended this model by considering single vendor–multi-buyer supply chain model, but still assuming deterministic demand.

When the assumption of deterministic demand is relaxed and demand coming to the buyer is assumed to be stochastic, it is possible for the buyer to experience out of stock for some period of time. To reduce the probability of having out of stock, the buyer has to have a certain level of safety stock. A solution procedure is developed for solving the proposed model and numerical examples are used to illustrate its application. Also, we explore the effect of changes in the value of key parameters on lot size, safety factor and the expected total cost.

This paper is organised as follows. In Section 2, we review the related literature. In Section 3, we develop our integrated model for single vendor–single buyer incorporating stochastic demand. In Section 4, we present the solution procedure. Numerical examples from the mathematical model are presented in Section 5. Finally, Section 6 concludes this paper.

2 Literature review

There have been plenty of published works on integrated inventory model. The essence of the models is generally to show that by integrating inventory decisions across the supply chain, firms will receive benefits in terms of reduced costs. Goyal (1976) was one of the first researchers that developed integrated vendor–buyer inventory problem. The result of the model was that integrated lot sizing model reduces the total relevant cost which include inventory holding and ordering costs. Banerjee (1986) investigated the lot-for-lot policy in which the vendor manufactures a lot at a finite rate of production and delivers equal to the production batch size. By relaxing Banerjee's lot-for-lot assumption,

Goyal (1988) proposed a more general joint economic lot size model that provided a lower-joint total relevant cost. He argued that producing a batch which is made up of equal shipments generally produced lower cost but the whole batch must be completed before the first shipment is made. Lu (1995), in considering heuristics for the single vendor–multi-buyer problem, gave an optimal solution to the single vendor–single buyer problem, again based on the assumption of a batch providing an integral number of equal shipments. A review of related literature is given by Goyal and Gupta (1989).

A number of researchers, including Goyal (1995), Hill (1997), Hill (1999), Goyal and Nebebe (2000), Hoque and Goyal (2000), Hill and Omar (2006) and Zhou and Wang (2007) developed a model with unequal-sized shipments, in contrast to the previous models that assumed equal-sized shipment policy. Mathematically, it has been shown that allowing shipment size to vary from one shipment to the next results in lower total cost compared to the case when shipment size is restricted to a constant value overtime. However, although the unequal-sized shipment models give lower cost than others, they have some deficiencies. Goyal and Szendrovits (1986) pointed out that the capacity of the handling, packing and shipping equipment must be at least equal to the largest shipment size and hence, becomes under-utilised for smaller shipment sizes, which leads to idle-capacity costs. Furthermore, Agrawal and Raju (1986) suggested that supply and receipt of unequal shipment size associated with order interval of different length cause a prohibitive operational planning and control effort for the vendor and the buyer.

The above practical factors have led the present researchers to solve single vendor–single buyer with equal shipment size in many different directions. It is beyond the scope of this paper to discuss all works in detail. Sarmah (2007) developed a model where both vendor and buyer have a certain amount of target profit. Ertogral et al. (2007) integrated transportation cost explicitly into integrated inventory model under equal shipment policy. Apichai and Ferrel (2007) incorporated cost of quality or rework cost into the model. And more recently, David and Eben-Chaime (2008) developed continuous model in integrated vendor–buyer problem with assuming demand and production to be continuous. However, the equal-sized shipments model still received a lot of attentions in recent years.

Interesting studies in equal-sized shipments model were done by Pujawan and Kingsman (2002) and Kelle et al. (2003). They proposed a model to study the benefit of coordinating supply chain inventories through synchronising the order times and agreeing on the delivery lot size. Significant savings on total relevant cost could be gained from determining frequency of delivery and production batch. Chan and Kingsman (2007) then developed Pujawan and Kingsman's (2002) model by considering single vendor–multi-buyer supply chain model. The synchronisation was achieved by scheduling the actual buyer's delivery days and coordinating them with the vendor's production cycle. The results of this model showed that the synchronised-cycles policy works better than an independent optimisation and restricts buyers to adopt a common cycle. This model gave the flexibility to buyers in choosing their lot sizes and order cycle independently. This policy may be useful for the buyer in planning delivery based on some constraints related to inventory management in practical environments.

Sarmah et al. (2006) reviewed literature dealing in vendor–buyer coordination under deterministic environment. They investigated the quantity discount mechanism in vendor–buyer coordination model. Finally, some of future directions of the research was suggested, including the relaxation of deterministic demand. Furthermore, Ben-Daya et al. (2008) presented a more comprehensive review on deterministic single vendor–single buyer problem. They provided general formulation of the problem and conducted a comparative empirical study among the policies. They suggested some extensions on the previous model. One of their suggestions is extending the model by relaxing the assumptions of deterministic demand.

This paper reconsiders the equal-sized shipments policy in single vendor–single buyer integrated system. We consider stochastic demand and giving flexibility to buyer in choosing frequency of delivery independently. The model is also extended to the situation with shortages permitted to occur in a buyer side. A complete and detailed explanation of the model development will be given in Section 3.

3 Development of the model

3.1 Notations

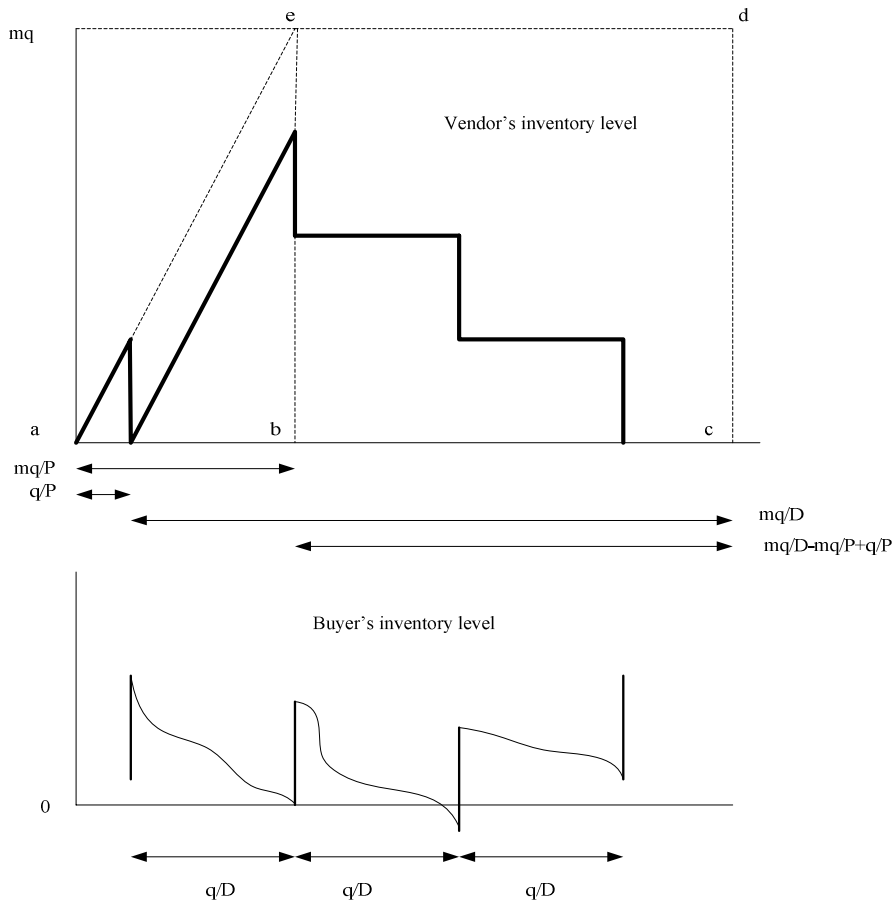
The following notations will be used to develop the model:

D	demand in units per unit time
σ	standard deviation of demand per unit time
P	production rate in units per unit time
K	production setup cost
A	order cost incurred by the buyer for each order size of nq
F	transportation cost for the buyer incurred with each shipment of size q
k	safety factor
SS	safety stock for the buyer
ES	expected number of backorder
h_b	holding cost per unit per unit time for buyer
h_v	holding cost per unit per unit time for vendor
π	backorder cost
n	shipment lot size factor, which is a positive integer
m	production lot size factor, which is a positive integer
q	the size of equal shipments from the vendor to the buyer
TC_B	total expected cost per unit time for the buyer
TC_V	total expected cost per unit time for the vendor
TC	integrated vendor–buyer expected total cost per unit time

3.2 Problem description

We consider a single item in a single buyer and single vendor inventory problem. The buyer sells items to the end customers whose demand follows a normal distribution with a mean of D per year. The buyer orders the item to the vendor in a constant lot of size nq (in a constant interval of nq/D). Each time an order is placed, a fixed ordering cost A incurs. The vendor manufactures the product in a lot of size mq with a finite rate P and incurs a fixed setup cost K . The buyer determines n (the number of shipment) individually and incurs a transportation cost F with each shipment of size q . Each shipment size will be delivered from vendor to buyer in an interval of q/D period. Partial backordering is not permitted. This means that if an order can not be satisfied fully, the whole quantity is assumed to be backordered and incurs a backorder cost π . In this model, we use a lot streaming policy, assuming an uninterrupted production run. Any shipments as long as the quantity is sufficient can be made before the production of the whole batch is completed. Hence, we use the basic model of Pujawan and Kingsman (2002) that considered lot streaming policy. The inventory profile for vendor and buyer is depicted in Figure 1.

Figure 1 The inventory pattern of vendor and buyer



The demand during period q/D is assumed to be normally distributed with mean $D(q/D)$ and standard deviation $\sigma\sqrt{q/D}$. We use Hadley–Within’s (1963) expression ($q/2$ + safety stock) to approximate average inventory level in period q/D . Average inventory level can be approximated by the average net inventory if the backorder condition during a replenishment cycle is small compared with the cycle length (Johnson and Montgomery, 1974, p.60). If we assume linier decrease over the cycle (period q/D) then,

$$\text{Average inventory} = \frac{q}{2} + k\sigma\sqrt{\frac{q}{D}}$$

Since shortage is permissible, the expected demand shortage at the end of period q/D is given by

$$\text{ES} = \sigma\sqrt{\frac{q}{D}}\psi(k) \quad (1)$$

where,

$$\psi(k) = \{f_s(k) - k[1 - F_s(k)]\}$$

$f_s(k)$ is probability density function of standard normal distribution and $F_s(k)$ is cumulative distribution function of standard normal distribution. The derivation of Equation (1) is shown in Appendix.

Considering buyer’s ordering cost is A and the number of order per unit time is D/nq , the expected ordering cost per unit time is given by DA/nq . Vendor will deliver a lot size q to the buyer and incurs transportation cost F . By formulating frequency of delivery is D/q , the transportation cost per unit time is given by DF/q .

Thus, the total expected cost per unit time for the buyer can be represented by:

$\text{TC}_B = \text{ordering cost} + \text{transportation cost} + \text{holding cost} + \text{backorder cost}$

$$\text{TC}_B = \frac{DA}{nq} + \frac{DF}{q} + h_b \left(\frac{q}{2} + k\sigma\sqrt{\frac{q}{D}} \right) + \left(\frac{D}{q} \right) \pi \text{ES} \quad (2)$$

On the other hand, for the vendor, since K is the production setup cost and the production quantity for a vendor in a lot mq , the expected setup cost per unit time is given by DK/mq . During the production period, once the first q units are produced, the vendor delivers them to the buyer, and then continuous making the delivery on average q/D units of time until the inventory level falls to zero. The vendor’s inventory level is shown in Figure 1. From this figure, we know that the vendor’s inventory level is given by the area below the bold lines. The calculation of vendor’s inventory level can be done by subtracting the area above the bold lines, which represents the cumulative delivery quantity, from the area $a-c-d-e$, which is the cumulative production for one production cycle. The cumulative delivery area can be represented by:

$$\frac{q^2}{D} + \frac{2q^2}{D} + \frac{3q^2}{D} + \dots + \frac{mq^2}{D} = \frac{m(m+1)q^2}{2D} \quad (3)$$

The cumulative production area consists of two areas, that is, the $a-b-e$ area and the $b-c-d-e$ area. The $a-b-e$ area is a triangle, then

$$\frac{1}{2}mq \frac{mq}{P} = \frac{m^2q^2}{2P} \quad (4)$$

The $b-c-d-e$ area is given by

$$mq \left(\frac{mq}{D} - \frac{mq}{P} + \frac{q}{P} \right) \quad (5)$$

Thus, the $a-c-d-e$ area can be calculated by adding Equation (4) into Equation (5)

$$\frac{m^2q^2}{2P} + mq \left(\frac{mq}{D} - \frac{mq}{P} + \frac{q}{P} \right) = mq \left(\frac{mq}{D} - \frac{(m-2)q}{2P} \right) \quad (6)$$

The vendor's inventory level for one cycle can be calculated by subtracting Equation (3) from Equation (6)

$$mq \left(\frac{mq}{D} - \frac{(m-2)q}{2P} \right) - \frac{m(m+1)q^2}{2D} = mq \left(\frac{(m-1)q}{2D} - \frac{(m-2)q}{2P} \right) \quad (7)$$

Finally, vendor's inventory level per unit time is given by

$$mq \left(\frac{(m-1)q}{2D} - \frac{(m-2)q}{2P} \right) \left(\frac{D}{mq} \right) = \frac{q}{2} \left((m-1) - (m-2) \frac{D}{P} \right) \quad (8)$$

By considering vendor's inventory level and the number of production setup (D/mq), we can formulate the total expected cost per unit time for the vendor

$TC_V =$ holding cost+setup cost

$$TC_V = \frac{q}{2}h_v \left((m-1) - (m-2) \frac{D}{P} \right) + \frac{DK}{mq} \quad (9)$$

The integrated vendor-buyer expected total cost per unit time is given by Equation (10) which is the total of cost incurred to the buyer (2) and the vendor (9):

$TC(m, q, k) =$ total expected cost for buyer+total expected cost for vendor

$$TC(m, q, k) = \frac{D}{nq} (A + Fn) + h_b \left(\frac{q}{2} + k\sigma \sqrt{\frac{q}{D}} \right) + \left(\frac{D}{q} \right) \pi\sigma \sqrt{\frac{q}{D}} \psi(k) + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq} \quad (10)$$

Consequently, the integrated vendor-buyer expected total cost per unit time can be rewritten as

$$TC(m, q, k) = \frac{D}{nq} (A + Fn) + h_b \left(\frac{q}{2} + k\sigma \sqrt{\frac{q}{D}} \right) + \left(\frac{D}{q} \right) \pi\sigma \sqrt{\frac{q}{D}} \{ f_s(k) - k[1 - F_s(k)] \} + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq} \quad (11)$$

Taking the first partial derivatives of $TC(m, q, k)$ with respect to k and q and equating them to zero, we obtain:

$$F_s(k) = 1 - \frac{h_b q}{\pi D} \tag{12}$$

$$q^* = \frac{2D \left(\left(\frac{A}{n} + F \right) + \frac{K}{m} + \pi \sigma \psi(k) \sqrt{\frac{q}{D}} \right)}{\sqrt{h_b + h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{h_b \sigma}{D \sqrt{\frac{q}{D}}} \left(k + \frac{\psi(k)}{1 - F_s(k)} \right)}} \tag{13}$$

The derivation of Equations (12) and (13) is shown in Appendix.

Proposition 1: For fixed q and k , $TC(m, q, k)$ is convex in m .

Proposition 2: For fixed m and k , $TC(m, q, k)$ is convex in q .

Proposition 3: For fixed q and m , $TC(m, q, k)$ is convex in k .

Therefore, for fixed m , $TC(m, q, k)$ is a convex function in (q, k) . Thus, the minimum value of $TC(m, q, k)$ occurs at the point (q^*, k^*) which satisfies $\partial TC(m, q, k) / \partial q = 0$ and $\partial TC(m, q, k) / \partial k = 0$, simultaneously. In Section 4, we develop an iterative procedure to find the minimum total cost and the optimal value of q and k .

4 Solution methodology

In this section, we develop an iterative procedure to determine the optimal values of m, q and k which minimises total cost per unit time. In previous section, we known that the minimum value of total cost can be found from q^* and k^* that satisfied the first partial derivatives of $TC(m, q, k)$ with respect to q and k , simultaneously. From Equations (12) and (13), we known that the optimal value of q is a function of k and the optimal value of k is a function of q . According to this condition, we need iterative procedure to find the convergence values of q and k . We use the basic idea of Ouyang et al. (2004) to solve this problem. We propose a heuristic method which is simple, easy to apply and computationally efficient. The algorithm to solve the above problem is as follows

- 1 set $m = 1$ and $TC(q_{m-1}^*, k_{m-1}^*, m-1) = \infty$
- 2 start with shipment size

$$q = \frac{2D \left(\left(\frac{A}{n} + F \right) + \frac{K}{m} \right)}{\sqrt{h_b + h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\}}}$$

- 3 substitute q into Equation (12) to find k
- 4 compute q using Equation (13)
- 5 repeat steps 3–4 until no change occurs in the values of q and k

- 6 set $q^* = q$ and $k^* = k$ and compute $TC(q_m^*, k_m^*, m)$ using Equation (11)
- 7 if $TC(q_m^*, k_m^*, m) \leq TC(q_{m-1}^*, k_{m-1}^*, m-1)$ repeat steps 1–5 with $m = m+1$, otherwise go to step 8
- 8 compute $TC(q^*, k^*, m^*) = TC(q_{m-1}^*, k_{m-1}^*, m-1)$, then (q^*, k^*, m^*) is the optimal solution

In our problem, the buyer has flexibility in determining the number of delivery. Our procedure accommodates this condition by giving a chance to user to determine the number of delivery before using this algorithm. We use $m = 1$ as an initial value of m and $TC(q_{m-1}^*, k_{m-1}^*, m-1) = \infty$ as an initial value of total cost. $m = 1$ means that the vendor will produce a production batch q in each production run. Hence, it is the minimum value of production batch. In step 2, we use optimal value of q in deterministic problem (See Pujawan and Kingsman, 2002, p.102) as our initial value. The q value in deterministic problem will always lower than the value in probabilistic problem. We use it as minimum value of q in our procedure. To solve in similar problem, Ben-Daya and Hariga (2004) used a deterministic value as an initial value of q .

Step 3 until step 5 will find the convergence value of q and k . We use the procedure that was developed by Ouyang et al. (2004). Then, procedure convergence can be proved by adopting a similar graphical technique used in Hadley and Within (1963). Step 6 use the value of q and k that was found in step 5 to calculate our new total cost $TC(q_m^*, k_m^*, m)$ and set them as our new value (q^* and k^*). In step 7, we compare the new total cost with the previous one. If the new total cost is less than our previous value, we replace the previous one with the new one. Then increase m by 1, which means that we use the bigger value of production batch, and repeat the above process of determining the minimal total cost and their associated values (m, q, k) until no change occurs in total cost. The final value of m, q and k give the minimal cost solution.

5 Numerical example

To illustrate the above solution procedure, let us consider a basic case with the data used in Ben-Daya and Hariga (2004):

- D 1,000 unit/year
- σ 5 unit/year
- P 3,200 unit/year
- A \$50/order
- F \$25/shipment
- h_b \$5/unit/year
- h_v \$4/unit/year
- π \$15/unit
- K \$400/setup

As we assume that demand is uncertain, it is interesting to explore how demand uncertainty affect performance of the system. In Table 1, we explore the effects of changes in standard deviation of demand on costs incurred to vendor and buyer. As the table shows, safety stock increases with the standard deviation of demand. Increasing demand uncertainty also results in higher stockout frequency. The table also shows that higher standard deviation of demand leads to lower shipment size and hence, higher shipment frequency, and higher cost incurred to the buyer. On the other hand, the cost to the vendor is relatively constant as the standard deviation of demand increases. This is understandable because the vendor delivers in equal size and intervals, so the buyer is the only party that is directly affected by the uncertainty in demand.

A range of other problems are generated from a basic case above to explore the effects of changes in key parameters on buyer cost, vendor cost and total cost. We develop 14 sets problem with $n = 1$ and $n = 5$ to explore the model behaviour. The results of the problems are summarised in Table 2. When an ordering cost (A) increases with the values other model parameters (D , P , F , hb , hv and K) fixed at a particular level, it is found that in all cases ($A = 50, 100$), buyer cost and total cost increase while vendor cost decreases. This is logical because the buyer will order less frequently but with larger quantity leading to higher inventory to obtain a new balance between order cost and inventory holding cost, but obviously that balance is achieved at higher total cost. On the other side, supplier will receive larger, but less frequent orders from the buyer which is an advantage because the supplier can satisfy orders with less frequent production setup. However, the decrease of cost at vendor side can not meet the increase of cost at the buyer side, thus the total cost increases.

With the increase of holding cost of buyer (hb), buyer will keep lower inventory level (q and k become smaller). The increase in vendor's holding cost does not affect much buyer's decision, but it leads to higher costs incurred to the vendor. Furthermore, a larger production rate results a larger vendor cost, buyer cost and total cost. However, vendor cost increases significantly due to the increase in inventory level. When vendor uses a larger production rate, production for a certain lot completed sooner and hence, inventory will sits for a longer period.

Table 1 Computation results for various standard deviation of demand values

<i>Parameters</i>	<i>Standard deviation of demand</i>					
	<i>0</i>	<i>5</i>	<i>10</i>	<i>20</i>	<i>30</i>	<i>40</i>
Safety stock		3.29	6.57	13.12	19.66	26.19
Q	175.65	176.15	175.65	174.69	173.76	172.85
Inventory level	87.83	91.36	94.39	100.47	106.54	112.62
Backorder		0.3	0.6	1.19	1.78	2.37
Buyer						
Number of order	2.84	2.84	2.85	2.86	2.89	2.89
Number of shipment	5.69	5.68	5.69	5.72	5.75	5.78
Buyer cost	724.67	745.15	765.61	806.46	847.24	887.92
Inventory level	148.2	148.63	148.21	147.39	146.61	145.84
Vendor						
Number of setup	1.89	1.89	1.9	1.91	1.92	1.93
Vendor cost	1,351.03	1,351.44	1,351.9	1,352.84	1,353.78	1,354.75
Total cost	2,075.7	2,096.59	2,117.51	2,159.3	2,201	2,242.67

Table 2 The sensitivity analysis with respect to A, F, h_b, h_v, K and P

Parameters	Value	n	m	k	q	Buyer cost	Vendor cost	Total cost
A	50	1	3	1.53	188	890	1,344	2,234
		5	4	1.69	136	617	1,381	1,998
	100	1	2	1.34	268	1,160	1,282	2,442
		5	3	1.57	174	714	1,354	2,068
F	25	1	3	1.53	188	890	1,344	2,234
		5	4	1.69	136	617	1,381	1,998
	50	1	2	1.37	258	1,055	1,292	2,347
		5	3	1.55	181	805	1,348	2,153
h_b	5	1	3	1.53	188	890	1,344	2,234
		5	4	1.69	136	617	1,381	1,998
	10	1	4	1.35	134	1,262	1,383	2,646
		5	6	1.56	90	869	1,416	2,284
h_v	2	1	4	1.53	189	891	978	1,869
		5	6	1.72	127	612	1,001	1,613
	4	1	3	1.53	188	890	1,344	2,234
		5	4	1.69	136	617	1,381	1,998
K	300	1	2	1.44	223	916	1,119	2,035
		5	3	1.64	151	629	1,172	1,801
	600	1	3	1.46	216	909	1,655	2,564
		5	5	1.7	134	615	1,716	2,332
P	1,600	1	5	1.64	151	894	1,172	2,066
		5	7	1.81	105	614	1,148	1,762
	4,800	1	2	1.39	247	944	1,304	2,248
		5	3	1.6	166	646	1,398	2,045

It is also informative to compare the integrated model with independent model. In integrated model, as our model, vendor and buyer agree to share cost information and determine their ordering, shipping and production decisions jointly while in independent model they make decisions individually. The results of each model are presented in Table 3. We find a number of interesting points when comparing the two models. Firstly, the integrated model always results a lower total cost comparing to independent model, but the cost savings are not always substantial. The average cost savings over all the values for n is 1.66%. The buyer is worse off by 1.39% on average whilst the vendor is better off by 3.13% when moving from independent model to integrated model. The buyer, however, is at a disadvantage position, since its annual cost increases. The increase in buyer's cost is always smaller than the decrease in vendor's cost, so there is an improvement in total cost. Nevertheless, in view of the greater total cost efficiency of the integrated policy, the total savings can and should be shared in some equitable manner through the mechanism of a side payment to the buyer from the vendor, or a price discount scheme (See Banerjee, 1986; Goyal, 1976). For example, if the vendor makes a side payment of \$10/year to the buyer in order to induce it to adopt the integrated model, yet the vendor is about 2.42% better off in comparison to independent model. Moreover,

in other models, such as the case of vendor managed inventory (see Yao et al., 2007), the immediate solutions bring the disadvantages to the vendor.

The ratio of production batch between integrated model and independent model is an average of 0.94 across all the values of n . The average production batch is 543 for integrated model and 512 for independent model. Table 3 shows that the vendor always aiming to produce constant production batches to minimise the total cost. The vendor's production batches might vary slightly because of integer requirement for m and n . In integrated model, we find that the vendor cost increases as there are increases in n , but when the vendor use $m = 4$, his cost is almost constant.

Table 3 Comparison of independent model and integrated model

n	<i>Independent model</i>					<i>Integrated model</i>					<i>Saving (%)</i>
	m	q	<i>Vendor cost</i>	<i>Buyer cost</i>	<i>Total cost</i>	m	q	<i>Vendor cost</i>	<i>Buyer cost</i>	<i>Total cost</i>	
1	3	110	1,587	887	2,474	3	188	1,344	890	2,234	9.69
2	4	108	1,437	727	2,164	3	176	1,351	745	2,097	3.12
3	4	128	1,389	665	2,054	4	139	1,380	667	2,046	0.36
4	4	121	1,401	631	2,032	4	137	1,381	636	2,016	0.75
5	5	117	1,400	610	2,011	4	136	1,381	617	1,998	0.63
6	5	114	1,400	596	1,996	4	135	1,382	604	1,986	0.50
7	5	112	1,400	585	1,986	4	135	1,382	595	1,977	0.43
8	5	111	1,401	577	1,978	4	134	1,383	588	1,970	0.39
9	5	109	1,401	571	1,972	4	134	1,383	582	1,965	0.36
10	5	108	1,402	566	1,968	4	134	1,383	578	1,961	0.35

6 Conclusions

In this study, we analyse the single vendor–single buyer lot sizing problem under equal-size shipment policy. Previous works on this problem mostly focused on the production shipment schedule in terms of the number and size of batches transferred between both parties under deterministic demand. Here, we assume that demand is stochastic. Besides, we allow the occurrence of shortages and it is assumed to be fully backordered. We seek to minimise the total cost by simultaneously optimising shipment size, safety factor and production lot size factor. A simple procedure is suggested to obtain an approximate solution of our proposed model. The results of the numerical examples consistently show that the integrated model results in lower total costs compared to the independent model across various different parameters' values. As demand uncertainty increases, we found that there is a significant increase in the buyer's cost but interestingly, the vendor can maintain approximately the same level of total costs. This is due to the fact that demand uncertainty from the end customer is absorbed entirely by the buyer (the buyer passed on to the vendor a deterministic and constant demand).

For future developments, it would be interesting to extend this model to incorporate the influence of variable lead time on the model. Another extension of this work may be conducted by considering the deteriorating items into the integrated inventory model. Furthermore, the results obtained here under a simplistic scenario, involving a single buyer, a single vendor and a single product, may provide valuable insights in analysing more complex inventory replenishment situations, dealing with multiple buyers, vendors and products within a supply network. As an immediate extension of this paper, the case of multiple buyers for a single product is not likely to pose serious analytical problems. Further extensions dealing with multiple buyers, as well as multiple products, even for a single vendor, are, however, likely to present some interesting and challenging computational problems. Finally, we suggest that future investigations in this area consider economic and other aspects of set-up cost/time reduction in conjunction with lot sizing issues.

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Appendix

Derivations and proofs

Derivation of Equation (1)

Let x denote continue random variable with normal distribution with mean μ and standard deviation $\sigma > 0$. Hence, the probability density function of x is formulated as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad (14)$$

If demand in period q/D is formulated as $D(q/D)$ with standard deviation $\sigma\sqrt{q/D}$, then inventory level in that period is given by

$$P = q + k\sigma\sqrt{\frac{q}{D}} \quad (15)$$

Shortage occurs in period q/D when $x > P$. The expected number of shortages in period q/D can be formulated as:

$$ES = \int_{x=P}^{\infty} (x-P)f(x)dx \quad (16)$$

Substitute Equations (14) and (15) into Equation (16), we have

$$ES = \int_{x=q+SS}^{\infty} (x-q-SS) \frac{1}{\sqrt{2\pi}\sigma\sqrt{q/D}} e^{-\frac{(x-q)^2}{2(\sigma\sqrt{q/D})^2}} dx \quad (17)$$

Substitute $z = \frac{(x-q)}{\sigma\sqrt{q/D}}$ and $dx = \sigma\sqrt{q/D}dz$ into Equation (17), then we have

$$\begin{aligned} ES &= \int_{x=SS/(\sigma\sqrt{q/D})}^{\infty} \left(z\sigma\sqrt{\frac{q}{D}} - SS\right) \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \\ ES &= -SS \int_{z=SS/(\sigma\sqrt{q/D})}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz + \sigma\sqrt{\frac{q}{D}} \int_{z=SS/(\sigma\sqrt{q/D})}^{\infty} z \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \end{aligned} \quad (18)$$

Recall that $F_s(.)$ cumulative distribution function and $f_s(.)$ is probability density function with mean 0 and standard deviation 1. Using $f_s(.)$ formula and definition of standard normal distribution, we have

$$\begin{aligned} 1 - F_s(y) &= \int_{z=y}^{\infty} f_s(z) dz \\ &= \int_{z=y}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz \end{aligned}$$

Substituting $w = z^2 / 2$ into Equation (18), we have

$$ES = -SS \left[1 - F_s \left(SS / \sigma \sqrt{\frac{q}{D}} \right) \right] + \sigma \sqrt{\frac{q}{D}} \int_{w=SS^2 / (2(\sigma \sqrt{q/D})^2)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-w} dw$$

$$ES = -SS \left[1 - F_s \left(SS / \sigma \sqrt{\frac{q}{D}} \right) \right] + \sigma \sqrt{\frac{q}{D}} f_s \left(SS / \sigma \sqrt{\frac{q}{D}} \right)$$

$$ES = \sigma \sqrt{\frac{q}{D}} \{ f_s(k) - k [1 - F_s(k)] \}$$

$$ES = \sigma \sqrt{\frac{q}{D}} \psi(k)$$

Derivation of Equation (12)

We formulated in Equation (11), the integrated vendor–buyer expected total cost per unit time as

$$\begin{aligned} TC(m, q, k) = & \frac{D}{nq} (A + Fn) + h_b \left(\frac{q}{2} + k \sigma \sqrt{\frac{q}{D}} \right) + \left(\frac{D}{q} \right) \pi \sigma \sqrt{\frac{q}{D}} \{ f_s(k) - k [1 - F_s(k)] \} \\ & + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq} \end{aligned}$$

The optimal value of k can be formulated by taking the first partial derivatives of $TC(m, q, k)$ with respect to k and equating it to zero

$$\frac{\partial TC(m, q, k)}{\partial k} = 0$$

From Silver and Peterson (1985), we found that $(\partial (f_s(k) - k [1 - F_s(k)])) / \partial k = F_s(k) - 1$, than the derivation of Equation (11) with respect to k becomes

$$\sigma \sqrt{\frac{q}{D}} h_b + \frac{\pi D \sigma \sqrt{q/D} (F_s(k) - 1)}{q} = 0$$

$$F_s(k) = 1 - \frac{h_b q}{\pi D}$$

Derivation of Equation (13)

For the given total cost function,

$$\begin{aligned} TC(m, q, k) = & \frac{D}{nq} (A + Fn) + h_b \left(\frac{q}{2} + k \sigma \sqrt{\frac{q}{D}} \right) + \left(\frac{D}{q} \right) \pi \sigma \sqrt{\frac{q}{D}} \{ f_s(k) - k [1 - F_s(k)] \} \\ & + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq} \end{aligned}$$

The optimal value of q can be formulated by taking the first partial derivatives of $TC(m, q, k)$ with respect to q and equating it to zero

$$\begin{aligned} \frac{\partial TC(m, q, k)}{\partial q} = 0 \\ \frac{-D}{nq^2} (A + Fn) + \frac{h_b}{2} + \frac{h_b k \sigma}{2D\sqrt{q/D}} + D\pi\sigma\psi(k) \left(\frac{\frac{q}{2D\sqrt{q/D}} - \sqrt{q/D}}{q^2} \right) \\ + \frac{h_v}{2} \left((m-1) - (m-2) \frac{D}{P} \right) - \frac{DK}{mq^2} = 0 \end{aligned} \quad (19)$$

Rearranging Equation (19), we obtain

$$\begin{aligned} \frac{2D}{q^2} \left\{ \left(\frac{A}{n} + F \right) + \frac{K}{m} + \pi\sigma\psi(k)\sqrt{q/D} \right\} = h_b + h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} \\ + \frac{h_b k \sigma}{D\sqrt{q/D}} + \frac{\pi\sigma\psi(k)}{q\sqrt{q/D}} \end{aligned}$$

Substituting Equation (12) into Equation (18) we have

$$\begin{aligned} \frac{2D}{q^2} \left\{ \left(\frac{A}{n} + F \right) + \frac{K}{m} + \pi\sigma\psi(k)\sqrt{q/D} \right\} = h_b + h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} \\ + \frac{h_b \sigma}{D\sqrt{q/D}} \left(k + \frac{\psi(k)}{1 - F_s(k)} \right) \end{aligned} \quad (20)$$

From Equation (20), we can find the optimal value of q as

$$q^* = \sqrt{\frac{2D \left(\left(\frac{A}{n} + F \right) + \frac{K}{m} + \pi\sigma\psi(k)\sqrt{q/D} \right)}{h_b + h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{h_b \sigma}{D\sqrt{q/D}} \left(k + \frac{\psi(k)}{1 - F_s(k)} \right)}}$$

Proof of Proposition 1:

For the given total cost function,

$$\begin{aligned} TC(m, q, k) = \frac{D}{nq} (A + Fn) + h_b \left(\frac{q}{2} + k\sigma\sqrt{q/D} \right) + \left(\frac{D}{q} \right) \pi\sigma\sqrt{q/D} \\ \times \left\{ f_s(k) - k[1 - F_s(k)] \right\} + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq} \end{aligned}$$

Taking second partial derivatives of $TC(m, q, k)$ with respect to m , we have

$$\frac{\partial^2 TC(m, q, k)}{\partial^2 m} = \frac{2DK}{m^3 q} > 0$$

Therefore, $TC(m, q, k)$ is convex in m for fixed q and k . This completes the proof of proposition 1.

Proof of Proposition 2:

For the given total cost function,

$$TC(m, q, k) = \frac{D}{nq}(A + Fn) + h_b \left(\frac{q}{2} + k\sigma\sqrt{q/D} \right) + \left(\frac{D}{q} \right) \pi\sigma\sqrt{q/D} \\ \times \left\{ f_s(k) - k[1 - F_s(k)] \right\} + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq}$$

Taking second partial derivatives of $TC(m, q, k)$ with respect to q , we have

$$\frac{\partial^2 TC(m, q, k)}{\partial q^2} = \frac{2D}{nq^3}(A + Fn) - \frac{h_b k \sigma}{4q\sqrt{qD}} + \frac{2D\pi\sigma(F_s(k) - 1)}{4q^2\sqrt{qD}} + \frac{2DK}{mq^3} > 0$$

Therefore, $TC(m, q, k)$ is convex in q for fixed m and k . This completes the proof of Proposition 2.

Proof of Proposition 3:

For the given total cost function,

$$TC(m, q, k) = \frac{D}{nq}(A + Fn) + h_b \left(\frac{q}{2} + k\sigma\sqrt{q/D} \right) + \left(\frac{D}{q} \right) \pi\sigma\sqrt{q/D} \\ \times \left\{ f_s(k) - k[1 - F_s(k)] \right\} + \frac{q}{2} h_v \left\{ (m-1) - (m-2) \frac{D}{P} \right\} + \frac{DK}{mq}$$

Taking second partial derivatives of $TC(m, q, k)$ with respect to k , we have

$$\frac{\partial^2 TC(m, q, k)}{\partial^2 k} = \frac{\pi\sigma\sqrt{qD}f_s(k)}{q} > 0$$

Therefore, $TC(m, q, k)$ is convex in k for fixed q and m . This completes the proof of Proposition 3.