# PARAMETER SENSITIVITY MAPS OF SURFACE GEOELECTRIC ARRAYS 

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#### Abstract

Parameter sensitivity maps allow a better understanding of various geoelectric responses, and they are also helpful in designing optimal new arrays for specific problems. We constructed systematic parameter sensitivity maps for various geoelectric arrays, and in this paper several examples are presented, among others for non-linear and focussed arrays. Our parameter sensitivity values are computed from the response of a small-size cube in a homogeneous subsurface at three different depths. Instead of 3D numerical modelling results we consider the small cube as a superposition of three electric dipoles, corresponding to the electric charge accumulation at the opposite cube faces. We apply simple analytical formulas and we present the parameter sensitivity values separately for the individual dipoles. Several theoretical and practical aspects are discussed. We recommend a methodical use of parameter sensitivity maps in geoelectric prospecting.


## Introduction

Parameter sensitivity means how the measured signal responses to a small change in some model parameter. There are two approaches to compute parameter sensitivity values. In the first (more frequent, more theoretical) approach the response due to a small body in a homogeneous half-space is studied (Roy and Apparao 1971, Barker 1979). In the second (less frequent, but more practical) approach the effect of a change in one parameter of the given subsurface model is studied (Gyulai 1989).

As it was stated by Spitzer (1998): „The sensitivities play key roles in understanding the physics of the DC potential and its response to subsurface resistivity changes. This is of fundamental concern for the interpretation of any kind of geoelectrical depth investigation." He explained a fundamental magnetotelluric (!) feature (the „static shift") by using DC sensitivity maps.

One of the convincing results of parameter sensitivity maps is the offset Wenner array (Barker 1981), which was constructed directly from the parameter sensitivity map of the Wenner array. In spite of such successes, the parameter sensitivity maps have been relatively rarely used in geoelectrical practice. In order to promote their methodological use, here we describe our simple technique.

## Parameter-sensitivity maps

Our parameter sensitivity maps show the response due to a small cube in the subsurface, put in any $\mathrm{x}, \mathrm{y}, \mathrm{z}$ positions around the current- and potential electrodes. The cube has a side length of 0.1 R , where R is the characteristic array length, i.e. the distance between the two outer electrodes (not at infinity), or the distance between the transmitter and receiver dipoles. The values, varying in the horizontal ( $\mathrm{x}, \mathrm{y}$ ) plane, show directly the electric potential difference in the given array due to the cube, in the percentage of the corresponding potential difference measured by using a Wenner array over a homogeneous halfspace, and normalised by the resistivity contrast.


The physical sources of the anomaly are electrical charges at the interfaces. As far as the cube is small, the charge effects can be approximated by electric dipoles $\mathrm{p}_{\mathrm{xc}}, \mathrm{p}_{\mathrm{yc}}$ and $\mathrm{p}_{\mathrm{zc}}$, as shown in Figure 1. It is easy to compute the responses analytically, since it was shown (Szalai and Szarka 1999) that the analytical and numerical responses due to a small-size cube are practically the same.

Parameter sensitivity maps are calculated for three different depths: $0.1 \mathrm{R}, 0.2 \mathrm{R}$ and 0.3 R .

Figure 1. Notations in the derivation to compute parameter sensitivity

## How to use parameter sensitivity maps?

Due to the limited abstract size, only four parameter sensitivity maps are shown: the traditional Wenner array, as a basic array (in Figure 2), the square- $\alpha$ array, which has a nonlinear geometry (in Figure 3), the Schlumberger null array, representing a special group of arrays, where the response over a homogeneous half space is zero ("null") (Figure 4) and the unipole- $\alpha$ array, which is a current focussed array (in Figure 5). (Parameter sensitivity maps of non-linear-, null-, and focussed arrays had not been reported previously.)

The dashed lines separate the zones with positive and negative parameter sensitivity values from each other. (The positive values mean an enhacement of the apparent resistivity, the negative values mean a decrease in the apparent resistivity, when the subsurface inhomogeneity is of high resistivity.)

Figure 2. In case of the Wenner array, the dominating electric dipol is x-directed, and the effect is the most significant in central position. Deep bodies appear only with positive sign, but near-surface small bodies appear either with positive or with negative sign, depending on their $x, y$ position. Barker (1981) used this feature for the construction of his offset Wenner array as follows: the same near-surface inhomogeneity by using two Wenner arrays (when the second one is shifted by the distance between the current electrodes), is detected with the same sensitivity values, but with opposite signs. Consequently the arithmetic mean value of the two measurements eliminates near-surface effects, and the resulting mean values are more appropriate for soundings.

The y-directed dipoles have influence only in off-side position; the z-directed dipoles only in the two outer $\mathrm{R} / 3$ sections.


Figure 2: Parameter-sensitivity map series of the Wenner array, with the current electrode (stars) and potential electrode (circles) positions. $x, y, z$ components illustrate the effect of electrical charges accumulated at the corresponding opposite cube faces, while „total" means their superposed effect. The maps were made at three various depths/characteristic length values ( $0.1,0.2$ and 0.3 ).


Figure 3. Parameter-sensitivity map series of the square- $\alpha$ array. Notations are the same as in Figure 2.


Figure 4. Parameter-sensitivity map series of the Schlumberger null array. Notations are the same as in Fig. 2.


Figure 5. Parameter-sensitivity map series of the unipole- $\alpha$ array. Notations are the same as in Fig. 2. (The two current electrodes are of the same polarity.)

Figure 3. In case of the square- $\alpha$ array, for the y and z dipoles the high sensitivity zones are those between the closest potential electrode - current electrode pairs, while the x dipole has a higher sensitivity in the central zone. When the so-called square- $\alpha$ array is rotated by 90 degrees, and when a mean values from the two measurements, we arrive to the offsetsquare array (Barker 1981).

Figure 4. The null arrays in general have antisymmetry features. The Schlumberger null-array (Szalai et al. 2002) has two antisymmetry axes: the $y=0$ and the $x=0.5$ lines. Due to this fact, any resistivity inhomogeneity (either 2D or 3D), having a symmetry axis at either $y=0$ or $x=0.5$, is simply not detected by using this array! With other words: the effect of the symmetrical inhomogeneity element pairs just eliminate each other, as it is shown in Figure 6 a . This is the basic feature of polar diagrams shown in 6 b and 6 c .

Over an elongated body, and when the $A B$ line is rotated in different directions, the whole length of the body is effected by sensitivity values of the same sign. Such a feature directly leads to flower-petal like polar diagrams, enabling determining very precisely the strike direction.

Figure 5. In the parameter sensitivity maps of the unipole- $\alpha$ array, the depth-focussing effect, that is a relatively enhanced sensitivity of $z$ dipoles (the horizontal cube surfaces) over the horizontal ones, is directly seen.


Figure 6: a.) Parameter-sensitivity map of the Schlumberger null array with the surface projections of the highly conductive layer;
b.) Polar diagram constructed with the help of Fig 6a;
c.) Polar diagram measured in the field above a fissure filled by clay. In figures $\mathbf{6 b}$ and $\mathbf{c}$ the absolute values of potential differences are shown

## Conclusions

We have presented parameter sensitivity maps of various geoelectric arrays, among other non-linear-, null- and current focussed ones. (Such parameter sensitivity maps had not been reported before.)

Our parameter sensitivity maps are results of simple analytical formula, and its advantages are as follows: (1) the parameter sensitivity values obtained for different arrays can be directly compared to each other, (2) the physical source of anomaly (the electrical charges) at various cube faces is directly separated from each other, (3) DC anomaly due to small-size 3D bodies can be directly computed.

A detailed study of parameter sensitivity maps helps in understanding the physics of the resulting anomalies and basic features of various arrays. With the help of parameter sensitivity maps it is possible to construct new arrays. We recommend their methodological use in geoelectric prospecting.

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