Dynamical generation of the weak and Dark Matter scale

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Chapter 1

Introduction

One year ago, the first LHC run ended. The most important result of this experimental work is certainly the discovery of a particle, with a mass of about 126 GeV, that is fully compatible with the Higgs boson of the Standard Model (SM) [1]. This is an important step of the modern physics because this particle confirms that the predictions on the Standard Model spectrum were true. For the physicists, there is another "hidden" result that may be more important than the success of the Standard Model: in the data collected over the last years we can observe the complete absence of new physics.

Maybe LHC will never show us unexpected phenomena. Although it can be quite disappointing, this result means that we have to revise the approach to new physics.

1.1 Hierarchy problems

In the '70s, the guideline for theoretical physics was gauge symmetry: physicists started to assume that particle physics is well described by gauge theories, and the experiments confirmed this hypotesis. This way of thinking led to the definition of the Standard Model. Though the SM explains a wide range of physical phenomena, it has still some unsatisfactory aspects. First of all, it doesn't take in account cosmological phenomena, like the Dark Matter (DM), or the expansion of the universe, or the theory of gravitation: this make us think that it is a low energy effective theory of a more complete theory. The idea is that the dominant terms of the complete theory at low energies are the SM ones, while the new terms, that describe new physics, are non renormalizable. If we call $\mathcal{L}_D$ the lagrangian with terms of mass dimension D, we could write the expression for a general lagrangian as

$$\mathcal{L} = \Lambda^4 \mathcal{L}_0 + \Lambda^2 \mathcal{L}_2 + \Lambda \mathcal{L}_3 + \mathcal{L}_4 + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \ldots$$

For example, $\mathcal{L}_2$ can be the Higgs mass term, while the gauge couplings are an $\mathcal{L}_4$ term. The renormalizable terms, that are the terms with $D \leq 4$, have a parameter that gets big corrections proportional to Λ^{4-D}, while the non-renormalizable terms, with $D > 4$, are suppressed by an energy scale that we call Λ: above this scale the new physics must be considered. For simplicity, we have supposed only one scale Λ, but every term can have his own scale, and so the relative effect can appear at different energies. To be more precise, when there are symmetries, the correspondence between the dimension of the term and the power of the correction is no longer true. For example, given the absence of a fermionic mass term in the lagrangian because of the gauge symmetry, we observe that the fermion masses don't get a correction proportional to Λ, but only a correction proportional to $v \log \Lambda$. Unfortunately, there is not such symmetry for the Higgs mass parameter.

At this point, we should introduce another problem of the SM, known with the name of "hierarchy problem" [2]. One can ask why the dimensional parameters of the SM, that is the Higgs mass, has to get the value that we can measure experimentally. The SM doesn't explain the link between the Higgs mass and other fundamental energy scales, e.g. the Planck mass. To describe our world, we need to "fine-tune" the parameters, that is, we have to set precisely their bare value to reproduce the experimental value.

As we said before, to make things worse, the SM implies quadratically divergent corrections to the Higgs mass (explained in [3], p.292). If we write $M_h^2 = M_{h\text{bare}}^2 + \delta M_h^2$, the bigger one-loop correction to the Higgs mass comes from the Top quark loop [5]:

$$\delta M_h^2(\text{top}) \approx \frac{12\lambda_t^2}{(4\pi)^2} \int \frac{dk^4}{k^2} \approx \frac{12\lambda_t^2}{(4\pi)^2} \Lambda_{\text{UV}}^2,$$

where Λ_{UV} is the ultra-violet cut-off of the integral over the momenta. This makes the fine-tuning even more difficult, because, with the systematics of renormalization, we absorb the divergences in the bare parameters. In this way we have to set a big bare mass such that the cancellations between the bare mass and the correction gives the experimental value of the parameter. The only solution to this problem seems to be that the new physics is at low energies (about at the weak scale), such that the corrections to the mass of the scalar are not greater than the value itself.

1.2 Naturalness

A new guideline for the physics beyond the SM may be naturalness [2]. Fundamentally this idea consists in replacing the brutal cut-off of the quadratic divergences with some new physics, such that the correction to the dimensional quantities of the SM are smaller than the quantity itself. Therefore, new physics should explain the origin of the values of the parameters of the Standard Model. In other words, naturalness suggests the existence of new physics at a certain scale Λ_{nat} such that the corrections $\delta m_h^2 \sim {\Lambda_{\text{nat}}}^2$ are less than m_h^2.

Following this guideline, some popular theories has been introduced.

In the past, the scientific community studied the difference between the charged pion and the neutral pion masses, getting QED quadratic divergences. This problem has been solved saying that the fundamental particles are the quarks and the mesons are composite. Perhaps following this idea, in the '90s a lot of articles about Technicolor and similar theories have been published [4]. These models consider the Higgs boson as a composite particle, and the divergences for the Higgs mass are counteracted by the exchange of vector-like particles between the Higgs components. In this case the Higgs mass depends of his components, and so there is not a mass term affected by quadratic divergences.

Another old problem, now solved, is about the electron mass: the result of the classical computation gives a linear divergence for this quantity. The chiral symmetry and the vacuum polarization have been the solution of this problem, and the positron was the new physics that has been discovered. In a similar way, SuperSymmetry (SUSY) introduces superpartners for the SM particles, that is, for every fermion of the SM there is a bosonic superpartner and vice-versa [6]. This model solves the hierarchy problem preventing quadratic corrections: each divergent contribution of fermionic loops cancels with the contribution of its superpartners. In this way, the decoupling occurs: the high energy physics does not affect the Higgs mass at low energies because it is "protected" by this mechanism and gets only logarithmic corrections. A SUSY solution to the hierarchy problem implies, in absence of fine tuning, a $\Lambda_{\text{SUSY}} \lesssim 100$ GeV and new particles around this weak scale. Since the SUSY correction to the Higgs mass is proportional to the difference between the Top quark mass and the Stop particle mass, if we want a small correction we need a light enough Stop mass. Therefore, SUSY provides us a particle, the neutralino, that is stable and it can describe the Dark Matter.

Looking at the data of the last period, i.e. the absence of these particles around the weak scale, the scale of the new SUSY particles had to move toward greater energy values. So SUSY models can no longer provide a fully natural solution to the hierarchy problem: in most popular models the fine-tuning is at the ≈ 100 level.

1.3 Finite naturalness

At this point, we could think that the naturalness guideline is wrong and the naturalness criterion has to be abandoned. Maybe the solution to the hierarchy problem is anthropic: our universe is just one of the many possibilities and our existence just "requires" these values of the parameters. Almost all the reviews on this argument are theoretical, because it's very difficult to imagine an experiment to prove these ideas. Instead, we think that some aspects of the naturalness can be recovered.

Briefly, "finite naturalness" consists in ignoring the quadratic divergences. The idea is that we can neglect these divergences exactly as we do in dimensional regularization computations. In this case the only remaining divergences are the logarithmic ones; they don't give a big contribution if there are no particles much heavier than the Higgs. For example, the reliability of finite naturalness for the SM has been studied in [7]. At one loop they redefine this parameter considering all the one particle irreducible Feynman diagrams with one loop. Following the the standard renormalization procedure of absorbing the $\sim 1/\epsilon$ poles in the Passarino-Veltman functions expansion, they observe that Higgs mass value doesn't change so much up to the Planck mass scale. At this point, the greater contribution comes from the quark top loop, that doesn't weight a lot more than Higgs. They obtain a fine-tuning of about 10^{-1}.

In this work we will address the Higgs hierarchy problem and the description of the Dark Matter as a particle. As in [8], the fundamental idea is to start from a model with a lagrangian that doesn't have any mass terms for scalar particles. The masses will arise from the quantum corrections to the theory and they won't depend of the renormalization scale used. The Coleman-Weinberg mechanism provides us a method to explain a non-zero value of the masses, also if the mass term is null, considering the radiative corrections of the theory. In [9] Coleman and Weinberg explain that the spontaneous symmetry breaking is not necessarily driven by a negative mass term for the scalar particle, but it can arise because of high-order processes involving virtual particles. They show how to compute the effective action, that is the functional generator of all the One particle irreducible (1PI) Green functions. To understand better, let's consider the simple case of a single scalar field: we call it ϕ.

If we expand the effective action in powers of the field, we get

$$\Gamma = \sum_n \int dx_1 \dots dx_n \Gamma^{(n)} \phi(x_1) \dots \phi(x_n),$$

where each $\Gamma^{(n)}$ is the sum of all the Feynman diagrams with n external legs. Therefore, expanding the effective action in powers of the momenta, about the point where the momenta are null, we get

$$\Gamma = \int d^4x \left(-V(\phi) + \frac{1}{2} Z(\phi) \partial_\mu \phi \partial^\mu \phi + \dots \right)$$

and so we define the effective potential V as the term of order zero of this expansion. We can say that it is the generator of all the 1PI Feynman diagrams with vanishing momenta, and at tree-level it coincides with the lagrangian potential of the theory. Now, if we are interested in the vacuum expectation values (VEV), we have to minimize this effective potential, so we have to impose $dV/d\phi = 0$.

In our model, spontaneous symmetry breaking doesn't occur at tree-level, because the tree-level potential doesn't have a negative mass term for the scalar. Depending on the values of the parameters, it can have a minimum in the origin or can't have a minimum at all. However, if we consider the one-loop effective potential, new minima arise and SSB occurs.

In this way we get rid of the presence of dimensional parameters that make the lagrangian not scale invariant, and we prevent quadratic divergences for the running of these parameters.

One of the new particles introduced in our model is a good candidate to represent the Dark Matter.

1.4 What is the Dark Matter?

An important evidence for the existence of new physics is Dark Matter. Some phenomena, at different scales, strongly suggest us the existence of this new type of matter [10], [11]. First, we call it "dark" because the interactions with the photons or with the other SM particles are negligible, while it interacts essentially through the gravitational force. One of the evidences comes from the observation of the rotation curves of the galaxies: we can say that they are not described by a solid-body rotation, nor by a Keplerian rotation. In fact, the tangent component of the velocity becomes flat at high distances from the center. We can explain this phenomenon with a simple idea: we suppose that in the galaxies there is a DM radial density profile such that the velocity distribution is reproduced. Another phenomena that can be explained with the DM is the weak lensing: there are some processes, like the collisions between galaxy clusters, in which the spatial off-set between the visible matter and the gravity has been measured looking at the deviation of the path of the light. Furthermore, DM is required in cosmological models to explain, for example, the formation of the structures in our universe.

We don't know what is the Dark Matter, because we can "see" it only through gravitational interactions. We don't even know if DM is made of astro-physical objects or by particles. Some physicists have analyzed the case of the Dark Matter as ultra-heavy objects like dead stars, planets or black holes. Other ideas involve new particles, for example the ultra-light scalars, like the axions. If we want to describe the Dark Matter as a particle, none of the particles we already know are good candidates, because we know that DM interacts with SM particles only gravitationally. The lightest neutrino, that have neglectable weak interactions, is excluded because Dark Matter has to be non-relativistic.

We know that in the first period of the history of the universe everything was in thermal equilibrium because of the scatterings between the particles. During this phase the particle density of the Dark Matter was, in the non-relativistic limit, the Boltzmann distribution, so

$$n_{\mathrm{DM}}^{\mathrm{eq}} = g \left(\frac{mT}{2\pi} \right)^{3/2} \mathrm{e}^{-m_{\mathrm{DM}}/T}.$$

During the evolution of the universe the temperature started to drop. We find that scatterings with Dark Matter became less and less: at this point, only annihilation processes between DM particles continue to occur. The variation in time of the DM number density can be obtained from the Boltzman equation:

$$\frac{\mathrm{d}n}{\mathrm{d}t} + 3Hn = -\langle \sigma v \rangle \left(n^2 - n_{\mathrm{eq}}^2 \right)$$

where H is the Hubble rate, $\langle \sigma v \rangle$ is the thermal averaged cross section for the annihilation times the relative velocity of the particles, and n_{eq} is the number density at thermal equilibrium. The temperature continued to decrease, until its value went well below the rest mass of the DM particle. The Dark Matter stopped every interaction, went out of equilibrium and became stable, because the interaction rate Γ became slower than the expansion of the universe, described by the Hubble rate. This phenomenon is called "freeze-out". If we call σ the cross section for the scatterings, we can say that for $T \lesssim m_{\mathrm{DM}}$ we have that

$$\Gamma \sim \langle n_{\mathrm{DM}} \sigma \rangle \lesssim H \sim \frac{T^2}{M_{\mathrm{Pl}}}.$$

In this formula we used angular parenthesis to indicate the average over the energies, while M_{Pl} is the Planck mass. According to this, we think DM is a thermal relict, that is it could not reach thermal equilibrium, so it did not annihilate completely. In a very rough approximation of the observed DM density, we can assume that this DM density it is about the same of the photons, and so, if we suppose that $\sigma \sim (g/m_{\mathrm{DM}})^2$, where g is the coupling constant, we can get an estimate for the mass of the Dark Matter particle: it is about a TeV.

$$\frac{m_{\mathrm{DM}}}{g} \sim \sqrt{T_{\mathrm{now} M_{\mathrm{Pl}}}} \approx 1 \ \mathrm{TeV}$$

With similar computations we can estimate when the freeze-out happened. The important result is that the temperature of the DM particles at that moment was almost an order of magnitude less than their mass, so these particles are in a non-relativistic regime. The cosmological DM abundance $\Omega_{\rm DM} h^2 \approx 0.11$ is reproduced for

$$\sigma v \approx 2.2 \times 10^{-26} \mathrm{cm}^3/\mathrm{s} = 1.83 \times 10^{-9} \mathrm{GeV}^{-2}$$

Another property of this DM particle is the stability: some models require the introduction of an *ad hoc* Z_2 symmetry to ensure the stability of the DM particle. For example, one of the elements of the SUSY theory is the conservation of the R-parity [6]. Every particle of the SUSY model has an R parity number. To summarize, we can say that SM particles have $R = 1$ while the new supersymmetric particles has $R = -1$ and the stability of the lightest supersymmetric particle, that is the neutralino, is given exactly by this conservation law. To be more precise, this law has not been introduced to allow the stability of the neutralino, but to explain the small decay rate of the proton. Anyhow, we are going to describe a model that implies the stability of the DM particle without introducing new symmetries on this purpose.

1.5 A new model for the Dark Matter

Because of the gauge symmetries, we can say that the SM has some "accidental" symmetries, like the baryonic number and the leptonic number conservation (neglecting instanton effects). These symmetries arise simply from the particle content of the model and from the charges associated to the particles. In fact, in the SM the photon is stable because it is massless, the electron is stable because it's the lightest charged particle, the lightest neutrino is stable because it is the lightest fermion and the proton is stable because of the conservation of the baryonic number. In [12], Hambye followed the same principle, supposing that the stability of the DM particle is not given by an *ad hoc* symmetry, but only because the gauge symmetry of the lagrangian and because of the particle content of the theory.

In this model, the DM particle is a multiplet of vector particles. Actually, if in the SM we don't mix $\mathrm{SU(2)_L}$ with $\mathrm{U(1)_Y}$ and if we don't consider fermions, we have that the three $\mathrm{SU(2)_L}$ vector bosons are automatically stable and degenerate in mass. They can't decay because the only vertices are the cubic gauge vertex, the quartic vertices of the gauge bosons and the quartic vertices with the Higgs; they have the same mass because the Weinberg angle is null. Following this idea, he introduces a new hidden sector of the lagrangian that is connected to the SM only through the so-called "Higgs portal", that is a scalar quartic interaction with the Higgs boson. For this purpose, a new scalar particle S has been introduced: it will be the only particle that interacts with the Higgs, and so with the SM.

He supposes that there is a new non-abelian symmetry group G' that has some gauge vectors X. The lagrangian of this new model will be invariant under the symmetry group of the SM and under G' at the same time. He supposes also that all the SM particles are singlets under G'. We should observe there can be no mixing between SM vectors and G' ones, because every tensor $F^{\mu\nu}$ has an index relative to its own symmetry group, so we can't construct interaction terms between vectors that are invariant under the two groups at the same time. The condition that G′ is non-abelian is fundamental, because if it is abelian, we can mix the only vector X of G′ with the vector boson of $\mathrm{U}(1)_{\mathrm{Y}}$: every tensor $F_X^{\mu\nu}$ or $F_Y^{\mu\nu}$ would be invariant under its own symmetry group and their contraction would be Lorentz-invariant.

In the following chapters, we will study if this model, without approximations, can be confirmed by the experimental data, maybe in the next phase of the work of LHC. We will analyze the general properties of the model in Chapter 2, we will study this model in an approximated case in Chapter 3, in Chapter 4 we show all the computations done in the most general case, while in Chapter 5 we show the results in both case and we discuss them. In the appendix we report the Feynman rules of the model.

Chapter 2

The model

In this chapter we will briefly study the properties of this model. In particular we will explain why should we study the one-loop effective potential of the theory.

2.1 Lagrangian and particle content

Let's set $G' = \mathrm{SU}(2)_{\mathrm{X}}$, so the symmetry group of the entire model becomes $\mathrm{U}(1)_{\mathrm{Y}}\times\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{SU}(3)_{\mathrm{c}}\times\mathrm{SU}(2)_{\mathrm{X}}$. The particle content is given by the SM particle content; plus we define the doublet S of the group $\mathrm{SU}(2)_{\mathrm{X}}$, that is a Lorentz scalar and a singlet under the SM symmetry group. To keep the gauge invariance, we need to define also $\mathrm{SU}(2)_{\mathrm{X}}$ gauge bosons, and we call them X_μ. These particles are, according to the model, the ones that constitute the Dark Matter. X_μ bosons, naturally, can be described as $X_\mu = X_\mu^a T^a$, where T^as are the generators of the new symmetry group, and they have a kinetic lagrangian term $\frac{1}{4}F^X_{\mu\nu}F_X^{\mu\nu}$, where $F^X_{\mu\nu} = [D_\mu, D_\nu]$. The kinetic term of the new scalar field is $|D_\mu S|^2$, where $D_\mu = \partial_\mu + \mathrm{i}\, g_X X_\mu$.

2.2 Tree-level potential

Instead of the SM potential we write a new potential:

$$V_0 = \lambda_H |H^\dagger H|^2 - \lambda_{HS}|H^\dagger H||S^\dagger S| + \lambda_S |S^\dagger S|^2.$$

We observe that there isn't a mass term for the Higgs field nor for the new scalar boson: as we said before, they will get their mass through the Coleman-Weinberg mechanism, considering one loop contributes to the theory. We want that spontaneous symmetry breaking down to $\mathrm{U}(1)_{\mathrm{em}}\times\mathrm{SU}(3)_{\mathrm{c}}$ occurs, and so the degrees of freedom represented by the six Goldstone bosons of the theory are absorbed into the longitudinal polarizations of all the gauge bosons. We can expand the scalar field in components as

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x)\end{pmatrix}, \qquad S(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ w + s(x)\end{pmatrix}.$$

An important remark is that also $SU(2)_X$ is broken by the VEV w of the doublet S, so every X_μ boson gets the same mass $M_X = g_X w/2$ from the interaction with the S field.

2.3 Parameters of the theory

The lagrangian parameters introduced are λ_S, λ_H, λ_{HS} and g_X. We want to fix their values starting from the experimental data that we know. In this specific case, experimental values of the Higgs mass, of the Dark Matter abundance in the universe and of the decay rate of the muon will be used. Therefore, since in this model the spontaneous symmetry breaking occurs, we have to find the minimum of the one-loop effective potential, imposing two relations that fix the expectation value of the two scalar fields. In this way also the vacuum expectation values become parameters we have to determine. We choose g_X to be the only free parameter of the theory, so every quantity will be studied as this parameter changes.

2.4 One-loop potential

If we search for the minimum of the tree-level lagrangian potential, we obtain only one minimum point in the origin in the case $4\lambda_H\lambda_S - \lambda_{HS}^2 > 0$, so the symmetry is exactly realized (i.e. no symmetry breaking). In the opposite case, the origin becomes a saddle point and there are four directions where the potential diverges negatively. At this point, to have spontaneous symmetry breaking, we need to consider one-loop corrections to the theory.

The condition for the SSB is $4\lambda_H\lambda_S - \lambda_{HS}^2 < 0$, and this condition can be dynamically verified. If we don't want to take in account the wavefunction renormalization, the one-loop potential is obtained replacing the parameters of the lagrangian with the running ones and setting the energy scale of the RGE equal to the generic VEV of the field. In other words, the VEV is the value of the energy where the condition turns to be satisfied.

Let's take the parameter λ_S: the result for the β_{λ_S} reported in the article by Hambye and Strumia is

$$\beta_{\lambda_S}(\mu) = \frac{d\lambda_S}{d\ln\mu} = \frac{1}{(4\pi)^2}\left[\frac{9}{8}g_X^4 - 9g_X^2\lambda_S + 2\lambda_{HS}^2 + 24\lambda_S^2\right].$$

This result has been obtained considering both the one-loop potential and the wave-function renormalization. We observe that β is always positive, in fact it doesn't cancel for any value of the constants. It can be verified quickly neglecting λ_{HS} term. Since λ_S becomes smaller and smaller at low energies, the symmetry breaking condition depends of the renormalization scale.

Therefore, the search of the minimum of the potential will depend of the perturbative expansion. To consider the one-loop contributions we need to start from the effective action, that is the generating functional of all the one particle irreducible Green functions. If we expand the effective action in powers of the derivatives of the fields, the effective potential is the zeroth order of this expansion. The new potential doesn't depend of the energy scale used to regularize loops computations.

Quantitatively, the effective potential is the sum of the tree-level potential and of the contributions given by scalar, fermion or vector loops. To compute it, we consider that the tadpoles with an external leg of a certain field are exactly the derivative of the potential with respect to that field. The complete derivation is in [3].

Chapter 3

Previous approximated computations

In this chapter we will study this model with some approximations. An useful approximation is to consider λ_{HS} small. It's simple understanding why the analysis simplifies: the "portal" between the SM and the new pieces of the lagrangian, represented by the $\lambda_{HS}|H|^2|S|^2$ term, is smaller. This case has already been studied in [13].

The SSB condition becomes simply $\lambda_S < 0$, and we can approximate the expression for λ_S with

$$\lambda_S \simeq \beta_{\lambda_S} \ln \frac{s}{s^*}.$$

Making this substitution, we obtain an approximate expression for the one-loop effective potential:

$$V^{1\text{loop}} \simeq \lambda_H |H^\dagger H|^2 - \lambda_{HS}|H^\dagger H||S^\dagger S| + \beta_{\lambda_S} \ln \frac{s}{s^*}|S^\dagger S|^2.$$

We know that, because of the running of λ_S, there is an energy scale s^* where λ_S goes to zero. We observe that for the energies near to the SSB scale, that is for $s \simeq s^*$, we have $\beta_{\lambda_S} \sim \lambda_S$, so the condition for λ_{HS} to be negligible becomes $\lambda_{HS}^2 \ll \beta_{\lambda_S}\lambda_H$.

To justify this expression for λ_S, we study the vectors loop contributions to the one-loop effective potential. If there is no mixing between H and S fields, the vector mass depends only on the S field. To be more specific,

$$M_X^2 = \frac{\partial^2 V}{\partial X_\mu \partial X_\nu} \sim g_X^2 s^2$$

The computation of the one-loop potential leads to

$$V^{1\text{loop}} = V_0 + (\text{const}) g_X^4 s^4 \log \frac{s}{\mu},$$

where all the cut-off dependent terms have been absorbed in the parameters of the potential V_0. It's simple to verify that a change of the renormalization scale doesn't lead to a modification of the potential. In fact, changing the renormalization scale from μ_1 to μ_2, we obtain a variation of the coupling constant of the quartic term:

$$\lambda_S(\mu_1) = \lambda_S(\mu_2) + (\text{const})g_X^4 \log\left(\frac{\mu_1}{\mu_2}\right)$$

In this way, we choose to renormalize all the potential to the scale s^*: the quartic terms will be

$$\left(\lambda_S(s^*) + (\text{const})g_X^4 \ln\frac{\mu}{s^*}\right)s^4 + \left((\text{const})g_X^4 s^4 \log\frac{s}{s^*} + (\text{const})g_X^4 s^4 \log\frac{s^*}{\mu}\right)$$

We defined s^* as the energy scale where renormalized λ_S cancels, and because of this only the logarithmic term remains.

If we consider also S loops and wavefunction renormalization, we obtain that the quartic term gets a factor $\beta_{\lambda_S}\log\frac{s}{s^*}$. Beacuse of this, the one-loop potential can be written as before in the approximation of small λ_{HS}.

Now we can find the minimum of this potential, so we can compute the vacuum expectation values of the two scalar fields, H and S. We impose that the first derivatives with respect to the fields cancels simultaneously. We obtain

$$\begin{cases} 2\lambda_H|H|^2 - \lambda_{HS}|S|^2 = 0 \\ -2\lambda_{HS}|H|^2 + \beta_{\lambda_S}|S|^2 + 4\beta_{\lambda_S}|S|^2\ln\frac{S}{s^*} = 0. \end{cases}$$

From the first equation we obtain the condition

$$v = \sqrt{\frac{\lambda_{HS}}{2\lambda_H}}w,$$

while from the second we obtain

$$-\frac{\lambda_{HS}^2}{\lambda_H} + \beta_{\lambda_S} + 4\beta_{\lambda_S}\ln\frac{s}{s^*} = 0$$

that, considering $\lambda_{HS}^2 \ll \beta_{\lambda_S}\lambda_H$, leads to the expression for the minimum:

$$w = s^*\mathrm{e}^{-\frac{1}{4}}$$

At this point we should study the mass terms of the h and s fields. The quadratic terms of the potential have the form

$$(h,s)v^2\begin{pmatrix} 2\lambda_H & -\sqrt{2\lambda_H\lambda_{HS}} \\ -\sqrt{2\lambda_H\lambda_{HS}} & \lambda_{HS} + 2\beta_{\lambda_S}\lambda_H/\lambda_{HS} \end{pmatrix}\begin{pmatrix} h \\ s \end{pmatrix}$$

We diagonalize the mass matrix and we obtain, in the limit of small λ_{HS}, the eigenvalues

$$m_1^2 \simeq 2v^2\left(\lambda_H - \frac{\lambda_{HS}^2}{\beta_{\lambda_S}}\right) \qquad m_2^2 \simeq 2v^2\frac{\beta_{\lambda_S}\lambda_H}{\lambda_{HS}}$$

The mass eigenvectors h_1 and h_2 mix with an angle α:

$$\sin 2\alpha = v^2\frac{\sqrt{8\lambda_H\lambda_{HS}}}{m_2^2 - m_1^2}.$$

In the limit of small λ_{HS} we get $m_2 \gg m_1$, and $\sin\alpha \simeq \alpha$, so

$$\alpha \simeq \frac{\lambda_{HS}^{\frac{3}{2}}}{\sqrt{2\lambda_H}\beta_{\lambda_S}}.$$

We identify the h_1 eigenstate with the physical Higgs boson, so $m_1 \approx 125.6$ GeV. We observe that the other eigenvector interacts like the Higgs does with the SM particles, but its interaction must be rescaled by a factor $\sin\alpha$. Therefore, we can write the Higgs mass as $m_1^2 \simeq w^2\lambda_{HS}$, this means that Higgs mass value can be written as a function of the vacuum expectation value of this new doublet and of the mixing constant λ_{HS}.

Now we have to study, in the approximation of small λ_{HS}, if this model for the DM reproduces the experimental data of the DM abundance in the universe. As we said in the introduction, DM went through a freeze-out, so we compute, in this approximation, the expressions for the non-relativistic cross sections for the annihilation and the semiannihilation processes of the DM. In this approximation the only relevant interactions are the gauge interactions, and the mixing between the two scalars is negligible. As a consequence, The annihilation process has two DM particle in the initial state and two s particles in the final state. The semiannihilation has again two X particles in the initial state, but in the final state there is an s and an X. If we call v the relative velocity of the two initial particles and we suppose that $M_X \gg M_s$ we have these results:

$$\sigma v_{\text{ann}} = \frac{11 g_X^2}{1728\pi w^2} \qquad \sigma v_{\text{semiann}} = \frac{g_X^2}{32\pi w^2}.$$

where we have already averaged over the polarization and over the gauge components. The DM abundance is reproduced for

$$\sigma_{\text{ann}} v + \frac{1}{2}\sigma_{\text{semiann}} v = 2.2 \times 10^{-26}\text{cm}^3/\text{s} = 1.83 \times 10^{-9}\text{GeV}^{-2},$$

where the factor $1/2$ for the semiannihilations indicates that the number of DM particles that annihilate is just one. From this relation, we can observe that the coupling constant g_X and the VEV of the S boson are linked by

$$g_X \simeq \frac{w}{2.0 \text{ TeV}}.$$

To be thorough, we skip the computation of the corrections to the VEV of the Higgs. This quantity, however, depends on the correction to the propagator of the W, so in this approximation there are no new terms, compared to the SM.

The results of all the computations of this approximation are reported in Section 5.1.

Chapter 4

Complete computation

In this chapter we will explain the computations that have been done. In the Introduction we said that we improve the SM introducing some new lagrangian terms, so new parameters arise in our model. The new parameters that we can find directly in the lagrangian are λ_H, λ_{HS}, λ_S and g_X. We have to consider also the scalar VEVs v and w between the unknown parameters. We aim to find the values of these parameters in terms of some known experimental data, so we compute some appropriate observables like the Higgs mass, the annihilation and semiannihilation cross sections of the DM, the muon decay amplitude.

In section 4.1 we describe the result of the one-loop potential of our model. Since in this model the spontaneous symmetry breaking occurs, we have to find the minimum of the potential. Thus, we describe the equations to minimize the one-loop potential in section 4.2. In Section 4.3 we explain how to compute the masses of the two scalar particles, considering that they are not mass eigenstates, and we report the result of the one-loop propagator of the Higgs boson of our model. In the section 4.4 we study the processes that led to the annihilation of the Dark Matter before the freeze-out, so we report the computations relative to DM annihilation and semi-annihilation cross sections. In the last section of this chapter (Section 4.5) we will consider the one-loop corrections to the Higgs VEV of our model, studying the amplitude of the muon decay process and considering that the value of the Fermi constant is well known.

4.1 One-loop potential

We observe that the potential V_0, depending of the parameter values, can have a minimum in the origin or not having a minimum at all, so, if we want SSB in this model, we can't consider only the tree-level potential. To find a minimum point different from the origin we have to consider the one-loop contributions computing the one-loop potential. The result for this theory is

$$V^{\mathrm{1loop}} = V_0 + V_1$$

$$V_1 = \frac{1}{64\pi^2}\left[3f_{5/6}(m_Z^2) - f_{3/2}(\xi_Z m_Z^2) + 6f_{5/6}(m_W^2) - 2f_{3/2}(\xi_W m_W^2) + \right.$$
$$\left. + 9f_{5/6}(m_X^2) - 3f_{3/2}(\xi_X m_X^2) - 12f_{3/2}(m_t^2) + \sum_i f_{3/2}(m_i)\right]$$

where Z, W and X loops (with longitudinal polarization for every vector), quark top loops (we suppose that this quark is the only fermion that gives a contribution), scalar particles and ghosts loops have been considered. In this expression, ξ_Z, ξ_W and ξ_X are the parameters that determine the gauge fixing for the Z, W and X sectors, respectively. We will choose the Landau gauge for the next computation, so we will take $\xi_Z = \xi_W = \xi_X = 0$. The sum is over all the scalar particles of the theory, that is the six Goldstone bosons and the two scalars h and s. The expression for the f function is

$$f_c(x) = x^2\left(\frac{1}{\epsilon} + \ln\frac{x}{\mu^2} - c\right),$$

where μ is the energy scale where we are renormalizing the theory. The three expressions for the mass of the Goldstone bosons related to the H field are $m_{1,2} = v^2\lambda_H - w^2\lambda_{HS}/2 + \xi_W M_W^2$ and $m_3 = v^2\lambda_H - w^2\lambda_{HS}/2 + \xi_Z M_Z^2$, while for the three Goldston bosons of the S field we have $m_{4,5,6} = w^2\lambda_S - v^2\lambda_{HS}/2 + \xi_X M_X^2$. Regarding of the mass of the two physical scalars, we observe the tree-level mass matrix is not diagonal:

$$M_0 = \begin{pmatrix} 3v^2\lambda_H - w^2\lambda_{HS}/2 & -vw\lambda_{HS} \\ -vw\lambda_{HS} & 3w^2\lambda_S - v^2\lambda_{HS}/2 \end{pmatrix}.$$

Since we want to describe scalar fields using the eigenstates of this matrix, the eigenvalues are their masses:

$$\begin{aligned} m_{1,2} =& \frac{1}{4}\left[v^2(6\lambda_H - \lambda_{HS}) - w^2(\lambda_{HS} - 6\lambda_S)\right. \\ & \pm \left(-2v^2w^2(\lambda_{HS}(6\lambda_H - 7\lambda_{HS}) + 6\lambda_S(6\lambda_H + \lambda_{HS}))\right. \\ & \left.\left. + v^4(6\lambda_H + \lambda_{HS})^2 + w^4(\lambda_{HS} + 6\lambda_S)^2\right)^{1/2}\right]. \end{aligned}$$

The interaction eigenstates don't coincide with mass eigenstates: we will see that this fact is true also considering one-loop corrections of the theory. In the Section 4.3 a mixing angle that correlates the two basis will be introduced. In our computation, for simplicity, we choose the Landau gauge for the expression of the effective one-loop potential, so we set $\xi_Z = \xi_W = \xi_X = 0$

4.2 Minimum equations

Since SSB occurs, we put the origin in a minimum point of the effective potential. Since the one-loop potential is scale invariant, we can choose freely the energy scale of the renormalized theory. To simplify calculations, we choose the critical scale where $4\lambda_H\lambda_S - \lambda_{HS}^2 = 0$. The existence of this scale is reasonable, in fact we can see in Figure 4.1 the running of the parameters of our model up to the Planck mass scale, fixing $g_X = 1$ at a scale of 100 GeV [13]. We observe that there is a scale where λ_S becomes negative, so the scale where $4\lambda_H\lambda_S - \lambda_{HS}^2 = 0$ exists certainly. In this situation, the tree-level potential has minima on two straight lines passing for the origin:

$$\frac{v}{w} = \left(\frac{\lambda_H}{\lambda_S}\right)^{1/4}.$$

This choice is possibile because, if we study the running of the constants as a function of the energy [13], we can see there is an energy μ where this condition is satisfied. At this point, the parameters of the theory become λ_H, λ_{HS} and the critical scale energy μ.

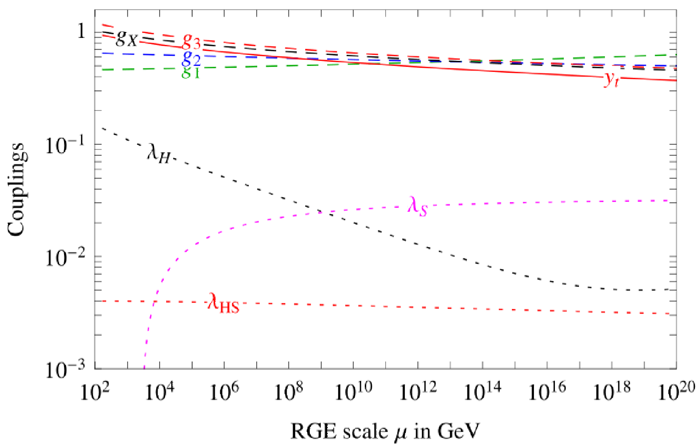


Figure 4.1: Running of the parameter of the model, up to Planck mass scale, fixing $g_X = 1$ for $\mu = 100$ GeV.

Now we switch to the effective potential. Since we have to find the minimum, we impose that the first derivatives of the potential with respect to the fields cancel:

$$\frac{\partial V}{\partial v} = v(\lambda_H v^2 - w^2\lambda_{HS}/2) + T_h = 0,$$
$$\frac{\partial V}{\partial w} = w(\lambda_S w^2 - v^2\lambda_{HS}/2) + T_s = 0,$$

where T_h and T_s represent the tadpoles related to the two scalars. By definition, tadpoles are the one particle irreducible diagrams with only one external leg and they correspond to the first derivative of the one-loop contributions to the effective potential with a minus sign.

4.3 Higgs mass

The mass matrix has this form:

$$M^2_{1\text{loop}} = \begin{pmatrix} \tilde{m}_1^2 + \Pi_{11} & \Pi_{12} \\ \Pi_{12} & \tilde{m}_2^2 + \Pi_{22} \end{pmatrix},$$

where $\tilde{m}_{1,2}$ represent the tree-level masses of the scalars, while Π represents the one-loop corrections to the propagator. If we want to consider only the one-loop approximation, off-diagonal terms are not important, and can be neglected, so we have

$$M_1^2 = \tilde{m}_1^2 + \Pi_{11}(\tilde{m}_1^2), \qquad M_2^2 = \tilde{m}_2^2 + \Pi_{22}(\tilde{m}_2^2)$$

In the critical condition we have chosen, one of the tree-level masses of the two scalars cancels, so the correction to it wouldn't be a small perturbation anymore, but it would constitute the entire value of the observable. Because of this, we compute the one-loop correction of the masses in two subsequent steps. We split $\Pi(p^2)$ in two parts:

$$\Pi(p^2) = \Pi(0) + \Delta\Pi(p^2).$$

For each scalar field we can obtain $\Pi(0)$ computing the second derivatives with respect to the field, for example

$$\Pi_{hh}(0) = \partial^2 V/\partial h^2.$$

Then, we do the same thing for $\Pi_{ss}(0)$ and for the off-diagonal term and we construct a matrix mass. We will call the eigenstates of this matrix h_1 and h_2, while the eigenvalues are a good approximation for the scalar masses. We call them m_1 and m_2. We observe that the one-loop potential doesn't take into account the renormalization of the wavefunction. To compute this correction we have to start from the one-loop correction to the propagators of h_1 and h_2. More precisely, we can write $\Delta\Pi(p^2) = \Pi(p^2) - \Pi(0)$, so we compute the one-loop contributions to the two-points Green function of each mass eigenstate for a generic p^2 and for $p = 0$ and than we do the subtraction. As we said before, the off-diagonal terms of these corrections are not important, so the final expressions for the masses of the scalars are:

$$M_1^2 = m_1^2 + \Delta\Pi_{11}(m_1^2), \qquad M_2^2 = m_2^2 + \Delta\Pi_{22}(m_2^2)$$

We observe that the one-loop contributions to the propagators of the scalar particles are similar to those of the Higgs propagator of the Standard Model. It is convenient to expand the computation of the SM, because we should describe the interaction of the scalars between them, the interaction with $\mathrm{SU}(2)_\mathrm{X}$ gauge bosons, and the mixing between the scalars.

Regarding of the mixing between the scalars, we need to introduce the mixing angle α, that is the rotation angle needed to diagonalize the one-loop mass matrix. Following the notation of the article written by Hambye and Strumia, it is defined by the relations

$$h_1 = h\cos\alpha + s\sin\alpha \quad \text{and} \quad h_2 = s\cos\alpha - h\sin\alpha.$$

The Feynman rules of this model are similar to those of the SM: one should consider that the Higgs boson field corresponds to one of the eigenstates h_1 and h_2. Regarding of the interactions with the gauge bosons of the $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry group and with the fermions, one should take the SM vertices and consider, for every Higgs line present in the diagrams, two similar diagrams that show respectively a line of h_1 or a line of h_2 in its place. The first of them takes a factor $\cos\alpha$, while the second a factor $\sin\alpha$. All the other interactions, that are substantially modified, are collected in the appendix. In Figure 4.2 we report all the one-loop diagrams that contribute to the Higgs propagator of this model.

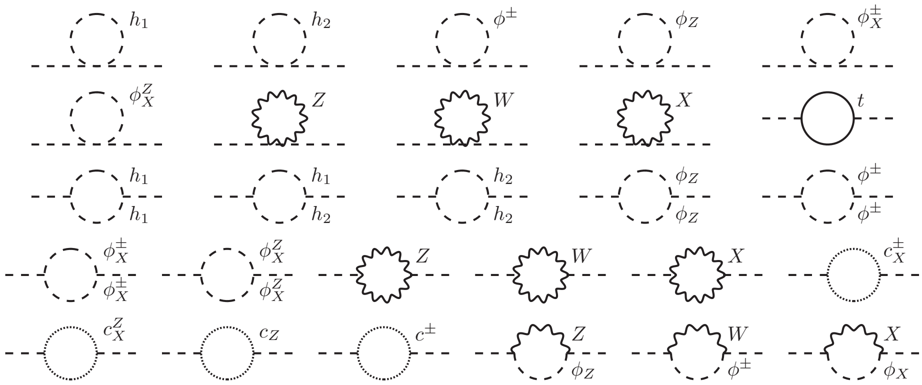


Figure 4.2: Contributi ad un loop al propagatore dell'Higgs.

We indicate with ϕ the Goldstone bosons of the Standard Model, while ϕ_X are the Goldstone bosons of the new symmetry group. In the same way we call respectively c and c_X the ghost fields of the SM and of the group $SU(2)_X$.

The expression for the Higgs one-loop propagator reported below is the sum of all these diagrams, in the same order as in Figure 4.2:

$$
\begin{aligned}
\Pi_{hh}(p^2) =& \frac{3A_0\left(M_h^2\right)\left(\lambda_H\cos^4\alpha - \lambda_{HS}\sin^2\alpha\cos^2\alpha + \lambda_S\sin^4\alpha\right)}{16\pi^2} \\
&+ \frac{A_0\left(M_s^2\right)\left((6\lambda_H + 4\lambda_{HS} + 6\lambda_S)\cos^2\alpha\sin^2\alpha - \lambda_{HS}\left(\cos^4\alpha + \sin^4\alpha\right)\right)}{32\pi^2} \\
&+ \frac{A_0\left(M_W^2\xi\right)\left(2\lambda_H\cos^2\alpha - \lambda_{HS}\sin^2\alpha\right)}{16\pi^2} \\
&+ \frac{A_0\left(M_Z^2\xi\right)\left(2\lambda_H\cos^2\alpha - \lambda_{HS}\sin^2\alpha\right)}{32\pi^2} \\
&+ \frac{3A_0\left(M_X^2\xi_X\right)\left(2\lambda_S\sin^2\alpha - \lambda_{HS}\cos^2\alpha\right)}{32\pi^2} \\
&+ \left(-\frac{g_2^2 M_Z^4}{32 M_W^2\pi^2} + \frac{3g_2^2 A_0\left(M_Z^2\right)M_Z^2}{64 M_W^2\pi^2} + \frac{g_2^2\xi A_0\left(M_Z^2\xi\right)M_Z^2}{64 M_W^2\pi^2}\right)\cos^2\alpha \\
&+ \left(-\frac{g_2^2 M_W^2}{16\pi^2} + \frac{3g_2^2 A_0\left(M_W^2\right)}{32\pi^2} + \frac{\xi g_2^2 A_0\left(M_W^2\xi\right)}{32\pi^2}\right)\cos^2\alpha \\
&+ 3\left(-\frac{g_X^2 M_X^2}{32\pi^2} + \frac{3g_X^2 A_0\left(M_X^2\right)}{64\pi^2} + \frac{\xi_X g_X^2 A_0\left(M_X^2\xi_X\right)}{64\pi^2}\right)\sin^2\alpha \\
&+ \left(-\frac{3M_t^2 A_0\left(M_t^2\right)g_2^2}{16 M_W^2\pi^2} - \frac{3\left(4M_t^4 - M_t^2 p^2\right)B_0\left(p^2, M_t^2, M_t^2\right)g_2^2}{32 M_W^2\pi^2}\right)\cos^2\alpha \\
&+ \frac{9B_0\left(p^2, M_h^2, M_h^2\right)\left(2v\lambda_H\cos^3\alpha + w\lambda_S\sin^3\alpha - \lambda_{HS}\left(w\sin\alpha\cos^2\alpha + v\sin^2\alpha\cos\alpha\right)\right)^2}{32\pi^2} \\
&+ \left(v\lambda_{HS}\cos^3\alpha - 2w\lambda_{HS}\sin\alpha\cos^2\alpha - 6w\lambda_S\sin\alpha\cos^2\alpha\right. \\
&\quad \left. -6v\lambda_H\sin^2\alpha\cos\alpha - 2v\lambda_{HS}\sin^2\alpha\cos\alpha + w\lambda_{HS}\sin^3\alpha\right)^2 \frac{B_0\left(p^2, M_h^2, M_s^2\right)}{32\pi^2} \\
&+ \left(v\lambda_{HS}\cos^3\alpha + 6v\lambda_H\sin\alpha\cos^2\alpha + 2v\lambda_{HS}\sin\alpha\cos^2\alpha\right. \\
&\quad \left. -2w\lambda_{HS}\sin^2\alpha\cos\alpha - 6w\lambda_S\sin^2\alpha\cos\alpha - w\lambda_{HS}\sin^3\alpha\right)^2 \frac{B_0\left(p^2, M_s^2, M_s^2\right)}{32\pi^2} \\
&+ \frac{B_0\left(p^2, M_W^2\xi, M_W^2\xi\right)\left(2v\lambda_H\cos\alpha - w\lambda_{HS}\sin\alpha\right)^2}{16\pi^2} \\
&+ \frac{B_0\left(p^2, M_Z^2\xi, M_Z^2\xi\right)\left(2v\lambda_H\cos\alpha - w\lambda_{HS}\sin\alpha\right)^2}{32\pi^2} \\
&+ \frac{3B_0\left(p^2, M_X^2\xi_X, M_X^2\xi_X\right)\left(2w\lambda_S\sin\alpha - v\lambda_{HS}\cos\alpha\right)^2}{32\pi^2} \\
&- \frac{g_2^2 M_W^2\xi^2 B_0\left(p^2, M_W^2\xi, M_W^2\xi\right)\cos^2\alpha}{32\pi^2} \\
&- \frac{g_2^2 M_Z^4\xi^2 B_0\left(p^2, M_Z^2\xi, M_Z^2\xi\right)\cos^2\alpha}{64 M_W^2\pi^2} \\
&- \frac{3g_X^2 M_X^2\xi_X^2 B_0\left(p^2, M_X^2\xi_X, M_X^2\xi_X\right)\sin^2\alpha}{64\pi^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\frac{\left(M_W^2(\xi-1)-p^2\right)A_0\left(M_W^2\right)g_2^2}{32M_W^2\pi^2}+\frac{\left((1-2\xi)M_W^2+p^2\right)A_0\left(M_W^2\xi\right)g_2^2}{32M_W^2\pi^2}\right.\\
&\quad+\frac{\left((\xi-1)^2M_W^4-2p^2(\xi+1)M_W^2+p^4\right)B_0\left(p^2,M_W^2,M_W^2\xi\right)g_2^2}{32M_W^2\pi^2}\\
&\quad\left.-\frac{\left(p^2-M_W^2\xi\right)^2B_0\left(p^2,M_W^2\xi,M_W^2\xi\right)g_2^2}{32M_W^2\pi^2}\right)\cos^2\alpha\\
&+\left(\frac{\left(M_Z^2(\xi-1)-p^2\right)A_0\left(M_Z^2\right)g_2^2}{64M_W^2\pi^2}+\frac{\left((1-2\xi)M_Z^2+p^2\right)A_0\left(M_Z^2\xi\right)g_2^2}{64M_W^2\pi^2}\right.\\
&\quad+\frac{\left((\xi-1)^2M_Z^4-2p^2(\xi+1)M_Z^2+p^4\right)B_0\left(p^2,M_Z^2,M_Z^2\xi\right)g_2^2}{64M_W^2\pi^2}\\
&\quad\left.-\frac{\left(p^2-M_Z^2\xi\right)^2B_0\left(p^2,M_Z^2\xi,M_Z^2\xi\right)g_2^2}{64M_W^2\pi^2}\right)\cos^2\alpha\\
&+3\left(\frac{\left(M_X^2(\xi_X-1)-p^2\right)A_0\left(M_X^2\right)g_X^2}{64M_X^2\pi^2}+\frac{\left((1-2\xi_X)M_X^2+p^2\right)A_0\left(M_X^2\xi_X\right)g_X^2}{64M_X^2\pi^2}\right.\\
&\quad+\frac{\left((\xi_X-1)^2M_X^4-2p^2(\xi_X+1)M_X^2+p^4\right)B_0\left(p^2,M_X^2,M_X^2\xi_X\right)g_X^2}{64M_X^2\pi^2}\\
&\quad\left.-\frac{\left(p^2-M_X^2\xi_X\right)^2B_0\left(p^2,M_X^2\xi_X,M_X^2\xi_X\right)g_X^2}{64M_X^2\pi^2}\right)\sin^2\alpha\\
&+\left(\frac{g_2^2(\xi-1)A_0\left(M_W^2\xi\right)}{32\pi^2}+\frac{g_2^2\left(12M_W^4-4p^2M_W^2+p^4\right)B_0\left(p^2,M_W^2,M_W^2\right)}{64M_W^2\pi^2}\right.\\
&\quad+\frac{g_2^2\left(p^2-2M_W^2\xi\right)^2B_0\left(p^2,M_W^2\xi,M_W^2\xi\right)}{64M_W^2\pi^2}-\frac{g_2^2M_W^2}{8\pi^2}-\frac{g_2^2(\xi-1)A_0\left(M_W^2\right)}{32\pi^2}\\
&\quad\left.-\frac{g_2^2\left((\xi-1)^2M_W^4-2p^2(\xi+1)M_W^2+p^4\right)B_0\left(p^2,M_W^2,M_W^2\xi\right)}{32M_W^2\pi^2}\right)\cos^2\alpha\\
&+\left(\frac{g_2^2(\xi-1)A_0\left(M_Z^2\xi\right)M_Z^2}{64M_W^2\pi^2}+\frac{g_2^2\left(12M_Z^4-4p^2M_Z^2+p^4\right)B_0\left(p^2,M_Z^2,M_Z^2\right)}{128M_W^2\pi^2}\right.\\
&\quad+\frac{g_2^2\left(p^2-2M_Z^2\xi\right)^2B_0\left(p^2,M_Z^2\xi,M_Z^2\xi\right)}{128M_W^2\pi^2}-\frac{g_2^2M_Z^4}{16M_W^2\pi^2}-\frac{g_2^2(\xi-1)A_0\left(M_Z^2\right)M_Z^2}{64M_W^2\pi^2}\\
&\quad\left.-\frac{g_2^2\left((\xi-1)^2M_Z^4-2p^2(\xi+1)M_Z^2+p^4\right)B_0\left(p^2,M_Z^2,M_Z^2\xi\right)}{64M_W^2\pi^2}\right)\cos^2\alpha\\
&+3\left(g_X^2\frac{(\xi_X-1)A_0\left(M_X^2\xi_X\right)}{64\pi^2}+\frac{g_X^2\left(12M_X^4-4p^2M_X^2+p^4\right)B_0\left(p^2,M_X^2,M_X^2\right)}{128M_X^2\pi^2}\right.\\
&\quad+\frac{g_X^2\left(p^2-2M_X^2\xi_X\right)^2B_0\left(p^2,M_X^2\xi_X,M_X^2\xi_X\right)}{128M_X^2\pi^2}-\frac{g_X^2M_X^2}{16\pi^2}-\frac{g_X^2(\xi_X-1)A_0\left(M_X^2\right)}{64\pi^2}\\
&\quad\left.-\frac{g_X^2\left((\xi_X-1)^2M_X^4-2p^2(\xi_X+1)M_X^2+p^4\right)B_0\left(p^2,M_X^2,M_X^2\xi_X\right)}{64M_X^2\pi^2}\right)\sin^2\alpha.
\end{aligned}
$$

In this formula A_0 and B_0 are the Passarino-Veltman functions:

$$A_0(m^2) = \frac{1}{\mathrm{i}\,\pi^{D/2}} \int \mathrm{d}q^D \frac{1}{q^2 - m^2 + \mathrm{i}\,\epsilon}$$

$$B_0(p^2, m_1^2, m_2^2) = \frac{1}{\mathrm{i}\,\pi^{D/2}} \int \mathrm{d}q^D \frac{1}{(q^2 - m^2 + \mathrm{i}\,\epsilon)((q+p)^2 - m_2^2 + \mathrm{i}\,\epsilon)}.$$

We can observe that the diagrams with a loop of the charged Goldstone of $\mathrm{SU}(2)_\mathrm{X}$ and the analogous with the neutral one give the same result; this cause the factor 3 before some of the contributions. We can make a similar argument for the vectors: if we define

$$X_\mu^+ = \frac{X^1 - \mathrm{i}\,X^2}{\sqrt{2}} \qquad X_\mu^- = \frac{X^1 + \mathrm{i}\,X^2}{\sqrt{2}},$$

the X^3 boson, in these diagrams, gives exactly the same result of the $X^\pm$ bosons. We have to observe that in these expressions, the plus or minus sign doesn't represent the electrical charge of the particle, this is just a convenient reparametrization of the fields.

4.4 Dark Matter abundance

In the introduction we introduced the DM as a thermal relic. To be more precise, we are going to describe which processes are important before the freeze-out of these particles. If we consider that X vectors interacts only between them and with the s boson, we can say that the fundamental processes are the annihilation processes, like $XX \to ss$ and the semiannihilation processes, like $XX \to Xs$. In Figure 4.3 we collected all the diagrams that describe annihilations. From the Feynman diagrams for the annihilation, we see that the final state can be a couple of h_1, a couple of h_2 or one of each. A couple of X vectors can annihilate via a direct quartic interaction, via an intermediate h_1 or h_2 in the s-channel, via an intermediate X in the t-channel or via an intermediate Goldstone boson, also in the t-channel. In Figure 4.5 there are all the diagrams relative to the semiannihilations. The final state is composed by a vector particle X and a scalar particle, that can be h_1 or h_2. Precisely, if we call σ_ann and σ_semiann the non-relativistic cross sections of these processes, and we say v is the relative velocity between the particles, we can say the experimental Dark Matter abundance is reproduced if

$$\sigma_\mathrm{ann} v + \frac{1}{2}\sigma_\mathrm{semiann} v = 2.2 \times 10^{-26} \mathrm{cm}^3/\mathrm{s} = 1.83 \times 10^{-9} \mathrm{GeV}^{-2}.$$

We added a factor 1/2 for the semi-annihilations because the number of DM particles drops only by one unit, so their contribution to the total annihilation of the DM is just one half of the contribution of the annihilations. Since we have no informations about which of the three X bosons is annihilating and about their polarizations, we have to average these cross sections over the polarizations of the vectors and over their $\mathrm{SU}(2)_\mathrm{X}$ index.

To do the computation of these cross sections it's useful to consider the known analogous annihilations and semiannihilations of the SM vector boson

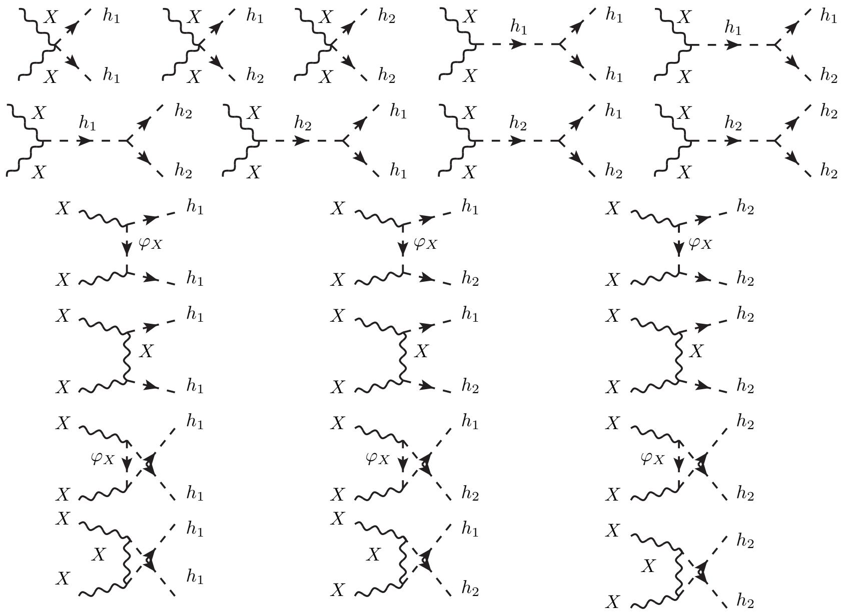


Figure 4.3: Feynman diagrams for the annihilation process of the DM with scalars in the final state.

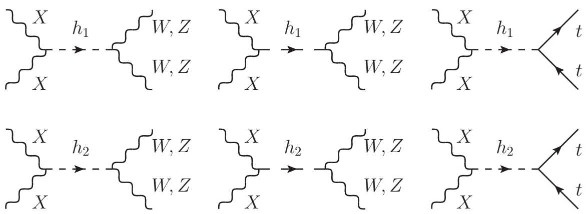


Figure 4.4: Feynman diagrams for the annihilation process of the DM with W, Z or Top quark in the final state.

into Higgs bosons and adopt these results: in our model we need to take into account the presence of two scalars and their mixing.

These cross sections are gauge-invariants but, to simplify this computation, we choose the unitary gauge, that is the gauge in which diagrams with Goldstone particles don't give any contribution. There are some contributions to the cross sections depending on the final state: for each piece we compute the amplitude, that is the sum of all the Feynman diagrams with that final state. Then, to get the cross section of this process, we compute the squared modulus of each amplitude and we multiply by the phase space factor of the process itself. To get the total cross section we sum all the contributions. In this computation we can consider only the non-relativistic limit, so the initial particles are about at rest. To get the final expressions for all the cross sections we used the application Mathematica, that automatically compute the cross sections, given the value of each Feynman diagram amplitude. These are the resuls for the annihilation cross sections. There are six contributions: the first one has two h_1 particles

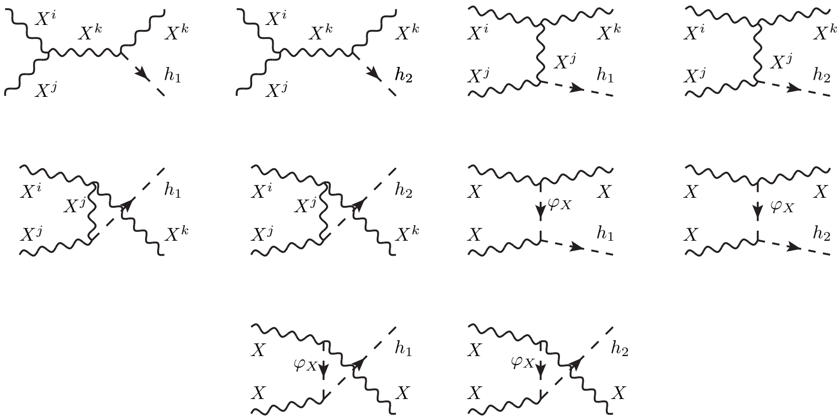


Figure 4.5: Feynman diagrams for the semiannihilation process of the DM.

in the final state, the third one has two h_2 particles, while the second one has one of each scalar. The last three contributions are related respectively to the production of a couple of W, a couple of Z or a couple of Top quarks.

$$
\begin{aligned}
\sigma v_{\text{ann}}^{h_1,h_1} &= \sqrt{M_X^2 - M_s^2} \\
\times &\left(\frac{g_X^4 \left(11M_s^4 - 28M_X^2M_s^2 + 44M_X^4\right)\cos^4(\alpha)}{1152M_X^3\left(M_s^2 - 2M_X^2\right)^2\pi} \right. \\
&+ \frac{g_X^3\left(M_s^2 - 10M_X^2\right)\cos^2(\alpha)}{2304M_X^2\left(M_s^2 - 4M_X^2\right)\left(M_s^2 - 2M_X^2\right)\left(4M_X^2 - M_h^2\right)\pi} \\
&\quad \times \left(3w\lambda_{HS}M_h^2 - 18w\lambda_S M_h^2 + 12v\lambda_H\sin(2\alpha)M_h^2 - 6v\lambda_{HS}\sin(2\alpha)M_h^2\right. \\
&\qquad - 6v\lambda_H\sin(4\alpha)M_h^2 - 3v\lambda_{HS}\sin(4\alpha)M_h^2 + M_s^2w\lambda_{HS} - 16M_X^2w\lambda_{HS} \\
&\qquad - 6M_s^2w\lambda_S + 96M_X^2w\lambda_S - 4w\left(6\lambda_S M_h^2 + M_s^2\lambda_{HS} - 4M_X^2(\lambda_{HS} + 6\lambda_S)\right)\cos(2\alpha) \\
&\qquad - 3\left(M_h^2 - M_s^2\right)w(\lambda_{HS} + 2\lambda_S)\cos(4\alpha) - 12M_s^2v\lambda_H\sin(2\alpha) \\
&\qquad \left. -2M_s^2v\lambda_{HS}\sin(2\alpha) + 32M_X^2v\lambda_{HS}\sin(2\alpha) + 6M_s^2v\lambda_H\sin(4\alpha) + 3M_s^2v\lambda_{HS}\sin(4\alpha)\right) \\
&+ \frac{g_X^2}{6144M_X\left(M_h^2 - 4M_X^2\right)^2\left(M_s^2 - 4M_X^2\right)^2\pi} \\
&\times \left(-3w\lambda_{HS}M_h^2 + 18w\lambda_S M_h^2 - 12v\lambda_H\sin(2\alpha)M_h^2 + 6v\lambda_{HS}\sin(2\alpha)M_h^2\right. \\
&\qquad + 6v\lambda_H\sin(4\alpha)M_h^2 + 3v\lambda_{HS}\sin(4\alpha)M_h^2 - M_s^2w\lambda_{HS} + 16M_X^2w\lambda_{HS} \\
&\qquad + 6M_s^2w\lambda_S - 96M_X^2w\lambda_S + 4w\left(6\lambda_S M_h^2 + M_s^2\lambda_{HS} - 4M_X^2(\lambda_{HS} + 6\lambda_S)\right)\cos(2\alpha) \\
&\qquad + 3\left(M_h^2 - M_s^2\right)w(\lambda_{HS} + 2\lambda_S)\cos(4\alpha) + 12M_s^2v\lambda_H\sin(2\alpha) \\
&\qquad \left.\left. +2M_s^2v\lambda_{HS}\sin(2\alpha) - 32M_X^2v\lambda_{HS}\sin(2\alpha) - 6M_s^2v\lambda_H\sin(4\alpha) - 3M_s^2v\lambda_{HS}\sin(4\alpha)\right)^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\sigma v_{\text{ann}}^{h_1,h_2} = \sqrt{M_X^2 - M_h^2} \Bigg(& \frac{g_X^4 \left(11M_h^4 - 28M_X^2 M_h^2 + 44M_X^4\right) \sin^4(\alpha)}{1152 M_X^3 \left(M_h^2 - 2M_X^2\right)^2 \pi} \\
& + \frac{g_X^3 \left(M_h^2 - 10M_X^2\right)}{1152 M_X^2 \left(M_h^2 - 4M_X^2\right) \left(M_h^2 - 2M_X^2\right) \left(4M_X^2 - M_s^2\right) \pi} \\
& \times (-24 \left(M_s^2 - 4M_X^2\right) w\lambda_S \sin^6(\alpha) - 4 \left(M_h^2 - 3M_s^2 + 8M_X^2\right) v\lambda_{HS} \cos(\alpha) \sin^5(\alpha) \\
& - 4w \left(2(\lambda_{HS} + 3\lambda_S) M_h^2 - 3M_s^2 \lambda_{HS} + 4M_X^2 (\lambda_{HS} - 6\lambda_S)\right) \cos^2(\alpha) \sin^4(\alpha) \\
& + 4 \left(M_h^2 - 4M_X^2\right) w\lambda_{HS} \cos^4(\alpha) \sin^2(\alpha) \\
& + v \left((3\lambda_H + \lambda_{HS}) M_h^2 - 3M_s^2 \lambda_H - 4M_X^2 \lambda_{HS}\right) \sin^3(2\alpha)) \\
& + \frac{g_X^2}{6144 M_X \left(M_h^2 - 4M_X^2\right)^2 \left(M_s^2 - 4M_X^2\right)^2 \pi} \\
& \big(w\lambda_{HS} M_h^2 - 6w\lambda_S M_h^2 + 12v\lambda_H \sin(2\alpha) M_h^2 + 2v\lambda_{HS} \sin(2\alpha) M_h^2 \\
& + 6v\lambda_H \sin(4\alpha) M_h^2 + 3v\lambda_{HS} \sin(4\alpha) M_h^2 + 3M_s^2 w\lambda_{HS} - 16M_X^2 w\lambda_{HS} \\
& - 18M_s^2 w\lambda_S + 96M_X^2 w\lambda_S + 4w \left(\lambda_{HS} M_h^2 + 6M_s^2 \lambda_S - 4M_X^2 (\lambda_{HS} + 6\lambda_S)\right) \cos(2\alpha) \\
& + 3 \left(M_h^2 - M_s^2\right) w(\lambda_{HS} + 2\lambda_S) \cos(4\alpha) - 12M_s^2 v\lambda_H \sin(2\alpha) + 6M_s^2 v\lambda_{HS} \sin(2\alpha) \\
& - 32M_X^2 v\lambda_{HS} \sin(2\alpha) - 6M_s^2 v\lambda_H \sin(4\alpha) - 3M_s^2 v\lambda_{HS} \sin(4\alpha)\big)^2 \Bigg)
\end{aligned}
$$

$$\sigma v_{\rm ann}^{h_2,h_2} = \sqrt{M_h^4 - 2\left(M_s^2 + 4M_X^2\right)M_h^2 + \left(M_s^2 - 4M_X^2\right)^2}$$
$$\times \Bigg(\frac{g_X^4}{36864 M_X^8 \left(M_h^2 + M_s^2 - 4M_X^2\right)^2 \pi}$$
$$\times \big(M_h^8 - 4\left(M_s^2 + 3M_X^2\right)M_h^6 + 2\left(3M_s^4 + 6M_X^2 M_s^2 + 46M_X^4\right)M_h^4$$
$$- 4\left(M_s^6 - 3M_X^2 M_s^4 + 2M_X^4 M_s^2 + 56M_X^6\right)M_h^2 + M_s^8 + 704M_X^8$$
$$-224M_s^2 M_X^6 + 92M_s^4 M_X^4 - 12M_s^6 M_X^2\big)\sin^2(2\alpha)$$
$$+ \frac{g_X^3\left(M_h^4 - 2\left(M_s^2 + M_X^2\right)M_h^2 + M_s^4 + 40M_X^4 - 2M_s^2 M_X^2\right)\sin(2\alpha)}{18432 M_X^5\left(M_h^2 - 4M_X^2\right)\left(M_h^2 + M_s^2 - 4M_X^2\right)\left(4M_X^2 - M_s^2\right)\pi}$$
$$\times \big(6v\lambda_H M_h^2 - v\lambda_{HS}M_h^2 + 2w\lambda_{HS}\sin(2\alpha)M_h^2 + 12w\lambda_S\sin(2\alpha)M_h^2$$
$$+ 3w\lambda_{HS}\sin(4\alpha)M_h^2 + 6w\lambda_S\sin(4\alpha)M_h^2 - 6M_s^2 v\lambda_H + M_s^2 v\lambda_{HS}$$
$$- 4\left(M_h^2 + M_s^2 - 8M_X^2\right)v\lambda_{HS}\cos(2\alpha) - 3\left(M_h^2 - M_s^2\right)v(2\lambda_H + \lambda_{HS})\cos(4\alpha)$$
$$+ 2M_s^2 w\lambda_{HS}\sin(2\alpha) - 16M_X^2 w\lambda_{HS}\sin(2\alpha) + 12M_s^2 w\lambda_S\sin(2\alpha)$$
$$-96M_X^2 w\lambda_S\sin(2\alpha) - 3M_s^2 w\lambda_{HS}\sin(4\alpha) - 6M_s^2 w\lambda_S\sin(4\alpha)\big)$$
$$+ \frac{g_X^2}{12288 M_X^2\left(M_h^2 - 4M_X^2\right)^2\left(M_s^2 - 4M_X^2\right)^2\pi}$$
$$\times \big(6v\lambda_H M_h^2 - v\lambda_{HS}M_h^2 + 2w\lambda_{HS}\sin(2\alpha)M_h^2 + 12w\lambda_S\sin(2\alpha)M_h^2$$
$$+ 3w\lambda_{HS}\sin(4\alpha)M_h^2 + 6w\lambda_S\sin(4\alpha)M_h^2 - 6M_s^2 v\lambda_H + M_s^2 v\lambda_{HS}$$
$$- 4\left(M_h^2 + M_s^2 - 8M_X^2\right)v\lambda_{HS}\cos(2\alpha) - 3\left(M_h^2 - M_s^2\right)v(2\lambda_H + \lambda_{HS})\cos(4\alpha)$$
$$+ 2M_s^2 w\lambda_{HS}\sin(2\alpha) - 16M_X^2 w\lambda_{HS}\sin(2\alpha) + 12M_s^2 w\lambda_S\sin(2\alpha)$$
$$-96M_X^2 w\lambda_S\sin(2\alpha) - 3M_s^2 w\lambda_{HS}\sin(4\alpha) - 6M_s^2 w\lambda_S\sin(4\alpha)\big)^2\Bigg)$$

$$\sigma v_{\rm ann}^{WW} = \frac{g_X^2\sin^2(2\alpha)\left(M_h^2 - M_s^2\right)^2\sqrt{M_X^2 - M_W^2}\left(3M_W^4 - 4M_W^2 M_X^2 + 4M_X^4\right)}{288\pi M_X v^2\left(M_h^2 - 4M_X^2\right)^2\left(M_s^2 - 4M_X^2\right)^2}$$

$$\sigma v_{\rm ann}^{ZZ} = \frac{g_X^2\sin^2(2\alpha)\left(M_h^2 - M_s^2\right)^2\sqrt{M_X^2 - M_Z^2}\left(4M_X^4 - 4M_X^2 M_Z^2 + 3M_Z^4\right)}{576\pi M_X v^2\left(M_h^2 - 4M_X^2\right)^2\left(M_s^2 - 4M_X^2\right)^2}$$

$$\sigma v_{\rm ann}^{TT} = \frac{g_X^2\sin^2(2\alpha)\left(M_h^2 - M_s^2\right)^2\sqrt{M_X^2 - M_T^2}\left(5M_T^2 - 8M_X^2\right)M_T^2}{2304\pi M_X v^2\left(M_h^2 - 4M_X^2\right)^2\left(M_s^2 - 4M_X^2\right)^2}$$

Therefore, we compute in the same way the result for the semiannihilation cross section. There are two contributions in this case: the first of them comes from the processes with h_1 in the final state, the second comes from the processes with h_2. The expression below is the sum of these two contributions.

$$\sigma_{\rm semiann} v = \frac{g_X^4\left(M_s^4 - 10M_s^2 M_X^2 + 9M_X^4\right)^{3/2}(\sin(\alpha) + \cos(\alpha))^2}{128\pi M_X^4\left(M_S^2 - 3M_X^2\right)^2}.$$

We observe that in the limit of small λ_{HS}, we get the same result of the approximate computation in Chapter 3.

4.5 Corrections to the VEV of the Higgs

The VEV of the Higgs is fixed by the amplitude of the muon decay process. In the Feynman diagrams for this decay there is a W propagator, so, if we want to study our model at one-loop level in perturbation theory, we have to consider the one-loop corrections of this propagator too. We can find the relation between the Higgs VEV and G_F, that is the Fermi constant.

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}(1+\Delta r),$$

where Δr encloses all the contributions given by the corrections to the W boson propagator. At tree-level approximation, as in the SM computations, we have $\Delta r = 0$. The experimental value of the Fermi constant is $1.16637 \times 10^{-5}\text{GeV}^{-2}$, so, considering only tree-level diagrams we obtain $v \simeq 246.22$ GeV from the previous relation.

In our model Δr is slightly different from the known result of the Standard Model. As in the previous computations, in our work we need to consider the presence of two scalars and their mixing. The contributions where Higgs doesn't enter are the same of the SM, so the result of this computation is well-known, and we don't compute it again. To understand better how to improve the SM to take in account the new scalar and the mixing, we have to consider the terms of the SM where the Higgs boson enters the computation. In the SM there are three diagrams, giving each a contribution to Δr:

$$\Delta r^{\text{SM}}_{\text{seagull}} = \frac{1}{(4\pi v)^2} A_0(m_h^2)$$

$$\Delta r^{\text{SM}}_{\text{rainbow } h/W} = \frac{1}{(4\pi v)^2}\left[-\frac{M_W^2+M_h^2}{2} + \frac{M_W^2 A_0(M_W^2) - M_h^2 A_0(M_h^2)}{M_h^2 - M_W^2}\right]$$

$$\Delta r^{\text{SM}}_{\text{rainbow } h/\varphi} = \frac{1}{(4\pi v)^2}\left[4M_W^2 \frac{A_0(M_h^2) - A_0(M_W^2)}{M_h^2 - M_W^2}\right]$$

We consider that for every Higgs line of the SM we have to draw two copies of the same diagram in our model, adding a factor $\cos^2\alpha$ for the Higgs boson contributions and a factor $\sin^2\alpha$ for the s boson. We get:

$$\Delta r^{(1\text{loop})} = \Delta r^{\text{SM}}(M_h \to M_{h_1})\cos^2\alpha + \Delta r^{\text{SM}}(M_h \to M_{h_2})\sin^2\alpha.$$

The complete result is

$$
\begin{aligned}
\Delta r^{(1\text{loop})} &= \frac{1}{16\pi^2 v^2} \\
&\times \Bigg[3M_T^2 - \frac{M_W^2 + M_Z^2}{2} - 6A_0\left(M_T^2\right) + A_0(M_h^2)\cos^2\alpha + A_0\left(M_s^2\right)\sin^2\alpha \\
&+ \left(9 - \frac{3M_W^2}{M_W^2 - M_Z^2}\right) A_0\left(M_W^2\right) + \left(\frac{6M_W^2 - 3M_Z^2}{M_W^2 - M_Z^2}\right) A_0\left(M_Z^2\right) \\
&+ \frac{4M_W^2\left(A_0\left(M_h^2\right) - A_0\left(M_W^2\right)\right)}{M_h^2 - M_W^2}\cos^2\alpha + \frac{4M_W^2\left(A_0\left(M_s^2\right) - A_0\left(M_W^2\right)\right)}{M_s^2 - M_W^2}\sin^2\alpha \\
&+ \left(\frac{M_W^2 A_0\left(M_W^2\right) - M_h^2 A_0\left(M_h^2\right)}{M_h^2 - M_W^2} - \frac{M_h^2 + M_W^2}{2}\right)\cos^2\alpha \\
&+ \left(\frac{M_W^2 A_0\left(M_W^2\right) - M_s^2 A_0\left(M_s^2\right)}{M_s^2 - M_W^2} - \frac{M_s^2 + M_W^2}{2}\right)\sin^2\alpha\Bigg]
\end{aligned}
$$

To make the computations simpler, we have consistently chosen the Landau gauge $\xi = 0$ for the expressions of the scalar propagators and for the correction to the VEV of the Higgs. In the computation of the annihilation and semi-annihilation cross sections we used the unitary gauge, since the cross sections are themselves gauge invariant.

Chapter 5

Results

In the previous chapter we have considered the computation of some observables in our model. Now we have to write a system of equation, imposing that our observables agree with the experimental data. As we said before, we introduced six parameters in this model, but the observable that we computed are only five. So we will determine the values of all the parameters, except for g_X: we choose it as the only free parameter. In this chapter we will show the prediction of the model about some observables, like the production cross section of the new scalar or the direct detection cross section for the DM particle. In Section 5.1 we consider the approximation of small λ_{HS}, while in Section 5.2 we show the results for the complete model.

5.1 Small λ_{HS} approximation

As we said in Chapter 3, the bulk of the correction of the theory is given by the $\mathrm{SU(2)_X}$ gauge interactions, so in this first case we will consider only these contribution to the one-loop potential. The first simplified system to be solved takes in account only three equations and three unknowns: we are going to find λ_H, λ_{HS} and μ^2, setting the values for the VEVs as $v = 246.22$ GeV and $w = 2.0$ TeV $\times\, g_X$. The last relation comes from the approximate case studied by Hambye and Strumia in [13], where they computed the annihilation and semi-annihilation cross sections for the gauge-only model.

$$\begin{cases} \frac{\partial V^{\mathrm{1loop}}}{\partial h} = 0 \\ \frac{\partial V^{\mathrm{1loop}}}{\partial s} = 0 \\ m_h^2 = (125.6\mathrm{GeV})^2 \end{cases}$$

In the previous chapter, we have seen that there are two mass eigenstates, but we don't know which of them is the Higgs boson and which is the s boson. Thus, in our computation we have to consider both the cases. In the first case we choose the first eigenstate to be the Higgs particle, so we use its eigenvalue in the third equation; in the second case, we make the same computation but the Higgs is the second eigenstate.

For each value of the free parameter g_X we solve the system and we get a point in the space of the parameters. With these data we draw some plots showing several interesting quantities. First of all, we plot the production cross section of the new scalar as a function of its mass and as a function of g_X. The expression for this cross section is similar to that of the Higgs: the only factor to take in account is $\sin(\alpha)^2$, where α is the mixing angle between the scalars. Therefore, a convenient choice is to plot the cross section in SM Higgs unit. In the first diagram of Figure 5.1 we can see two branches, on the left the branch for a new scalar lighter than the Higgs, and on the right the case in which the new scalar weights more than 125.6 GeV. We observe that there is a discontinuity: there are no points with a mass for the s boson in the range between about 105 GeV and 145 GeV. To understand why, we need to consider that the mass matrix is not diagonal, so the eigenvalues are never degenerate. In this diagram we report also the bounds set by LEP or ATLAS and CMS experiments, so the points in the grey areas are not acceptable. We can see that for a big range of the free parameter the predictions for the masses and for the cross sections of the new scalar give values compatible with the bounds of LEP and LHC experiments. In the second diagram of Figure 5.1 we plotted σ_{SI}, that is the spin-independent cross section for DM direct detection, as a function of the DM mass, with the change of g_X. Its expression is:

$$\sigma_{\mathrm{SI}} = \frac{m_N^4 f^2}{16\pi v^2}\left(\frac{1}{m_1^2} - \frac{1}{m_2^2}\right)^2 g_X^2 \sin^2(2\alpha)$$

where f is the nucleon mass matrix, $f \approx 0.295$, and m_N is the nucleon mass. From this diagram, we can see that this model is compatible with the experimental data for the direct detection when $g_X \gtrsim 0.8$.

Our approximated results reproduce those of [13]. We can now add the more precise comutation performed in this thesis. To start, we modify the one-loop potential, taking in account other interaction. The following plots (Figure 5.2) were made adding new contributes to the potential, like scalar loops, Top quark loops, SM vector loops, Goldstone loops. In each diagram we leave the result of the X-loops-only case as small points. From the comparison of these plots we can observe that the biggest correction to the production cross section of the new scalar is given by Top loops.

5.2 The complete model

Finally, we present the results for the complete model. In this section we consider the contributions of all the interactions to the one-loop potential of the theory. Furthermore, we consider also the corrections to the Higgs mass and to the s mass given by wave-function renormalization (not taken in account by the effective potential), the exact relation between w and g_X given by the annihilation and the semiannihilation cross sections, the correction to the Higgs VEV through the value of the Fermi constant.

To do this we want to solve a system of five equations with five unknowns.

We are going to find the values of λ_H, λ_{HS}, μ^2, v and w:

$$\begin{cases} \frac{\partial V^{1\text{loop}}}{\partial h} = 0 \\ \frac{\partial V^{1\text{loop}}}{\partial s} = 0 \\ M_h^2 = m_h^2 + \Delta\Pi(p^2) = (125.6\text{GeV})^2 \\ \frac{1}{v^2\sqrt{2}}(1 + \Delta r^{(1\text{loop})}) = G_F = 1.16637 \times 10^{-5}\text{GeV}^{-2} \\ \sigma_{\text{ann}} v + \frac{1}{2}\sigma_{\text{semiann}} v = 2.2 \times 10^{-26}\text{cm}^3/\text{s} = 1.83 \times 10^{-9}\text{GeV}^{-2} \end{cases}$$

Again, we will consider that this system has to be solved in two cases, because we don't know which of the eigenvalues m_h of the one-loop mass matrix corresponds to the Higgs. The only free parameter will be g_X, so we have to compute the solution for every value of it.

As in the section above, we have a new set of solutions showing us the values of the parameters of the model as function of g_X. With these data we build the diagram of the production cross section of the new scalar as a function of the mass of the scalar itself (Figure 5.3, above). Also in this complete case we observe that there is a discontinuity of about about 20 GeV around the Higgs mass, and so the wavefunction renormalization that we have considered for both the scalars doesn't give a big contribution in this sense. The plots report the bounds given by LEP experiments for energies lower than the Higgs mass and by ATLAS and CMS experiments for greater energies. Below, in Figure 5.3, we present the spin independent cross section for direct detection in the complete case. The model is not excluded by LUX2013 data for $g_X \gtrsim 0.8$

Furthermore, for completeness, in Figure 5.5 we plot how the parameters of the theory depend on g_X.

In the Conclusions we make further comments and observations about the results given above.

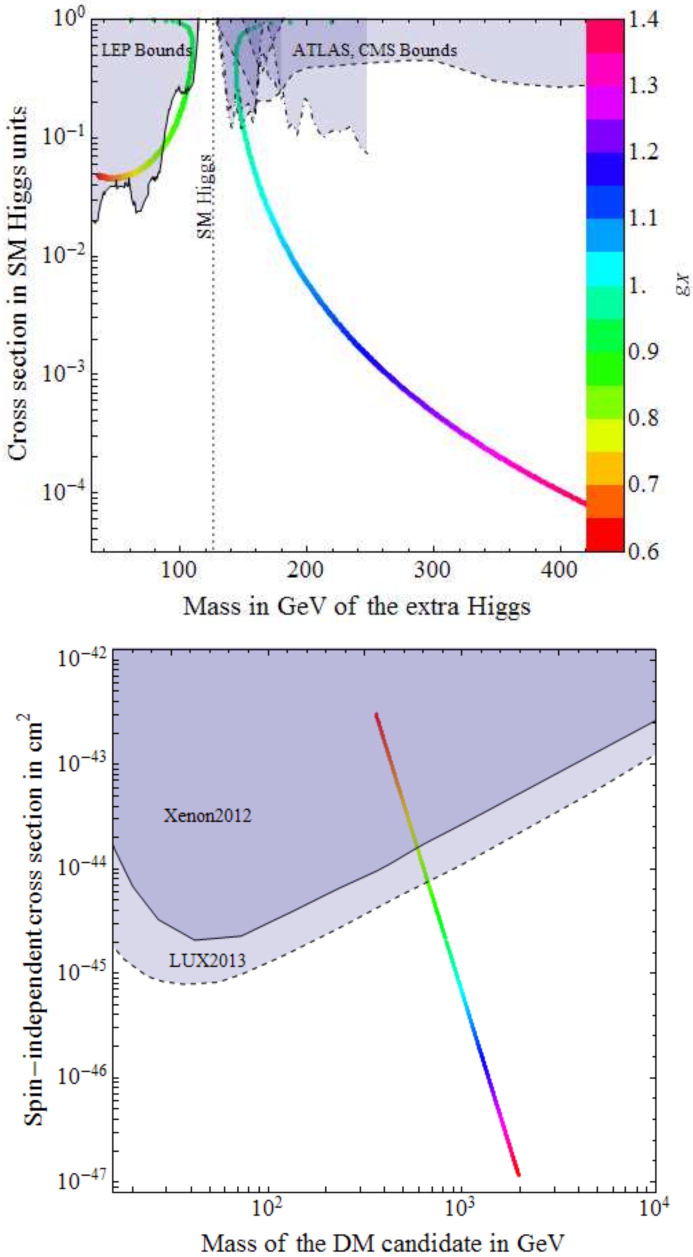


Figure 5.1: Above, the prediction of the gauge-only approximation about the cross section of the new scalar. The grey areas are excluded by LEP or CMS and ATLAS experiments. Below, the prediction of the σ_{SI}, the gray areas are excluded by XENON2012 and LUX2013 experiments. Everything is computed as a function of the parameter g_X, that varies as shown in the colour legend.

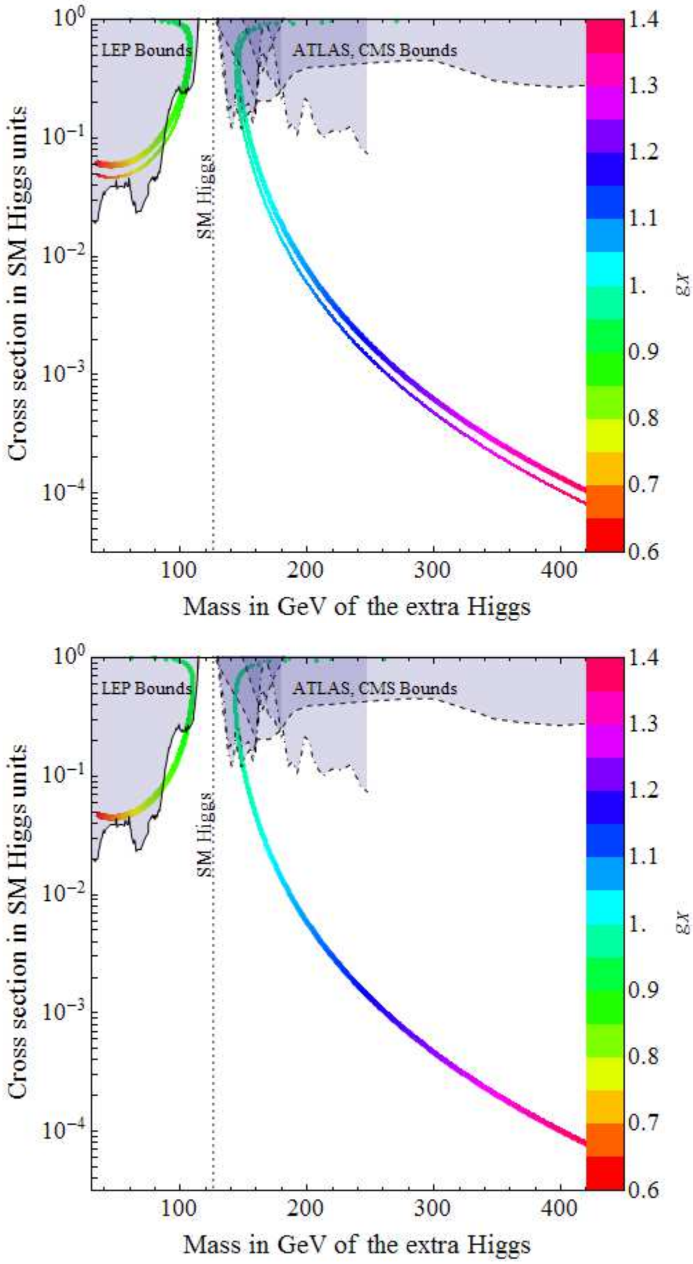


Figure 5.2: Cross section of the new scalar as a function of the parameter g_X, considering also Top loops (above) or considering W, Z, scalars and Goldstone loops (below)

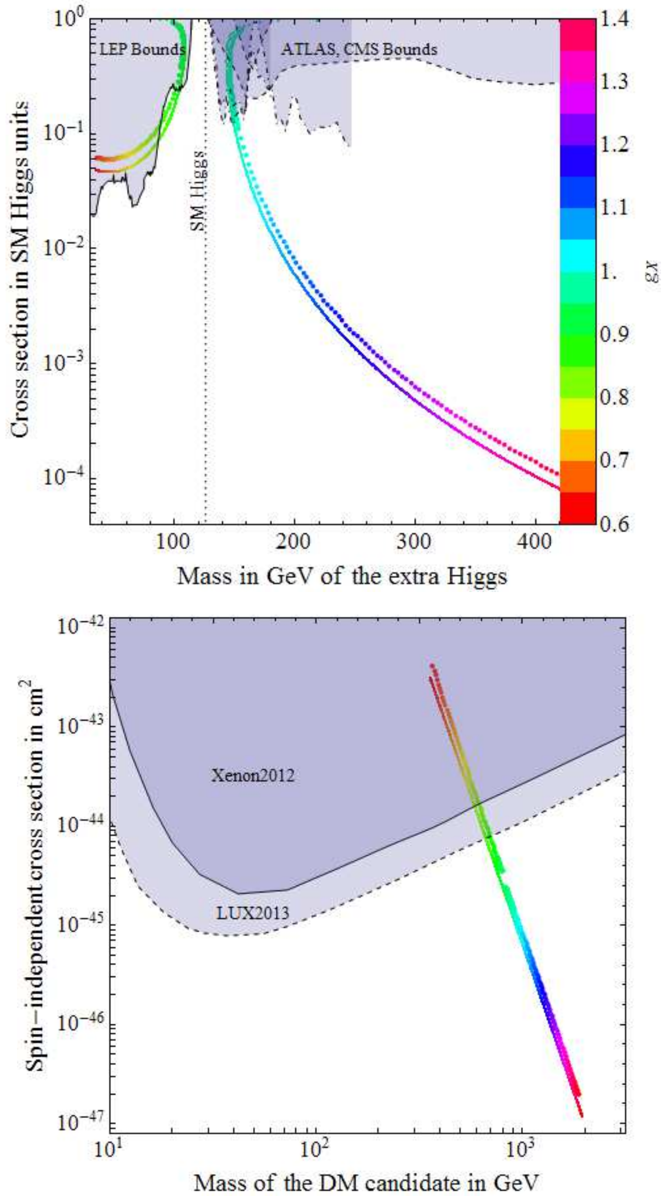


Figure 5.3: Our final result: above, the prediction of the complete model for the production cross section of the new scalar. Below we report the prediction for the cross section for DM direct detection. These quantities are plotted as a function of the parameter g_X, that varies accordingly to the colors on the legend. For a comparison, in these diagrams we leave the data of the approximated case as smaller points. As in the approximated case, in Figures 5.1 and 5.2, the grey areas are excluded by LEP or CMS and ATLAS experiments for the diagram above, while the bounds comes from XENON2012 and from LUX2013 experiments for the diagram below.

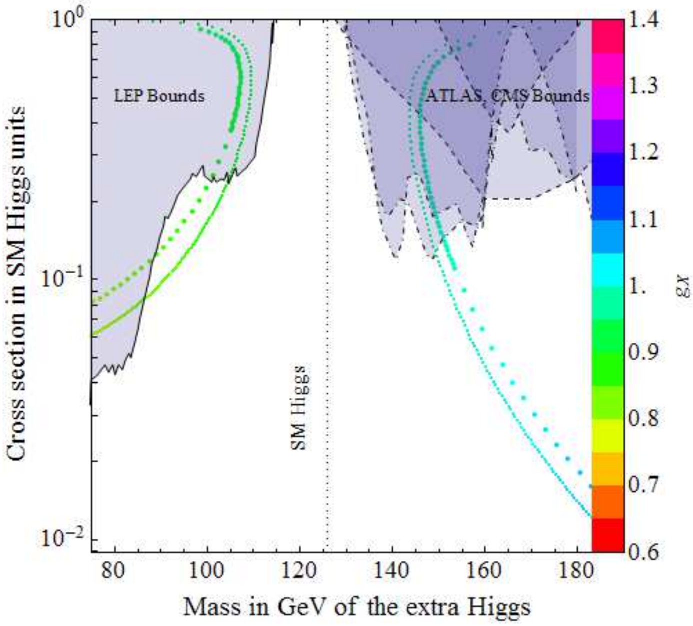


Figure 5.4: We plot again the first diagram of Figure 5.3, enlarging on the area where the data show similar masses for the scalars.

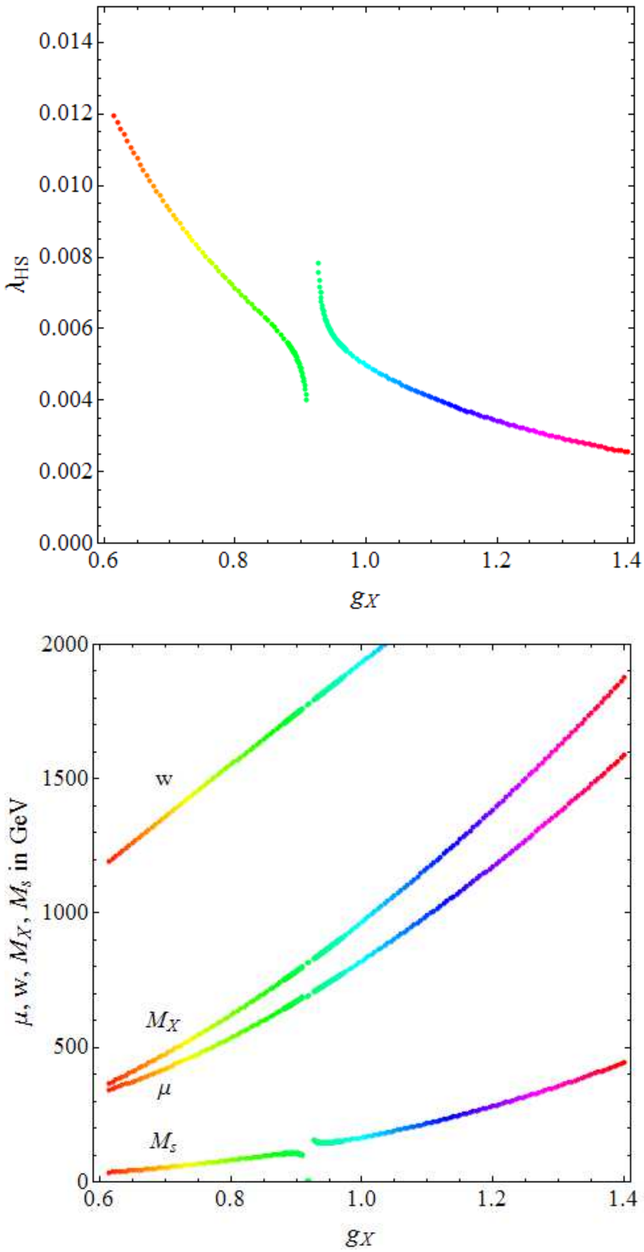


Figure 5.5: Predictions of the complete model: above, the parameter λ_{HS}, that is the coefficient of the "portal" term. Below, the parameters w and μ, the mass of the extra scalar M_s and the mass of the DM particle. w is the vacuum expectation value of the new scalar s, while μ indicate the critical scale at which $4\lambda_H\lambda_S - \lambda_{HS}^2 = 0$, so the scale μ in our computations effectively replaced the parameter λ_S. All these quantities are plotted as a function of the parameter g_X.

Chapter 6

Conclusion

We considered an extension of the SM that describes the Dark Matter and proposes a solution to the hierarchy problem.

In the introduction we analized the hierarchy problem and the presence of quadratically divergent corrections to the dimensional parameter of the SM. In the context of "finite naturalness", we introduced a model without a mass term for the Higgs. The masses of the particles arise from a Coleman-Weinberg mechanism, so spontaneous symmetry breaking does not occur at tree-level, but is generated by the radiative corrections to the theory. We supposed that there is a new particle S, scalar doublet under an extra group $\mathrm{SU}(2)_X$, and new vector bosons X of the same gauge group. The only communication between this new sector and the SM is through the so-called "Higgs portal", that is the quartic vertex between two Higgs fields and two S fields. The VEVs of the two scalars are fixed by the one-loop potential. The interactions with the scalars give mass to all the particles of the model, so all the scales are related and exponentially suppressed with respect to the Planck scale.

Astrophisical and cosmological experiments demand the presence of Dark Matter. We don't know, as we wrote in the introduction, what it is, and there are a lot of hypoteses on it. We think it is a particle, and in our model we introduced the vector boson X of $\mathrm{SU}(2)_X$, that is a good candidate to represent the Dark Matter. It has a mass of about 1 TeV, and if we make a rough estimate, as we did in the introduction, this is the order of magnitude of the scale where the mass of the DM particle is expected, assuming it is a thermal relict. Furthermore, this particle has to be stable. Some theories have to introduce special symmetries with the specific purpose of keeping the DM particle stable. In our model, X vectors are automatically stable, because of the gauge symmetry and because of the particle content of the theory.

Another peculiarity of this simple model is the presence of only one free parameter. The other parameters introduced in the model are fixed by the experimental values of the DM cosmological abundance, of the Fermi constant and of the Higgs mass.

The original work presented in this thesis consisted in performing for the first time a precise computation of the predictions of the model for the LHC and for direct detection experiments.

The new computation includes for the first time a full one-loop computation of the scalar masses and of the effective potential, and a full tree-level computation of the DM annihilations and semi-annihilations relevant for the thermal DM abundance. We find that:

- there are new solutions missed in the previous computation; however they are in the already excluded area.
- in [13] the diagrams the predictions for the DM mass and direct detection cross section show a "gap". In our computations, this discontinuity disappeared, being an artifact of the previous approximated computation.
- given the mass M_s of the extra scalar, the cross section for its production for LHC increases by a factor $\approx$1.3 with respect to the approximated computation. Anyhow, this cross section is compatible with the experiments in a small range around $g_X \approx 0.9$ when s is lighter than Higgs, and for $g_X \gtrsim 1.0$ when s is heavier.
- the prediction for the DM direct detection is compatible with LUX2013 and XENON2012 bounds for $g_X \gtrsim 0.8$.

Appendix A

Feynman rules of the model

A.1 Overview

The lagrangian of the theory is:

$$\mathcal{L} = \mathcal{L}_{\text{SM}}^{m_h=0} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + |D_\mu S|^2 + \lambda_{HS}|HS|^2 - \lambda_S|S|^4.$$

This lagrangian is invariant under $\text{U}(1)_\text{Y}\times\text{SU}(2)_\text{L}\times\text{SU}(3)_\text{c}\times\text{SU}(2)_\text{X}$. We introduced a new symmetry group, $\text{SU}(2)_\text{X}$, and S, that is a doublet under this group. In this model there is the spontaneous symmetry breaking, so we write directly the H and S fields as a sum of a vacuum expectation value and a physical field:

$$H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \qquad S = \begin{pmatrix} 0 \\ \frac{w+s}{\sqrt{2}} \end{pmatrix}.$$

Therefore, we observe that the mass matrix for h and s is not diagonal: we call h_1 and h_2 the mass eigenstates and we define a mixing angle α:

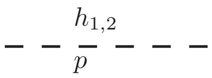
$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

The expression $D_\mu S$ represents the covariant derivative of the field: $(\partial_\mu + \text{i}\, g_X X_\mu)S$, where the X_μ fields are the vector bosons of the new symmetry group $\text{SU}(2)_\text{X}$. The Feynman rules of this model are similar to those of the SM: we should consider that the Higgs boson field is not simply h, but it is rotated, so it is a combination between h_1 and h_2. To write the rules for the interactions with the W and Z bosons and with fermions, we just take the SM vertices and we consider, for every line of the Higgs field, two similar diagrams: in each of them the h line is replaced with a h_1 line or a h_2 line respectively. The first of them takes a $\cos\alpha$ factor, while the second takes a $\sin\alpha$ factor. All the other interactions, that are substantially modified, are listed below.

A.2 Scalar and vector interactions

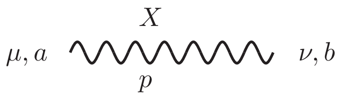
A.2.1 Propagators

The propagators of the h_1 and h_2 scalars are



$$= \frac{\mathrm{i}}{p^2 - m_{h_{1,2}}^2 + \mathrm{i}\,\epsilon},$$

while the vector boson propagators, that have all the same mass $m_X = \frac{g_X w}{2}$, become



$$= \frac{-\mathrm{i}\,\delta^{ab}}{p^2 - m_X^2 + \mathrm{i}\,\epsilon}\left[g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m_X^2}\right]$$

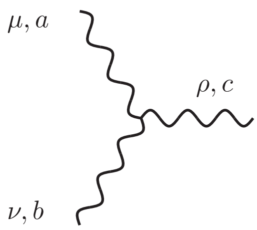
We can express the vector bosons in this way:

$$X_\mu^+ = \frac{X^1 - \mathrm{i}\,X^2}{\sqrt{2}} \qquad X_\mu^- = \frac{X^1 + \mathrm{i}\,X^2}{\sqrt{2}},$$

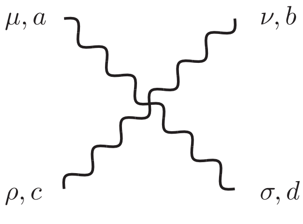
where the plus or minus doesn't represent the electrical charge of the particle, this is just a convenient reparametrization of the fields.

A.2.2 Gauge vertices

Gauge bosons only

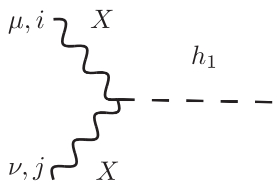


$$\begin{aligned}&= g_X \epsilon^{abc}\\ &\times [g^{\mu\nu}(p_1 - p_2)^\rho + g^{\nu\rho}(p_2 - p_3)^\mu + g^{\rho\mu}(p_3 - p_1)^\nu]\\ &p_1 + p_2 + p_3 = 0\end{aligned}$$

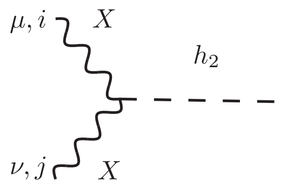


$$\begin{aligned}= -\mathrm{i}\,g_X^2 [&\epsilon^{eab}\epsilon^{ecd}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})\\ &+ \epsilon^{eac}\epsilon^{edb}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\nu}g_{\sigma\rho})\\ &+ \epsilon^{ead}\epsilon^{ebc}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma})]\\ &p_1 + p_2 + p_3 + p_4 = 0\end{aligned}$$

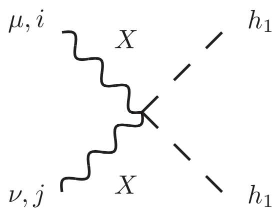
Vertices involving scalars



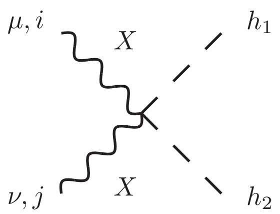
$$= \mathrm{i}\, g^{\mu\nu}\delta^{ij} g_X m_X \sin\alpha = \frac{\mathrm{i}}{2} g^{\mu\nu}\delta^{ij} g_X^2 w \sin\alpha$$



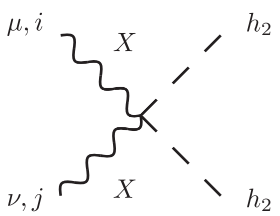
$$= \mathrm{i}\, g^{\mu\nu}\delta^{ij} g_X m_X \cos\alpha = \frac{\mathrm{i}}{2} g^{\mu\nu}\delta^{ij} g_X^2 w \cos\alpha$$



$$= \frac{\mathrm{i}}{2} g^{\mu\nu}\delta^{ij} g_X^2 \sin^2\alpha$$



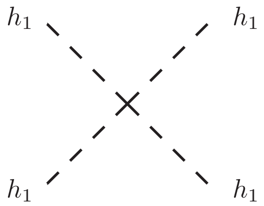
$$= \frac{\mathrm{i}}{4} g^{\mu\nu}\delta^{ij} g_X^2 \sin 2\alpha$$



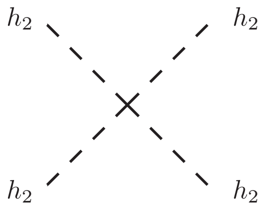
$$= \frac{\mathrm{i}}{2} g^{\mu\nu}\delta^{ij} g_X^2 \cos^2\alpha$$

A.2.3 Scalar vertices

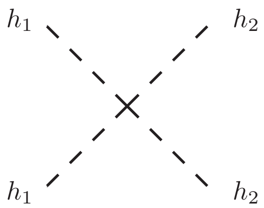
Quartic



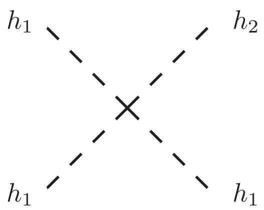
$$= -6\,\mathrm{i}(\lambda_H \cos^4\alpha - \lambda_{HS}\cos^2\alpha\sin^2\alpha + \lambda_S \sin^4\alpha)$$



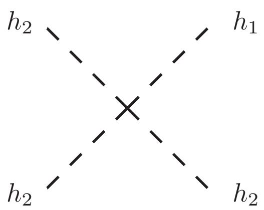
$$= -6\,\mathrm{i}(\lambda_S \cos^4\alpha - \lambda_{HS}\cos^2\alpha\sin^2\alpha + \lambda_H \sin^4\alpha)$$



$$\begin{aligned} = &-\mathrm{i}[(6\lambda_H + 4\lambda_{HS} + 6\lambda_S)\cos^2\alpha\sin^2\alpha + \\ &- \lambda_{HS}(\cos^4\alpha + \sin^4\alpha)] \end{aligned}$$

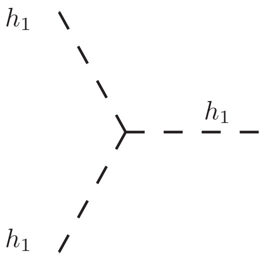


$$= 3\,\mathrm{i}(2\lambda_H \cos^3\alpha\sin\alpha + \lambda_{HS}\frac{\sin 4\alpha}{4} - 2\lambda_S\cos\alpha\sin^3\alpha)$$

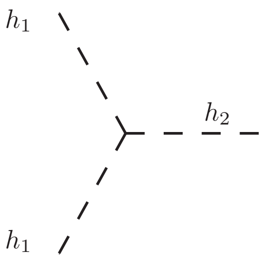


$$= 3\,\mathrm{i}(2\lambda_H \cos\alpha\sin^3\alpha - \lambda_{HS}\frac{\sin 4\alpha}{4} - 2\lambda_S\cos^3\alpha\sin\alpha)$$

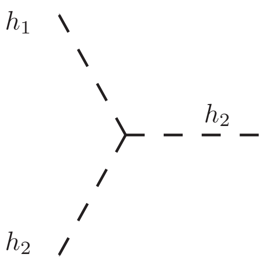
Cubic



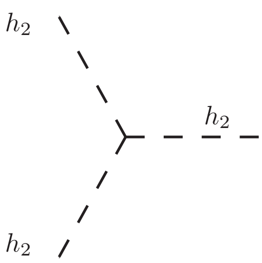
$$= -3\,\mathrm{i}(2\lambda_H v\cos^3\alpha - \lambda_{HS}(w\cos^2\alpha\sin\alpha + v\cos\alpha\sin^2\alpha) + 2\lambda_S w\sin^3\alpha)$$



$$= \mathrm{i}(\lambda_{HS} w\cos^3\alpha + 6\lambda_H v\cos^2\alpha\sin\alpha + 2\lambda_{HS} v\cos^2\alpha\sin\alpha \\ - \lambda_{HS} v\sin^3\alpha - 6\lambda_S w\cos\alpha\sin^2\alpha - 2\lambda_{HS} w\cos\alpha\sin^2\alpha)$$



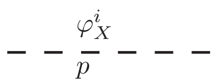
$$= \mathrm{i}(\lambda_{HS} v\cos^3\alpha - 6\lambda_H v\cos\alpha\sin^2\alpha - 2\lambda_{HS} v\cos\alpha\sin^2\alpha + \\ + \lambda_{HS} w\sin^3\alpha - 6\lambda_S w\cos^2\alpha\sin\alpha - 2\lambda_{HS} w\cos^2\alpha\sin\alpha)$$



$$= -3\,\mathrm{i}(2\lambda_S w\cos^3\alpha + \lambda_{HS}(v\cos^2\alpha\sin\alpha - w\cos\alpha\sin^2\alpha) - 2\lambda_H v\sin^3\alpha)$$

A.2.4 Goldstone bosons interactions

Besides the Goldston bosons of the Higgs, there are three new Goldstone bosons relative to the S field, and we call them φ_X^i. Their propagator is



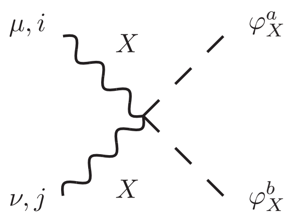
$$= \frac{\mathrm{i}}{p^2 - \xi_X m_X^2 + \mathrm{i}\,\epsilon}$$

A.2.5 Goldstone vertices

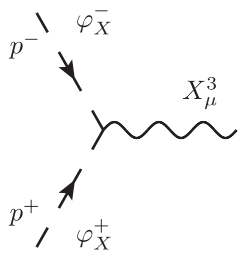
An useful way to describe Goldstone bosons is:

$$\varphi_X^+ = \frac{\varphi_X^1 - \varphi_X^2}{\sqrt{2}} \qquad \varphi_X^- = \frac{\varphi_X^1 + \varphi_X^2}{\sqrt{2}} \qquad \varphi_X^Z = \varphi_X^3$$

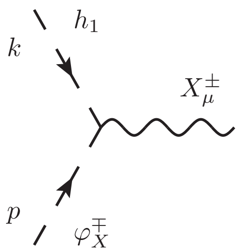
Goldstone-vectors vertices



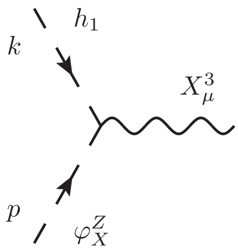
$$= \frac{\mathrm{i}}{2} g^{\mu\nu} \delta^{ij} \delta^{ab} g_X^2$$



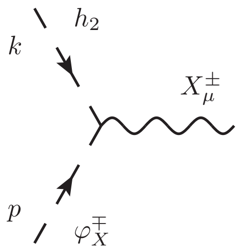
$$= -\mathrm{i}\,\frac{g_X}{2}(p_\mu^+ - p_\mu^-)$$



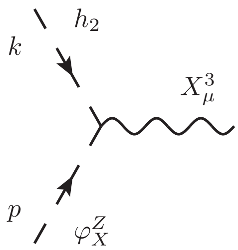
$$= -\mathrm{i}\,\frac{g_X}{2}(p_\mu - k_\mu)\sin\alpha$$



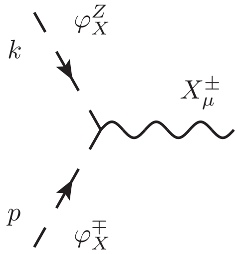
$$= -\,\mathrm{i}\,\frac{g_X}{2}(p_\mu - k_\mu)\sin\alpha$$



$$= -\,\mathrm{i}\,\frac{g_X}{2}(p_\mu - k_\mu)\cos\alpha$$

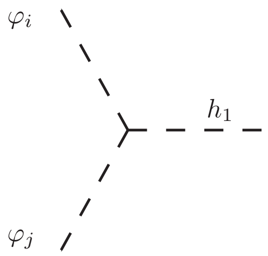


$$= -\,\mathrm{i}\,\frac{g_X}{2}(p_\mu - k_\mu)\cos\alpha$$

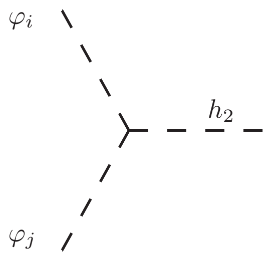


$$= -\,\mathrm{i}\,\frac{g_X}{2}(p_\mu - k_\mu)$$

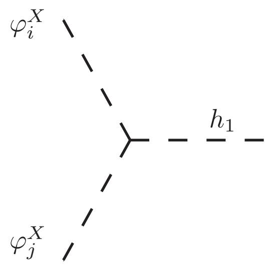
Goldstone-scalars vertices



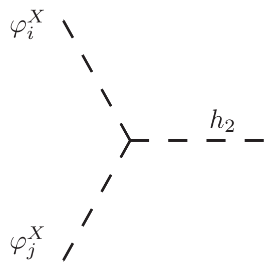
$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_H v\cos\alpha - \lambda_{HS} w\sin\alpha)$$



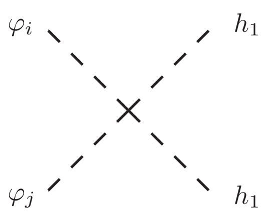
$$= \mathrm{i}\,\delta^{ij}(2\lambda_H v\sin\alpha + \lambda_{HS} w\cos\alpha)$$



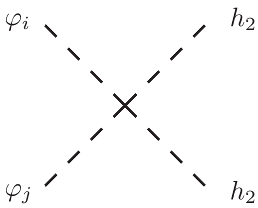
$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_S w\sin\alpha - \lambda_{HS} v\cos\alpha)$$



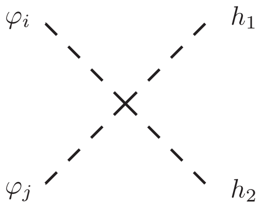
$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_S v\cos\alpha + \lambda_{HS} w\sin\alpha)$$



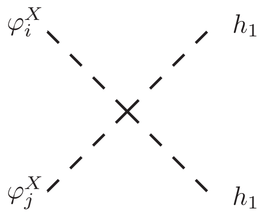
$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_H \cos^2\alpha - \lambda_{HS}\sin^2\alpha)$$



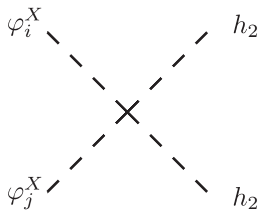
$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_H \sin^2\alpha - \lambda_{HS}\cos^2\alpha)$$



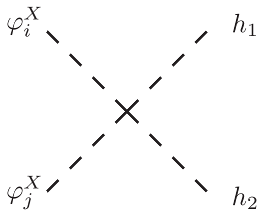
$$= \mathrm{i}\,\delta^{ij}(2\lambda_H + \lambda_{HS})\sin\alpha\cos\alpha$$



$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_S \sin^2\alpha - \lambda_{HS}\cos^2\alpha)$$

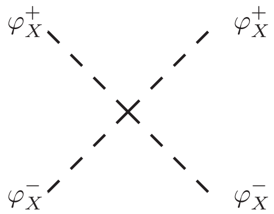


$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_S \cos^2\alpha - \lambda_{HS}\sin^2\alpha)$$

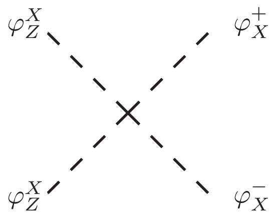


$$= -\,\mathrm{i}\,\delta^{ij}(2\lambda_S + \lambda_{HS})\sin\alpha\cos\alpha$$

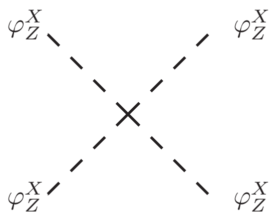
A.2.6 Goldstone only vertices



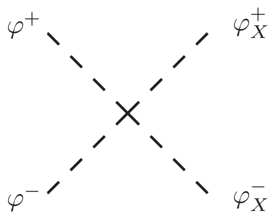
$= -4\,\mathrm{i}\,\lambda_S$



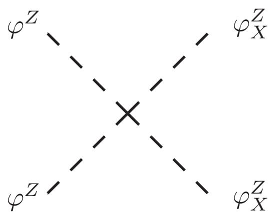
$= -2\,\mathrm{i}\,\lambda_S$



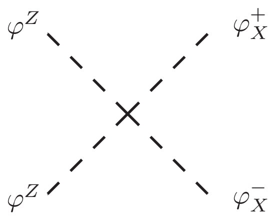
$= -3\,\mathrm{i}\,\lambda_S$



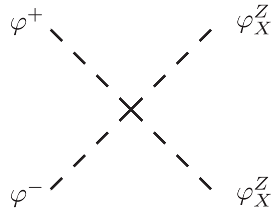
$= \mathrm{i}\,\lambda_{HS}$



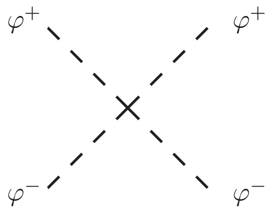
$= \mathrm{i}\,\lambda_{HS}$



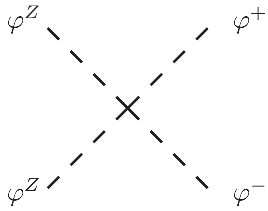
$= \mathrm{i}\,\lambda_{HS}$



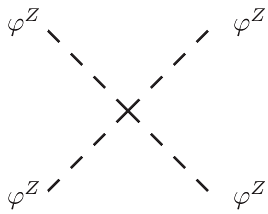
$$= \mathrm{i}\,\lambda_{HS}$$



$$= -4\,\mathrm{i}\,\lambda_H$$



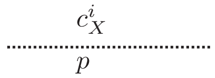
$$= -2\,\mathrm{i}\,\lambda_H$$



$$= -3\,\mathrm{i}\,\lambda_H$$

A.3 Ghost fields

To fix the gauge over the new vector bosons, we introduce three new couples of ghost and anti-ghost fields. We call them c^i_X and $\bar{c}^i_X$. Their propagator is



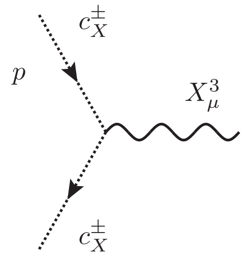
$$= \frac{\mathrm{i}}{p^2 - \xi_X m_X^2 + \mathrm{i}\,\epsilon}$$

A.3.1 Ghost vertices

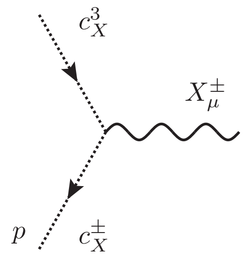
Also for the ghosts, a useful way to describe them is

$$c^+_X = \frac{c^1_X - c^2_X}{\sqrt{2}} \qquad c^-_X = \frac{c^1_X + c^2_X}{\sqrt{2}},$$

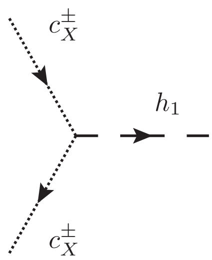
considering that antighost fields are defined with the opposite signs. Similarly to the case of the vector bosons, the signs are not the electric charge of the particle.



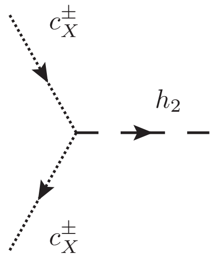
$$= \mp\,\mathrm{i}\,g_X p_\mu$$



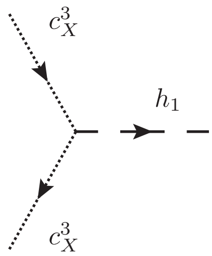
$$= \pm\,\mathrm{i}\,g_X p_\mu$$



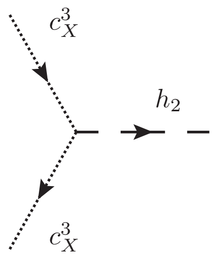
$$= -\frac{\mathrm{i}}{4}\xi g_X^2 w \sin\alpha$$



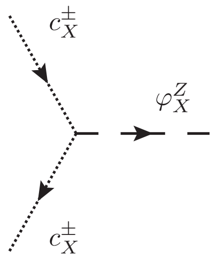
$$= -\frac{\mathrm{i}}{4}\xi g_X^2 w \cos\alpha$$



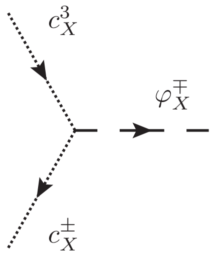
$$= -\frac{\mathrm{i}}{4}\xi g_X^2 w \sin\alpha$$



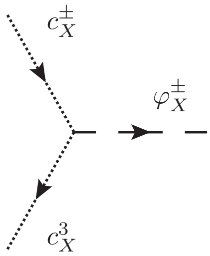
$$= -\frac{\mathrm{i}}{4}\xi g_X^2 w \cos\alpha$$



$$= \pm\frac{\mathrm{i}}{4}\xi g_X^2 w$$



$$= \frac{\mathrm{i}}{4}\xi g_X^2 w$$



$$= -\frac{\mathrm{i}}{4}\xi g_X^2 w$$

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