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Emissions trading in a network of linked markets

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Emissions trading in a network of linked markets*

Abstract

In this article we construct a model to analyse permit markets which are connected by a network of links. A link between markets means that participants of one emissions trading system can use the permits of another. In a linked network of markets, domestic policy outcomes can be influenced by foreign regulators even without a direct link. We apply graph theory to study the dependencies between permit markets and develop a method to determine who can affect domestic emissions and prices. The results help to avoid unexpected interference with domestic policy outcomes and secure the effectiveness of climate change policies.

JEL Classification: L14, F13, Q54, Q58, D41

Keywords: networks, graph theory, emissions trading, trade theory

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1 Introduction

Climate change is a global externality that is best solved through international coordination. Unfortunately, international efforts to mitigate climate change by reducing greenhouse gases with a binding global agreement have been unsuccessful so far. Instead, many countries have moved forward with unilateral climate policies, with a hope of coordinating efforts in the future (Stavins, 2010; Newell et al., 2013).

Emissions trading, despite its controversies, has been one of the main policy tools used to reduce emissions (Grubb, 2012). Many countries and regions have implemented local emissions trading systems with the ability to link them to other systems. Linking means that the regulator of one emissions trading systems allows its participants to use permits of another system. Open trade tends to result in a common price level for permits and equalize marginal abatement costs for emitters, which is commonly considered a requirement for efficiency (Montgomery, 1972). Thus, in principle, it is possible to achieve a globally efficient solution by first creating local emissions trading systems and later linking them.

However, a common concern related to linking is that links make domestic permit market outcomes dependent on policy decisions of foreign regulators. When two permit markets are linked, one regulator might unilaterally create and sell new permits, and thus affect both markets and increase the aggregate amount of emissions, while making a profit.¹ Preventing this, and other similar harmful actions, requires an agreement and trust between the regulators.

Furthermore, it is well known that dependencies arise indirectly when two unlinked systems trade with a common third system (Anger, 2008; Flachsland et al., 2009a; Newell et al., 2013). In the general case, considering a network of several emissions trading systems arbitrarily linked to each other, dependencies can be conveyed thorough several links. To our knowledge, no previous study has been devoted to the general case, which is understandable, as one could easily imagine that it is simply sufficient and necessary to have a path of links between two systems to make them dependent. Somewhat surprisingly, we find this is not the case.

Our main questions is, given a network of arbitrarily linked emissions trading systems (like in Figure 1), which systems will be affected by a change in some other system? Knowing this is crucial for the policymaker, as the domestic policy outcomes could be undermined not only through its own

¹Rehdanz and Tol (2005) and Itkonen (2009) show that linking gives an additional incentive to print more permits.

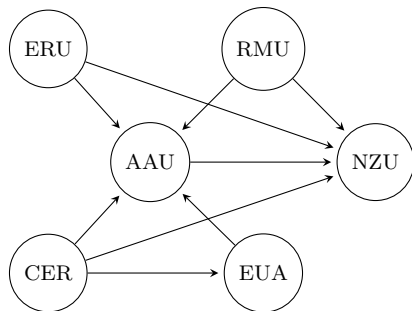


Figure 1: The graph depicts links between emissions trading systems at the end of the Kyoto period. The systems are identified by their emissions unit: Emission Reduction Unit (ERU), Removal Unit (RMU), Assigned Amount Unit (AAU), New Zealand Unit (NZU), Certified Emission Reductions (CER), EU emission allowance (EUA). Loops have been omitted from the figure.

links, but also through the links of its partners and their partners. In more technical terms, we ask how will marginal changes in exogenous variables of one system, e.g. the endowment of permits, change endogenous variables of other systems, with an emphasis on which systems will be affected and which will not.

Alongside the literature on emissions trading in general (Goulder, 2013), the idea of linking emissions trading system became a topical issue when the Kyoto protocol established multiple emissions trading mechanisms with the possibility of using permits from different mechanisms to meet the emissions targets set by the agreement. Since then, linking has been extensively studied in policy papers and technical reports, which cover diverse issues relevant for the implementation of linking policies, such as cost-effectiveness, distributional effects, and the compatibility of different design features (Haites, 2001; Ellis and Tirpak, 2006; Jaffe and Stavins, 2007; Itkonen, 2009; Mehling and Haites, 2009; Flachsland et al., 2009b; Tuerk et al., 2009; Hare et al., 2010; Newell et al., 2013). Some studies focus on legal issues (Jaffe et al., 2009) or sectoral perspectives (Aasrud et al., 2009; Anger, 2010; Marschinski et al., 2012), while others consider linking as a part of an international policy architecture (Flachsland et al., 2009a; Hare et al., 2010; Olmstead and Stavins, 2012). Cason and Gangadharan (2011) even perform a laboratory experiment where they test the efficiency of different linking structures.

In academically oriented literature, the idea of viewing legal rights as factors of production originates from Coase (1960), while Dales (1968) and

Crocker (1966) refined the idea into permits markets. Montgomery (1972) gave a proof for efficiency in a partial equilibrium model. In more recent literature, Copeland and Taylor (2005) framed linking as an application of trade theory with an emphasis on the general equilibrium effects which show that benefits can be ambiguous (see also Chichilnisky, 1994; Marschinski et al., 2012). Some studies use numerical simulations to analyse the costs and benefits of linking in general equilibrium (Böhringer et al., 2005; Klepper and Peterson, 2006) or partial equilibrium (Anger, 2008) models. In a related study Rehdanz and Tol (2005) analyse linking with a particular focus on how it affects the incentives of regulators to uphold emissions targets that were set prior to linking.

In this paper, we follow the tradition of partial equilibrium analysis and set up a model with emitting firms that participate in an emissions trading system which may be linked to other systems. We restrict the analysis to the partial equilibrium in order to focus on permit markets and the links between them. Trade is incentivized by allowing firms and endowments to be heterogeneous. We set up the model to allow for both cap-and-trade and baseline-and-credit types of emissions trading schemes, that is, firms either need permits for their emissions or they receive credits from their emissions reductions.

To address the policymaker's concern for indirect influence, we use graph theoretic tools to derive a dependency structure from the equilibrium conditions. We show that the network is partitioned into segments, which we call supply and demand components. The members of these components are connected by a specific type of alternating path of supply and demand, and they face the same price. We show that two systems are dependent if and only if there is such an alternating path between them, that is, they belong to the same component. By identifying the supply and demand components, we get a subset of equilibrium conditions that allows us to study the comparative statics of the equilibrium and show how changing endowments and production possibilities of one system affects other systems.

For a policymaker, who wishes to avoid unexpected interference by a foreign regulator to domestic policy outcomes, the theory provides an easy tool: if there is an alternating supply or demand path between the domestic and foreign systems, then the foreign regulator can interfere with the domestic market, and the policymaker should take precautionary measures.

Even though we apply the theoretic framework solely for the analysis of permit markets, the model shows potential for more general use. Considering trade theory, the framework generalizes the comparison between free trade (all markets are linked) and autarky (no markets are linked) by allowing bilateral trade between some markets while disallowing it between others.

From this point of view, one could ask, for example, what is the sufficient set of links needed to achieve an efficient outcome? Or which links need to be removed to achieve an effective embargo? Our theory predicts, that only after removing all the alternating paths between the embargoed country and the rest of the world, would the global market tear apart into separate supply and demand components, forcing the embargoed country to lose its gains from trade.

To our knowledge, this is the first study devoted to the network structure of dependencies in a market equilibrium with a network of trading possibilities and restrictions.² Even though the model setup we use is very conventional in economics (e.g. compare with Baumol and Oates, 1975), the network of constraints opens up a host of new questions and a need for graph theory, which enables us to contribute rather fundamental results.

In the next section we define necessary graph theoretic tools and set up the economic model. In Section 3, we solve the equilibrium, show some basic properties of the equilibrium, and derive a network to describe the equilibrium's dependency structure. In Section 5, we analyse comparative statics. In the final section we conclude. In the appendix we give examples.

2 Preliminaries

Next, we define the necessary graph theoretic concepts and set up the economic model. Graph theory will not only help us visualize the dependencies between markets, but it also provides us with a tool for rigorous deduction, and turns out to be extremely useful as we prove key propositions in the following sections.³

2.1 Graphs and connectivity

A graph (S, A) consists of a set of vertices S and a set of edges $A \subset \{\{s, r\} \mid s, r \in S\}$. A directed graph (S, A) consists of a set of vertices S and set of arcs $A \subset \{(s, r) \mid s, r \in S\}$. A (directed) graph (S', A') is a subgraph of

²Mathematically the setting resembles network flow problems (see for instance Boyd and Vandenberghe, 2004), which are studied in operations research, even though the structure of the problem is very different to ours. Also, Ostrovsky (2008); Hatfield and Kominers (2012); Hatfield et al. (2013) study the existence of equilibria in a trade network of agents with predetermined roles and indivisible goods in a matching model. With divisible goods, as in our case, existence is easy to show and we can focus on the structure of dependencies.

³For a similar approach, see De Benedictis and Tajoli (2011) who apply tools of network analysis to study international trade.

(directed) graph (S, A) if $S' \subset S$ and $A' \subset A$. If (S', A') is a subgraph of (S, A) then (S, A) is said to be a supergraph of (S', A') .

Vertices s_0 and s_k are connected by a path in graph (S, A) if there is a sequence of vertices $s_1, \dots, s_{k-1} \in S$, $k \in \mathbb{N}$ such that $(s_{i-1}, s_i) \in A$ for all $i = 1, \dots, k$.⁴ Subgraph (S', A') of graph (S, A) is a connected component if (1) any vertices $s, r \in S'$ are connected by a path in (S', A') when $s \neq r$, (2) there are no $s \in S'$ and $r \in S \setminus S'$ which are connected by a path in (S, A) , and (3) if $s, r \in S'$ and $(s, r) \in A$ then $(s, r) \in A'$.

2.2 The model

We construct a partial equilibrium model that focuses on dependencies between permit markets. We give a very simple description of the production side, in order to make the analysis tractable.

Consider a set of emissions trading systems S . Each system $s \in S$ has an endowment of permits $\omega^s > 0$ and regulates a set of firms which we call participants.⁵ As our focus is on the relationships between the systems and not on what happens inside the the systems, we assume each emissions trading system $s \in S$ can be described by a representative firm whose choices are equivalent to the sum of choices of the individual firms which participate in the system and trade permits under perfect competition. The representative firm produces output $y_s \geq 0$ using emissions $c_s \geq 0$ as a factor. The production possibilities of the representative firm s are described by a strictly concave, twice continuously differentiable production function $f_s: \mathbb{R}_+ \rightarrow \mathbb{R}$ for which $y_s = f_s(c_s)$.⁶

These assumptions allow for a saturation point \hat{c}_s above which there is no gain from further emissions, that is $f'_s(\hat{c}_s) = 0$, where f'_s is the derivative of f_s . In the emissions trading literature the saturation point is often called the *baseline* emissions level.⁷

The choice of firms is constrained by an obligation to buy permits for the

⁴Note that in some texts this would be referred to as a walk, with the distinction that a path has no repeated vertices or edges, but in our case this contrast makes no difference.

⁵Each firm is a participant of exactly one emissions trading system. See Goulder and Stavins (2011) for an account of the problems that arise from overlapping policy measures. For the same reason we omit emissions taxes from the analysis.

⁶Alternatively, one could begin by deriving the choices of the representative firm by explicitly defining a set of participating firms P_s which have such production functions f_i , $i \in P_s$, that $y_s = \sum_{i \in P_s} f_i(c_i)$ and the sum has the properties assumed for the representative firm, and also $c_s = \sum_{i \in P_s} c_i$.

⁷In our application it would not be unreasonable to assume that all firms have saturation point, i.e. firms would emit a finite amount even without regulations, but we do not wish to waste generality.

emissions they make. The rules of emissions trading dictate that participants can only emit an amount less or equal to the amount of permits they have acquired. That is, firms must have enough permits to cover their emissions.

In this specification a credit-and-baseline type of emissions trading system is a special case where the endowments equal the baseline emissions levels.⁸ Note that in order to have a positive price, this type of permits must have demand outside the system through a link.

When several emissions trading systems exist, their rules might allow for participants to use permits of other systems. When it is not explicitly allowed, the regulator will simply not accept foreign permits. In other words, the rules of an emissions trading system determine which permits its participants can use to comply with their obligations. We call the set of emissions trading systems and the description of which permits they allow, a trading network:

Definition. A *trading network* is a directed graph (S, A) , where the set of vertices $S = \{1, \dots, m\}$ refers to a set of m emission trading systems and the set of arcs $A \subset S \times S$ specifies which permits are allowed in each system: $(s, r) \in A$ indicates that system r allows its participants to use permits of system s . We use binary variable $a_r^s \in \{0, 1\}$ to indicate that $(s, r) \in A$ and vector $a_s = (a_s^1, \dots, a_s^m)$ to summarise which permits are allowed by system $s \in S$.

Even though in principle, firms can buy emissions permits from any emissions trading system, for compliance they can only use permits that are allowed by the system they participate in. The *emissions permit vector* of representative firm s describes how many permits of each system is held by the firm, and it is denoted by $e_s = (e_s^1, \dots, e_s^m)$, where $e_s^r \geq 0$ is the amount of emissions permits of system r held by firm s . The *obligations constraint* requires that firms emit an amount less or equal to the amount of acquired permits which are allowed by their system.

The *resource constraint* requires that sum of permits used cannot exceed the amount of permits issued.

Given a trading network, prices are denoted by a non-negative vector $p = (p_1, \dots, p_m)$. We assume prices are taken as given by the firms.

The *profit maximization problem* of a firm that represents system $s \in S$ of network (S, A) is to choose emissions $c_s \in \mathbb{R}_+$ and emissions permit vector

⁸In real-life implementations the number of credits generated equals the difference between the baseline and emissions, $\omega_s - c_s$, while the emissions themselves require no permits, but using this type of formulation would lead to identical results but a less concise presentation.

$e_s \in \mathbb{R}_+^m$ to maximize its profits

$$f_s(c_s) + p_s \omega^s - p e_s,$$

subject to the obligations constraint

$$c_s \leq a_s e_s,$$

while prices $p \in \mathbb{R}_+^m$ are taken as given. Note that the value of the endowment $p_s \omega^s$ is also taken as given by the firms.

3 Equilibrium

The decentralized perfect competition equilibrium in a trading network is equivalent with the solution of a social planner's problem of maximizing the sum of outputs. To show this, we solve the social planner's problem and show that its solution is equivalent with the competitive equilibrium conditions.

The social planner's problem is to choose emissions c_s for each $s \in S$ and emissions permits e_s^r for each $s, r \in S$ to maximize the sum of outputs while satisfying the obligations and resource constraints, that is, to solve

$$\max_{\substack{c_1, \dots, c_m, \\ e_1, \dots, e_m}} \sum_{s \in S} f_s(c_s),$$

so that resource constraints

$$\sum_{r \in S} e_r^s \leq \omega^s$$

and obligations constraints

$$c_s \leq a_s e_s$$

are satisfied for all $s \in S$.

As the optimization problem is convex and satisfies appropriate regularity conditions, such as the Slater's condition, the Karush-Kuhn-Tucker theorem gives the necessary and sufficient conditions for the solution. The conditions can be expressed with the help of a Lagrangian function

$$\begin{aligned} & L(e_1, \dots, e_m, c_1, \dots, c_m, \lambda_1, \dots, \lambda_m, p_1, \dots, p_m) \\ &= \sum_{s \in S} \left(f_s(c_s) - \lambda_s (c_s - a_s e_s) - p_s \left(\sum_{r \in S} e_r^s - \omega^s \right) \right), \end{aligned}$$

where $\lambda_1, \dots, \lambda_m, p_1, \dots, p_m$ are the non-negative Lagrangian multipliers.

Karush-Kuhn-Tucker conditions are as follows:
for all $s, r \in S$ such that $a_s^r = 1$

$$\frac{\partial L}{\partial e_s^r} = \lambda_s - p_r \leq 0, \quad (1a)$$

$$e_s^r \frac{\partial L}{\partial e_s^r} = e_s^r (\lambda_s - p_r) = 0, \quad (1b)$$

$$e_s^r \geq 0, \quad (1c)$$

for all $s, r \in S$ such that $a_s^r = 0$

$$\frac{\partial L}{\partial e_s^r} = -p_r \leq 0, \quad (2a)$$

$$e_s^r \frac{\partial L}{\partial e_s^r} = e_s^r (-p_r) = 0, \quad (2b)$$

$$e_s^r \geq 0, \quad (2c)$$

for all $s \in S$

$$\frac{\partial L}{\partial c_s} = f'_s(c_s) - \lambda_s \leq 0, \quad (3a)$$

$$c_s \frac{\partial L}{\partial c_s} = c_s (f'_s(c_s) - \lambda_s) = 0, \quad (3b)$$

$$c_s \geq 0, \quad (3c)$$

$$\frac{\partial L}{\partial \lambda_s} = c_s - a_s e_s \leq 0, \quad (4a)$$

$$\lambda_s \frac{\partial L}{\partial \lambda_s} = \lambda_s (c_s - a_s e_s) = 0, \quad (4b)$$

$$\lambda_s \geq 0, \quad (4c)$$

$$\frac{\partial L}{\partial p_s} = \sum_{r \in S} e_r^s - \omega^s \leq 0, \quad (5a)$$

$$p_s \frac{\partial L}{\partial p_s} = p_s \left(\sum_{r \in S} e_r^s - \omega^s \right) = 0, \quad (5b)$$

$$p_s \geq 0. \quad (5c)$$

An *equilibrium* of the model is a $(m^2 + 3m)$ -tuple

$$(e_1^1, \dots, e_m^1, \dots, e_1^m, \dots, e_m^m, c_1, \dots, c_m, \lambda_1, \dots, \lambda_m, p_1, \dots, p_m)$$

that satisfies conditions (1–5). We interpret the Lagrangian multipliers p_1, \dots, p_m as the competitive market equilibrium prices and $\lambda_1, \dots, \lambda_m$ as the marginal costs of regulation.

Note that the Karush-Kuhn-Tucker conditions for the social planners problem can also be derived by first solving the firms' problems individually, while taking prices as given, to get equation (1–4), and then postulating the market clearing conditions (5). Hence the conditions characterize an equilibrium for the representative firms of the trading network under perfect competition, as no firm can increase its profits by deviating from the social planner's optimum.

We can see, that the equilibrium conditions include inequality constraints and not all equilibrium variables might appear in each system's constraint, so it is possible that a subset of equilibrium variables can be solved with a subset of the equilibrium conditions. In such cases, the value of the equilibrium variables will depend only on the exogenous variables that appear in the subset of equations. Our goal in the following chapters is to make an exact account of such subsets and show that they are essential in answering our main question: which systems will be interdependent in the equilibrium. Subsequently, this will allow us to study the comparative statics of changes made by foreign regulators.

3.1 Equilibrium properties

First, we show some descriptive properties of the equilibrium using the Karush-Kuhn-Tucker conditions (1–5).

Proposition 1. *In equilibrium, a representative firm*

1. *uses only the cheapest permits among the allowed,*
2. *never emits beyond its saturation point, and*
3. *holds unallowed permits only if their price is zero.*

Proof. Let $s \in S$. First, to state the first part of the proposition precisely, we aim to show, that if $c_s > 0$ and $r' \in S$ is such that $(r', s) \in A$ and $p_{r'} > \min_{(r,s) \in A} p_r$ then $e_s^{r'} = 0$. By rearranging inequalities (1a) and (3a), we see that the marginal product $f'_s(c_s)$ is a lower bound for all prices among the allowed permits, that is

$$f'_s(c_s) \leq \lambda_s \leq p_r$$

for all r such that $(r, s) \in A$. Inequality (4a) implies $a_s e_s \geq c_s > 0$, so there is some r for which $e_s^r > 0$ and $a_s^r = 1$. For such r equation (1b)

implies that also inequality (1a) must be binding, and equation (3b) implies that inequality (3a) is binding, i.e. $f'_s(c_s) = \lambda_s$, therefore $f'_s(c_s) = \lambda_s = p_r$. Because $f'_s(c_s)$ is a lower bound,

$$f'_s(c_s) = \min_{(r,s) \in A} p_r. \quad (6)$$

For all r' for which $p_{r'} > p_r = \lambda_s$, that is $\lambda_s - p_{r'} < 0$, equation (1b) implies $e_s^{r'} = 0$.

Second, let there be a saturation point \hat{c}_s . Suppose, contrary to our claim, that $c_s > \hat{c}_s > 0$. Strict convexity and the definition of a saturation point implies that $f'_s(c_s) < f'_s(\hat{c}_s) = 0$. Now equation (3b) and inequality (3a) imply that $\lambda_s = f'_s(c_s) < 0$ which is a contradiction with inequality (4c), and hence $c_s \leq \hat{c}_s$.

Third, let $e_s^r > 0$ for some $r \in S$ such that $(r, s) \notin A$. Then equation (2b) implies $p_r = 0$. \square

We denote by p_s^* the lowest price available to system s , as in equation (6).

Note that the profit maximization problem does not generally have a unique solution with respect to the emissions permit vectors e_s , even though the solution for emissions c_s is unique. This is because allowed permits are perfect substitutes, and when their prices are equal, firms are indifferent between them. As the production function f_s is strictly concave, its derivative is also strictly decreasing and therefore it has an inverse function $f_s'^{-1}$, which we can use with equation (6) to express equilibrium emissions $c_s > 0$ in terms of the lowest price available:

$$c_s = f_s'^{-1}(p_s^*) = f_s'^{-1}\left(\min_{(r,s) \in A} p_r\right) \quad \forall s \in S.$$

This gives a unique amount of emissions for every price vector p .

3.2 Equilibrium network

Next, we apply the graph theoretic tools defined in Subsection 2.1 to analyse the dependencies further, with an aim to determine which equilibrium prices and emissions will be affected by changes in the endowment of a given system.

In the proof of Proposition 1, we noticed that equilibrium emissions c_s are directly dependent only from prices that equal the lowest price among the permits which are allowed in system s . That is, equilibrium conditions relate specific emissions choices to specific prices according to the network structure and the ordering of prices. Part 1 of Proposition 1 implies that permits whose price is higher than lowest price will not be used in equilibrium. This

suggest we can focus on a smaller set of connections. To study the equilibrium further, we define the subgraph that characterises the relevant dependencies between the equilibrium prices and quantities:

Definition. The *equilibrium network* of trading network (S, A) is the directed subgraph (S, M) , where $M = \{(r, s) \in A \mid p_r = p_s^*\} \subset A$, and p_r and p_s^* are equilibrium prices.

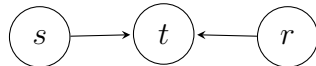
It is worth noting that, in equilibrium, the system's own permits might be too expensive for its participants to buy. More specifically, it is possible that $p_s > p_s^*$ and hence $(s, s) \notin M$, even if $(s, s) \in A$.

3.3 Supply and demand

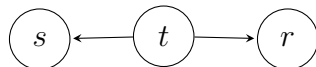
Next, we use the network equilibrium to define two adjacency properties, and in the following lemma we show that these properties imply an equivalence relation for prices.

Definition. Systems s and r are *adjacent sellers* in equilibrium network (S, M) if there is a system t such that $(s, t) \in M$ and $(r, t) \in M$. Similarly, s and r are *adjacent buyers* in equilibrium network (S, M) if there is a system t such that $(t, s) \in M$ and $(t, r) \in M$.

That is, systems s and r are adjacent sellers if there is a subgraph



and adjacent buyers if there is a subgraph



for some t .

The practical interpretation of the adjacency concepts is that when two firms sell to the same market or two firms buy from the same markets, under perfect competition, prices tend to be the same. The following lemma states this in more exact terms:

Lemma 1. *In an equilibrium network, the permits of adjacent sellers have an equal price, and adjacent buyers use permits with an equal price. That is, adjacent sellers sell for the same price and adjacent buyers buy at the same price.*

Proof. First, let s and r be adjacent sellers. Now there is a system t such that $(s, t) \in M$ and $(r, t) \in M$, and by definition $p_s = p_t^*$ and $p_r = p_t^*$. Hence $p_s = p_r$.

Second, let s and r be adjacent buyers. Now there is a system t such that $(t, s) \in M$ and $(t, r) \in M$, and by definition $p_t = p_s^*$ and $p_t = p_r^*$. Since systems s and r have the same minimum price among the allowed permits, Proposition 1 implies that the permits they use have the same price. \square

Due to the transitivity of the equivalence relation, prices are equated beyond adjacent pairs. Next we aim to find the largest set of system among which prices are equated.⁹

First, we note that the relationship of being adjacent sellers or buyers is symmetric, so all such relationships found in the equilibrium network can be summarized as an undirected graph.

Definition. The *adjacent seller graph* is the undirected graph (S, M_S) , where $M_S = \{\{s, r\} \mid s, r \in S, \exists t: (s, t) \in M \text{ and } (r, t) \in M\}$. The *adjacent buyer graph* is the undirected graph (S, M_D) , where $M_D = \{\{s, r\} \mid s, r \in S, \exists t: (t, s) \in M \text{ and } (t, r) \in M\}$.

The adjacent seller and buyer graphs can be used to partition the set of systems of the trading network into connected components that are essential for characterizing the dependencies between between systems.

Definition. A *supply component* of equilibrium network (S, M) is a connected component in adjacent seller graph (S, M_S) . Similarly, a *demand component* of (S, M) is a connected component in adjacent buyer graph (S, M_D) .

Basic graph theory tells us that the connected components of a graph induce a unique partition for the set of vertices, and in a finite graph there is a finite number of components.

We can now show that both supply and demand components constitute a set of systems which have an equal prices.

Proposition 2. *In equilibrium,*

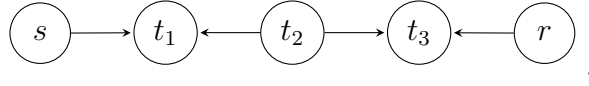
1. *permit prices of systems that belong to the same supply component are equal, and*
2. *prices of permits bought by firms in systems belonging to the same demand component are equal.*

⁹That is, we aim to construct the equivalence classes for the equal permit price relation.

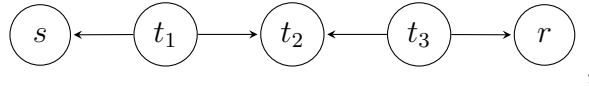
Proof. First, let s and r be members of a supply component. Now there is a path of adjacent sellers between s and r . According to Lemma 1, each consecutive pair in the path must sell at the same price. Due to the transitivity of the equivalence relation, s and r sell at the same price. The proof of the second part is similar. \square

Proposition 2 allows us to define a single price for all permit in a supply component. We denote the price of permits in supply component S_i by p_{S_i} . Later we will show that this is the price faced by all systems in a particular demand component.

To illustrate the idea behind Proposition 2, consider a supply component of equilibrium network (S, M) . If systems s and r are separate members of the same supply component, then by definition, there must a particular type of path connecting them in (S, M) . For example, consider a subgraph



where s and t_2 , as well as t_2 and r are adjacent sellers. Both pairs have a common system to which their permits are sold to, in this case t_1 and t_3 , respectively. Similarly, in subgraph



systems s and t_2 , as well as t_2 and r are adjacent buyers, which buy permits from t_1 and t_3 , respectively.

The illustration suggest another way of stating that two systems belong to the same supply or demand component: by defining an appropriate form of connectivity in the underlying equilibrium network.

Definition. Vertices s_0 and s_k are connected by an *alternating supply path* in equilibrium network (S, M) if there is a sequence of vertices s_1, \dots, s_{k-1} , where $k \geq 2$ is an even number, such that $(s_{i-1}, s_i) \in M$ and $(s_{i+1}, s_i) \in M$ for all odd $i = 1, \dots, k-1$. Similarly, vertices s_0 and s_k are connected by an *alternating demand path* in equilibrium network (S, M) if there is a sequence of vertices s_1, \dots, s_{k-1} , where k is an even integer, such that $(s_i, s_{i-1}) \in M$ and $(s_i, s_{i+1}) \in M$ for all odd $i = 1, \dots, k-1$.

Based on the definitions, it is clear that two systems are connected by an alternating supply path or demand path if and only if they are members of the same supply component or demand component, respectively. As we will

later show that supply and demand components are essential for specifying dependencies between equilibrium variables, alternating supply and demand paths will help determine whether two systems are dependent.

Next, we will show that each supply component is related to a particular demand component, and conversely each demand component is related to a particular supply component. We say that such components are *matching*.

Lemma 2. *Let (S, M) be an equilibrium network.*

1. *Let S_i be the set of systems in a supply component of (S, M) and $D_i = \{r \in S \mid \exists s \in S_i: (s, r) \in M\}$. Then D_i is the set of systems of a demand component of (S, M) .*
2. *Let D_i be the set of systems in a demand component of (S, M) and $S_i = \{s \in S \mid \exists r \in D_i: (s, r) \in M\}$. Then S_i is the set of systems of a supply component of (S, M) .*

Proof. Let S_i be the set of systems in a supply component of (S, M) and $D_i = \{r \in S \mid \exists s \in S_i: (s, r) \in M\}$. Let $s, r \in D_i$. Now there are vertices $t_0 \in S_i$ and $t_l \in S_i$ such that $(t_0, s) \in M$ and $(t_l, r) \in M$. If $t_0 = t_l$, then s and r are adjacent buyers. Suppose $t_0 \neq t_l$. Since S_i is a supply component, t_0 and t_l are connected by a path of adjacent sellers, which we denote by sequence of vertices $(t_0, t_1, \dots, t_{l-1}, t_l)$. For each $j = 1, \dots, l$, subsequent system (t_{j-1}, t_j) are adjacent sellers, and hence there exists a $q_j \in D_i$ such that $(t_{j-1}, q_j) \in M$ and $(t_j, q_j) \in M$. Now vertices q_{j-1} and q_j are adjacent buyers for all $j = 2, \dots, l$, as are (s, q_1) and (q_l, r) , so s and r are connected by a path in the adjacent buyer graph.

Also note that vertices in D_i are not connected to other vertices of the supergraph outside of D_i : if $s \in D_i$ were connected to $r \in S \setminus D_i$, we could use the same strategy as above to show, that $r \in D_i$, which would be a contradiction.

The proof for part 2 is similar. □

Lemma 2 shows us that for each supply component there is a unique matching demand component, and vice versa. We denote the sets of systems of the supply component by S_i and D_i , where $i = 1, \dots, k$ is index for the k components.

Finally, we show that the set of arcs having an initial vertex in the supply component equals the set of arcs having a terminal vertex in the demand component. This result can be interpreted as a type of completeness property of matching supply and demand components and it will be pivotal in the next section where we analyse the comparative static results of the equilibrium.

Proposition 3. *If S_i and D_i are matching supply and demand components of equilibrium network (S, M) , then*

$$\{(s, r) \in M \mid s \in S_i, r \in S\} = \{(s, r) \in M \mid s \in S, r \in D_i\}.$$

Proof. Let S_i and D_i be matching supply and demand components of equilibrium network (S, M) .

Suppose $(s, r) \in \{(s, r) \in M \mid s \in S_i, r \in S\}$. Now $s \in S_i$ and $r \in S$, and part 1 of lemma 2 implies $r \in D_i = \{r \in S \mid \exists s \in S_i: (s, r) \in M\}$, therefore $(s, r) \in \{(s, r) \in M \mid s \in S, r \in D_i\}$.

Suppose $(s, r) \in \{(s, r) \in M \mid s \in S, r \in D_i\}$. Now $s \in S$ and $r \in D_i$, and part 2 of lemma 2 implies $r \in S_i = \{s \in S \mid \exists r \in D_i: (s, r) \in M\}$, therefore $(s, r) \in \{(s, r) \in M \mid s \in S_i, r \in S\}$. \square

Proposition 3 means that all permits that are sold from the supply component are bought somewhere within the matching demand component. And the other way around, the permits bought by firms in the demand component are sold from the matching supply component.

4 Comparative statics

Next, we will study the comparative statics of the equilibrium. First, we define a concept, which we use to restrict to non-trivial equilibria. Second, we apply Propositions 1–3 to derive a (sub)system of equations, which characterizes the equilibrium locally and allows us to analyse the comparative statics. Finally, two comparative statics results are presented.

4.1 Slackless equilibrium

Some constraints can be unbinding in the equilibrium if the endowment of permits is so large that there is no scarcity or the endowment is so small that firms stop emitting (i.e. exit the permit market). In such cases, marginal changes in the involved exogenous variables would have no effect on the equilibrium. Also, in the special case where constraints are binding in only one direction, analysing marginal effects would have to be restricted to semiderivatives. We will exclude these equilibria from the analysis and focus on the ones where constraints are binding within an open set. To be exact, we restrict to what we call slackless equilibria¹⁰:

¹⁰Note that to avoid confusion we do not use the term non-boundary as the equilibria are indeed on the boundary in the sense that the related constraints are binding.

Definition. An equilibrium is *slackless* for matching supply and demand components S_i and D_i if for all $s \in S_i$ and $r \in D_i$ there exists $\varepsilon > 0$ such that $c_r > \varepsilon$, $\lambda_r > \varepsilon$, $p_{S_i} > \varepsilon$ and $e_r^s > \varepsilon$ when $(s, r) \in M$.

The slackless assumption refers to the complementary slackness conditions of the optimization problem and implies that the resource and obligations constraints are binding. If there are permits with no value or non-emitting firms, then marginal changes in the exogenous variables will have either one-sided effects or no effect at all on the equilibrium variables. By restricting the analysis of comparative statics to matching components with a slackless equilibrium, we are guaranteed that within an open neighbourhood of the exogenous variables the endogenous variables have an open neighbourhood in which the relevant equilibrium constraints are binding and fully characterize the equilibrium. This allows us to apply the implicit function theorem to analyse the marginal effects on the equilibrium prices and emissions of changes in the production functions and endowments.

4.2 Subset of equations

Finally, we will apply the theory and concepts we have developed to derive a subset of equilibrium conditions that are necessary and sufficient to describe the equilibrium locally.

Consider a slackless equilibrium for matching supply and demand components S_i and D_i .

The slacklessness of the equilibrium and Propositions 2 imply that $p_{S_i} = p_s > 0$ for each $s \in S_i$, so the related resource constraints (5) are binding and we can use part 1 of Proposition 1 to represent the constraints as

$$\sum_{(s,r) \in M} e_r^s = \omega^s \quad \forall s \in S_i. \quad (7)$$

Similarly, slacklessness means that $\lambda_r > 0$ for each $r \in D_i$, so the related obligations constraints (4) are binding and we can use part 1 of Proposition 1 to restate the constraints as

$$\sum_{(s,r) \in M} e_r^s = c_r \quad \forall r \in D_i. \quad (8)$$

Using equations (7) and (8), together with Proposition 3, we can equate the sum of emissions demands of demand component D_i and the endowments of supply component S_i :

$$\sum_{r \in D_i} c_r = \sum_{r \in D_i} \sum_{(s,r) \in M} e_r^s = \sum_{s \in S_i} \sum_{(s,r) \in M} e_r^s = \sum_{s \in S_i} \omega^s.$$

This can be summarized as

$$\sum_{r \in D_i} c_r - \sum_{s \in S_i} \omega^s = 0. \quad (9)$$

Also for all $s \in S_i$ and $r \in D_i$ such that $(s, r) \in M$ slacklessness implies $e_r^s > 0$, which means constraint (1) is binding so that $\lambda_r = p_s$. Following Proposition 2, all prices of the supply component are bound to be equal. As $c_r \geq e_r^s > 0$ and constraints (3) are binding, equation $f'_r(c_r) = \lambda_r = p_s = p_{S_i}$ applies for all $s \in S_i$ and $r \in D_i$. This allow us to restate equation (6) as

$$f'_r(c_r) - p_{S_i} = 0 \quad \forall r \in D_i. \quad (10)$$

Equations (9) and (10) are necessary and sufficient conditions for an equilibrium that is slackless for matching supply and demand components with sets of systems S_i and D_i within some neighbourhood.

Equations (9) and (10) can be interpreted as implicit functions that describe the relationship between exogenous parameters and the endogenous variables, which are uniquely defined, within matching supply and demand components. The slackless assumptions guarantee that equations (9) and (10) are equivalent to the Karush-Kuhn-Tucker conditions in an open set containing the equilibrium. Because the production functions are twice continuously differentiable, the implicit functions defined by equations (9) and (10) are continuously differentiable. Hence, equations (9) and (10) characterize the system of equations that determine the endogenous variables related to the matching supply and demand components.

Finally, in the next two subsections we use comparative statics to assess the economic outcomes resulting from (1) a change in endowments and (2) an additional non-emitting factor that substitutes emissions.

4.3 Endowment effect

Let $s \in S_i$ be a system that changes its endowment ω^s , i.e. it issues more permits. We can use the implicit function derived from equations (9) and (10) to get the system of equations that specifies the implicit function's partial derivatives:

$$\sum_{r \in D_i} \frac{\partial c_r}{\partial \omega^s} - 1 = 0 \quad (11)$$

and

$$f''_r(c_r) \frac{\partial c_r}{\partial \omega^s} - \frac{\partial p_{S_i}}{\partial \omega^s} = 0 \quad \forall r \in D_i. \quad (12)$$

Equation (12) can be rearranged to get

$$\frac{\partial c_r}{\partial \omega^s} = f_r''(c_r)^{-1} \frac{\partial p_{S_i}}{\partial \omega^s} \quad \forall r \in D_i \quad (13)$$

and plugged into equation (11) to get

$$\frac{\partial p_{S_i}}{\partial \omega^s} = \left(\sum_{r \in D_i} f_r''(c_r)^{-1} \right)^{-1} < 0, \quad (14)$$

which is negative because the second derivative of a strictly concave function is negative. Plugging this into equation (13) gives

$$\frac{\partial c_r}{\partial \omega^s} = f_r''(c_r)^{-1} \left(\sum_{r' \in D_i} f_{r'}''(c_{r'})^{-1} \right)^{-1} > 0 \quad \forall r \in D_i. \quad (15)$$

This means that increasing the endowment of any system of a supply component has an effect on the supply component's price level and on the emissions of the matching demand component. Inequality (14) implies by that increasing the endowment, the price of all permits in the supply component will decrease, and inequality (15) implies that emissions will increase in all systems of the demand component.

4.4 Effect of cleaner technology

Let $r' \in D_i$ be a system that has a production function of form $f_{r'}(c_{r'} + \beta)$, where $\beta \in \mathbb{R}$ is a factor of production that can be used to replace emissions. We assume $f_{r'}$ has all the properties required from other production functions in the model.

Now the implicit function defined by equations (9) and (10) has a derivative that is determined by equations

$$\sum_{r \in D_i} \frac{\partial c_r}{\partial \beta} = 0, \quad (16)$$

$$f_r''(c_r) \frac{\partial c_r}{\partial \beta} - \frac{\partial p_{S_i}}{\partial \beta} = 0 \quad \forall r \in D_i \setminus \{r'\}, \quad (17)$$

$$f_{r'}''(c_{r'} + \beta) + f_{r'}''(c_{r'} + \beta) \frac{\partial c_{r'}}{\partial \beta} - \frac{\partial p_{S_i}}{\partial \beta} = 0, \quad r' \in D_i. \quad (18)$$

We evaluate the derivative at $\beta = 0$, and rearrange equations (17) and (18) to get

$$\frac{\partial c_r}{\partial \beta} = f_r''(c_r)^{-1} \frac{\partial p_{S_i}}{\partial \beta} \quad \forall r \in D_i \setminus \{r'\} \quad (19)$$

and

$$\frac{\partial c_{r'}}{\partial \beta} = f''_{r'}(c_{r'})^{-1} \frac{\partial p_{S_i}}{\partial \beta} - 1, \quad r' \in D_i.$$

Plugging these into equation (16) gives

$$\frac{\partial p_{S_i}}{\partial \beta} = \left(\sum_{r \in D_i} f''_r(c_r)^{-1} \right)^{-1} < 0, \quad (20)$$

which is negative as the second derivatives of a strictly concave function is negative. Plugging equation (20) into equation (19) gives

$$\frac{\partial c_r}{\partial \beta} = f''_r(c_r)^{-1} \left(\sum_{r \in D_i} f''_r(c_r)^{-1} \right)^{-1} > 0 \quad \forall r \in D_i \setminus \{r'\}, \quad (21)$$

while equation (16) is equivalent to

$$\frac{\partial c_{r'}}{\partial \beta} = - \sum_{r \in D_i \setminus \{r'\}} \frac{\partial c_r}{\partial \beta} < 0, \quad (22)$$

where the expressions are negative due to inequality (21).

The analysis implies that introducing clean technology to any firm in a demand component has an effect on the emissions of other firms in the demand component and on the price of the matching supply component. Inequalities (21) and (22) tell us that cleaner technology in one firm decreases its own emissions while allowing others to emit more. Inequality (20) tells us that cleaner technology decreases the price of permits in the matching supply component.

5 Conclusions

We have set up a simple model to analyse trade given an underlying network of trade possibilities. We have shown that the restrictions will partition the market into areas of equal price. These supply and demand components constitute market areas, within which agents are dependent of each other even if they are not directly connected, while agents remain unaffected by events outside the market area.

The concepts of alternating supply and demand paths provide a convenient tool for finding out whether systems are interdependent. If we observe permits being sold from one system to another, it is sufficient to conclude that they are connected in the equilibrium network. If the observed connections form an alternating path between two markets, we can infer that they

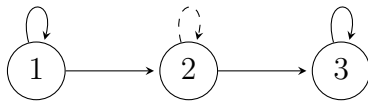
are interdependent. In such a case, the policymaker should acknowledge that their policy outcome will depend on the actions of the foreign regulator, even if there is no direct link between the systems.

To offer a rather abstract idea, alternating paths could be understood as pathways for price signals, which are necessary to equate prices and to achieve gains from trade. The theory shows that it is not necessary for agents to operate directly in the same market to equate prices. The essential requirement is to have a path of agents who share buyers and sellers. Without such paths the perfectly competitive markets are unable to generate the efficient outcome.

One could also interpret the framework as a generalization of a simple trade model. When all systems are linked, the model reduces to a simple partial equilibrium model of production with open trade. When no systems are linked, the model is equivalent with autarchy. In these extreme cases, the tools developed here are trivial. However, they allow us to study the intermediate cases. With intermediate linking structures, one could study how the equilibrium breaks in to separate markets, while losing benefits from trade. Or one could study which links are necessary to establish the open trade equilibrium, or which links need to be removed in order to cut the interdependence. Clearly the model and theory constructed in this paper shows various possibilities for further investigation.

A A numerical example

Consider emissions a trading network (S, A) with systems $S = 1, 2, 3$ and arcs $A = \{(1, 1), (1, 2), (2, 2), (2, 3), (3, 3)\}$. The corresponding graph is depicted below.



Let $f_s(c_s) = \beta_s c_s - c_s^2$ and $\omega^s = 1/4$ for $s = 1, 2, 3$, and $\beta_1 = \beta_2 = 2$ and $\beta_3 = 5$. We can derive the derivatives $f'_s(c_s) = \beta_s - 2c_s$ and the saturation points $\hat{c}_s = \beta_s/2$. Given the arcs, the resource constraints are $e_1^1 + e_2^1 \leq \omega^1$, $e_2^2 + e_3^2 \leq \omega^2$, and $e_3^3 \leq \omega^3$.

Consider the equilibrium of the specified model. First, note that part b of Proposition 1 implies that in equilibrium $f'_s(c_s) > 0$ when $c_s < \hat{c}_s$. Because $c_s \leq a_s e_s \leq \sum_{r \in S} \omega^r = 3/4 < \beta_s/2 = \hat{c}_s$ for all $s \in S$ applies $f'_s(c_s) > 0$. Now inequality (3a) implies $\lambda_s > 0$, therefore equation (4b) implies that obligation constraint (4a) is binding, and inequality (1a) implies that $p_r > 0$ and resource constraint (5a) is binding for all $s, r \in S$.

Given that the obligation constraint (4a) are binding, inequalities (1a) (3a) implies that

$$2 - 2e_1^1 \leq p_1,$$

$$2 - 2(e_2^1 + e_2^2) \leq p_1 \quad \text{and} \quad 2 - 2(e_2^1 + e_2^2) \leq p_2,$$

and

$$5 - 2(e_3^2 + e_3^3) \leq p_2 \quad \text{and} \quad 5 - 2(e_3^2 + e_3^3) \leq p_3.$$

If $p_3 > p_2$, then $e_3^3 = 0$ and $e_3^3 \neq \omega^3$, which would contradict the binding resource constraint for system 3. If $p_3 < p_2$, then $e_3^2 = 0$. As $e_2^1 \geq 0$ and $c_2 \geq e_2^2 = \omega^2 = 1/4$, which means $p_2 = \lambda_2 = f_2(c_s)$, we get

$$3/2 \geq 2 - 2(e_2^1 + e_2^2) = p_2 > p_3 = 5 - 2(e_3^2 + e_3^3) = 9/2,$$

which is a contradiction. Therefore $p_3 = p_2$.

If $c_2 = 0$, then $c_1 = e_1^1 = 1/4 > 0$ and inequalities (1a) and (3a) imply that $3/2 = 2 - 2e_1^1 = p_1 \geq 2 - 2(e_2^1 + e_2^2) = 2$, which is a contradiction. If $c_1 = 0$, then $c_2 \geq e_2^1 = 1/4 > 0$ and inequalities (1a) and (3a) imply that $2 = 2 - 2e_1^1 \leq p_1 = 2 - 2(e_2^1 + e_2^2) \leq 3/2$, which is a contradiction. Therefore $c_1 > 0$ and $c_2 > 0$. Also $c_3 \geq e_3^3 = \omega^3 > 0$

Now equation (3b) implies that $f_s(c_s) = \lambda_s$ for $s = 1, 2, 3$.

Suppose $p_1 = p_2 = p_3$. Now $e_1^1 = c_1 > 0$, $e_3^3 > 0$, and either $e_2^1 > 0$ or $e_2^2 > 0$, which implies $\lambda_1 = \lambda_2 = \lambda_3 = p_1 = p_2 = p_3$. Hence the marginal products must be equal:

$$2 - 2e_1^1 = 2 - 2(e_2^1 + e_2^2) = 5 - 2(e_3^2 + e_3^3).$$

By plugging in the resource constraints, we get

$$2 - 2(\omega^1 - e_2^1) = 2 - 2(e_2^1 + e_2^2) = 5 - 2((\omega^2 - e_2^2) + \omega^3). \quad (23)$$

Both sides of the first equation in (23) can be reduced by 2 and divided by -2 to get $\omega^1 - e_2^1 = e_2^1 + e_2^2$ and solved to get $e_2^1 = \frac{1}{2}\omega^1 - \frac{1}{2}e_2^2$. The second equation in (23) can be rearranged to get, $e_2^1 = \omega^2 + \omega^3 - \frac{3}{2} - \frac{3}{2}e_2^2$. Equating these gives $\frac{1}{2}\omega^1 - \frac{1}{2}e_2^2 = \omega^2 + \omega^3 - \frac{3}{2} - \frac{3}{2}e_2^2$, which can be solved to get

$$e_2^2 = \omega^2 + \omega^3 - \frac{3}{2} - \frac{1}{2}\omega^1 = -\frac{9}{8} < 0,$$

which is a contradiction.

Suppose $p_1 > p_2 = p_3$. Now $e_2^1 = 0$, $e_1^1 = \omega^1$, and $p_1 = \lambda_1$, which implies $3/2 = 2 - 2e_1^1 = p_1 > p_2 = 5 - 2(e_3^2 + e_3^3) \geq 4$, which is a contradiction.

Therefore in equilibrium the prices must have order $p_1 < p_2 = p_3$. Now, because $p_2 > \lambda_2$, equation (1b) implies that $e_2^2 = 0$. As the marginal products

of systems 1 and 2 are equal, $2 - 2e_1^1 = 2 - 2e_2^1$, we can solve their permit usage: $e_1^1 = e_2^1 = c_1 = c_2 = \omega^1/2 = 1/8$. The price of permit 1 is set to the marginal product, $p_1 = 2 - \frac{2}{8} = \frac{7}{4}$. Given the resource constraints $e_3^2 = e_3^3 = \omega^2 = \omega^3 = 1/4$ and emissions are $c_3 = 1/2$. Also the prices of permit 2 and 3 are set to the marginal product of system 3:

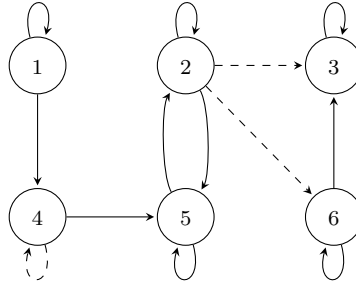
$$p_2 = p_3 = 5 - \frac{2}{2} = 4.$$

The equilibrium network is determined as (S, M) , where $S = 1, 2, 3$ and $M = \{(1, 1), (1, 2), (2, 3), (3, 3)\}$. This depicted by the solid arcs in the previous graph. Note that arc $(2, 2)$ is missing which indicates that system 2 is not using its own permits in equilibrium.

The equilibrium network contains 2 pairs of demand and supply components: components with low price, $D_1 = \{1, 2\}$ and $S_1 = \{1\}$, and components with high price, $D_2 = \{3\}$ and $S_2 = \{2, 3\}$. We can denote the supply components' prices by $p_{S_1} = p_1$ and $p_{S_2} = p_2 = p_3$, respectively.

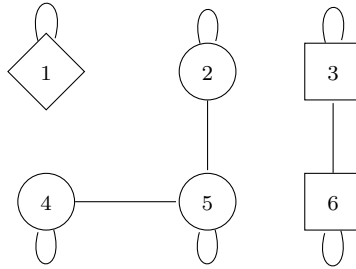
B A graphical example

Consider a trading network of six systems whose links can be depicted by the following directed graph:

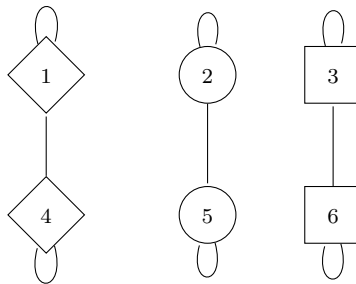


Suppose the dashed arcs are part of the trading network but not the equilibrium network. That is, the dashed links are removed because the lowest price available to the system at the arc's terminal vertex is cheaper than the price of the permit at the initial vertex.

Now we can directed graph to determine vertices are adjacent sellers and construct the adjacent seller graph:



We identify the supply components with different vertex shapes. Similarly we can determine the adjacent buyers and construct the adjacent buyer graph:



We identify the matching demand component with the corresponding shapes. Sets $\{1\}$, $\{2, 4, 5\}$, and $\{3, 6\}$ indicate supply components and sets $\{1, 4\}$, $\{4, 5\}$, and $\{3, 6\}$ indicate their matching demand components, respectively.

Now we can easily determine which systems will be affected by changes in exogenous variables. For example, a change in the endowment of system 4, which is a member of the "circle component", will affect the price of permits 2, 4, and 5 and emissions of systems 2 and 5. Similarly, replacing emissions by a non-emitting factor in system 4 will affect the price of permit 1, decrease emissions in system 4 and increase emissions in system 1.

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