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# Characterization of Dielectric Electroactive Polymer transducers

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## ABSTRACT

This paper analysis the small-signal model of the Dielectric Electro Active Polymer (DEAP) transducer. The DEAP transducer have been proposed as an alternative to the electrodynamic transducer in sound reproduction systems. In order to understand how the DEAP transducer works, and provide guidelines for design optimization, accurate characterization of the transducer must be established. The small signal model of the DEAP transducer is derived and verified. Impedance measurements are shown for a push-pull DEAP based loudspeaker, and the dependency of the biasing voltage is explained. A measuring setup is proposed, which allows the impedance to be measured, while the DEAP transducer is connected to its biasing source.

**Keywords:** Dielectric electroactive polymers, lumped equivalent model, loudspeaker

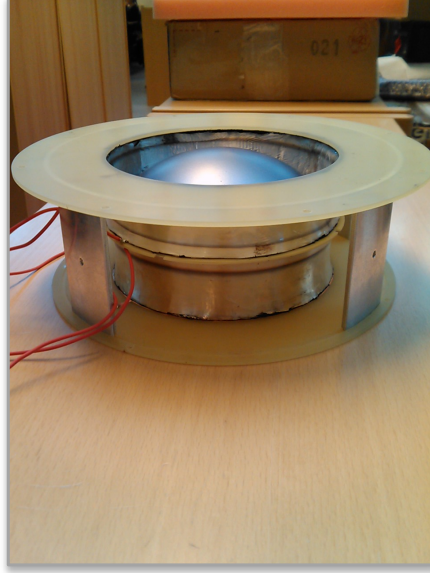
## 1. INTRODUCTION

Electrodynamic transducers have dominated the market of sound reproduction for a century. The electrostatic transducer is proposed as a very interesting alternative to these inefficient and bulky transducers. Electrostatic transducers are most known from their usage in electrostatic loudspeakers, however Dielectric Electro Active Polymers (DEAP) can also be used to form an electrostatic transducer [1, 2, 3, 4]. DEAP transducers are constructed by printing compliant electrodes on both sides of a silicone membrane. In order to design not only the loudspeaker itself, but also the amplifier driving the DEAP transducer, the small-signal model of the DEAP transducer must be known. Such a model will allow for the frequency response, efficiency and input impedance of the DEAP transducer to be analyzed.

The paper analyzes the small-signal model of the (DEAP) transducer through its input impedance. Electrodynamic transducers and their input impedance are well documented [6]. Also models of the electrostatic loudspeaker can be found in the literature [5], however with little experimental data backing the theory. The theory of the electrostatic loudspeaker shows, that the loudspeaker is not be considered as a pure capacitance at high bias voltage. As the high voltage is increased the mechanical resonance of the loudspeaker appears in the impedance. This information is key for the design of the amplifier driving the electrostatic loudspeaker. While the electrostatic loudspeaker and the DEAP transducer based loudspeaker both rely on the forces of electrostatic, and those have some similarities, the change in capacitance is not comparable. Accordingly a new model must be derived for the DEAP transducer based loudspeaker. The model is validated using measurements of the input impedance. A measuring-setup is proposed, allowing for the impedance of the DEAP transducer to be measured under the influence of a biasing voltage. Measurements are presented under different biasing conditions.

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**Figure 1:** Push-pull DEAP transducer based loudspeaker.

## 2. THEORY

The DEAP transducer is nonlinear, and the pressure increases with the square of the applied voltage [7, 8, 5]

$$\sigma(t) = \epsilon_0 \epsilon_r \left( \frac{v_c(t)}{h} \right)^2 \quad (1)$$

with  $\epsilon_0$  being the permittivity of vacuum,  $\epsilon_r$ , the relative permittivity of the Dielectric Electro (DE) material and  $h$  the undeformed distance between the electrodes.

The undeformed capacitance is

$$C_{E0V} = \frac{\epsilon_r \epsilon_0 V}{Y} \quad (2)$$

The static capacitance of the DEAP transducer changes with biasing voltage [9]

$$C_{EBiasing} = C_{Eo}(1 + s(t))^{k_A} \quad (3)$$

with  $s(t)$  being the strain

$$s(t) = \frac{l(t)}{d} \quad (4)$$

$d$ , the undeformed length,  $l(t)$ , the change of length, and  $k_A$ , the degree of anisotropy.

For a pre-strain below 10%, the change in compliance can be neglected [10]. Linearity is those preserved

$$C_{EBiasing} = C_{Eo}(1 + s_{Pre} + s)^{k_A} \quad (5)$$

with  $s_{Pre} = \frac{p_{Pre}}{100}$ , and  $p_{Pre}$  being the applied pre-strain in percent.

The mechanical compliance of a DEAP transducer

$$C_M = \frac{d}{YA_0} \quad (6)$$

with  $Y$  being the Young's modulus and  $A_0$  the cross sectional area of the undeformed transducer.

### 3. ANALYSIS

The charge and energy balance is used in Section 6 to derive the small signal model of the DEAP transducer. It is shown, that two linear equations in two unknown can be used to describe the coupling between the electrical and mechanical domain. These are

$$v(t) = \frac{i(t)}{sC_{EOV}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A} - \frac{V_{Bias}k_A u(t)}{sl_0(1+x)} \quad (7)$$

and

$$f(t) = \frac{k_A V_{Bias}}{sl_0(1+x)} i(t) - \frac{u(t)}{sC_{MEq}} \quad (8)$$

Using Equation (7) and Equation (8) allows for the establishment of a fully coupled model from the electrical to the mechanical domain of the DEAP transducer. The analogous circuit model is shown in figure 2. The model includes the electrical capacitance and mechanical compliance. These two components are coupled through current depended current sources. The depended current sources utilizes the coefficients of the undeformed length,  $d$ , the deformed length,  $l_0$ , the biasing voltage, the electrical capacitance and the mechanical compliance, respectively. For simplicity the combined mechanical and acoustical load are assumed to have the impedance  $Z_M$ .

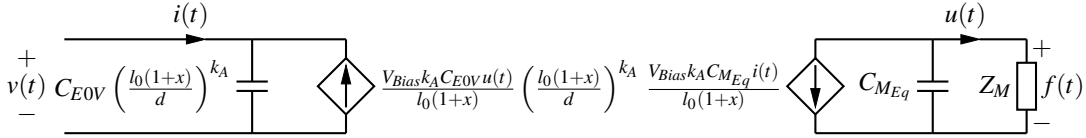


Figure 2: DEAP transducer small signal model.

The push-pull DEAP transducer based loudspeaker of figure 1 is a three terminal device. A membrane is placed in the middle of a tubular DEAP construction. The linear model of the push-pull DEAP transducer based loudspeaker are shown in figure 3.

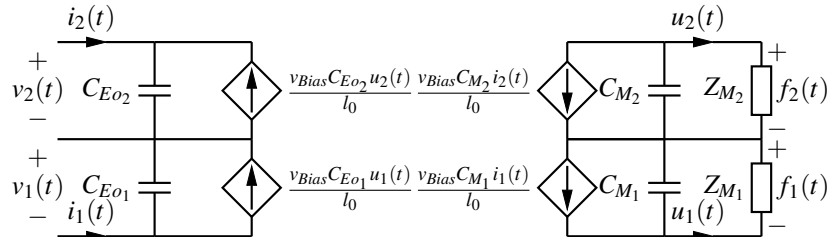


Figure 3: Push-pull DEAP transducer small signal model.

**Table 1: Parameters of the push-pull loudspeaker**

	Designator	Value
Biasing voltage	$v_{Bias}$	0–1.8 kV
Constant	$k_A$	1.5
Young's modulus	Y	2 MPa
Relative permittivity		3
Vacuum permittivity		$8.854 \frac{pF}{m}$
Average film thickness	$h_0$	$40 \mu m$
Length of un-rolled DEAP	$b$	3 m
Active height of DEAP cylinder	$d$	3 cm

**Table 2: Calculated parameters of the push-pull loudspeaker**

	Designator	Value
Compliance	$C_M$	$31 \frac{\mu m}{N}$
Unbiased capacitance	$C_{E0V}$	50 nF
Moving mass	$L_M$	30 g

The input impedance of the analogous circuit model in figure 2 is

$$Z_{In} = \frac{1}{sC_{E0}} \left( 1 - \frac{V_{Bias}^2 C_{E0}}{sI_0^2 (sC_M + Z_M)} \right) \quad (9)$$

Equation (9) consists of two terms. The first is the static capacitance at a certain bias level (mechanical and voltage). The second term is a function of the bias voltage. This term will cause the mechanical resonance to appear in the electrical impedance as the bias voltage is increased. A single mechanical resonance,  $Z_M = R_M + sL_M$ , at

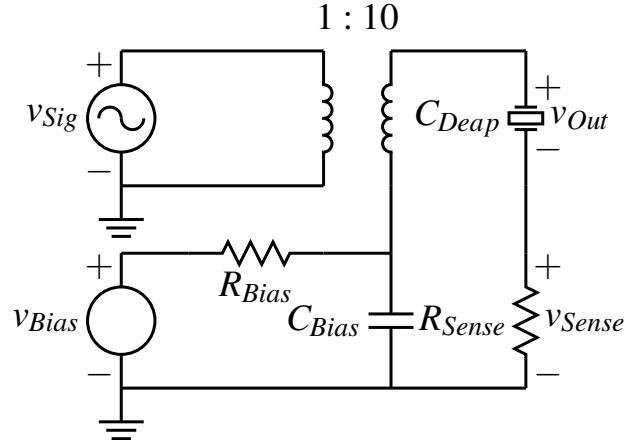
$$f_M = \frac{1}{2\pi\sqrt{C_M L_M}} \quad (10)$$

will be assumed throughout the paper.

#### 4. EXPERIMENTAL RESULTS

In order to accurately characterize the DEAP transducers, its input impedance must be measured with a biasing source connected to the transducer. The measuring setup used for this paper is shown in figure 4. A frequency response analyzer, AP300, of Ridley Engineering connected to an isolation transformer couples the test signal to the DEAP transducer. A Matsusada AU-5R60 high voltage supply ensures the bias voltage, with the  $R_{Bias}C_{Bias}$  circuit providing a low impedance return path for the test signal. For the measurements  $C_{Bias} = 10C_{E0}$  and  $R_{Sense} = 10\Omega$ . A 1.4 kV 1/200 differential probe was used to measure the voltage  $v_{Out} + v_{Sense}$ . An alternative measuring setup is proposed in [12] using a capacitive balanced bridge to measure the motional current of the DEAP transducer. This measuring setup is proposed by Peter Walker for characterization of the electrostatic loudspeaker. The setup relies on a capacitive balanced bridge, and is highly sensitive to component tolerances, making it unsuitable for most practical applications.

Measurements are presented on the push-pull DEAP transducers based loudspeaker of figure 1. The push-pull loudspeaker is characterized using a 5 % pre-strain. Figure 5(a) gives the measured impedance for a selection of bias voltages. Notice that the coupling between the electrical and mechanical domain is so pure, that no change in impedance can be observed in the log-log plot. In order to make the effects of the bias voltage visible, the normalized measured impedances are shown in figure 5(b). The plot is constructed by dividing the measured impedance at a certain biasing level with the measured unbiased impedance. Plotting the results on a linear y-axis shows that the biasing voltage indeed influences the impedance of the DEAP transducer. According to equation 3 the static capacitance should increase with biasing voltage. Another key observation is the mechanical resonances. The push-pull loudspeaker has its first mechanical resonance at 160

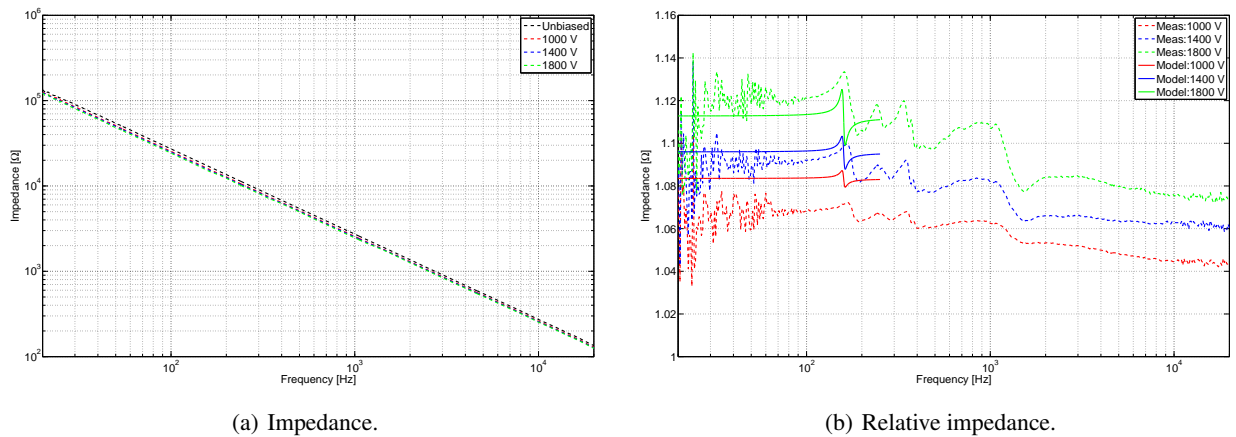


**Figure 4:** Setup for measuring impedance under different bias voltages.

Hz. As the biasing voltage is increase this resonance peak clearly becomes visible in the electrical domain.

The calculated relative impedance using Equation (9) are also plotted in figure 5(b) for the parameters of Table 1 and Table 2. This it seen, that the model accurate predict the behavior around the first resonance of the push-pull loudspeaker. Note that parameter tolerances easily can reach 10 %. Especially the estimated moving or active mass is a rough estimate, assuming that half of the DEAP mass is moving [13]. Finite element analysis can be used to determine the actual moving mass, but this is beyond the scope of this paper. For frequencies above the first mechanical resonance, the small signal model of figure 3 is not valid.

Figure 5(b) shows significant noise at frequencies below 100 Hz. This is due to the limited bandwidth used for the measurements (10 Hz), and poor signal to noise ratio at these very low currents. Better signal to noise ratio can be achieved by increasing  $R_{Sense}$ . This is however at the expense of the bandwidth of the measuring setup.



(a) Impedance.

(b) Relative impedance.

**Figure 5:** Impedance measurements of push-pull loudspeaker.

## 5. CONCLUSION

This paper has derived the small-signal model of the DEAP transducer. Impedance measurements are used to validate the model. A measuring setup is proposed allowing for the DEAP impedance to be recorded under different biasing voltages. The experimental work is performed on a push-pull DEAP transducer based loudspeaker. It shown, that the coupling

between the mechanical and electric domain is a function of the biasing voltage. The mechanical resonance of the DEAP transducers appears in the impedance of the DEAP transducer as the biasing voltage increases.

## 6. APPENDIX

This section derives the linearized small-signal model of the DEAP transducer. The charge and energy balance are utilized to derive a set of linear coupled equations (two equations in two unknown). Using Equation (5) and Equation (4), the capacitance can be related to a static displacement,  $l_0$ , together with a time dependent displacement in direction of the elongation,  $l(t)$

$$C_E = C_{E0V} \left( \frac{l_0(1+x) + l(t)}{d} \right)^{k_A} \quad (11)$$

### 6.1 Electrical charge

The total voltage are related to the total charge

$$V_{Bias} + v(t) = \frac{Q + q(t)}{C_E} \quad (12)$$

$$= \frac{Q + q(t)}{C_{E0V}} \left( \frac{l_0(1+x) + l(t)}{d} \right)^{-k_A} \quad (13)$$

with  $Q$  being the static charge at a bias voltage,  $V_{Bias}$ ,  $q(t)$  the time dependent charge, and  $v(t)$  the time dependent voltage.

First order Taylor expansion around  $l(t) = 0$  and neglecting cross terms yields

$$V_{Bias} + v(t) = \frac{Q}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A} - \frac{k_A Q}{d C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A - 1} l(t) + \frac{q(t)}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A} \quad (14)$$

Using  $V_{Bias} = \frac{Q}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A}$

$$v(t) = \frac{q(t)}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A} - \frac{V_{Bias} k_A l(t)}{l_0(1+x)} \quad (15)$$

Assuming linear and harmonic signals ( $q(t) = \frac{i(t)}{s}$  and  $l(t) = \frac{u(t)}{s}$ )

$$v(t) = \frac{i(t)}{s C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A} - \frac{V_{Bias} k_A u(t)}{s l_0(1+x)} \quad (16)$$

where  $i(t)$  is the time dependent current through the static capacitance, and  $u(t)$  is the velocity in the direction of elongation.

## 6.2 Energy

The total energy of the system is a sum of the electrical and mechanical energy

$$W = \frac{1}{2} \frac{(Q+q(t))^2}{C_E} + \frac{1}{2} \frac{(l_0(1+x)+l(t)-d)^2}{C_M} \quad (17)$$

$$\text{Using } V_{Bias} = \frac{Q}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A}$$

$$W = \frac{1}{2} \frac{(Q+q(t))^2}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A} + \frac{1}{2} \frac{(l_0(1+x)+l(t)-d)^2}{C_M} \quad (18)$$

The net force is found using the principal of virtual work,  $f = -\frac{dW}{dl}$

$$f = \frac{k_A(Q+q(t))^2}{2dC_{E0V}} \left( \frac{l_0(1+x)+l(t)}{d} \right)^{-k_A-1} - \frac{l_0(1+x)+l(t)-d}{C_M} \quad (19)$$

Taylor expansion around  $l(t) = 0$ , and neglecting cross terms yields

$$\begin{aligned} f &= \frac{k_A Q^2}{2dC_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A-1} - \frac{l_0(1+x)-d}{C_M} + \frac{k_A Q q(t)}{dC_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A-1} \\ &\quad - \frac{l(t)}{C_M} - \frac{k_A(k_A+1)Q^2}{2d^2C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A-2} l(t) \end{aligned} \quad (20)$$

The first and second term on the right hand side of the equation are the quiescent electrostatic force and the quiescent mechanical restoring force, which are equal but of opposite direction and thus cancels each other. Introducing  $V_{Bias} = \frac{Q}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A}$ ,  $q(t) = \frac{i(t)}{s}$  and  $l(t) = \frac{u(t)}{s}$  yields

$$f = \frac{k_A V_{Bias}}{s l_0(1+x)} i(t) - \frac{u(t)}{s} \left( \frac{1}{C_M} + \frac{k_A(k_A+1)V_{Bias}^2 C_{E0V}}{2l_0^2(1+x)^2} \left( \frac{l_0(1+x)}{d} \right)^{k_A} \right) \quad (21)$$

The equivalent mechanical compliance is defined as

$$C_{MEq} = \frac{1}{\frac{1}{C_M} + \frac{k_A(k_A+1)V_{Bias}^2 C_{E0V}}{2l_0^2(1+x)^2} \left( \frac{l_0(1+x)}{d} \right)^{k_A}} \quad (22)$$

Substitution Equation (22) into Equation (21) yields

$$f(t) = \frac{k_A V_{Bias}}{s l_0(1+x)} i(t) - \frac{u(t)}{s C_{MEq}} \quad (23)$$

In order to calculate  $l_0$  at a given  $V_{Bias}$ , the problem of static elongation must be solved. The quiescent electrostatic and mechanical forces in Equation (20) combine to describe the equilibrium

$$0 = \frac{k_A Q^2}{2dC_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A-1} - \frac{l_0(1+x)-d}{C_M} \quad (24)$$

$$\text{Using } V_{Bias} = \frac{Q}{C_{E0V}} \left( \frac{l_0(1+x)}{d} \right)^{-k_A}$$

$$0 = \frac{k_A V_{Bias}^2 C_{E0V}}{2l_0(1+x)} \left( \frac{l_0(1+x)}{d} \right)^{k_A} - \frac{l_0(1+x)-d}{C_M} \quad (25)$$



Linearization by Taylor expansion around  $l_0 = d$  yields

$$0 = l_0^2 \frac{2(1+x)^2}{k_A V_{Bias}^2 C_{E0V} C_M} + l_0 \left( \frac{-2d(1+x)}{k_A V_{Bias}^2 C_{E0V} C_M} - \frac{k_A(1+x)^{k_A}}{d} \right) + k_A(1+x)^{k_A} - (1+x)^{k_A} \quad (26)$$

Equation (26) is a second order equation, and can be solved for  $l_0$ . Note, that  $l_0 \leq d$ .

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