One Approach to Adaptive Control of a Tubular Chemical Reactor

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Abstract: – The paper deals with continuous-time adaptive control of a tubular chemical reactor with the countercurrent cooling as a nonlinear single input – single output process. The mean reactant temperature and the output reactant temperature are chosen as the controlled outputs, and, the coolant flow rate as the control input. The nonlinear model of the reactor is approximated by an external linear model with a structure chosen on the basis of controlled outputs step responses. Its parameters are estimated via corresponding delta model. The control system structure with two feedback controllers is considered. The resulting controllers are derived using polynomial approach. The method is tested on a mathematical model of the tubular chemical reactor.

Key-Words: – Nonlinear system, tubular chemical reactor, approximate linear model, parameter identification, polynomial approach, pole assignment.

1 Introduction

Tubular chemical reactor are units frequently used in chemical industry. From the system theory point of view, tubular chemical reactors belong to a class of nonlinear distributed parameter systems with mathematical models described by sets of nonlinear partial differential equations (NPDRs). The methods of modelling and simulation of such processes are described e.g. in [1] – [5].

It is well known that the control of chemical reactors, and, tubular reactors especially, often represents very complex problem. The control problems are due to the process nonlinearity, its distributed nature, and high sensitivity of the state and output variables to input changes. Evidently, the process with such properties is hardly controllable by conventional control methods, and, its effective control requires application some of advanced methods. Here, various efficient methods may be used as the predictive control, e.g. [6], [7], [8], the robust control, e.g. [9], the fuzzy nonlinear control, e.g. [10], the model reference control, e.g. [11], or nonlinear control, e.g. [12], [13] and [14]. Some others methods are described in [15].

One possible method to cope with this problem is using adaptive strategies based on an appropriate choice of a continuous-time external linear model (CT ELM) with recursively estimated parameters. These parameters are consequently used for parallel updating of controller's parameters. Some results obtained in this field were presented by authors of this paper e.g. in [16] and [17].

For the CT ELM parameter estimation, either the direct method [18] and [19] or application of an external delta model with the same structure as the CT model can be used. The basics of delta models have been described in e.g. [20] and [21]. Although delta models belong into discrete models, they do not have such disadvantageous properties connected with shortening of a sampling period as discrete *z*-models. In addition, parameters of delta models can directly be estimated from sampled signals. Moreover, it can be easily proved that these parameters converge to parameters of CT models for a sufficiently small sampling period (compared to the dynamics of the controlled process), as shown in [22].

This paper deals with continuous-time adaptive control of a tubular chemical reactor with a countercurrent cooling as a nonlinear single input – single output process. With respect to practical possibilities of a measurement and control, the mean reactant temperature and the output reactant temperature are chosen as the controlled outputs, and, the coolant flow rate as the control input. The nonlinear model of the reactor is approximated by a CT external linear model with a structure chosen on the basis of computed controlled outputs step responses. The parameters of the CT ELM then are estimated via corresponding delta model. The control structure with two feedback controllers is considered, e.g. [23]. The resulting controllers are derived using the polynomial approach [24] and the pole assignment method (see, e.g. [25]). The method is tested on a mathematical model of a tubular chemical reactor.

2 Model of the Reactor

An ideal plug-flow tubular chemical reactor with a simple exothermic consecutive reaction $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ in the liquid phase and with the countercurrent cooling is considered. Heat losses and heat conduction along the metal walls of tubes are assumed to be negligible, but dynamics of the metal walls of tubes are significant. All densities, heat capacities, and heat transfer coefficients are assumed to be constant. Under above assumptions, the reactor model can be described by five PDRs in the form

$$\frac{\partial c_A}{\partial t} + v_r \frac{\partial c_A}{\partial z} = -k_1 c_A \quad (1)$$

$$\frac{\partial c_B}{\partial t} + v_r \frac{\partial c_B}{\partial z} = k_1 c_A - k_2 c_B \quad (2)$$

$$\frac{\partial T_r}{\partial t} + v_r \frac{\partial T_r}{\partial z} = \frac{Q_r}{(\rho c_p)_r} - \frac{4U_1}{d_1(\rho c_p)_r}(T_r - T_w) \quad (3)$$

$$\frac{\partial T_w}{\partial t} = \frac{4}{(d_2^2 - d_1^2)(\rho c_p)_w}\left[d_1 U_1 (T_r - T_w) + d_2 U_2 (T_c - T_w)\right] \quad (4)$$

$$\frac{\partial T_c}{\partial t} - v_c \frac{\partial T_c}{\partial z} = \frac{4 n_1 d_2 U_2}{(d_3^2 - n_1 d_2^2)(\rho c_p)_c}(T_w - T_c) \quad (5)$$

with initial conditions

$c_A(z,0) = c_A^s(z)$, $c_B(z,0) = c_B^s(z)$, $T_r(z,0) = T_r^s(z)$, $T_w(z,0) = T_w^s(z)$, $T_c(z,0) = T_c^s(z)$

and boundary conditions

$c_A(0,t) = c_{A0}(t)$ (kmol/m^3),

$c_B(0,t) = c_{B0}(t)$ (kmol/m^3), $T_r(0,t) = T_{r0}(t)$ (K),

$T_c(L,t) = T_{cL}(t)$ (K).

Here, t is the time, z is the axial space variable, c are concentrations, T are temperatures, v are fluid velocities, d are diameters, ρ are densities, c_p are specific heat capacities, U are heat transfer coefficients, n_1 is the number of tubes and L is the length of tubes. The subscript $(\cdot)_r$ stands for the reactant mixture, $(\cdot)_w$ for the metal walls of tubes, $(\cdot)_c$ for the coolant, and the superscript $(\cdot)^s$ for steady-state values.

The reaction rates and heat of reactions are nonlinear functions expressed as

$$k_j = k_{j0} \exp\left(\frac{-E_j}{RT_r}\right), \quad j = 1, 2 \quad (6)$$

$$Q_r = (-\Delta H_{r1}) k_1 c_A + (-\Delta H_{r2}) k_2 c_B \quad (7)$$

where k_0 are pre-exponential factors, E are activation energies, $(-\Delta H_r)$ are in the negative considered reaction entalpies, and R is the gas constant.

The fluid velocities are calculated via the reactant and coolant flow rates as

$$v_r = \frac{4 q_r}{\pi n_1 d_1^2}, \quad v_c = \frac{4 q_c}{\pi (d_3^2 - n_1 d_2^2)} \quad (8)$$

The parameter values with correspondent units used for simulations are given in Table 1.

Table 1. Used parameter values

L = 8 m	n_1 = 1200
d_1 = 0.02 m	d_2 = 0.024 m
d_3 = 1 m	
ρ_r = 985 kg/m^3	c_{pr} = 4.05 kJ/kg K
ρ_w = 7800 kg/m^3	c_{pw} = 0.71 kJ/kg K
ρ_c = 998 kg/m^3	c_{pc} = 4.18 kJ/kg K
U_1 = 2.8 kJ/m^{2}s K	U_2 = 2.56 kJ/m^{2}s K
k_{10} = 5.61·10^{16} 1/s	k_{20} = 1.128·10^{18} 1/s
E_1/R = 13477 K	E_2/R = 15290 K
$(-\Delta H_{r1})$ = 5.8·10^4 kJ/kmol	$(-\Delta H_{r2})$ = 1.8·10^4 kJ/kmol

From the system engineering point of view, $c_A(L,t) = c_{A\text{out}}$, $c_B(L,t) = c_{B\text{out}}$, $T_r(L,t) = T_{r\text{out}}$ and $T_c(0,t) = T_{c\text{out}}$ are the output variables, and, $q_r(t)$, $q_c(t)$, $c_{A0}(t)$, $T_{r0}(t)$ and $T_{cL}(t)$ are the input variables. Among them, for the control purposes, mostly the coolant flow rate can be taken into account as the control variable, whereas other inputs entering into the process can be accepted as disturbances. In this paper, the mean reactant temperature given by

$$T_m(t) = \frac{1}{L}\int_0^L T_r(z,t)\,dz \quad (9)$$

and the reactant output temperature $T_{r\text{out}}(t)$ are considered as the controlled outputs.

3 Computation Models

For computation of both steady-state and dynamic characteristics, the finite diferences method is employed. The procedure is based on substitution of the space interval $z \in \langle 0, L \rangle$ by a set of discrete node points $\{z_i\}$ for $i = 1, \dots, n$, and, subsequently, by approximation of derivatives with respect to the space variable in each node point by finite differences. Two types of finite differences are applied, either the backward finite difference

$$\left.\frac{\partial y(z,t)}{\partial z}\right|_{z=z_i} \approx \frac{y(z_i,t)-y(z_{i-1},t)}{h} = \frac{y(i,t)-y(i-1,t)}{h} \quad (10)$$

or the forward finite difference

$$\left.\frac{\partial y(z,t)}{\partial z}\right|_{z=z_i} \approx \frac{y(z_{i+1},t)-y(z_i,t)}{h} = \frac{y(i+1,t)-y(i,t)}{h} \quad . \quad (11)$$

Here, a function $y(z,t)$ is continuously differentiable in the interval $<0,L>$, and, $h = L/n$ is the diskretization step.

3.1 Dynamic model

Applying the substitutions (10), (11) in (1) – (5) and, omitting the argument t in parenthesis, PDRs (1) – (5) are approximated by a set of ODRs in the form

$$\frac{dc_A(i)}{dt} = -\left[b_0 + k_1(i)\right]c_A(i) + b_0\, c_A(i-1) \quad (12)$$

$$\frac{dc_B(i)}{dt} = k_1(i)c_A(i) - \left[b_0 + k_2(i)\right]c_B(i) + b_0\, c_B(i-1) \quad (13)$$

$$\frac{dT_r(i)}{dt} = b_1 Q_r(i) - (b_0 + b_2)T_r(i) + b_0\, T_r(i-1) + b_2\, T_w(i) \quad (14)$$

$$\frac{dT_w(i)}{dt} = b_3\left[T_r(i) - T_w(i)\right] + b_4\left[T_c(i) - T_w(i)\right] \quad (15)$$

$$\frac{dT_c(m)}{dt} = -(b_5 + b_6)T_c(m) + b_5\, T_c(m+1) + b_6\, T_w(m) \quad (16)$$

for $i = 1, \dots, n$ and $m = n - i + 1$, and, with initial conditions

$c_A(i,0) = c_A^s(i)$, $c_B(i,0) = c_B^s(i)$, $T_r(i,0) = T_r^s(i)$, $T_w(i,0) = T_w^s(i)$ and $T_c(i,0) = T_c^s(i)$ for $i = 1, \dots, n$.

The boundary conditions enter into Eqs. (12) – (14) and (16) for $i = 1$.

Now, nonlinear functions in Eqs. (12) – (16) take the discrete form

$$k_j(i) = k_{j0}\exp\left(\frac{-E_j}{RT_r(i)}\right),\ j = 1, 2 \quad (17)$$

$$Q_r(i) = (-\Delta H_{r1})\, k_1(i)\, c_A(i) + (-\Delta H_{r2})\, k_2(i)\, c_B(i) \quad (18)$$

for $i = 1, \dots, n$.

The parameters b in Eqs. (12) – (16) are calculated from formulas

$$b_0 = \frac{v_r}{h},\ b_1 = \frac{1}{(\rho c_p)_r},\ b_2 = \frac{4U_1}{d_1(\rho c_p)_r},$$
$$b_3 = \frac{4d_1U_1}{(d_2^2 - d_1^2)(\rho c_p)_w},\ b_4 = \frac{4d_2U_2}{(d_2^2 - d_1^2)(\rho c_p)_w} \quad (19)$$
$$b_5 = \frac{v_c}{h},\ b_6 = \frac{4n_1d_2U_2}{(d_3^2 - n_1d_2^2)(\rho c_p)_c}.$$

Here, the formulas for computation of T_m and $T_{r\,out}$ take the discrete form

$$T_m(t) = \frac{1}{n}\sum_{i=1}^{n} T_r(z_i,t),\ T_{r\,\mathrm{out}}(t) = T_r(z_n,t) \quad (20)$$

3.2 Steady-state model

Computation of the steady-state characteristics is necessary not only for a steady-state analysis but the steady state values $y^s(i)$ also constitute initial conditions in ODRs (12) – (16) (here, y presents some of the variable in the set (12) – (16)).

The steady-state model can simply be derived equating the time derivatives in (12) – (16) to zero. Then, after some algebraic manipulations, the steady-state model takes the form of difference equations

$$c_A^s(i) = \frac{b_0}{b_0 + k_1^s(i)}c_A^s(i-1) \quad (21)$$

$$c_B^s(i) = \frac{1}{b_0 + k_2^s(i)}\left[k_1^s(i)c_A^s(i) + b_0\, c_B^s(i-1)\right] \quad (22)$$

$$T_r^s(i) = \frac{1}{b_0 + b_2}\left[b_1 Q_r^s(i) + b_0 T_r^s(i-1) + b_2 T_w^s(i)\right] \quad (23)$$

$$T_w^s(i) = \frac{1}{b_3 + b_4}\left[b_3 T_r^s(i) + b_4 T_c^s(i)\right] \quad (24)$$

$$T_c^s(m) = \frac{1}{b_5 + b_6}\left[b_5 T_c^s(m+1) + b_6 T_w^s(m)\right] \quad (25)$$

for $i = 1, \dots, n$ and $m = n - i + 1$. Nonlinear functions accordant with a steady-state are

$$k_j^s(i) = k_{j0} \exp\left(\frac{-E_j}{R T_r^s(i)}\right), \; j = 1, 2 \quad (26)$$

$$Q_r^s(i) = (-\Delta H_{r1})\, k_1^s(i)\, c_A^s(i) + + (-\Delta H_{r2})\, k_2^s(i)\, c_B^s(i) \quad (27)$$

Now, the formulas for computation T_m and $T_{r\,out}$ have the form

$$T_m^s = \frac{1}{n}\sum_{i=1}^{n} T_r^s(z_i)\,, \quad T_{r\,\mathrm{out}}^s = T_r^s(z_n) \quad (28)$$

3.3 Steady-state and dynamic characteristics

Typical reactant temperature profiles along the reactor tubes computed for $c_{A0}^s = 2.85$, $c_{B0}^s = 0$, $T_{r0}^s = 323$, $T_{c0}^s = 293$ and $q_r^s = 0.15$ for various coolant flow rates are shown in Fig. 1. A presence of a maximum on the reactant temperature profiles is a common property of many tubular reactors with exothermic reactions.

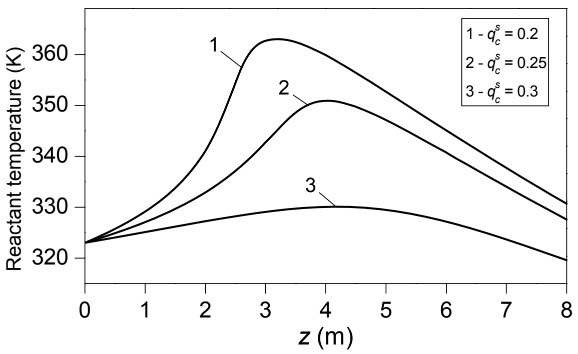

Fig. 1 Reactant temperature profiles for various coolant flow rates.

A dependences of the reactant mean temperature and the reactant output temperature on the coolant flow rate is shown in Fig. 2. The form of both curves documents a nonlinear relation between supposed controlled outputs and the coolant flow rate which is considered as the control input.

Dynamic charakteristics were computed in the neighbourhood of the chosen operating point $q_c^s = 0.27\,\mathrm{m}^3/\mathrm{s}$, $T_m^s = 334.44\,\mathrm{K}$, $T_{r\,\mathrm{out}}^s = 326.10\,\mathrm{K}$

For the dynamic analysis and subsequent control purposes, the controlled outputs are defined as

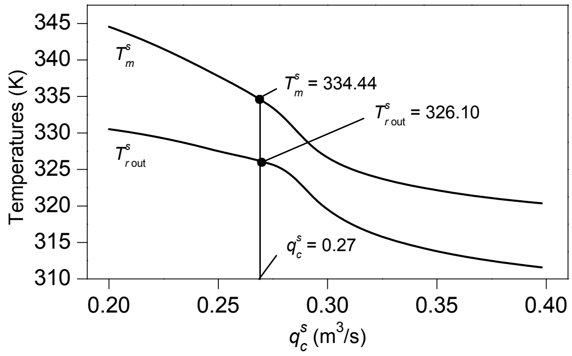

Fig. 2 Dependence of the reactant mean temperature on the coolant flow rates.

deviations from steady values

$$\begin{aligned} y_1(t) &= \Delta T_m(t) = T_m(t) - T_m^s \\ y_2(t) &= \Delta T_{r\,\mathrm{out}}(t) = T_{r\,\mathrm{out}}(t) - T_{r\,\mathrm{out}}^s \end{aligned} \quad . \quad (29)$$

Such form is frequently used in the control. The deviation of the coolant flow rate is denoted as

$$\Delta q_c = q_c(t) - q_c^s\,. \quad (30)$$

The responses of both outputs to the coolant flow rate step changes are shown in Figs. 3, 4.

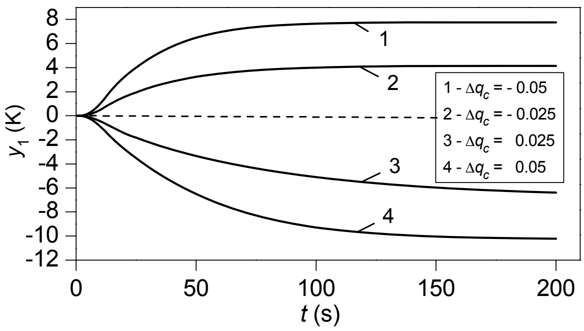

Fig. 3 Reactant mean temperature step responses.

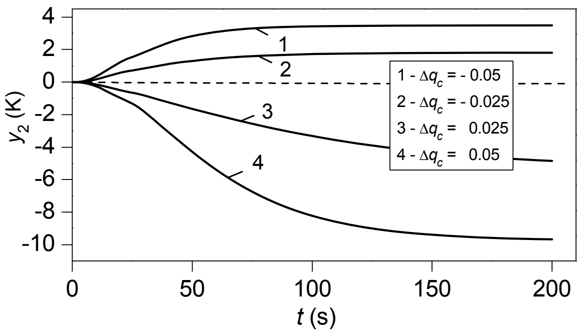

Fig. 4 Reactant output temperature step responses.

The above shown responses demonstrate more expressive nonlinear behaviour of the reactant output temperature to input changes than the reactant mean temperature. This fact is evident also from the gain values computed as

$$g_s = \lim_{t\to\infty} \frac{y(t)}{\Delta q_c} \qquad (31)$$

and presented in Tab. 2.

Tab. 2 Gains for various input hanges.

Δq_c	- 0.025	- 0.05	0.025	0.05
Main reactant temperature				
g_s	-155.4	-166.2	-263.5	-205.1
Output reactant temperature				
g_s	-69.6	-72.0	-194.3	-193.5

This fact predicates better properties of the reactant mean temperature as the controlled output than the reactant output temperature. Moreover, the dynamics of the reactant output temperature is slower in comparison with the dynamics of the reactant mean temperature.

4 CT and Delta ELM

For the control purposes, the control input variable are considered in the form

$$u(t) = 10\ \frac{q_c(t) - q_c^s}{q_c^s} \qquad (32)$$

This expression enables to obtain control input and controlled output variables of approximately the same magnitude.

A choice of the CT ELM structure does not stem from known structure of the model (1) – (5) but from a character of simulated step responses. It is well known that in adaptive control a controlled process of a higher order can be approximated by a linear model of a lower order with variable parameters. Taking into account profiles of curves in Figs. 3 and 4 with zero derivatives in $t = 0$, the second order CT ELM has been chosen for both controlled outputs in the form of the second order linear differential equation

$$\ddot{y}(t) + a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t) \qquad (33)$$

where $y = y_1$ or $y = y_2$, and, in the complex domain, as the transfer function

$$G(s) = \frac{b_0}{s^2 + a_1 s + a_0}. \qquad (34)$$

Establishing the δ operator

$$\delta = \frac{q-1}{T_0} \qquad (35)$$

where q is the forward shift operator and T_0 is the sampling period, the delta ELM corresponding to (33) takes the form

$$\delta^2 y(t') + a_1' \delta y(t') + a_0' y(t') = b_0' u(t') \qquad (36)$$

where t' is the discrete time.

When the sampling period is shortened, the delta operator approaches the derivative operator, and, the estimated parameters a', b' reach the parameters a, b of the CT model (33).

5 Delta Model Parameter Estimation

Substituting $t' = k - 2$, equation (36) can be rewriten to the form

$$\delta^2 y(k-2) + a_1' \delta y(k-2) + a_0' y(k-2) = \\ = b_0' u(k-2) \qquad (37)$$

In the paper, the recursive identification method with exponential and directional forgetting was used.

Establishing the regression vector

$$\mathbf{\Phi}_\delta^T(k-1) = \left(-\delta y(k-2) \quad -y(k-2) \quad u(k-2)\right) \qquad (38)$$

where

$$\delta y(k-2) = \frac{y(k-1) - y(k-2)}{T_0} \qquad (39)$$

the vector of delta model parameters

$$\mathbf{\Theta}_\delta^T(k) = \left(a_1'\ a_0'\ b_0'\right) \qquad (40)$$

is recursively estimated from the ARX model

$$\delta^2 y(k-2) = \mathbf{\Theta}_\delta^T(k)\mathbf{\Phi}_\delta(k-1) + \varepsilon(k) \qquad (41)$$

where

$$\delta^2 y(k-2) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_0^2} \qquad (42)$$

6 Controller Design

The control system with two feedback controllers is depicted in Fig. 5.

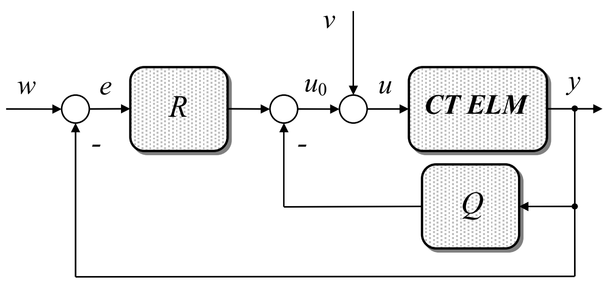

Fig. 5. Control system with two feedback controllers

In the scheme, w is the reference signal, v denotes the load disturbance, e the tracking error, u_0 output of controllers, u the control input and y the

controlled output. The transfer function $G(s)$ of the CT ELM is given by (34).
The reference w and the disturbance v are considered as step functions with transforms

$$W(s) = \frac{w_0}{s}, \quad V(s) = \frac{v_0}{s} \tag{43}$$

The transfer functions of both controllers are in forms

$$R(s) = \frac{r(s)}{\tilde{p}(s)}, \quad Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)} \tag{44}$$

where $\tilde{q}$, r and $\tilde{p}$ are coprime polynomials in s fulfilling the condition of properness $\deg r \le \deg \tilde{p}$ and $\deg q \le \deg \tilde{p}$.

The controller design described in this section appears from the polynomial approach. The general requirements on the control system are formulated as its internal properness and strong stability (in addition to the control system stability, also the controller stability is required), asymptotic tracking of the reference and load disturbance attenuation. The procedure to derive admissible controllers can be performed as follows:
Transforms of the basic signals in the closed-loop system take following forms (for simplification, the argument s is in some equations omitted)

$$Y(s) = \frac{b}{d}\left[r W(s) + \tilde{p} V(s) \right] \tag{45}$$

$$E(s) = \frac{1}{d}\left[(a\tilde{p} + b\tilde{q}) W(s) - b\tilde{p} V(s) \right] \tag{46}$$

$$U(s) = \frac{a}{d}\left[r W(s) + \tilde{p} V(s) \right] \tag{47}$$

Here,

$$d(s) = a(s)\tilde{p}(s) + b(s)\left[r(s) + \tilde{q}(s) \right] \tag{48}$$

is the characteristic polynomial with roots as poles of the closed-loop.
Establishing the polynomial t as

$$t(s) = r(s) + \tilde{q}(s) \tag{49}$$

and substituting (49) into (48), the condition of the control system stability is ensured when polynomials $\tilde{p}$ and t are given by a solution of the polynomial Diophantine equation

$$a(s)\tilde{p}(s) + b(s)t(s) = d(s) \tag{50}$$

with a stable polynomial d on the right side.
With regard to the transforms (43), the asymptotic tracking and load disturbance attenuation are provided by divisibility of both terms $a\tilde{p} + b\tilde{q}$ and $\tilde{p}$ in (46) by s. This condition is fulfilled when polynomials $\tilde{p}$ and $\tilde{q}$ have forms

$$\tilde{p}(s) = s\,p(s), \quad \tilde{q}(s) = s\,q(s). \tag{51}$$

Subsequently, the transfer functions (44) take forms

$$Q(s) = \frac{q(s)}{p(s)}, \quad R(s) = \frac{r(s)}{s\,p(s)} \tag{52}$$

and, a stable polynomial $p(s)$ in their denominators ensures the stability of controllers.
The control system satisfies the condition of internal properness when the transfer functions of all its components are proper. Consequently, the degrees of polynomials q and r must fulfil these inequalities

$$\deg q \le \deg p, \quad \deg r \le \deg p + 1. \tag{53}$$

Now, the polynomial t can be rewritten to the form

$$t(s) = r(s) + s\,q(s). \tag{54}$$

Taking into account the solvability of (50) and conditions (53), the degrees of polynomials in (50) and (52) can be easily derived as

$$\deg t = \deg r = \deg a, \quad \deg q = \deg a - 1,$$
$$\deg p \ge \deg a - 1, \quad \deg d \ge 2\deg a. \tag{55}$$

Denoting deg $a = n$, polynomials t, r and q have forms

$$t(s) = \sum_{i=0}^{n} t_i s^i, \quad r(s) = \sum_{i=0}^{n} r_i s^i, \quad q(s) = \sum_{i=1}^{n} q_i s^{i-1} \tag{56}$$

and, relations among their coefficients are

$$r_0 = t_0, \quad r_i + q_i = t_i \text{ for } i = 1, \ldots, n. \tag{57}$$

Since by a solution of the polynomial equation (50) provides calculation of coefficients t_i, unknown coefficients r_i and q_i can be obtained by a choice of selectable coefficients $\beta_i \in \langle 0,1 \rangle$ such that

$$r_i = \beta_i t_i, \quad q_i = (1 - \beta_i) t_i \text{ for } i = 1, \ldots, n. \tag{58}$$

The coefficients β_i distribute a weight between numerators of transfer functions Q and R.

Remark: If $\beta_i = 1$ for all i, the control system in Fig. 5 reduces to the 1DOF control configuration (Q = 0). If $\beta_i = 0$ for all i, and, both reference and load disturbance are step functions, the control system corresponds to the 2DOF control configuration.

For the second order model (34) with $\deg a = 2$, the controller's transfer functions take specific forms

$$Q(s) = \frac{q(s)}{p(s)} = \frac{q_2 s + q_1}{s + p_0}$$
$$R(s) = \frac{r(s)}{s\,p(s)} = \frac{r_2 s^2 + r_1 s + r_0}{s\,(s + p_0)}. \tag{59}$$

where

$$r_0 = t_0, \; r_1 = \beta_1 t_1, \; r_2 = \beta_2 t_2,$$

$$q_1 = (1-\beta_1)t_1\,,\; q_2 = (1-\beta_2)t_2\,. \qquad (60)$$

The controller parameters then result from a solution of the polynomial equation (50) and depend upon coefficients of the polynomial *d*. The next problem here is to find a stable polynomial *d* that enables to obtain acceptable stabilizing controllers.

In this paper, the polynomial *d* with roots determining the closed-loop poles is chosen as

$$d(s) = n(s)(s+\alpha)^2 \qquad (61)$$

where *n* is a stable polynomial obtained by spectral factorization

$$a^*(s)a(s) = n^*(s)n(s) \qquad (62)$$

and α is the selectable parameter.

Note that a choice of *d* in the form (61) provides the control of a good quality for aperiodic controlled processes.

The coefficients of *n* then are expressed as

$$n_0 = \sqrt{a_0^2}\,,\;\; n_1 = \sqrt{a_1^2 + 2n_0 - 2a_0} \qquad (63)$$

and, the controller parameters p_0 and t can be obtained from solution of the matrix equation

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ a_1 & b_0 & 0 & 0 \\ a_0 & 0 & b_0 & 0 \\ 0 & 0 & 0 & b_0 \end{pmatrix} \times \begin{pmatrix} p_0 \\ t_2 \\ t_1 \\ t_0 \end{pmatrix} = \begin{pmatrix} d_3 - a_1 \\ d_2 - a_0 \\ d_1 \\ d_0 \end{pmatrix} \qquad (64)$$

where

$$\begin{aligned} & d_3 = n_1 + 2\alpha,\, d_2 = 2\alpha n_1 + n_0 + \alpha^2 \\ & d_1 = 2\alpha n_0 + \alpha^2 n_1,\, d_0 = \alpha^2 n_0 \end{aligned} \qquad (65)$$

Now, it follows from the above introduced procedure that tuning of controllers can be performed by a suitable choice of selectable parameters β and α.

The controller parameters *r* and *q* can then be obtained from (60).

The adaptive control system is shown in Fig. 6.

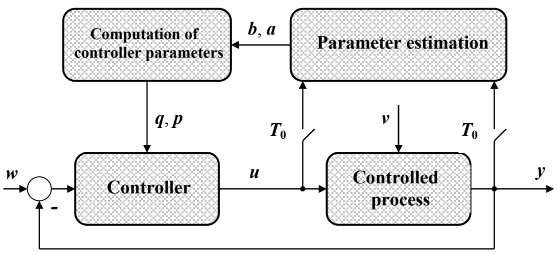

Fig. 6 Adaptive control scheme.

7 Control Simulation

Also the control simulations were performed in a neighbourhood of the operating point $q_c^s = 0.27\ \mathrm{m^3/s}$, $T_m^s = 334.44\,\mathrm{K}$, $T_{r\,\mathrm{out}}^s = 326.10\,\mathrm{K}$.

For the start (the adaptation phase), the P controller with a small gain was used in all simulations.

With respect to more expressive nonlinearity and slower dynamics of the reactant output temperature in comparison with the reactant mean temperature, the changes of references as well as the control running time intervals were chosen different for both outputs.

The effect of the pole α on the controlled responses is transparent from Figs. 7 and 8. For both outputs, two values of α were selected. The control simulations show sensitivity of controlled outputs to α. The higher values of this parameter speed the control, however, they provide greater overshoots (undershoots). Other here not mentioted simulations showed that a careless selection of the parameter α can lead to controlled output responses of a poor quality, to oscillations or even to the control instability.

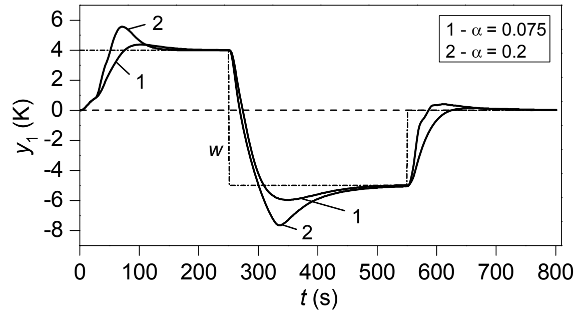

Fig. 7 Controlled output y_1 responses: effect of α ($\beta_1 = 1$, $\beta_2 = 0.5$).

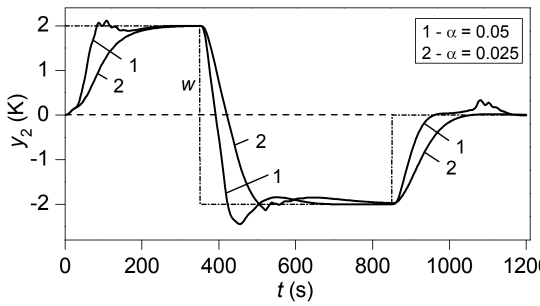

Fig. 8 Controlled output y_2 responses: effect of α ($\beta_1 = 1$, $\beta_2 = 0$).

Moreover, an increasing α leads to higher values and changes of the control input as shown in Fig. 9 and 10. This fact can be important in control of real technological processes.

The controlled output y_1 response for two values β_2 is shown in Fig. 11. It can be seen that an effect of this parameterer is insignificant.

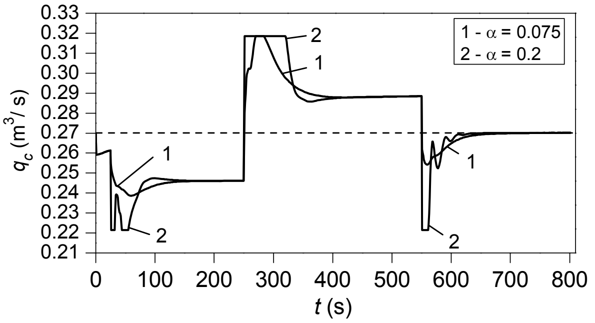

Fig. 9 Coolant flow rate responses in control of reactant mean temperature – effect of α ($\beta_1 = 1$, $\beta_2 = 0.5$).

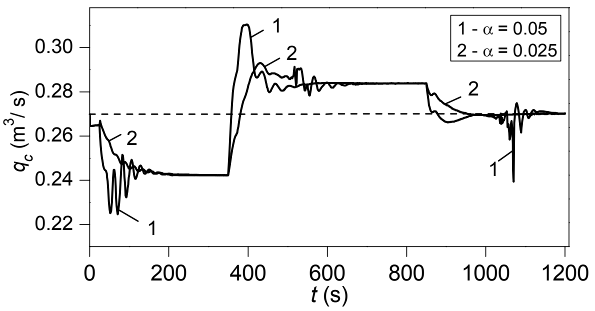

Fig. 10 Coolant flow rate responses in control of reactant output temperature – effect of α ($\beta_1 = 1$, $\beta_2 = 0$).

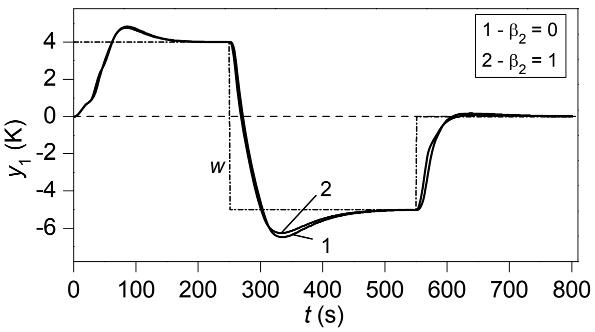

Fig. 11 Controlled output responses: effect of β_2 ($\alpha = 0.1$, $\beta_1 = 1$).

The controlled output responses documenting an effect of the parameter β_1 are in Figs. 12 and 13. In both cases, a higher value of β_1 results in greater overshoots (undershoots) whereas its influence on the speed of control is inexpressive.

Corresponding control input responses can be seen in Figs. 14 and 15. There, an increasing β_1 leads to greater values of inputs, however, it can reduce occured oscilattions, as shown in Fig. 15.

Of interest, the evolution of estimated CT ELM parameters in control of the reactant mean temperature is shown in Fig. 16.

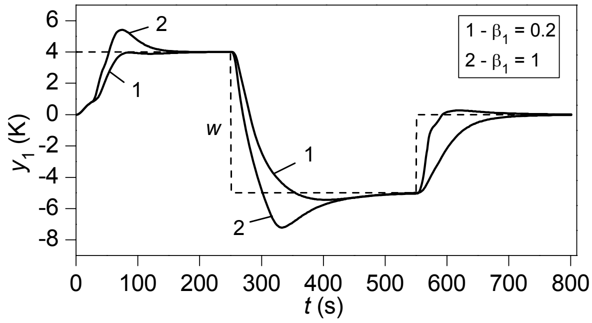

Fig. 12 Controlled output responses: effect of β_1 ($\alpha = 0.15$, $\beta_2 = 0$).

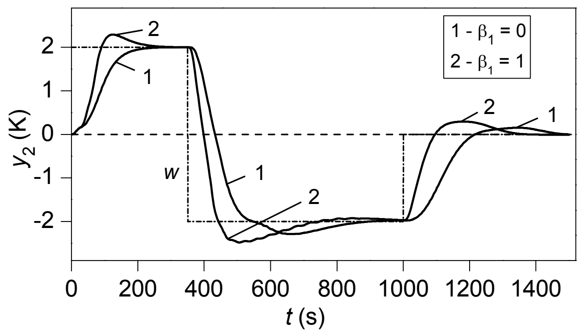

Fig. 13 Controlled output responses: effect of β_1 ($\alpha = 0.04$, $\beta_2 = 0$).

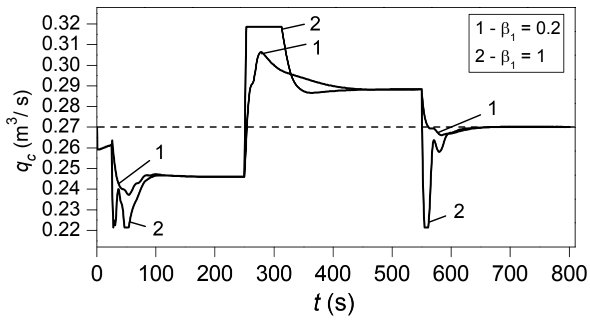

Fig. 14 Coolant flow rate responses in control of reactant mean temperature – effect of β_1 ($\alpha = 0.15$, $\beta_2 = 0$).

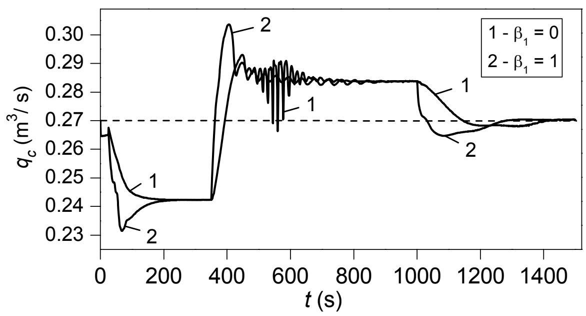

Fig. 15 Coolant flow rate responses in control of reactant output temperature – effect of β_1 ($\alpha = 0.15$, $\beta_2 = 0$).

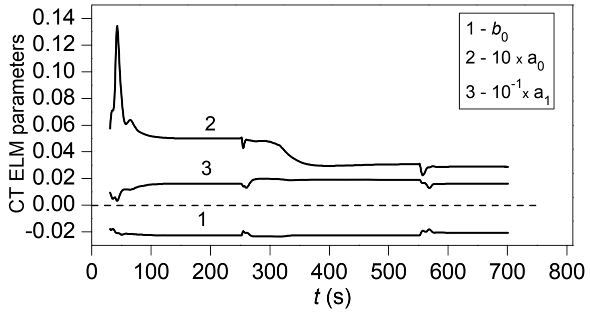

Fig. 16 CT ELM parameter evolution ($\alpha = 0.15$, $\beta_1 = 1$, $\beta_2 = 0$).

A presence of an integrating part in the controller enables rejection of various step disturbances entering into the process. As an example, step disturbances attenuation for the output y_1 is presented. Step disturbances $\Delta c_{A0} = 0.15\,\text{kmol/m}^3$, $\Delta q_r = -0.03\,\text{m}^3/\text{s}$ and $\Delta T_{r0} = 2\,\text{K}$ were injected into the nonlinear model of the reactor in times $t_v = 220\,\text{s}$, $t_v = 440\,\text{s}$ and $t_v = 640\,\text{s}$. The controller parameters were estimated only in the first (tracking) interval t < 200 s. The authors' experiences proved that an utilization of recursive identification using the delta model after reaching of a constant reference and in presence of step disturbances decreases the control quality. From this reason, during interval $t \geq 200$ s, fixed parameters were used. The controlled output responses y_1 are shown in Fig. 17.

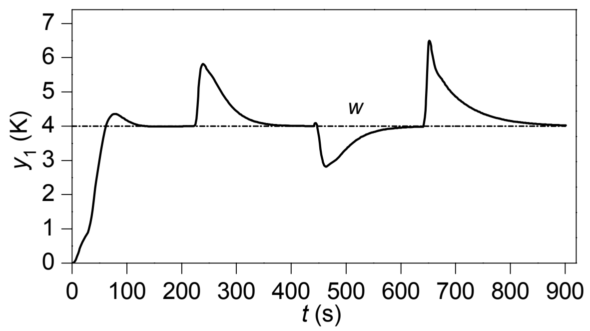

Fig. 17 Controlled output in presence of step disturbances ($\alpha = 0.15$, $\beta_1 = 0.5$, $\beta_2 = 0$).

To illustrate an effect of an additive random disturbance, the result of the controlled output y_1 simulation in a presence of the random signal $v(t) = c_{A0}(t) - c_A^s$ is shown in Fig. 18.

8 Conclusions

In this paper, one approach to continuous-time adaptive control of the mean and output reactant

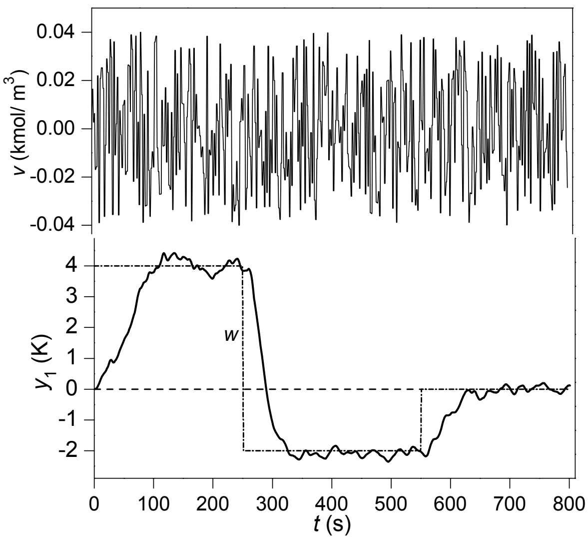

Fig. 18. Controlled output in the presence of random disturbance in c_{A0} ($\alpha = 0.15$).

temperatures in a tubular chemical reactor was proposed. The control strategy is based on the preliminary steady-state and dynamic analysis of the process and on the assumption of the temperature measurement along the reactor. The proposed algorithm employs an alternative continuous-time external linear model with parameters obtained through recursive parameter estimation of a corresponding delta model. The control system structure with two feedback controllers is considered. Resulting continuous-time controllers are derived using the polynomial approach and given by a solution of the polynomial Diophantine equation. Tuning of their parameters is possible via closed-loop pole assignment. The presented method has been tested by computer simulation on the nonlinear model of the tubular chemical reactor with a consecutive exothermic reaction. The simulation results demonstrate an applicability of the presented control strategy.

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