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Endogenous Network Effects, Platform Pricing and Market Liquidity

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Abstract

This paper examines a monopoly platform's two-sided pricing strategies in a setting with seller competition, which gives rise to not only positive cross-side network effects between buyers and sellers, but also a negative same-side network effect among sellers. We show that platform pricing depends crucially on the characteristics associated with market liquidity, which contrasts with the previous studies that point to the two sides' relative demand elasticities and/or network effects. A market is said to be more liquid when it has less friction, resulting in a larger total surplus for the platform economy and hence greater equilibrium entry on both sides. We find that in response to higher market liquidity, the platform raises the buyer entry fee and lowers the seller entry fee. The platform's subsidy strategy is consistent: market liquidity is conducive to seller subsidy but hinders buyer subsidy.

Keywords: *Two-sided platforms, network effects, market liquidity, subsidy, product differentiation*

JEL codes: L1, L11, L22, L86

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1 Introduction

A two-sided platform is a market where two distinctive groups of users transact such that each member benefits more from the transactions when more members from the other group are using the platform (Rysman 2009). For example, Microsoft Xbox is a gaming platform where gamers and game developers trade. The more game titles that developers provide, the more enjoyment a gamer derives from using Xbox; conversely, the more consumers purchasing Xbox consoles, the more profit a game developer can make. Another example is Airbnb which is a platform that joins hosts and guests for renting living spaces. Either side of Airbnb users derives more benefit when the opposite side becomes larger in size. A variety of other platforms are observed in practice and discussed in the literature (Rochet and Tirole 2003, Parker and Van Alstyne 2005).

A privately-owned platform needs to determine how to charge the two sides of users appropriately, as optimal pricing to the two sides are interconnected. A lower fee on one side can increase the network size of that side, which allows the platform to charge a higher fee on the other side as a result of the cross-side network effect. This leads to the key questions that have received much attention in the literature:[1] Under what condition should a platform subsidize one side by charging a below-cost fee and gain revenues from the other side? Which side to subsidize? How to adjust the two fees simultaneously in response to a parameter change?

The established answer to these questions is that a platform's optimal fee structure, in particular which side to subsidize, depends crucially on a comparison between the two sides' demand elasticity and/or network externalities. In their survey paper, Rochet and Tirole's (2006) summarized that "attracting one side by lowering price is particularly profitable for the platform if this side creates substantial externalities on the other side." Armstrong (2006) shows a consistent insight: "It is possible that the profit-maximizing outcome involves group 1, say, being offered a subsidized service... [T]his occurs if the group's elasticity of demand is high and/or the external benefit enjoyed by group 2 is large."

Although these conditions describe the equilibrium elegantly, applying them can be difficult because the key variables for making subsidy decisions—network effects and demand elasticities—are likely to be endogenous. For example, the magnitude of cross-side network effect can be sensitive to the numbers of users on both sides. When a side is increasingly crowded, each user on that side

[1] The pioneering studies of two-sided platforms such as Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Parker and Van Alstyne (2005), and Armstrong (2006) have devoted much attention to the pricing questions (see also the survey by Rysman 2009). In fact, Rochet and Tirole (2003) have gone so far as to regard the possibility of cross-subsidy as the defining feature of two-sided markets: "A market with network externalities is a two-sided market if platforms can effectively cross-subsidize between different categories of end users that are parties to a transaction." They went on to highlight the central question faced by platforms: "The platform owners in all these industries devote much attention to their price-allocation business model: Is it more important to woo one side or the other?"

may make a smaller contribution to the other side. If that is the case, bringing an additional user on the crowded side is unlikely to have a strong cross-side impact. Similarly, demand elasticity is likely to depend on the platform's equilibrium prices unless users' entry costs follow a specific distribution. Because endogenous variables are involved, often with the need to compare two or more endogenous variables, the subsidy conditions commonly accepted in the literature are hard to evaluate.

Our research intends to address this difficulty by endogenizing network effects and expressing the equilibrium conditions solely in exogenous parameters. We build a standard pricing model of monopoly platform, which is assumed to be the only place where buyers and sellers can trade. Each side's trading choices are explicitly analyzed based on market conditions and user and product characteristics. This allows cross-side network effects to arise endogenously in both directions. Other variables such as network sizes and demand elasticities are also endogenously determined.

By endogenizing users' trading decisions, we illustrate that network effects exists not only across the two sides, but also within the same side among sellers. In our model, sellers compete in price; a seller's profit drops when more sellers enter the platform. We refer to this property as negative same-side network effect. Most studies of two-sided platforms have assumed away user interactions within the same side (Caillaud and Jullien 2003, Rochet and Tirole 2003, 2006, Armstrong 2006), but as we will explain, same-side network effect introduces asymmetry between the two sides, leading to different conclusions and new insights for platform pricing. Due to the very two-sidedness of the platform business model, when a parameter changes to encourage the entry on one side, in equilibrium it will also encourage the entry on the other side. If rivalry mainly exists among sellers but not among buyers, a larger size of sellers benefits buyers but hurts sellers, while a larger size of buyers benefits sellers without hurting buyers. The joint effect is that the total surplus is increasingly generated from the buyer side as the platform network expands. This would have direct implications for the platform's optimal fee structures.

The most important finding arising from our endogenization exercise is that the platform's optimal pricing decisions depends critically on market liquidity. A market is said to be more liquid when it has less friction, resulting in a larger total surplus for the platform economy and hence greater equilibrium entry on both sides. In our model, greater liquidity may result from a larger number of potential users, lower entry costs, stronger buyer preference for product differentiation, and product quality that varies more and/or rises on average. By raising the total trading surplus, these factors lead to expanded entry scale on both sides in equilibrium.

We find that when the market is more liquid, the platform should raise buyer fees and reduce seller fees; when the market is less liquid, the platform does the opposite. The results hold even when a fee becomes negative, i.e., the platform is more likely to subsidize sellers in a more liquid market, and subsidize buyers in a less liquid market. The intuition can be understood as follows. A marginal cut of the seller fee reduces the platform's revenue from sellers, but the resulting greater number of sellers benefits buyers, which allows the platform to raise the buyer fee and collect more revenues there. A seller fee is optimal when the platform's buyer-side gain exactly offsets its seller-side loss. If the market becomes more liquid, the network sizes of sellers and buyers both increase. However, due to the asymmetric same-side effect—the rivalry among sellers but not among buyers—the buyer-side gain overweights the seller-side loss, making it optimal for the platform to reduce the seller fee and raise the buyer fee.

Our central finding has two important and closely related implications. First, platform pricing is related to characteristics of the market, i.e., the two sides combined, rather than each side considered in separation or a comparison between the two sides. As a result, how a parameter affects the optimal fee structure depends on how it affects liquidity (whether total surplus increases or decreases) regardless of the channel through which it exerts the impact (whether from the buyer or seller side). Second, the platform's response to increased liquidity is to favor sellers: the seller fee is reduced while the buyer fee is raised. Such discrimination between buyers and sellers does not arise in most existing papers, where subsidy is explained by greater demand elasticity or network externality on a particular side, so a side-specific response is linked to a side-specific parameter change. But as we have argued above, the platform should pay attention to the collective property of the two sides rather than a comparison between them. If so, what distinguishes between buyers and sellers when the platform decides which fee to raise and which fee to lower? Our results demonstrate that buyers and sellers may differ in important aspects. In our model, the difference is caused by negative same-side effect among sellers but not buyers. Given the ubiquitous price competition in real life, this is a reasonable assumption.[2] More importantly, it demonstrates the role of side asymmetry in platform pricing.

Our finding is different from the established conclusions, which usually "obtain comparative statics results that fit with standard intuition." (Rochet and Tirole 2006, p.659). An example concerns the impact of an exogenous shift of a side's demand function.[3] We show that a drop in

[2]Rivalry among sellers is often inevitable, whereas that among buyers is not always present or is likely to be weaker than seller-side rivalry. There may even be positive network effect among buyers, as examined in studies of traditional one-sided network effects (Katz and Shapiro 1985, 1986; Farrell and Saloner 1985, 1986).

[3]Another example involves captive users. Rochet and Tirole's (2006) survey cited their own study (Rochet and Tirole 2003): "[A] factor affecting elasticities on a given side is the size of the installed base of end-users on that side. When the number of, say, captive buyers increases, the buyer price naturally increases, and the seller price decreases as attracting sellers yields a higher collateral profit on the buyer's side." Our model would predict the same effect (higher buyer fee and lower seller fee) by captive buyers, but for a different reason (captive buyers increase market liquidity). Since Rochet and Tirole's (2003) two sides are symmetric, they would have predicted that captive sellers have the opposite effect (higher seller fee and lower buyer fee), but our model would predict that captive sellers have the same effect (higher buyer fee and lower seller fee).

seller entry cost induces the platform to reduce the seller fee (and raise the buyer fee). This result is counter intuitive, as standard (one-sided) monopoly pricing would suggest that a decrease in entry cost shifts up the demand function, so the platform should increase the seller fee. Viewed from the lens of market liquidity, however, our result is easy to understand: The seller fee drops because a lower entry cost on the seller side increases market liquidity. In comparison, Rochet and Tirole (2003) concludes that when the seller demand shifts up (due to, say, a larger seller surplus caused by the presence of marquee buyers), the seller price will increase while the buyer price will decrease. Similarly, Armstrong (2006) would have suggested that a drop in seller-side cost and a drop in buyer-side cost should pull the platform's optimal pricing in opposite directions. However, we show that a drop in either side's entry cost has the same effect on the platform's pricing strategy. Our conclusions differ from these papers because the endogenization enables us to construct the two sides' demands that are truly interconnected,[4] and to derive equilibrium conditions completely devoid of any endogenous variables.

Our research makes three contributions. First, we provide new insights to an old question of what a platform should consider in its pricing strategies and how the two fees should be adjusted. At the root of these insights are two powerful ideas: the interconnection between the two sides and the possible asymmetry between the two sides. Given the very nature of two-sided markets, the interconnection should be expected, and yet most papers have not fully accounted for it. While exactly how the two sides differ may depend on empirical relevance and modeling techniques, we show that the presence of side asymmetry can lead to very different conclusions in two-sided platforms. Second, we demonstrate the importance of endogenizing network effects, i.e., to derive user surpluses from first principles rather than from reduced-form assumptions. Endogenization has at least three benefits: all endogenous variables are excluded from equilibrium conditions so that the conclusions can be more definite; the two sides are connected in a natural and coherent way; same-side interactions can no longer be ignored. The last benefit is particularly useful because it is a natural way to generate side asymmetry, as same-side interactions can differ between the two sides in sign and/or magnitude, while cross-side effects are generally symmetric. Third, our findings are directly useful to practitioners. Containing no endogenous variables, our conditions (both necessary and sufficient) are expressed in six parameters with well defined economic meaning, and the conclusions are definite in terms of which fee to raise and which fee to reduce. Such comparative statics results are directly testable and can provide useful guidance to managers.

[4] Rochet and Tirole (2003) set up their model in such a way that the connection between the two sides' demands is relatively weak: "The surplus of a buyer, $(b^B - p^B)N^S$, depends on the number of sellers N^S, but the buyer 'quasi-demand function', $N^B = \Pr(b^B \geq p^B) = D^B(p^B)$, is independent of the number of sellers." (p.995).

2 Related Literature

Most studies of platforms have assumed away same-side interactions and arrived at conclusions that treat the two sides symmetrically.[5] In a pioneering work, Rochet and Tirole (2003) study a platform's optimal per-transaction charges in a setting with pure usage externalities between two sides that display symmetric and independent demands. They find that the two optimal fees are proportional to their respective demand elasticities. Armstrong (2006) focuses on pure membership externalities. Based on implicit equilibrium price formula, he concludes that a platform is more likely to subsidize the side with greater demand elasticity and/or network externality. Rochet and Tirole (2006) survey the literature and provide a general model that encompasses both usage and membership externalities. They conclude that the literature obtained "comparative statics results that fit with standard intuition." (p.659). They also discuss an interesting property of platform pricing "in the form of a simple 'seesaw principle': a factor that is conducive to a high price on one side, to the extent that it raises the platform's margin on that side, tends also to call for a low price on the other side as attracting members on that other side becomes more profitable." (p.659). Our findings confirm the seesaw principle despite many differences in modeling and conclusion. While the above three papers start at the utility level, Parker and Van Alstyne (2005) introduce exogenous cross-side network effects at the level of reduced-form demand functions. They show that which side is subsidized "depends on cross-price elasticities as well as the relative sizes of the two-sided network effects." (p.1496).

Several more recent studies have started to explore the asymmetry between the two sides of a platform. In the context of payment card platform, Wright (2012) points out an inherent asymmetry as the explanation of why retailers are charged too much: Through their endogenous pricing, merchants internalizes the benefit that cardholders enjoy when joining the platform, but consumers ignore such cross-side benefits. Baye and Morgan (2001) investigate sellers' incentive to advertise their prices on a platform ("information gatekeeper") and buyers' incentive to subscribe to the price information. They show that the platform tends to charge a relatively low fee to buyers and a relatively high fee to sellers. For both sides, the alternative to making use of the platform is to transact without the price information, so the benefit of joining and not joining the platform are both endogenized. This gives rise to an asymmetry between the two sides: price competition on

[5] A major purpose of many studies is to investigate the equilibrium and welfare consequences of platform competition. Since our paper deals only with a monopoly platform, we will focus on the monopoly part of the major papers in our review of the literature. Caillaud and Jullien (2003) suggested that an entrant intermediation service provider may successfully challenge an incumbent by charging a low membership fee to win the competition, while charging a high transaction fee to make profit. Similarly, when two platforms compete for multi-sided users with sequential pricing, Jullien (2011) demonstrated that the second mover can always find some weaker fronts in the first mover's pricing and win these customers with subsidies. Rysman (2009) surveyed many issues of two-sided platforms including pricing, price competition, openness, and public policy.

the seller side and free-riding problem on the buyer side. As a result of the asymmetry, a larger number of buyers intensifies seller competition, which in turn makes it more valuable for a buyer to stay off the platform.

One way to generate asymmetry is to introduce same-side network effect to a particular side. Some of the seminal papers mentioned above have briefly discussed the possibility of introducing the same-side effect.[6] In a recent work, Hagiu (2009) assumes that each seller's surplus decreases with the number of sellers. He finds that when consumers have stronger preference for variety, a platform increasingly relies on the seller side in generating revenue. While Hagiu looks at the distribution of total revenues between the two sides, we focus on per-user charges. In his model each consumer buys every product on the platform, so seller competition is not as fierce as in our location model, where each buyer buys only one product.

Some papers have explored other consequences of same-side rivalry. Ellison and Fudenberg (2003) and Ellison et al. (2004) both show that when platforms compete, negative same-side effect may serve as a counter force to the increasing returns effect that favors concentration. As a result, it is possible to have two active platforms coexisting in equilibrium. Belleflamme and Toulemonde (2009) show that when same-side negative externalities are either very strong or very weak relative to the cross-side positive externalities, a new platform can win customers from an existing platform with fees and subsidies. Lin et al. (2011) allow vertically differentiated sellers to compete on the platform and find that subsidizing buyers may be optimal if the buyers' willingness to pay for quality is sufficiently dispersed, as more variation in the buyers' quality preferences can mitigate the seller competition and enable the platform to generate revenues on the seller side.

3 Model Setup

Three types of players make decisions in this game: (the owner of) a platform, N potential sellers, and z potential buyers. Sellers offer products that buyers desire, but the two groups must use the platform to trade. The platform charges each seller and buyer an entry fee, denoted as R_s and R_b, respectively.[7] In addition to the entry fees, sellers and buyers incur entry costs. Buyers

[6] Rochet and Tirole (2006, p.664): "the independence of B^i relative to N^i excludes same-side externalities. Consider for example a software platform with N^S application developers and N^B consumers. Then, $B^S = b^S(N^S)N^B$, with $\dot{b}^S < 0$ if the applications are substitutes (rivalry effects) and $\dot{b}^S > 0$ if the applications are complements." Armstrong (2006): "In some contexts, there are intragroup externalities present, and a group-2 agent might be better off if there were fewer other group-2 agents on the same platform, in which case the formula (24) is not valid." (p.679). "A plausible hypothesis is: platforms will allow competition within the platform if consumers can be charged for entry, but if for exogenous reasons consumers have free entry, then platforms will restrict competition to drive up the revenues obtained from retailers." (p.686).

[7] A more general pricing scheme, which is also very common in practice, is two-part tariff. In our model, however, buyers are assumed to have unit demand, in which case a two-part tariff is equivalent to a lump sum entry fee. In other words, our model is set up in such a way that the platform displays membership externality but not usage

are heterogeneous in the entry cost, c, which follows a uniform distribution on $[0, C]$. We assume that all sellers have the same entry cost, f.[8] Entry fees and entry costs differ in three aspects. First, entry fee is a transfer payment between a user and the platform, whereas entry cost is a deadweight loss, to the three parties combined. Second, entry fees are endogenous and chosen by the platform, whereas entry costs are exogenous. Third, unlike positive entry costs, the entry fees can be negative, which suggests a subsidy provided by the platform.[9] Assume that the platform incurs a constant marginal cost in serving each buyer, and another constant marginal cost in serving each seller. Given that the two costs are constant, we may normalize them both to zero without any loss of generality. Under such normalization, a subsidy (i.e., a negative entry fee) simply means a below-cost entry fee.

Each seller offers only one product; thus, seller and product are conceptually equivalent, and seller heterogeneity and vertical differentiation are interchangeable. Products are assumed to differ both horizontally and vertically, which is modeled by introducing quality heterogeneity to Salop's (1979) circle. Horizontal differentiation is then represented by sellers' locations on the circle, while vertical differentiation is represented by quality heterogeneity among these sellers. Each seller's quality is modeled as an independent draw from an identical distribution on $[\underline{v}, \overline{v}]$ with mean μ and variance σ^2.

The game unfolds in three stages (Figure 1). In stage I, the platform sets the two entry fees R_s and R_b. In stage II, before qualities are realized, sellers and buyers simultaneously make entry decisions by paying their respective entry fees and incurring entry costs. Here we assume that sellers are uncertain about their quality before entering the platform. Many reasons, such as the serendipitous nature of R&D and unpredictable business environment, may lead to such uncertainties. Upon entry, sellers are located equidistantly based on a common expectation of quality,[10] and buyers are assigned uniformly on the circle. In stage III, sellers' quality levels are

externality (Armstrong 2006, Rochet and Tirole 2006). Rochet and Tirole (2006, p.651) have pointed out the relevance of membership externalities in many cases.

[8]Heterogeneous entry cost on the buyer side is assumed to ensure an interior solution of buyer entry scale, which will then adjust marginally as the buyer entry fee changes. Such heterogeneity is unnecessary on the seller side because a seller's expected profit decreases as more sellers enter the platform, which leads to an interior entry scale on the seller side even if their entry costs are identical, which we assume for simplicity. If sellers also differ in their entry costs, our findings continue to hold.

[9]Examples of entry fee may include the annual subscription fee the smartphone application developers pay to the application market owner (e.g., Apple) and the price of the smartphone paid by the users. When subsidizing the users, the platform sets the price of the smartphones such that they are sold at a loss. Entry cost can include learning cost. Application developers often incur learning cost when creating applications for a certain type of technology; similarly, users also face learning cost when adopting a new device.

[10]Typically, product features are determined first, and they reflect sellers' locations on the circle upon entry. Quality is revealed later after product is developed and introduced to the market. If product quality is known before entry, sellers may pay identical or different entry fees depending on the platform's information about each seller's quality and its capability of price discrimination. Furthermore, there is the question of how to model the location choice by heterogeneous sellers. All these issues are orthogonal to and non-essential for the questions that we study. For this

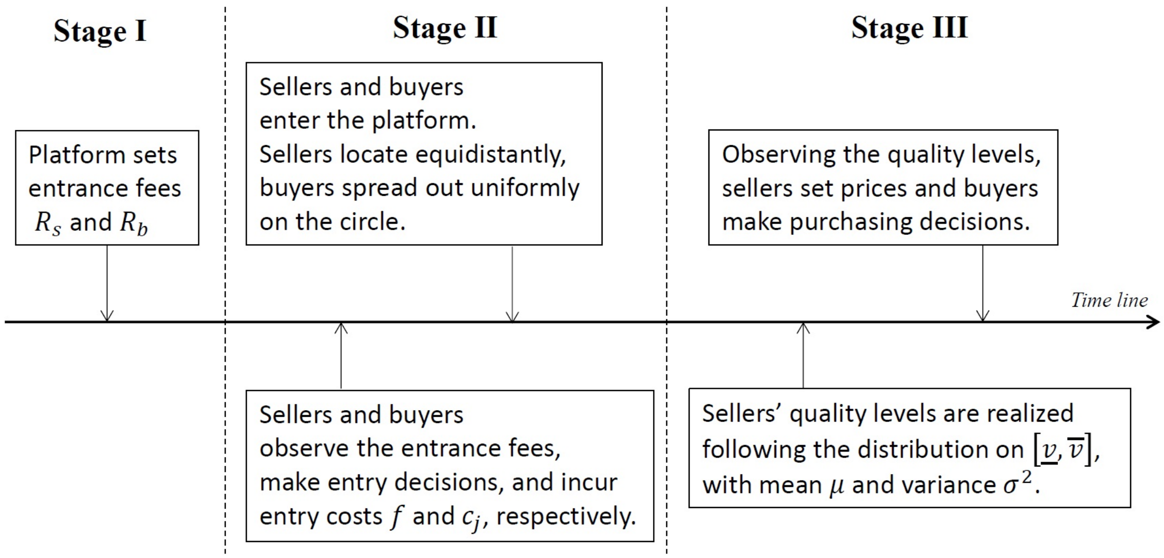

Figure 1: Timeline of the Game

realized and become public information, based on which the two sides trade. Sellers compete by setting prices, and buyers have unit demand. Buyer j receives surplus $v_i - p_i - td_{ij}$ by purchasing from seller i that has quality v_i and sets price p_i; d_{ij} is the distance between buyer j and seller i, and t is buyers' unit transportation cost. The distance can be interpreted as the degree of misfit between the buyer's horizontal preference and the seller's product. To ensure that all sellers obtain a positive market share (i.e., every seller sells to some buyers), we assume $\overline{v} - \underline{v} < \frac{t}{N}$, which is consistent with the condition for localized competition in Alderighi and Piga (2012) and Lin and Wu (2015).

4 Analysis

We solve the subgame-perfect Nash equilibrium by backward induction.

4.1 Stage III: Trading

We analyze the equilibrium trading decisions, taking the number of sellers and buyers who enter the platform, n_s and n_b, as given. We will focus on the equilibrium in which market is fully covered (i.e., every buyer buys from some seller). Without loss of generality, let the location of the ith seller

reason, we assume that quality is realized only after entry has taken place and locations have been chosen. This is a typical way of dealing with entry issues when users are uncertain about their herterogeneous trading surplus, and is widely used in the literature. See, for example, Ellison et al (2004), and Rochet and Tirole (2006, p.661).

be $\frac{i}{n_s}$ for $i = 0, 1, \cdots, n_s - 1$. A buyer at location x is indifferent between buying from either of the two nearest sellers indexed by i and $i+1$, if $v_i - p_i - t(x - \frac{i}{n_s}) = v_{i+1} - p_{i+1} - t\left(\frac{i+1}{n_s} - x\right)$. The location of the marginal buyer is then solved as $x^*_{i,i+1} = \frac{(v_i - p_i) - (v_{i+1} - p_{i+1})}{2t} + \frac{i}{n_s} + \frac{1}{2n_s}$. Similarly, the location of the marginal buyer between sellers i and $i-1$ is $x^*_{i-1,i} = \frac{(v_{i-1} - p_{i-1}) - (v_i - p_i)}{2t} + \frac{i-1}{n_s} + \frac{1}{2n_s}$. Then seller i's quantity sold is:

$$q_i = n_b(x^*_{i,i+1} - x^*_{i-1,i}) = n_b\left[\frac{1}{2t}(2v_i - v_{i+1} - v_{i-1} - 2p_i + p_{i+1} + p_{i-1}) + \frac{1}{n_s}\right].$$

Given that seller i's revenue is $\pi_i = q_i p_i$, the first-order condition (FOC) with respect to p_i gives,

$$p_i = \frac{2v_i - (v_{i+1} - p_{i+1}) - (v_{i-1} - p_{i-1})}{4} + \frac{t}{2n_s}.$$

From here, we obtain $q_i = n_b\left[\frac{2v_i - (v_{i+1} - p_{i+1}) - (v_{i-1} - p_{i-1})}{2t} + \frac{1}{n_s} - \frac{p_i}{t}\right] = \frac{n_b p_i}{t}$.

The optimal price of seller i depends on the prices charged by its two neighboring sellers, $i-1$ and $i+1$. In equilibrium, the prices of all n_s sellers must be solved simultaneously. Closed-form solution of the equilibrium price is derived as follows.

Lemma 1 *Given the number of buyers and sellers and the realization of qualities of all sellers, the equilibrium price for seller i is*

$$p^*_i = \frac{t}{n_s} + v_i - \sum_{j=0}^{n_s-1} b_j v_{i-j}, \tag{1}$$

where $b_j = \frac{\delta^{n_s-j} + \delta^j}{\sqrt{3}(\delta^{n_s} - 1)}$ and $\delta = 2 + \sqrt{3}$. Seller i's equilibrium quantity is then $q_i = \frac{n_b}{t} p^*_i$ and revenue is $\pi_i = \frac{n_b}{t} p^{*2}_i$.

Proof. The proof of Lemmas 1, 2, and 3 are collected in the technical appendix. All the other proofs can be found at the end of this paper. ■

4.2 Stage II: Entry

Seller i's expected profit from entering the platform is $E(\pi_i) - R_s - f$, where $E(\pi_i)$ is its expected trading surplus (or expected revenue) gross of the entry fee and entry cost. For simplicity, we refer to $E(\pi_i)$ as a seller's *expected surplus* in the rest of the paper. Since all sellers are ex ante identical, they have equal expected surplus, denoted by $E(\pi)$.

Lemma 2 *A seller's expected surplus is:*

$$E(\pi) = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \frac{\sigma^2}{n_s} g_s(n_s)\right], \tag{2}$$

where $g_s(n_s) = n_s\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) > 0$. *The seller surplus displays a positive cross-side network effect from buyers to sellers (i.e.,* $\frac{\partial E(\pi)}{\partial n_b} > 0$*), and a negative same-side network effect among sellers (i.e.,* $\frac{\partial E(\pi)}{\partial n_s} < 0$*).*

Two-sided markets are characterized by cross-side network effect, i.e., greater participation on one side increases the surplus of participating on the other side. Such effect is given exogenously in most studies. Here, we generate it endogenously by modeling the two sides' trading decisions. The endogenization also reveals something that is missing in many studies: A negative same-side network effect arises among sellers, as a seller's expected surplus decreases when more sellers enter the platform. In reality, many two-sided markets involve price competition among sellers, so seller rivalry is inevitable. As will be clear later, this negative same-side network effect plays a crucial role in the asymmetry between sellers and buyers and, therefore, in the platform's pricing strategies.

If buyer j enters the platform, her expected payoff is $E(u) - R_b - c_j$, where c_j is her entry cost, and $E(u)$ is her expected surplus from trading on the platform, with the expectation taken on her location and the product quality of the seller from whom she makes the purchase.

Lemma 3 *A buyer's expected surplus is:*

$$E(u) = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}g_b(n_s), \tag{3}$$

where $g_b(n_s) = n_s\left(\frac{1}{6\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} - \frac{n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) > 0$. *The buyer surplus displays a positive cross-side network effect from sellers to buyers (i.e.,* $\frac{\partial E(u)}{\partial n_s} > 0$*), but no direct same-side network effect exists among buyers (i.e.,* $\frac{\partial E(u)}{\partial n_b} = 0$*).*

A buyer's surplus increases with the number of sellers, indicating a positive cross-side network effect in the reverse direction of that in Lemma 2. The manifestation of positive cross-side network effects in both directions echoes the standard network effect assumptions. However, unlike the usual assumption, the effect is non-linear, indicating that the cross-side network effect cannot be captured by one single parameter. Rather, it depends endogenously on a whole set of parameters in a non-linear fashion, which in turn would directly affect the platform's optimal pricing strategies. Also note that the buyer surplus is independent of the number of buyers, showing no direct same-side network effect among buyers. This property is a direct consequence of our assumption of constant production cost by the sellers, so that when more buyers enter the circle, each seller simply increases its quantity supplied without having to raise the equilibrium price. In reality, rivalry among buyers may not exist (consider the example of yellow page users); even when it does, is usually less severe than that among sellers. There are also many cases of positive network effect among buyers (Katz and Shapiro 1985), e.g., networked video games, group-buying, and customer reviews. Such positive network effect among buyers is likely to only further sharpen the asymmetry between sellers and buyers.

Because $g_s(n_s) > 0$ and $g_b(n_s) > 0$, direct inspection of Eq. (2) and (3) reveals:

Corollary 1 $\frac{\partial E(\pi)}{\partial \sigma^2} > 0$ *and* $\frac{\partial E(u)}{\partial \sigma^2} > 0$*; that is, an increase in quality variation leads to higher surplus for both sellers and buyers.*

The corollary says that quality heterogeneity benefits both sellers and buyers. Fixing the number of users on the two sides, increased quality variation allows the higher-quality sellers to gain additional market share and charge higher prices; the opposite applies to the lower-quality sellers. Since the number of transactions made at a lower price is reduced, and those made at a higher price are raised, the total profit gain overweighs the total profit loss. Therefore, a seller's expected surplus increases with quality variation (i.e., $g_s(n_s) > 0$). Buyers' expected surplus also increases. Fixing the seller in a transaction, the buyer's expected surplus is not affected by the degree of quality variation. However, quality variation can enable a buyer to switch to a different seller in her equilibrium choice and raise her expected surplus. When quality varies more, switching is more likely, thus a buyer's expected surplus is higher (i.e., $g_b(n_s) > 0$).

The significance of Corollary 1 is to highlight the fact that some parameters may affect the two sides' surplus simultaneously. This would pose a particular challenge to the platform's two-sided pricing strategies: When σ^2 is larger so that both buyer surplus and seller surplus are greater, does the platform raise the entry fees on both sides? We will address this issue in Section 6.

Having derived sellers' and buyers' expected trading surplus, we are now ready to analyze the two sides' entry decisions. A seller enters the platform as long as his expected profit is non-negative. Therefore, the equilibrium number of the sellers on the platform, n_s, must satisfy the free-entry condition, $E(\pi) - R_s - f = 0$, or

$$R_s(n_s, n_b) = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \frac{\sigma^2}{n_s} g_s(n_s)\right] - f. \tag{4}$$

A buyer with entry cost c_j enters the platform if and only if her expected payoff is non-negative: $E(u) - R_b - c_j \geq 0$, or equivalently $c_j \leq E(u) - R_b$. If the equilibrium number of buyer entering the platform is n_b, the marginal buyer must have an entry cost of $c_b = \frac{C}{z} n_b$ given the uniform distribution of c_j. As a result, $R_b = E(u) - c_b$, or

$$R_b(n_s, n_b) = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t} g_b(n_s) - \frac{C}{z} n_b. \tag{5}$$

The two equations, (4) and (5), can be viewed as the demand function on the two sides, in the same spirit as in Armstrong (2006).

4.3 Stage I: Platform Pricing

The platform chooses R_s and R_b to maximize its total profit $\Pi = n_s R_s + n_b R_b$. Mathematically, this is equivalent to choosing n_s and n_b, with R_s and R_b being expressed as functions of n_s and n_b as in Eq. (4) and (5). Then the platform's optimization problem becomes:

$$\max_{n_s, n_b} \Pi(n_s, n_b) = n_s R_s(n_s, n_b) + n_b R_b(n_s, n_b). \tag{6}$$

The FOCs with respect to n_s and n_b are:

$$\frac{\partial \Pi}{\partial n_s} = R_s^* + n_s \frac{\partial R_s}{\partial n_s} + n_b \frac{\partial R_b}{\partial n_s} = 0, \tag{7}$$

$$\frac{\partial \Pi}{\partial n_b} = R_b^* + n_b \frac{\partial R_b}{\partial n_b} + n_s \frac{\partial R_s}{\partial n_b} = 0. \tag{8}$$

Assume that the second-order conditions are satisfied and that a unique solution exists for the two equations.

5 Subsidy and Liquidity

In this section, we examine the platform's pricing strategies, in particular the conditions under which the platform subsidizes the entry of either side. Consider Eq. (7), which captures the effect on the platform's profit by admitting one more seller when the buyer-side entry scale is fixed. The effect consists of three parts. First, the platform can collect an entry fee from the newly admitted seller, R_s^* (which is negative if sellers are being subsidized). Second, to admit this additional seller, the platform must lower the seller entry fee, which applies to all existing sellers given the assumption that the platform cannot discriminate among sellers ($n_s \frac{\partial R_s}{\partial n_s} < 0$). We call this *the same-side loss* (SSL), which is the platform's loss of revenues on the side of additional entry. Third, all buyers benefit from the increased number of sellers; thus, the platform can raise the buyer-side fee ($n_b \frac{\partial R_b}{\partial n_s} > 0$) without losing any buyers. This is termed *the cross-side gain* (CSG), which is the platform's gain of revenues on the side *across* from the side of the additional entry. When the seller entry scale is optimal, the three effects must add up to zero. Therefore, the platform subsidizes sellers (i.e., $R_s^* < 0$) if and only if the same-side loss from the sellers is outweighed by the cross-side gain from the buyers. Similarly, the platform subsidizes buyers (i.e., $R_b^* < 0$) if and only if the same-side loss from the buyers is outweighed by the cross-side gain from the sellers.

We now derive the specific conditions for subsidizing sellers and buyers. Given the complex forms of $g_s(n_s)$ and $g_b(n_s)$, approximation is needed to simplify these expressions to achieve analytical tractability.[11] Let $\delta_2 = 1 - \frac{4}{3\sqrt{3}}$, $\delta_3 = \frac{1}{6\sqrt{3}}$, and $\delta_1 = \delta_2 + \delta_3 = 1 - \frac{7}{6\sqrt{3}}$. Then with approximation,

[11] When n_s is not too small, $\frac{\delta^{n_s}+1}{\delta^{n_s}-1} \approx 1$ and $\frac{n_s \delta^{n_s}}{(\delta^{n_s}-1)^2} \approx 0$ (recall that $\delta = 2 + \sqrt{3} \approx 3.73$). See Appendix B for

$E(\pi) \approx \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \delta_2\sigma^2\right]$ and $E(u) \approx \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s$. Given that $R_s = E(\pi) - f$ and $R_b = E(u) - \frac{C}{z}n_b$, the platform's profit function, $\Pi = n_s R_s + n_b R_b$, becomes

$$\Pi = n_b\mu - \frac{t}{4}\frac{n_b}{n_s} + \delta_1\frac{\sigma^2}{t}n_s n_b - n_s f - \frac{C}{z}n_b^2. \tag{9}$$

The two first-order conditions are derived:

$$\frac{\partial\Pi}{\partial n_s} \equiv 0 \text{ leads to } n_b = \frac{4ftn_s^2}{t^2 + 4\delta_1\sigma^2 n_s^2}, \tag{10}$$

$$\frac{\partial\Pi}{\partial n_b} \equiv 0 \text{ leads to } n_b = \frac{z}{8tC}\frac{4\delta_1\sigma^2 n_s^2 + 4\mu t n_s - t^2}{n_s}. \tag{11}$$

We are now ready to report the first major result of our paper:

Proposition 1 *The platform subsidizes sellers' entry ($R_s^* < 0$) if and only if*

$$\frac{Ctf}{z} < \sigma^2(\delta_5\mu + \delta_6\sigma), \tag{12}$$

where δ_5 and δ_6 are positive constants. The platform subsidizes buyers' entry ($R_b^* < 0$) if and only if

$$\frac{Ctf}{z} > \delta_7(\mu + \theta)(\mu - \delta_8\theta)(\mu - \delta_9\theta), \tag{13}$$

where $\theta = \sqrt{\mu^2 - 9\delta_3\sigma^2}$, and δ_7, δ_8, and δ_9 are also positive constants.

The proposition identifies a sufficient and necessary condition for subsidizing sellers and that for subsidizing buyers. These conditions imply that seller subsidy is more likely if entry on either side is less costly (C or f is lower), buyers have weaker horizontal preferences (t is lower), more potential buyers are in the market (z is higher), the products have higher average quality (μ is higher), or they have greater quality variation (σ^2 is higher).[12] All these characteristics describe a *liquid* market, which we define as a setting where interactions on the platform generate a large total surplus so that many buyers and sellers enter the platform in equilibrium. Conversely, buyer subsidy is more likely under the opposite conditions, corresponding to an *illiquid* market where few buyers and sellers enter the platform in equilibrium. Proposition 1 states that the platform should subsidize sellers when facing a sufficiently liquid market, and subsidize buyers when facing a sufficiently illiquid market.

more details. Numerical calculations indicate that there is virtually no loss of accuracy from approximation in the derivation of the results. Also note that since $g_s(n_s)$ and $g_b(n_s)$ are coefficients of σ^2, the approximation makes little difference if σ^2 is small.

[12] We can show that the right hand side of Conditions (12) and (13) increases with both μ and σ^2. For the impact of the transportation unit cost, t, note that t measures the mismatch between a seller's product and its buyers' horizontal preference and is, therefore, a deadweight loss to the transactions. A weaker horizontal preference (in the sense of a smaller t) means greater surpluses for both buyers and sellers.

To understand the intuition, recall that users on one side is subsidized if and only if the same-side loss (SSL) from admitting one more user on that side is outweighed by the cross-side gain (CSG). In a liquid market with many sellers and many buyers, seller competition is sufficiently intense, such that it does not further intensify substantially if the platform brings in one more seller. Therefore, admitting another seller will result in a relatively small SSL from the seller side when many sellers already are on the platform. Meanwhile, the entry of the additional seller benefits every buyer through intensified seller competition. Because all buyers receive the benefit, the total CSG tends to be large when there are many buyers. In such a liquid market, therefore, bringing in an additional seller tends to generates a CSG that is greater than the SSL, which means the platform should subsidize sellers.

Now consider the opposite case of an illiquid market with few buyers and sellers. Since an additional buyer has no direct impact on the other buyers, the platform does not need to lower the buyer fee substantially. In other words, the SSL from the reduction in the buyer fee is less severe if fewer buyers are on the platform. At the same time, the CSG is greater if there are fewer sellers, as their competition is less intense, which helps to generate greater gains from the increased number of buyers. Therefore, in a more illiquid market, it is more likely for the additional buyer to generate a CSG that is greater than the SSL, which means it is optimal for the platform to subsidize buyers.

Proposition 1 highlights the connection between the platform's subsidy policy and market liquidity. At the root of the connection is the fundamental asymmetry between sellers and buyers: rivalry exists among sellers, but not among buyers. Intense seller competition leads to higher buyer-side surplus; mitigated seller competition helps to generate seller-side surplus. To subsidize any side, the platform must expect to more than recover the subsidies by gaining from the opposite side. In a liquid market with many sellers and many buyers, the platform mainly relies on the buyer side to generate its revenue. Then subsidizing sellers makes sense, as the expanded seller network benefits the buyers. And the same applies conversely.

The conditions for subsidizing sellers and buyers are clearly reversed: An increase in any of the six parameters is conducive to the subsidy of one side, and hinders the subsidy of the other side. Such asymmetry is curious because the well-known insight, that a platform is more inclined to subsidize the side that exerts a stronger cross-side network effect (Parker and Van Alstyne 2005, Armstrong 2006), does not distinguish between the two sides. When the magnitude of the network effect is endogenous, as in our model, any change in a parameter can alter the network effects on both sides. In that case, the previous findings based on relative network effects are ambiguous. In contrast, we are able to arrive at a definite conclusion through the endogenization. Our results also highlight the asymmetry between the two sides. This asymmetry can be traced to the negative same-side network effect resulting from sellers' price competition, which is not accounted for in many earlier works. To the extent that buyers usually exhibit a weaker degree of rivalry than sellers, we expect the asymmetry and the resulting connection between market liquidity and the platform's subsidy strategies to hold in a more general setting.

Due to the very nature of two-sidedness, whenever a condition encourages entry on one side, it is also conducive to the entry on the other side. It is therefore non-trivial to show that a parameter change, which moves the two sides' entry scales in the same direction, always causes their subsidies to move in opposite directions. Consider, for example, the role of entry costs in the platform's subsidy strategies. One may expect subsidy to mitigate the friction from entry cost, that is, an increase in the entry cost of any side should pull the optimality of subsidy to that side. That indeed would be the prediction when the network effect or demand elasticity is exogenous: A greater entry cost on one side tends to increase that side's demand elasticity, making it more likely for the platform to subsidize that side. An increase in the two sides' entry costs should therefore have opposite effects on the platform's subsidy choices. In contrast, we find that buyer and seller entry cost play the same role in the platform's subsidization strategy. A higher entry cost on either side encourages buyer subsidy, and a lower entry cost on either side encourages seller subsidy. Such result can be easily understood through the lens of liquidity. Entry costs on both sides have a consistent impact on the network size of the platform: When either cost is higher, the equilibrium entry scale on both sides shrinks, and the market becomes less liquid.

The subsidy conditions have been expressed in Eq. (12) and (13). They can be rearranged to highlight the role of σ^2 in relation to other parameters:[13]

Proposition 2 *The platform's subsidy strategies depend on quality heterogeneity in the following way:*

(1) There exists a unique $\sigma_s\left(\mu, \frac{Ctf}{z}\right) > 0$ *such that sellers are subsidized if and only if* $\sigma > \sigma_s$*. Furthermore,* σ_s *decreases with* μ *and increases with* $\frac{Ctf}{z}$*.*

(2) If $\frac{Ctf}{z} \leq \frac{16}{729}\mu^3$*, then buyers are never subsidized for any* σ*. If* $\frac{Ctf}{z} > \frac{16}{729}\mu^3$*, then there exists a unique* $\sigma_b\left(\mu, \frac{Ctf}{z}\right) > 0$ *such that buyers are subsidized if and only if* $\sigma < \sigma_b$*. Furthermore,* σ_b *decreases with* μ *and increases with* $\frac{Ctf}{z}$*.*

Proposition 2 says that quality variation is conducive to seller subsidy but hinders buyer subsidy. In particular, given any set of parameters C, t, f, z, and μ, the platform subsidizes sellers if and only if the quality variation is sufficiently large. The flip side is that the platform never subsidizes sellers if the quality varies little, including the case of no vertical differentiation ($\sigma^2 = 0$). Conversely, the platform subsidizes buyers only if the quality variation is sufficiently low. Note that some additional

[13] The coefficient $\frac{16}{729}$ in statement (2) is due to peculiar features of price competition on a circular city with quality variation.

conditions are needed for buyer subsidy. Even with zero quality variation, buyer subsidy would not be optimal unless the average product quality μ is sufficiently low and/or the entry and transaction are sufficiently costly (C, t, and f cannot be too low).

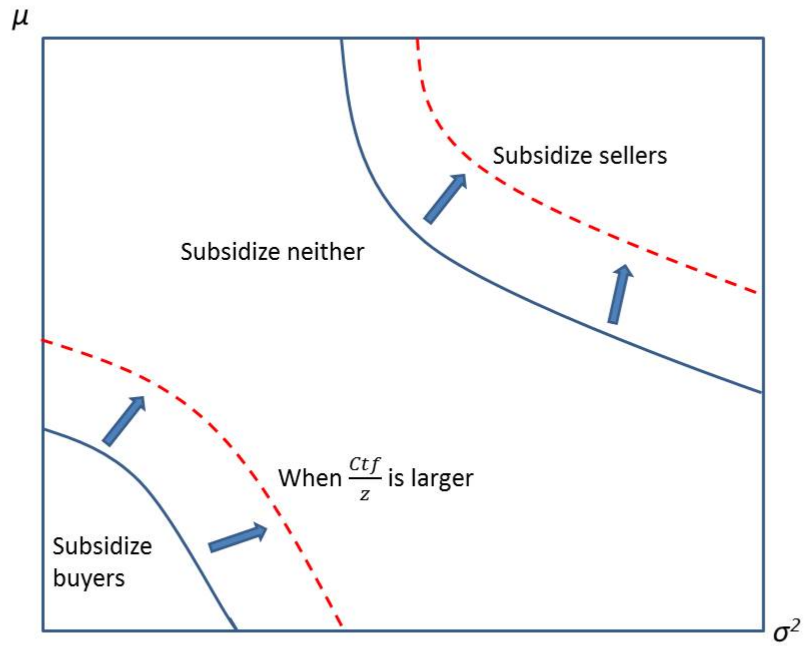

Figure 2: Subsidization Conditions in the σ^2-μ Space

Figure 2 demonstrates the results in Proposition 2 in the σ^2-μ space and completely captures how each of the six parameters affects the subsidy condition of either side. The two solid lines indicate the combinations of σ^2 and μ for which buyers are subsidized (the lower left corner), sellers are subsidized (the upper right corner), or neither group is subsidized (the area in the middle). It is clear that a higher σ^2 and/or μ is conducive to seller subsidy and hinders buyer subsidy. As indicated in Proposition 2, the remaining four parameters affect the subsidy conditions through a composite variable, $\frac{Ctf}{z}$. The dotted lines in Figure 2 show how $\frac{Ctf}{z}$ affects the boundary of the σ^2-μ combination: A greater $\frac{Ctf}{z}$ shifts both boundaries to northeast, meaning that a greater z is conducive to seller subsidy, while a greater C, t, or f is conducive to buyer subsidy.

To highlight the connection between the platform's subsidy strategies and the underlying parameters and to make our results more useful for practitioners, we derive the comparative statics results based on Propositions 1 and 2 and report them in the following corollary.

Corollary 2 *Seller-side subsidy is more likely when*

1. *entry on either side is less costly (C or f is lower);*

2. buyers' horizontal preferences are weaker (t is lower);

3. more potential buyers are present (z is higher);

4. the average quality is higher (μ is higher), or

5. sellers differ more in quality (σ^2 is higher).

Buyer-side subsidy is more likely under the opposite conditions.

6 Quality Heterogeneity, Liquidity, and Platform Pricing

In the previous section, we have demonstrated that the platform's subsidy strategy depends crucially on market liquidity. In this section, we will further investigate the role of liquidity in the platform's general pricing strategy. That is, instead of finding the conditions that result in a negative fee on either side, we focus on how liquidity affects the two fees regardless of whether they are positive or negative. Recall that liquidity captures the impacts of the six parameters in our model: the entry cost on each side (C and f), the potential buyer market size (z), buyers' horizontal preference (t), the average seller quality (μ), and quality heterogeneity (σ^2). Of particular interest is quality heterogeneity as represented by σ^2. Unlike most other parameters, σ^2 directly affects the two sides simultaneously, which poses a particular challenge on the comparison between the two sides. We will formally establish the connection between σ^2 and market liquidity as reflected in the equilibrium entry scale on both sides. We then proceed to investigate how σ^2 affects platform pricing. Further investigation of the remaining five parameters reveals that all parameters affect platform pricing through their impacts on liquidity.

6.1 Quality Heterogeneity and Liquidity

We now examine how σ^2 affects the equilibrium entry scales, n_s and n_b. Figure 3 illustrates the entry scales on the two sides in the n_s-n_b space. Recall the platform's two FOCs in Eq. (10) and (11), which are reproduced here: $n_b = \frac{4ftn_s^2}{t^2+4\delta_1\sigma^2 n_s^2}$ (derived from $\frac{\partial \Pi}{\partial n_s} \equiv 0$, and the curve is denoted as Π_s in Figure 3) and $n_b = \frac{z}{8tC}\frac{4\delta_1\sigma^2 n_s^2+4\mu t n_s - t^2}{n_s}$ (derived from $\frac{\partial \Pi}{\partial n_b} \equiv 0$, and the curve is denoted as Π_b). Figure 3 shows the two conditions (for any given level of σ^2) as two solid lines, where Π_s represents the optimal choice of n_s for any arbitrary n_b, while Π_b represents the optimal choice of n_b for any arbitrary n_s. The intersection of the two lines gives the platform's optimal choice of n_s and n_b. Both lines are upward sloping, which is to be expected for two-sided markets, as the platform will optimally choose to expand one side if the other side is larger. Furthermore, the second-order condition ensures that Π_s is steeper than Π_b. Now consider the impact of an increase in σ^2. Eq. (10) and (11) show that a greater σ^2 lowers Π_s and raises Π_b. The dotted lines in Figure 3 represent the entry scales at higher σ^2.

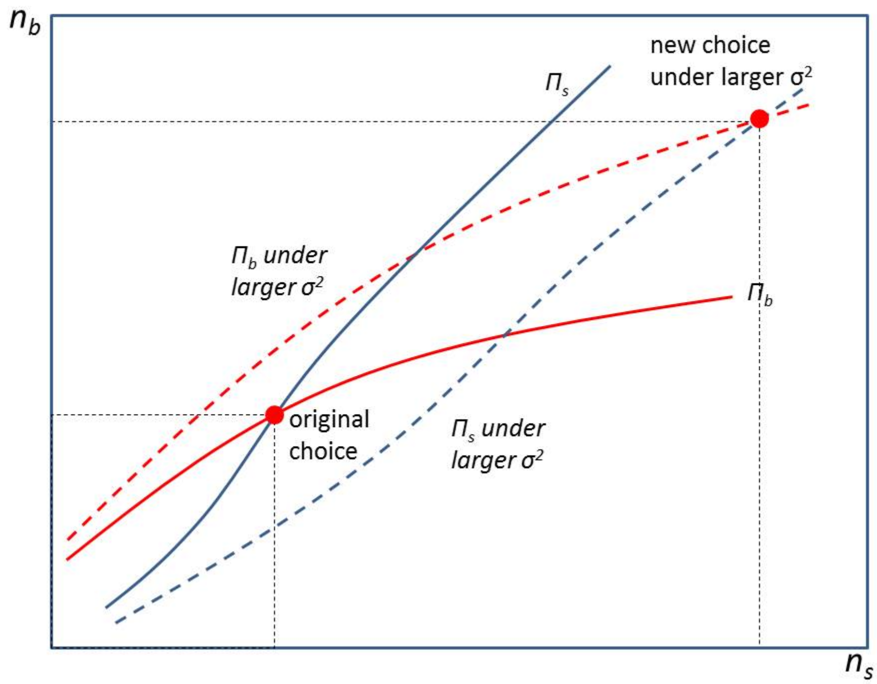

Figure 3: Impact of σ^2 on the Platform's Optimal Choices of n_s and n_b

Immediately we conclude:

Proposition 3 $\frac{dn_s^*}{d\sigma^2} > 0$ *and* $\frac{dn_b^*}{d\sigma^2} > 0$*; that is, the equilibrium* n_s *and* n_b *both increase with* σ^2*.*

The proposition says that greater quality variation induces the platform to allow more buyers and more sellers to enter.[14] Intuitively, when product quality varies more, both sellers and buyers benefit if the entry scales on both sides remain unchanged (Corollary 1). Fixing the buyer entry scale, the platform should optimally expand the seller entry scale (Π_s shifts to the right). Fixing the seller entry scale, the platform should optimally expand the buyer entry scale (Π_b shifts up). Combining the two effects, the platform will expand the entry scale of both sides. That is, a greater variation in product quality increases market liquidity.

Using the same method, we can easily show that n_s and n_b both increase with μ, and both

[14]The result that $\frac{dn_s^*}{d\sigma^2} > 0$ can also be understood as the relationship between vertical and horizontal product differentiation. In our model, the degree of vertical differentiation is represented by σ^2; the degree of horizontal differentiation is captured by the distance between neighboring sellers, $\frac{1}{n_s^*}$, so that a higher number of equilibrium sellers implies a lower degree of horizontal differentiation. Note that the degree of vertical differentiation is exogenous, whereas the degree of horizontal differentiation is endogenously determined by the platform's pricing strategy. Then the finding of $\frac{dn_s^*}{d\sigma^2} > 0$ implies that when sellers are more differentiated vertically, the platform admits more sellers, which reduces horizontal differentiation. Therefore, the vertical and horizontal dimensions of differentiation are substitutes in equilibrium.

decrease with t, f and $\frac{C}{z}$, so the market becomes more liquid when μ or z increases, and becomes less liquid when t, f or C increases.

6.2 Liquidity and Entry Fees

We now analyze how the liquidity parameters affect the optimal entry fees, with a particular focus on quality heterogeneity. The following proposition states the second major result of our paper.

Proposition 4 *As quality heterogeneity (σ^2) increases, the platform reduces the seller fee (R_s^*) and raises the buyer fee (R_b^*).*

Proposition 4 anchors strongly on the two-sided feature of the model. Recall that $R_s = \frac{n_b}{t}\left(\frac{t^2}{n_s^2} + \delta_2\sigma^2\right) - f$ and $R_b = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s - \frac{C}{z}n_b$. A greater σ^2 allows the platform to raise both R_s and R_b for fixed n_s and n_b. Intuition may suggest that in equilibrium when n_s and n_b are chosen optimally, the platform should still raise both R_s and R_b, albeit by smaller amounts. To see this, note that $R_s(n_s) = \frac{n_b}{t}\left(\frac{t^2}{n_s^2} + \delta_2\sigma^2\right) - f$ can be regarded as the seller-side demand function. When σ^2 is larger, the demand curve as a function of n_s moves up, so the optimal R_s should be higher. Similarly, a greater σ^2 moves up the buyer-side demand $R_b(n_b) = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s - \frac{C}{z}n_b$, and the platform should raise R_b. These arguments conform to the well established principle that a (one-sided) monopolist should adjust its pricing such that it shares with its customers the benefit or burden brought by a changing parameter. Such thinking, however, is flawed, as it treats the two sides separately. The very spirit of two-sided market is the connection between the two sides. Once this connection is accounted for, Proposition 4 demonstrates that in response to greater quality variation, the platform raises the buyer fee but reduces the seller fee.

The intuition can be understood as follows. Increased quality variation affects the two sides' surplus through both a direct effect and an indirect effect. The direct effect is that, fixing the number of sellers and buyers, both sides benefit from the increase in quality heterogeneity. The indirect effect is that increased quality variation induces the platform to attract more sellers and more buyers (Proposition 3), which tends to reduce the sellers' surplus but raise the buyers' surplus. On the buyer side, the two effects are both positive; thus, the platform raises the buyer fee. On the seller side, the two effects are in opposite directions. In the complex interactions between the two sides, the indirect effect dominates, so the platform reduces the seller fee.

To better understand the intuition and put the above result in perspective, we have also analyzed the impacts of the remaining five parameters (Table 1). We find that in equilibrium, *the seller fee R_s increases in t, C, and f, and decreases in z and μ. The exact opposite holds for the buyer*

Table 1: Comparative Statics

Parameter	n_s	n_b	R_s	R_b	n_sR_s	n_bR_b	$\frac{n_sR_s}{n_bR_b}$
σ^2	+	+	−	+	−	+	−
t	−	−	+	−	+	−	+
μ	+	+	−	+	−	+	−
C	−	−	+	−	+	−	+
f	−	−	+	−	+	−	+
z	+	+	−	+	−	+	−

fee R_b.[15] Out of the six parameters, four (C, f, z, μ) exert a direct effect on a single side. For instance, a higher C clearly reduces the buyers' surplus, whereas a higher μ raises the buyers' surplus. Given that $R_b = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s - \frac{C}{z}n_b$, when n_s and n_b are both fixed, the direct effect is that R_b decreases in C and increases in z and μ. It seems intuitive to hypothesize (which is also the standard intuition for one-sided monopoly) that the platform's optimal strategy is to raise the fee on the side where the parameter has a positive direct effect; or vice versa. According to Table 1, this is true for C, z, and μ. However, such a hypothesis cannot explain the impact of σ^2, which affects both sides simultaneously. Look at the expressions of R_b and R_s. Apparently the direct impact of σ^2 is to raise both R_b and R_s, so the hypothesis would have suggested that the platform should optimally raise both fees, and yet our Proposition 4 has shown that the platform should raise the buyer fee but reduce the seller fee. More strikingly, such hypothesis fails when applied to f. Given $R_s = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \delta_2\sigma^2\right] - f$, the direct effect of an increased f is to reduce R_s. However, we find the opposite: When f is higher, the platform raises R_s and reduces R_b in its optimal strategy.

The resolution for the incoherence of the hypothesis is the following: The platform should be "side-blind" about a parameter's direct effect and focus on how a parameter affects the total surplus from both sides as expressed in Eq. (9). The platform should raise the buyer fee (and reduce the seller fee) if and only if a parameter directly raises the total surplus, regardless of through which particular side it exerts such an impact. For example, a greater z, μ, or σ^2 all serve to raise the two sides' total surplus directly, and indeed the platform raises the buyer fee and lowers the seller fee in response to an increase in any of the three parameters. And the converse applies for a greater

[15] In other words, whenever one side's entry fee increases, the other side's fee decreases. This confirms the "seesaw principle" discussed in Rochet and Tirole (2006).

t, C, or f, which all serve to reduce the total surplus directly.[16] When a parameter increases the total surplus directly, the platform admits more buyers and more sellers under its optimal pricing strategy. Greater seller competition tends to reduce each seller's surplus, whereas no such rivalry exists on the buyer side. As a result, the platform relies increasingly on the buyer side to generate its revenue, which means the platform should raise the buyer fee and lower the seller fee.

In sum, two forces are at work: the interconnection between the two sides, and the same-side negative network effect among the sellers. Regardless of the source of a parameter change, as long as it (directly) raises the two sides' total surplus, the platform will expand the entry scale on both sides and increasingly focus on the buyer side by raising the buyer fee and lowering the seller fee. And the principle discussed here is consistent with the one governing the subsidy policy as highlighted in Proposition 1. This is hardly surprising, as subsidy policy is part of the general pricing strategy. So the consistent conclusion is: When a market becomes more liquid (as manifested by any of the six liquidity parameters), the platform should raise the buyer fee and lower the seller fee; seller subsidy also becomes more likely. Conversely, when a market becomes less liquid, the platform should raise the seller fee and lower the buyer fee; buyer subsidy becomes more likely.

6.3 Liquidity and Platform Profits

By the Envelope Theorem, it is straightforward to show that the platform's profit increases as products become more heterogeneous in quality. Similarly, any parameter change would raise the platform's profit if and only if it raises liquidity/total surplus. Therefore, an increase in z, μ and σ^2 would raise the platform's equilibrium profits, while an increase in t, C or f would reduce the platform's equilibrium profits. We also find that *as quality variation increases, the platform's revenue source shifts away from the seller side and toward the buyer side*; that is, $\frac{n_s^* R_s^*}{n_b^* R_b^*}$, decreases with σ^2. This follows intuitively from our previous results that the platform focuses less on charging sellers and more on charging buyers at an increased quality variation.

7 Welfare

In the traditional circular city model, Salop had shown that from social perspective, there is too much entry on the seller side (his buyer side is fixed). In our circular city with two-sided interaction, a natural question is whether the equilibrium entry on either side is above or below the socially optimal levels. Our concern is the efficiency of entry into the platform, not the efficiency of sellers'

[16] A greater t reduces the buyer surplus, but affects the seller surplus in a non-linear way. When t is larger, each buyer's net willingness to pay is reduced, which tends to reduce a seller's surplus, but the price competition between sellers is also mitigated, which tends to increase a seller's surplus. For our purpose, however, what matters is how t affects the two sides' combined total surplus, and the effect is unambiguous: the total surplus decreases with t.

pricing in trading. So we will allow a social planner to directly control n_b and n_s in order to maximize social welfare without changing the trading equilibrium in stage three. Post entry, we have calculated each seller's expected profit as $E(\pi)$, and each buyer's expected surplus as $E(u)$. Then the post-entry total social surplus is $n_sE(\pi) + n_bE(u)$. The total seller entry cost is n_sf. Given that the number of entrant buyers is n_b, the cutoff buyer entry cost must be $c_b = \frac{n_b}{z}C$, so the total buyer entry cost is $\int_0^{c_b} c_j \frac{z}{C} dc_j = \frac{n_b^2 C}{2z}$. The overall social welfare is therefore

$$W = n_sE(\pi) + n_bE(u) - n_sf - \frac{C}{2z}n_b^2.$$

The platform's total profit is $\Pi = n_sR_s + n_bR_b$, with $R_s = E(\pi) - f$, and $R_b = E(u) - \frac{C}{z}n_b$. That means the platform's profit function is

$$\Pi = n_sE(\pi) + n_bE(u) - n_sf - \frac{C}{z}n_b^2.$$

Given that $E(\pi) = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \delta_2\sigma^2\right]$ and $E(u) = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s$, we have already derived the platform's two first-order conditions, which are re-produced here:

$$\begin{aligned}
\frac{\partial \Pi}{\partial n_s} &\equiv 0 \text{ leads to } n_b = \frac{4ftn_s^2}{t^2 + 4\delta_1\sigma^2 n_s^2}, \text{ denoted as } \Pi_s \\
\frac{\partial \Pi}{\partial n_b} &\equiv 0 \text{ leads to } n_b = \frac{z}{8tC}\frac{4\delta_1\sigma^2 n_s^2 + 4\mu t n_s - t^2}{n_s}, \text{ denoted as } \Pi_b
\end{aligned}$$

Similarly, for the social planner, the two first-order conditions lead respectively to:

$$\begin{aligned}
\frac{\partial W}{\partial n_s} &= 0 \text{ leads to } n_b = \frac{4ftn_s^2}{t^2 + 4\delta_1\sigma^2 n_s^2}, \text{ denoted as } W_s \\
\frac{\partial W}{\partial n_b} &= 0 \text{ leads to } n_b = \frac{z}{4tC}\frac{4\delta_1\sigma^2 n_s^2 + 4\mu t n_s - t^2}{n_s}, \text{ denoted as } W_b
\end{aligned}$$

Apparently, $\Pi_s = W_s$ (meaning that given any arbitrary n_b, the platform and the social planner will choose the same optimal n_s), and $W_b = 2\Pi_b$ (meaning that given any arbitrary n_s, the social planner's optimal choice of n_b is greater than the platform's optimal choice). The curves representing the four FOCs are shown in Figure 4. Since Π_s coincides with W_s, there are only three curves in the graph. All these curves are upward sloping due to the positive cross-side network effect for both the platform and the social planner. Furthermore, the second-order condition ensures that Π_s and W_s are steeper than either Π_b or W_b.

The graph immediately proves:

Proposition 5 *The equilibrium entry scale on either side is lower than the socially optimal levels.*

The proposition says that the platform's optimal pricing strategy results in too few sellers and too few buyers from societal perspective. Intuitively, the platform and the social planner's objective

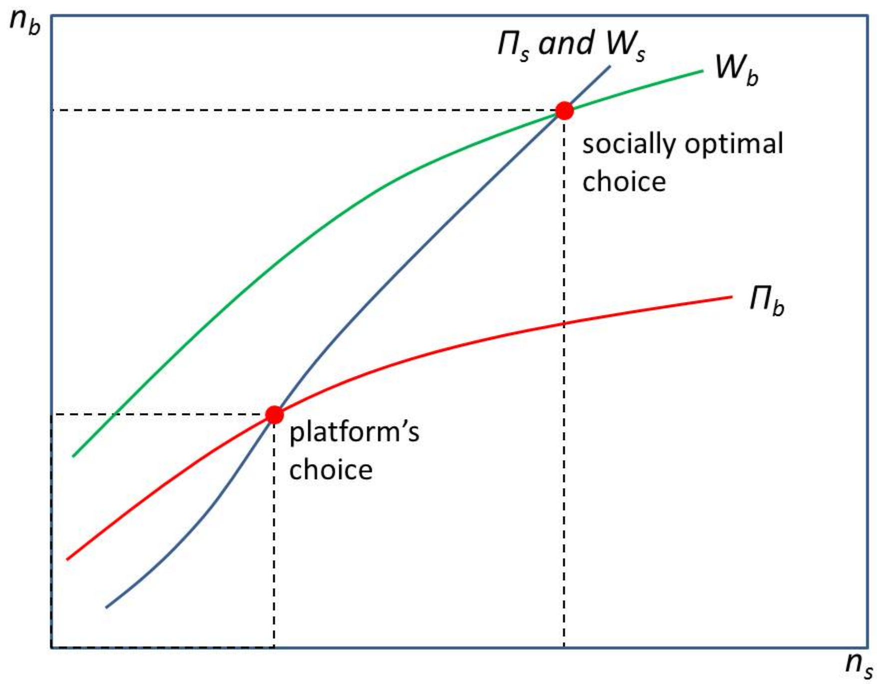

Figure 4: Platform's and Social Planner's Optimal Choices

functions largely coincide except for the buyer side's entry cost: The platform cares about $\frac{C}{z}n_b^2$ while the social planner cares about $\frac{C}{2z}n_b^2$. This is the typical divergence between a monopolist who cares about the marginal customer (the entry cost of the marginal buyer is $\frac{C}{z}n_b$), and a social planner who cares about the average customer (the entry cost of all entrant buyers ranging from 0 to $\frac{C}{z}n_b$). As a result, the platform admits too few buyers. By the very nature of two-sided markets, a smaller entry scale on the buyer side generates smaller profits for the sellers, so the platform also admits too few sellers as compared with the socially optimal level.

Economists care about two-sided markets mainly for their impacts on competition. Given our setting of monopoly platform, this paper is not positioned to study the competition between platforms. Nevertheless, it sheds some light on the welfare consequence when a monopoly platform controls both sides' entry through entry fees. The platform certainly plays a positive role, as it internalizes the positive cross-side network effects for both sides as well as the negative same-side network effects for buyers. Our results indicate that under conditions similar to our model setting, we need not worry too much about inefficient entry due to these externalities. Nevertheless, there is still a discrepancy between the platform's incentive and the social incentive, and the inefficiency stems from heterogeneity of buyers in terms of their entry costs.[17]

[17]In our model the sellers are assumed to have identical entry cost. If their entry costs are also heterogeneous, the

At this point it is helpful to compare our result of under-entry with the celebrated over-entry result obtained by Salop (1979). Both are based on circular city models. In Salop (1979), the buyer's entry scale is fixed exogenously, so sellers do not experience any positive network effect from buyers. In equilibrium, too many sellers enter in a "tragedy of commons" type of mechanism due to the negative network externality among sellers. In our model, by contrast, such negative externality is fully internalized by the monopoly platform as a gatekeeper. Our under-entry result is driven by the platform's failure to fully internalize the buyers' heterogeneous entry costs.

8 Conclusion and Discussion

This paper examines a monopoly platform's two-sided pricing problem in a setting where positive cross-side network effects are generated endogenously from buyers' and sellers' trading and entry decisions. Price competition then gives rise to a negative same-side network effect among sellers, so the two sides are intrinsically asymmetric. We find that the platform's pricing depends crucially on market liquidity, an indication of how much total surplus can be generated on the platform by the two sides' interactions. When a parameter change causes the market to become more liquid so that more buyers and more sellers enter the platform in equilibrium, the platform will reduce the seller entry fee and raise the buyer entry fee in its optimal strategy. Therefore, what matters for platform pricing is the market's collective property (i.e., the two sides combined) rather than comparisons between the two sides, as is the prevailing conclusion of the literature. Our results demonstrate two major forces in two-sided markets that seem to have received insufficient attention in the literature: a profound and fundamental interconnection between the two sides, and side asymmetry due to, say, the two sides' different degrees of same-side interactions.

These findings provide a new perspective for thinking about platform subsidies. Buyer-side subsidy is commonly observed in practice, especially when the networks are not large. For example, Microsoft subsidized Xbox by offering the console at a very low price. Although the strategy may be designed in part to compete with other console makers, not having high liquidity in the market of players and game developers allowed Microsoft to recover the consumer-side subsidy from the game developer side. This supports our result. In contrast to game consoles' buyer-side subsidy, computer operating systems often subsidize the sellers (software developers). Microsoft Windows is known to subsidize developers by offering free or low-cost software development kit (SDK) and support, while charging high prices on the user side (Eisenmann et al. 2006). Part of the reason may be that the quality of console games is not as variable as quality of software programs – the quality of software programs critically depends on developers' expertise on design and development and,

under-entry problem will be further exacerbated.

often, industry-specific knowledge. Trade fairs also come close to illustrating this case, including wedding expos, computer fairs, and many others. Some of these fairs, especially those that are large in scale and held in central locations (e.g., metropolitan areas) tend to charge an entrance fee on the buyer side and attract a large crowd nevertheless. Because of easy access for attendees and low setup costs for vendors, both sides incur low entry costs to participate. These are also consistent with our findings.

In Section 6, we explore quality heterogeneity in depth and derived a number of important results, which guide the platform's strategy in managing sellers' entry scale. In some cases, platform owners exercise limited or no control over the degree of quality heterogeneity on the seller side. For example, as Airbnb expands to serve over 33,000 cities in 192 countries, differences in income distribution, culture, and government regulations across these regions may create variations in quality of hosts. Hosts in certain regions might be more consistent in terms of lodging space quality and hospitality than those in other regions. Our finding suggests that the platform owner may be better off tailoring the network size of its hosts in different regions for quality heterogeneity based on regional characteristics. In other cases, the platform has access to various instruments to influence the outcomes of sellers' quality. Smartphone platform owners such as Apple and Google employ various contests and even directly screen applications to craft the distribution of application qualities. Our finding then offers insights into how platforms might consider balancing the number of applications while controlling quality variation.[18]

The findings on the impact of σ^2 on the two sides' entry scale and entry fees also provide a basis for how the platform can adjust its pricing when market condition changes. For example, if some smartphone application developers succeed in a particular technology innovation and release games of superior visual experience or applications that more seamlessly integrate with the operating system, the newly introduced applications then effectively increase quality variation in the market. The platform may then consider reducing the fee on the developer side while raising the price charged on the user side. The same strategy may be implemented in reverse. For instance, if Airbnb decides to exclude hosts below a certain quality to avoid legal disputes, the reduced quality variation would suggest a higher fee for the hosts. Given these strategies, the platform can benefit from including more types of sellers to create a more diverse marketplace. This advises against mechanisms to screen out lower quality sellers for the sole reason of improving overall seller quality. Understandably, some platform might still exclude sellers that may lead to other undesirable consequences (e.g., by reviewing applications for security or adverse selection problems). Our result

[18]Strictly speaking, in these cases the platform's control over quality would affect both the average quality μ and the quality variation σ^2; thus, our conclusions regarding the impact of σ^2 with fixed μ may require reinterpretation before they can be applied. This adjustment may be straightforward, at least for some specific functional forms of the quality distribution.

raises cautions regarding such exclusion and suggests an evaluation of screening criteria to avoid unintended narrowing of quality variation among sellers.

In sum, our research generates a number of predictions by linking at least six parameters to a platform's pricing strategies. All these are empirically testable and directly useful to managers. We have considered a setting with asymmetry between the two sides using same-same interaction among the sellers. It is interesting to consider other ways to generate side asymmetry. Another direction of future research is to extend the endogenization to competing platforms, which is necessary for understanding most business phenomena in real life as well as deriving useful antitrust policies. Also, platforms may be able to strategically inform buyers of sellers' products to reduce information asymmetry in the interaction between the two sides. For example, platforms' investments in technologies that improve buyers' shopping interface, facilitate product recommendations, or empower product search are instrumental for determining information and trading surplus for both sides. Endogenizing the related variables can be a fruitful direction.

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A Appendix: Proofs

Proof of Proposition 1

In the text we formulate the platform's optimization problem as choosing n_s and n_b simultaneously. In deriving the subsidy conditions, it is easier to turn the optimization problem into a sequential choice. To do this, we first use the first-order condition (10) to arrive at $n_b = \frac{4ftn_s^2}{t^2+4\delta_1\sigma^2 n_s^2}$, and then plug this expression into the platform's objective function. The platform will then solve the following optimization problem:

$$\max_{n_s} \Pi(n_s) = \frac{z}{4C}\left(\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t}\delta_1 n_s\right)^2 - n_s f, \tag{14}$$

for which the FOC is,

$$\Pi'(n_s) = \frac{z}{2C}\left(\mu - \frac{t}{4n_s} + \frac{\sigma^2}{t}\delta_1 n_s\right)\left(\frac{t}{4n_s^2} + \frac{\sigma^2}{t}\delta_1\right) - f = 0. \tag{15}$$

Assume that the second-order condition is satisfied, so the FOC is both necessary and sufficient to define a unique solution of n_s^*.

Now, $R_s < 0$ if and only if $E(\pi) < f$. Given that $n_b = \frac{4ftn_s^2}{t^2+4\delta_1\sigma^2 n_s^2}$, we have $E(\pi) = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \delta_2\sigma^2\right] = \frac{4fn_s^2}{t^2+4\delta_1\sigma^2 n_s^2}\left[\frac{t^2}{n_s^2} + \delta_2\sigma^2\right] > f$ if and only if $n_s > \frac{\delta_4 t}{\sigma}$, where $\delta_4 = \sqrt[4]{\frac{243}{4}}$. Because $\Pi''(n_s) < 0$, $n_s^* > \frac{\delta_4 t}{\sigma}$ if and only if $0 = \Pi'(n_s^*) < \Pi'(\frac{\delta_4 t}{\sigma})$, which leads to $\frac{Ctf}{z} < \sigma^2(\delta_5\mu + \delta_6\sigma)$, where $\delta_5 \equiv \frac{1}{2} - \frac{5}{27}\sqrt{3} > 0$ and $\delta_6 \equiv \frac{\sqrt[4]{12}}{324}(353 - 189\sqrt{3}) > 0$.

Turning to the buyer subsidy. $R_b < 0$ if and only if $E(u) < \frac{C}{z}n_b$. The first order condition (11) gives $n_b = \frac{z}{8tC}\frac{4\delta_1\sigma^2 n_s^2 + 4\mu t n_s - t^2}{n_s}$. Plug this into $E(u)$ and we find $E(u) < \frac{C}{z}n_b$ if and only if $n_s < \frac{9t}{2(\mu+\theta)} \equiv \bar{n}$, where $\theta = \sqrt{\mu^2 - 9\delta_3\sigma^2}$. Because $\Pi''(n_s) < 0$, $n_s^* < \bar{n}$ if and only if $0 = \Pi'(n_s^*) > \Pi'(\bar{n})$, which is reduced to $\frac{Ctf}{z} > \delta_7(\mu+\theta)(\mu - \delta_8\theta)(\mu - \delta_9\theta)$, where $\theta = \sqrt{\mu^2 - 9\delta_3\sigma^2}$, $\delta_7 = \frac{4(35+12\sqrt{3})}{729}$, $\delta_8 = \frac{22+\sqrt{3}}{26}$, and $\delta_9 = \frac{29+4\sqrt{3}}{61}$. It can be shown that the right hand side is increasing in both μ and σ^2, so the condition is less likely to be satisfied when σ^2 is larger.

Proof of Proposition 2

(1) The necessary and sufficient condition for seller subsidy has been derived as Eq. (12), which is re-produced here: $\frac{Ctf}{z} < \sigma^2(\delta_5\mu + \delta_6\sigma)$, where δ_5 and δ_6 are positive constants. Define a function of σ: $F(\sigma) \equiv \delta_6\sigma^3 + \delta_5\mu\sigma^2 - \frac{Ctf}{z}$. Because $F(0) < 0$, $F(\sigma) > 0$ when σ is sufficiently large, and $F'(\sigma) > 0$ for all $\sigma > 0$, there exists a unique $\sigma_s > 0$ such that $F(\sigma_s) = 0$. Seller subsidy requires $F(\sigma) > 0$, which is then equivalent to $\sigma > \sigma_s$. Note that σ_s is a function of the two parameters, μ and

$\frac{Ctf}{z}$. We can re-write the definition of σ_s as: $F\left(\sigma_s\left(\mu, \frac{Ctf}{z}\right), \mu, \frac{Ctf}{z}\right) \equiv 0$. Then $\frac{\partial F}{\partial \sigma}\frac{\partial \sigma_s}{\partial \mu} + \frac{\partial F}{\partial \mu} = 0$. Apparently $\frac{\partial F}{\partial \sigma} > 0$ and $\frac{\partial F}{\partial \mu} > 0$. Therefore $\frac{\partial \sigma_s}{\partial \mu} < 0$. Similarly, $\frac{\partial F}{\partial \sigma}\frac{\partial \sigma_s}{\partial(\frac{Ctf}{z})} + \frac{\partial F}{\partial(\frac{Ctf}{z})} = 0$. Because $\frac{\partial F}{\partial \sigma} > 0$ and $\frac{\partial F}{\partial(\frac{Ctf}{z})} < 0$, we have $\frac{\partial \sigma_s}{\partial(\frac{Ctf}{z})} > 0$.

(2) The necessary and sufficient condition for buyer subsidy has been derived as Eq. (13), which is re-produced here: $\frac{Ctf}{z} > \delta_7 (\mu+\theta)(\mu-\delta_8\theta)(\mu-\delta_9\theta)$, where $\theta = \sqrt{\mu^2 - 9\delta_3\sigma^2}$, $\delta_7 = \frac{4(35+12\sqrt{3})}{729} > 0$, $\delta_8 = \frac{22+\sqrt{3}}{26} > 0$, and $\delta_9 = \frac{29+4\sqrt{3}}{61} > 0$. It can be easily shown that the right hand side of the inequality increases in both μ and σ. Therefore, if we define $G(\sigma) = \delta_7(\mu+\theta)(\mu-\delta_8\theta)(\mu-\delta_9\theta) - \frac{Ctf}{z}$, we will have $G'(\sigma) > 0$ for all $\sigma > 0$. Note that $G(0) = \frac{16}{729}\mu^3 - \frac{Ctf}{z}$. If $\frac{Ctf}{z} \leq \frac{16}{729}\mu^3$, we have $G(\sigma) > G(0) \geq 0$ for any $\sigma > 0$. In that case, buyer subsidy never happens, as buyer subsidy requires $G(\sigma) < 0$. If $\frac{Ctf}{z} > \frac{16}{729}\mu^3$, then $G(0) = \frac{16}{729}\mu^3 - \frac{Ctf}{z} < 0$. Given that $G(\sigma) > 0$ for sufficiently large σ, and $G'(\sigma) > 0$ for all $\sigma > 0$, there exists a unique $\sigma_b > 0$ such that $F(\sigma_b) = 0$. Buyer subsidy requires $G(\sigma) < 0$, which is equivalent to $\sigma < \sigma_b$. Finally, we can prove $\frac{\partial \sigma_b}{\partial \mu} < 0$ and $\frac{\partial \sigma_b}{\partial(\frac{Ctf}{z})} > 0$ similarly as in (1).

Proof of Proposition 4

To prove that R_s decreases with σ^2: From $n_b = \frac{4ftn_s^2}{t^2+4\delta_1\sigma^2 n_s^2}$ we can solve $\frac{t^2}{n_s^2} = \frac{4ft}{n_b} - 4\delta_1\sigma^2$. Plug this into: $R_s = E(\pi) - f = \frac{n_b}{t}\left[\frac{t^2}{n_s^2} + \delta_2\sigma^2\right] - f = \frac{n_b}{t}\left[\frac{4ft}{n_b} + (\delta_2 - 4\delta_1)\sigma^2\right] - f = 4f + (\delta_2 - 4\delta_1)\frac{\sigma^2}{t}n_b - f$. Given the numerical values of δ_1 and δ_2, we know that $\delta_2 < 4\delta_1$. Then when σ^2 is larger, n_b will be larger, so R_s is smaller.

To prove R_b increases with σ^2: $R_b = E(u) - \frac{C}{z}n_b = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s - \frac{C}{z}\frac{z}{8tC}\frac{4\delta_1\sigma^2 n_s^2 + 4\mu t n_s - t^2}{n_s} = \mu - \frac{5t}{4n_s} + \frac{\sigma^2}{t}\delta_3 n_s - \frac{\sigma^2}{2t}\delta_1 n_s - \frac{\mu}{2} + \frac{t}{8n_s} = \frac{\mu}{2} - \frac{9t}{8n_s} + \frac{\sigma^2}{t}n_s\left(\delta_3 - \frac{\delta_1}{2}\right)$. But $\delta_3 = 0.096 < \frac{\delta_1}{2} = 0.163$, so we need further manipulation. Now we use $\frac{t^2}{n_s^2} = \frac{4ft}{n_b} - 4\delta_1\sigma^2$: $R_b = \frac{\mu}{2} + \frac{\sigma^2}{t}n_s\left[\left(\delta_3 - \frac{\delta_1}{2}\right) - \frac{9}{8}\frac{1}{\sigma^2}\frac{t^2}{n_s^2}\right] = \frac{\mu}{2} + \frac{\sigma^2}{t}n_s\left[\left(\delta_3 - \frac{\delta_1}{2}\right) - \frac{9}{8}\frac{1}{\sigma^2}\left(\frac{4ft}{n_b} - 4\delta_1\sigma^2\right)\right] = \frac{\mu}{2} + \frac{\sigma^2}{t}n_s\left(4\delta_1 + \delta_3 - \frac{9}{2}\frac{ft}{n_b\sigma^2}\right) = \frac{\mu}{2} + \frac{\sigma^2}{t}n_s\left(4\delta_1 + \delta_3 - \frac{9}{2}\frac{ft}{n_b\sigma^2}\right)$. We know that $4\delta_1 + \delta_3 > 0$, and that n_s and n_b both increase with σ^2, so when σ^2 is larger, $\frac{\sigma^2}{t}n_s$ is larger, and $\left(4\delta_1 + \delta_3 - \frac{9}{2}\frac{ft}{n_b\sigma^2}\right)$ is also larger, so R_b is larger.

B Numerical Approximation

When n_s is not too small, $\frac{\delta^{n_s}+1}{\delta^{n_s}-1} \approx 1$ and $\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2} \approx 0$. For example, when $n_s = 6$, $\frac{\delta^{n_s}+1}{\delta^{n_s}-1} = 1.00074$ and $\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2} = 0.00222$. A larger n_s leads to closer approximations (Table 2).

Our numerical study without applying approximation shows that the equilibrium n_s is easily well above 10. Thus, approximating $\frac{\delta^{n_s}+1}{\delta^{n_s}-1}$ at 1 and $\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2}$ at 0, which requires n_s to be sufficiently high, is consistent with the actual equilibrium.

Table 2: Numerical Examples

n_s	5	6	7	8	9	10	11
$\frac{\delta^{n_s}+1}{\delta^{n_s}-1}$	1.00277	1.00074	1.00020	1.00005	1.00001	1.00000	1.00000
$\frac{n_s\delta^{n_s}}{(\delta^{n_s}-1)^2}$	0.00693	0.00222	0.00069	0.00021	0.00006	0.00002	0.00001

Based on this approximation, we can rewrite the following expressions:

$$\begin{aligned} g_s(n_s) &= n_s\left(1 - \frac{4}{3\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} + \frac{2n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) \approx \delta_2 n_s \\ g_b(n_s) &= n_s\left(\frac{1}{6\sqrt{3}}\frac{\delta^{n_s}+1}{\delta^{n_s}-1} - \frac{n_s}{3}\frac{\delta^{n_s}}{(\delta^{n_s}-1)^2}\right) \approx \delta_3 n_s \end{aligned}$$

where $\delta_2 = 1 - \frac{4}{3\sqrt{3}}$ and $\delta_3 = \frac{1}{6\sqrt{3}}$.