APPROXIMATION OF ATTAINABLE LANDING AREA OF A MOON LANDER BY REACHABILITY ANALYSIS

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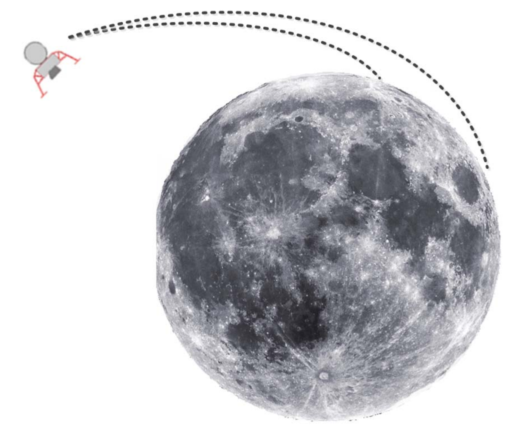
Introduction

- Developments in space technology have paved the way for more challenging missions which require **advanced guidance** and control algorithms for safely and autonomously landing on celestial bodies.
- Instant determination of hazards, automatic guidance during landing maneuvers and likelihood maximization of a safe landing are of paramount importance.
- **Reachability analysis** is used to obtain attainable landing areas for the final phase of interplanetary space missions given initial conditions, admissible control inputs and landing constraints.



Determination of Attainable Landing Area by Forward Reachability

Problem Statement

Equations of motion of the moon lander are taken from [1]. The vector of states and control inputs are defined as follows:

$$\mathbf{x}(t) = (\dot{d}, \dot{h}, \dot{c}, d, h, c, \beta, \chi, m)^T \qquad \mathbf{u}(t) = (T_u, T_s, T_q, \omega_\beta, \omega_\chi)^T$$

$\dot{d}$: Downrange Rate	d: Downrange	β: Pitch
$\dot{h}$: Altitude Rate	h: Altitude	χ: Yaw
$\dot{c}$: Crossrange Rate	c: Crossrange	m: Mass

T_u, T_s, T_q: RCS Thrusters
ω_β: Pitch Rate
ω_χ: Yaw Rate

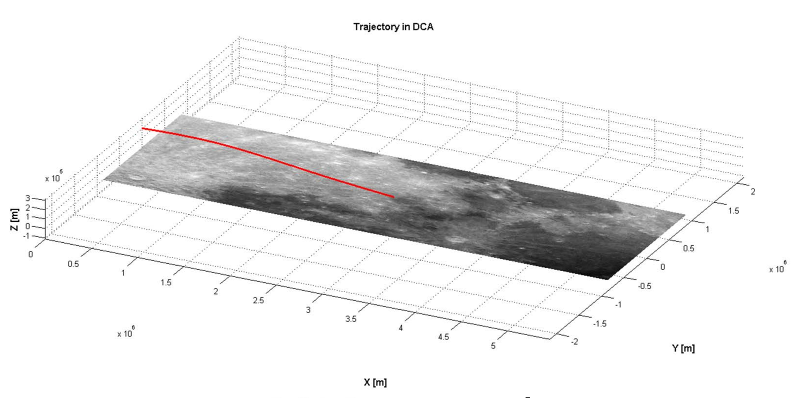
Initial condition and terminal conditions:

$$\mathbf{x}(0) = (\dot{d}_0, \dot{h}_0, \dot{c}_0, 0, h_0, 0, \beta_0, \chi_0, m_0)^T$$
$$\mathbf{x}(t_f) = (0,0,0, free, 0, free, -\frac{\pi}{2}, 0, free)^T$$

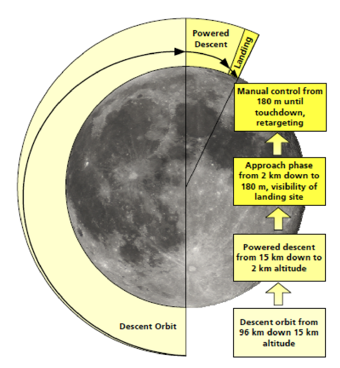
Condition for successful landing:

$$|\Delta x(t_f)| \le \Delta x_{max}$$
$$\Delta x_{max} = (1\,m/s\; 1\,m/s, 1\,m/s, free, 1m, free, 10°, 180°, free)^T$$

Reference Frame: Downrange-Crossrange-Altitude (DCA)



DCA Representation

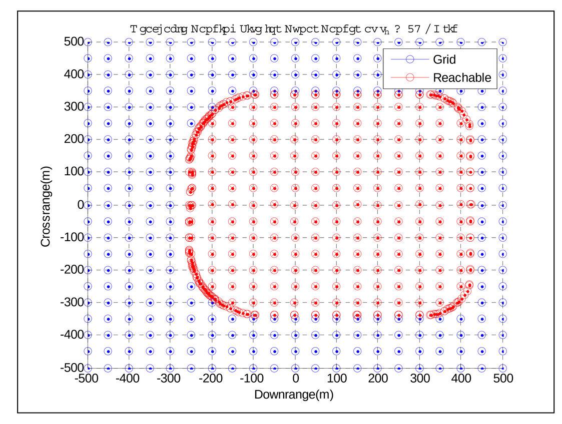


Generic Mission Profile for Lunar Lander

Method

In this study, we apply an **optimal-control-based algorithm** for approximating **nonconvex** reachable sets of **nonlinear** systems [2].

- Discretize region of interest
- Find optimal control law that steers the system from the initial condition to the target state
- Approximate the reachable set with an error of discretization step by solving following optimal control problem (OCP) for each grid points



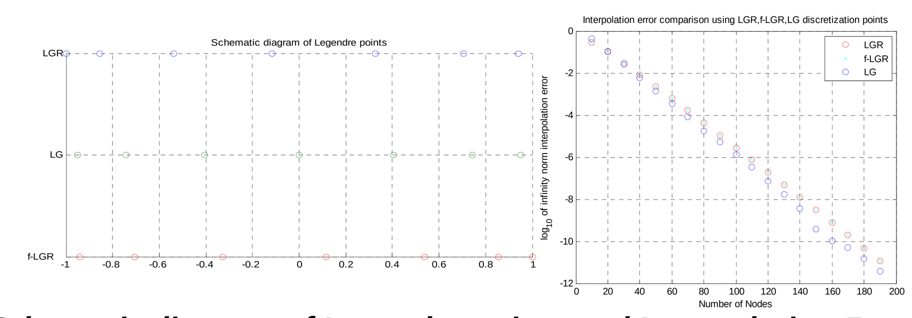
Discretization of State Space

$$\text{Min} \quad \frac{1}{2}\|x(t_f) - g_h\|_2^2$$
$$\text{s.t. } \dot{x}(t) = f(x(t), u(t)) \quad \text{a.e. in } [t_0, t_f]$$
$$x(t_0) = x_0$$
$$u(t) \in U_0 \text{ a.e. in } [t_0, t_f]$$

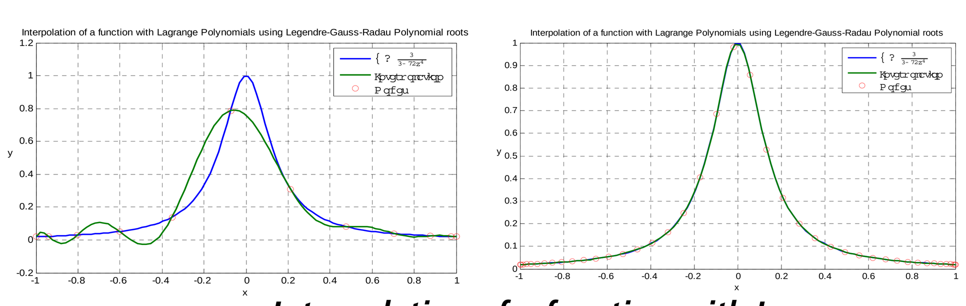
Discretization of Optimal Control Problem

OCP is transcribed into NLP by SPARTAN (SHEFEX-3 Pseudospectral Algorithm for Reentry Trajectory ANalysis) [3].

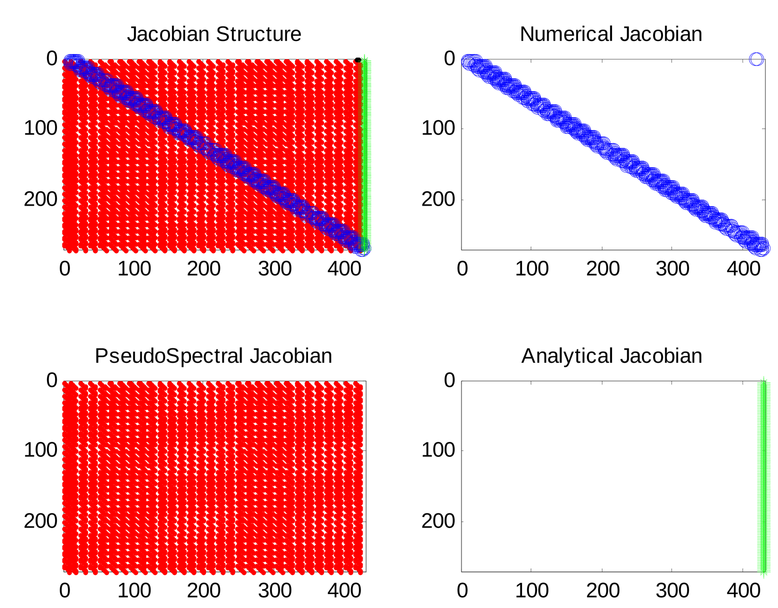
- Non-uniform collocation points to avoid the Runge phenomenon
- Exponential convergence w.r.t. number of collocation points
- Lagrange polynomials for interpolation of states,control inputs
- Jacobian of the associated NLP J is expressed as sum of 3 different contributors. Resulting sparse J is used by commercial off-the-shelf SQP solver.



Schematic diagram of Legendre points and Interpolation Error



Interpolation of a function with Lagrange polynomials using LGR roots



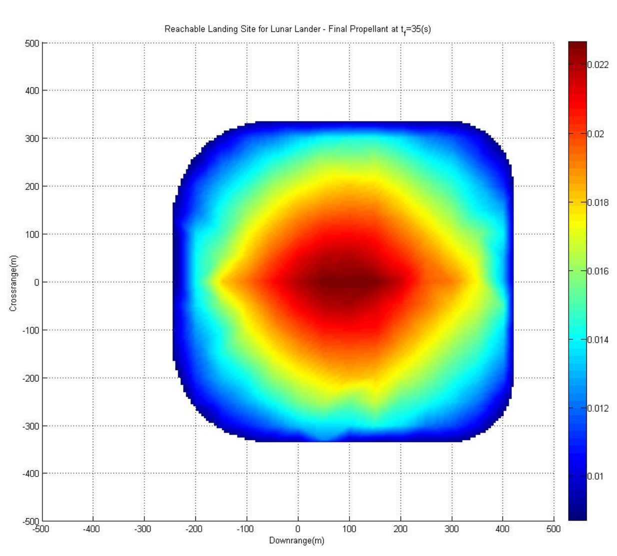
Jacobian structure of associated NLP

$$\dim(J) = [n(n_s + n_g) + 1] \times [(n+1)n_s + n n_c + 1]$$

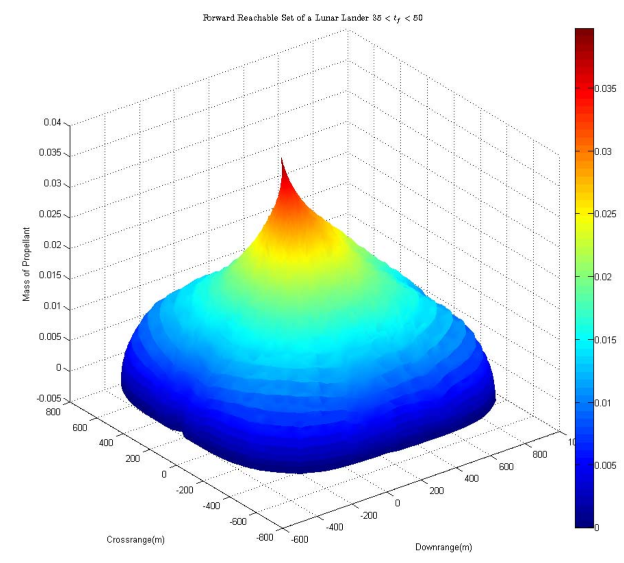
n : Number of discretization points	n_c: Number of control Inputs
n_s: Number of states	n_g: Number of constraints

Results

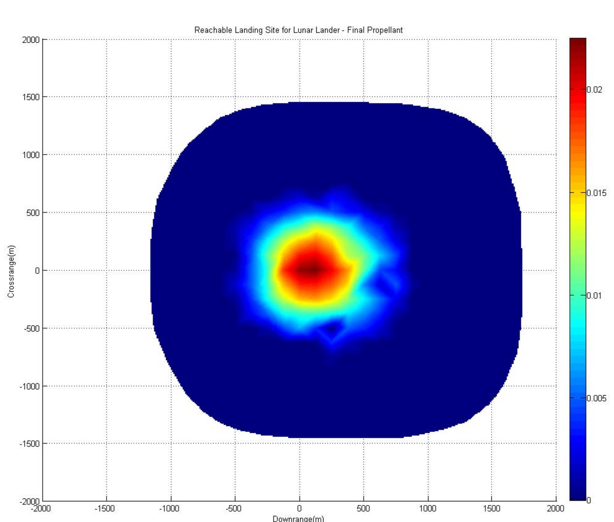
- **Attainable landing area** and **propellant&time** cost map of associated region is computed using reachability analysis



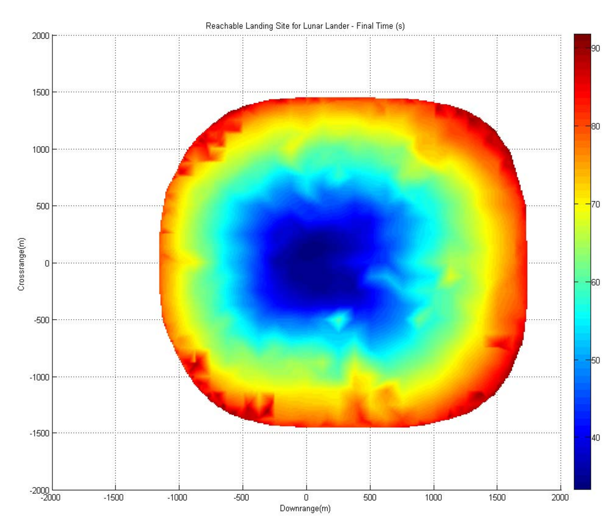
Reachable Set at t_f=35 (s) -Propellant Cost-



Reachable Landing Tunnel t_f<35 (s) -Propellant Cost-



Reachable Set with free final time -Propellant Cost-



Reachable Set with free final time -Time Cost-

References:

[1] *T. Oehlschlägel, S. Theil, H. Krüger, M. Knauer, J. Tietjen, C. Büskens, Optimal Guidance and Control of Lunar Landers with Non-throttable Main Engine, Advances in Aerospace Guidance, Navigation and Control, pp. 451-463, 2011*

[2] *R.Baier, M. Gerdts, I. Xausa, Approximation of Reachable Sets Using Optimal Control Algorithms, Numerical Algebra; Control and Optimization, Vol. 3, 2013*

[3] *M. Sagliano, S. Theil, Hybrid Jacobian Computation for Fast Optimal Trajectories Generation, AIAA Guidance, Navigation and Control(GNC) Conference, Boston, August 19-22,2013*

Acknowledgements:

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