

## An Equivalent Source Method for Modelling the Global Lithospheric Magnetic Field

Kother, Livia Kathleen; Hammer, Magnus Danel; Finlay, Chris; Olsen, Nils

*Publication date:*  
2014

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*

Kother, L. K., Hammer, M. D., Finlay, C., & Olsen, N. (2014). An Equivalent Source Method for Modelling the Global Lithospheric Magnetic Field. Poster session presented at European Geosciences Union General Assembly 2014, Vienna, Austria.

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## Summary

We produce a new model of the global lithospheric magnetic field based on 3-component vector field observations at all latitudes from the CHAMP satellite using an equivalent source technique.

## Method

A regularized iteratively reweighted least squares algorithm is applied. Data error covariance matrices are implemented, including both the latitude dependence of data error variances  $\sigma^2$  (Fig.1) and covariances  $\mathbf{C}$  between the vector field components due to unmodelled sources. The regularization norm  $\mathbf{R}$  is defined to be the Euclidean length of the model solution. Our scheme iteratively minimizes:

$$\Theta(\mathbf{m}_k) = (\mathbf{d} - \mathbf{G}\mathbf{m}_k)^T \mathbf{W}_{k-1} (\mathbf{d} - \mathbf{G}\mathbf{m}_k) + \lambda \mathbf{R}(\mathbf{m}_k)$$

$$\mathbf{W}_{k-1} = \mathbf{C}^{-1/2} \mathbf{H}_{k-1} \mathbf{C}^{-1/2}$$

Huber weighting ensures a robust solution in the presence of non-Gaussian data errors

$$\mathbf{H}_{k-1} = \min\left(\frac{1.5}{|\mathbf{d} - \mathbf{G}\mathbf{m}_{k-1}|/\sigma}, 1\right)$$

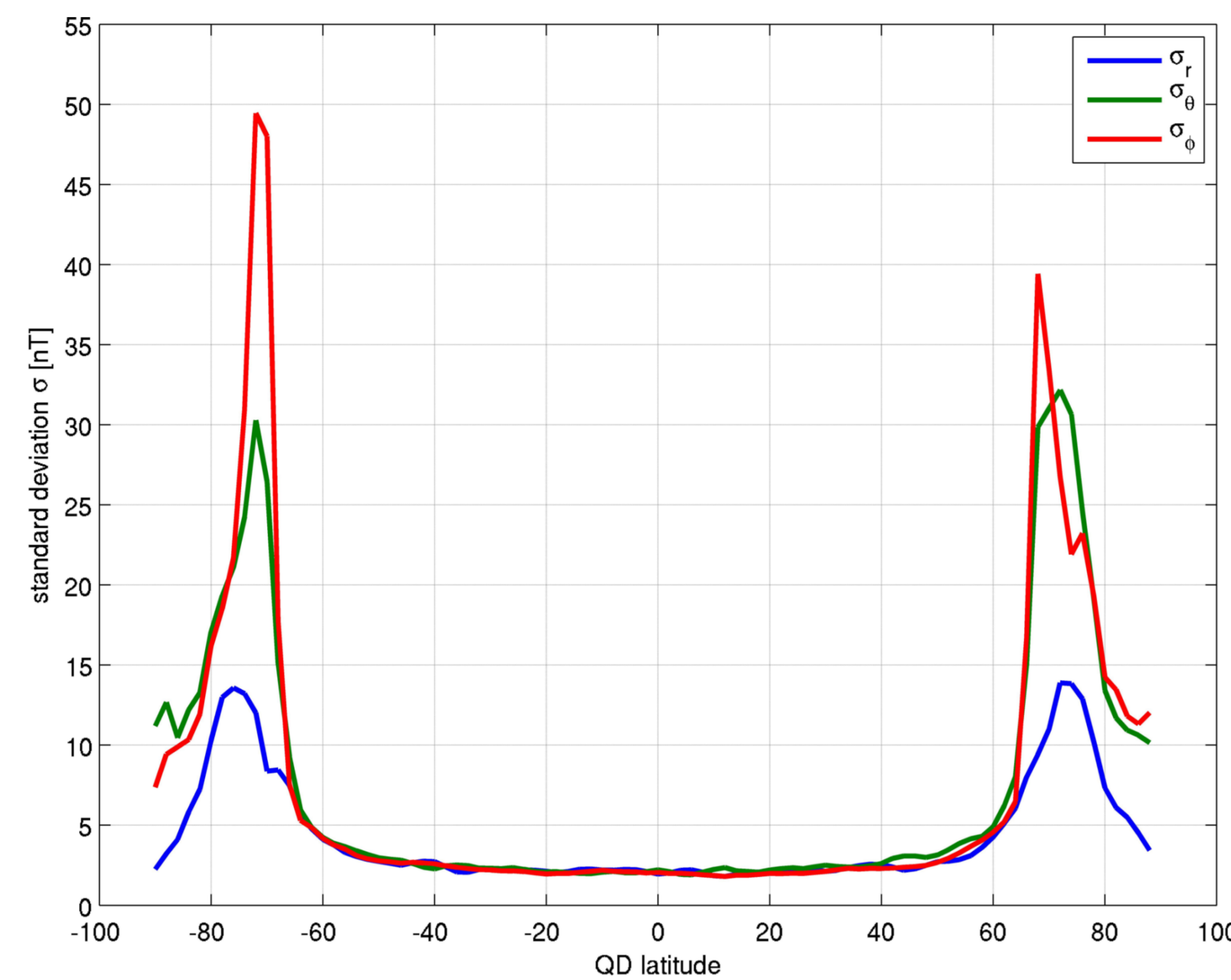


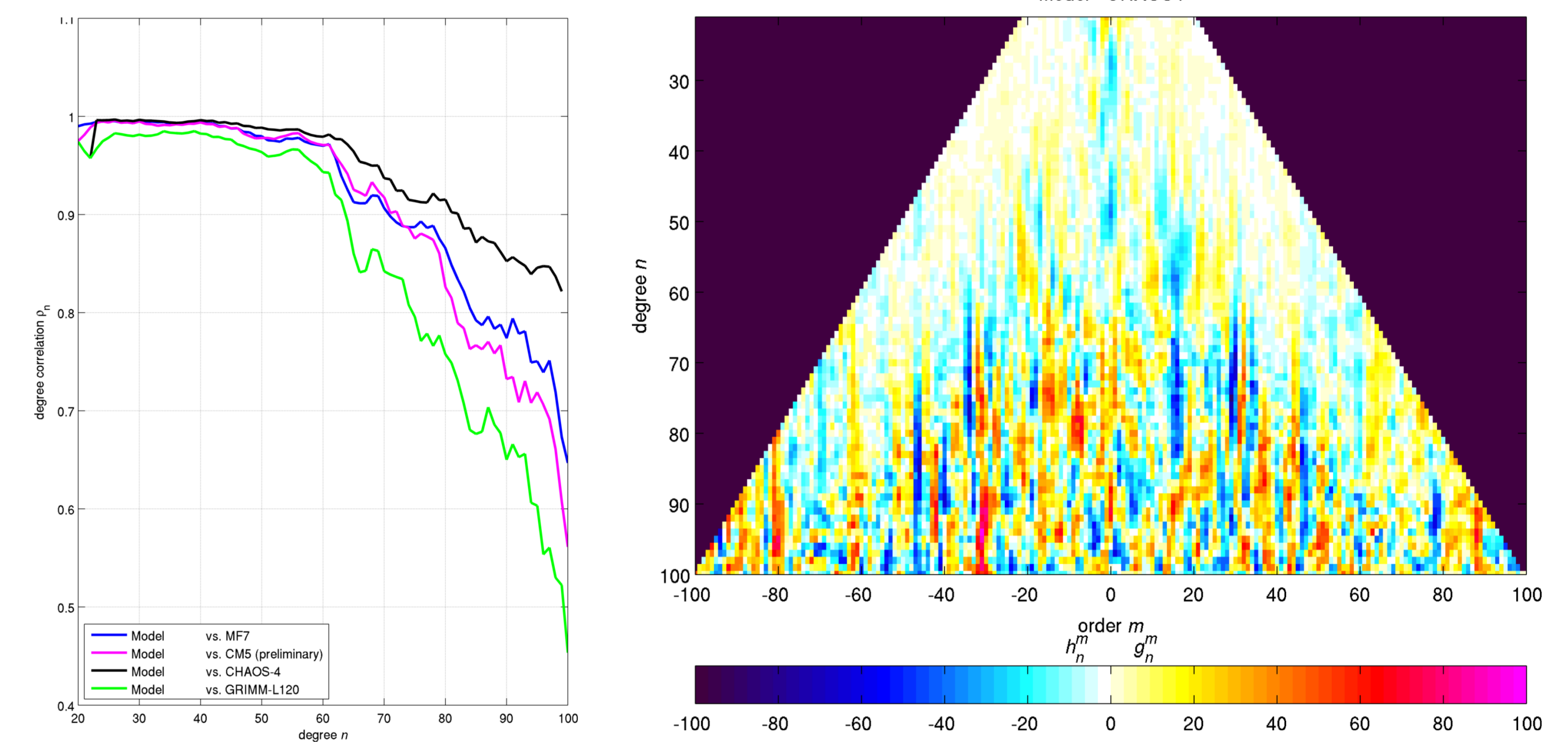
Fig.1: Latitude dependent standard deviations of the assigned errors due to unmodelled sources.

An initial unregularized ( $\lambda = 0$ ) model is derived using 10 iterations. The final model is obtained with 5 further iterations using quadratic regularization and  $\lambda = 3\text{E-}13$ .

## Results and Outlook

The presented model has a power spectrum that compares well to CHAOS-4, MF7 and CM5 (cf. Poster EGU2014-6883) models to degree  $n = 100$  (Fig.4). Ongoing investigations concern non-quadratic regularization using maximum entropy. Looking forward, we plan to explore local grid refinement options in order to incorporate aeromagnetic survey data.

Fig 3 left: Final model degree correlation with CHAOS-4, MF7, GRIMM-L120 and CM5. right: Sensitivity matrix between final model and CHAOS-4. The scale saturates at 100 nT.



## Equivalent Source Method

The equivalent potential field sources  $\mathbf{m}$  (monopoles) are arranged in an icosahedron grid (Fig.2), consisting of  $K = 30722$  vertices and midpoints, placed at a depth of 100km below the Earth's surface. The derived model can be transformed into a spherical harmonic representation by:

$$g_n^l = \sum_{k=1}^K \frac{r_k^n}{a^{n+2}} m_k P_n^l(\cos \theta_k) \cos(l\phi_k)$$

$$h_n^l = \sum_{k=1}^K \frac{r_k^n}{a^{n+2}} m_k P_n^l(\cos \theta_k) \sin(l\phi_k)$$

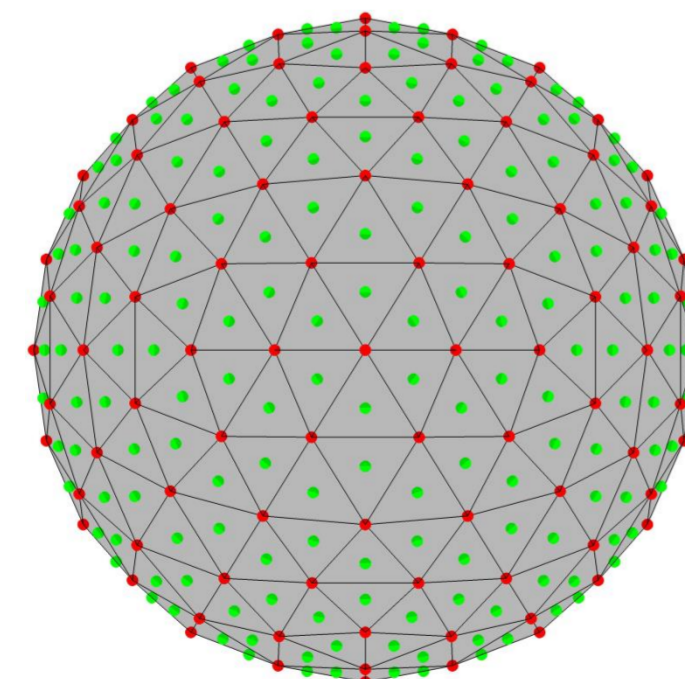


Fig.2: Right: Icosahedron grid with 483 of vertices (red) and midpoints (green). Left: The corresponding  $K$  equivalent potential field sources  $\mathbf{m}$  can be directly transformed into spherical harmonics of order  $l$  and degree  $n$ .  $a$  = Earth mean radius,  $(\theta, \phi)$  = source location.

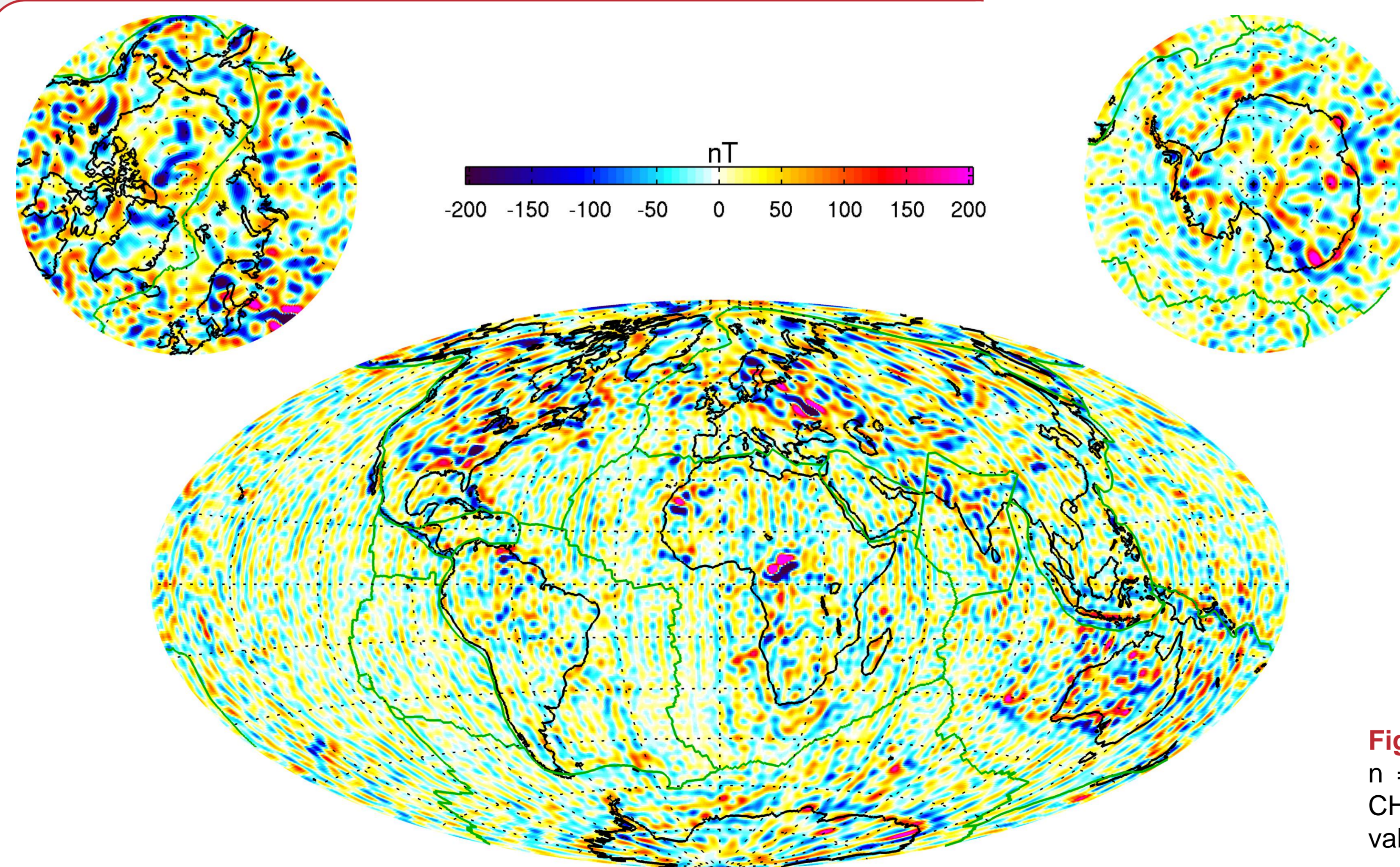


Fig 4 left: Modelled lithospheric radial magnetic field at the Earth's surface for degree  $n = 16-180$ . The scale saturates at 200 nT. right: Power spectrum for MF7, CM5 and CHAOS-4 models in comparison to model results with different regularization damping values. The chosen model is represented by the red line.