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USING O–D MATRICES FOR DECISION MAKING IN ROAD NETWORK MANAGEMENT

Borja Alonso[1], José Luis Moura[2], Ángel Ibeas[3], Luigi dell'Olio[4]

Dept of Transportation, University of Cantabria, Spain
E-mails: [1]alonsobo@unican.es (corresponding author); [2]mourajl@unican.es; [3]ibeasa@unican.es; [4]delloliol@unican.es

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Abstract. This article uses a real example to demonstrate the importance of mobility modelling (objective criteria) on correctly planned investments in infrastructure. Some radical conclusions are drawn which differ greatly from those suggested by more subjective mental models. Strategic developments on inter-urban networks are evaluated by applying a mobility model including a model for estimating O–D matrices based on traffic counts and a model for the optimal location of traffic counting stations at the same time as a population accessibility model. An important decision making tool is provided for planning investments in road infrastructure by rationalising the high public spending required for this type of work. The models are applied to various possible projects at determined points on a regional road network in the Autonomous Community of Cantabria (Northern Spain) following the proposals and demands of different social and political groups. The models find a 68% reduction in the number of links required to obtain O–D matrices. This provides considerable savings in data collection costs (approximately 50% less) involving future traffic counts on similar and especially regional road networks.

Keywords: optimization, transport planning, transport management, simulation, origin-destination matrix.

Introduction and Objectives

This article is a result of the application of different Interurban Mobility models (estimation of O–D matrices, optimal location of traffic counting stations and accessibility models) to be used as a planning aid for investments in infrastructure. In this case, it is applied to the road network under the jurisdiction of the Regional Government of Cantabria (Spain).

The combined application of these models supports the planning of any work on a road network and provides objective criteria for evaluating the repercussions of different possible scenarios in regional development. The analysis does not only have consequences for traffic, but also has a social bearing when the variations in journey time, accessibility and income per capita in each zone are quantified for each project.

The mobility model presented here is based on an O–D matrix representing the trip patterns in a determined area and will serve as a basic tool for evaluating future investments in infrastructure. It does not only provide Origin-Destination trip data, but by applying it to the network it can also predict daily traffic intensity, detect network congestion problems and quantify the number of people who will benefit or lose out as a result of the work causing changes to journey time, accessibility and income.

Many authors refer to the importance of being able to count on a good O–D matrix for planning and modelling. Examples are: Willumsen (1978) who points out the importance of estimating O–D matrices for modelling demand; Yang and Zhou (1998) state that it is essential to have a good O–D matrix available for efficient traffic management; Tsekeris and Stathopoulos (2003) explains a lot of applications of O–D matrices in the world of transport planning, traffic forecasts and providing users with dependable information. Nie *et al.* (2005), Doblas and Benitez (2005) and Kwon and Varaiya (2005) state that the estimates of O–D matrices are an essential source of traffic demand information for most transportation planning processes, and for the management and control of the transportation system.

One of the most famous practical cases which reflected this took place in Santiago de Chile, where a series of information packages were created: ESTRAUS (De Cea *et al.* 2003), VERDI and MODEM for the advance planning and evaluation of future work on the road network, land use and public transport (SECTU 1989). Another clear example is provided by the Massachusetts Department of Conservation and Recreation (Beta Group Inc. 2006) in their work on remodelling the tunnels between two locations in Boston using an O–D matrix to model user behaviour when several important routes were closed due to the tunnelling work.

The main objective of this article is to use a real example to demonstrate the importance of mobility and traffic studies in rationalising public resources when planning investments in infrastructure.

The introduction is followed by section 1 which presents the calibrated mobility model and emphasises the importance of estimating the O–D matrices. Section 2 shows how the model is developed for finding the optimal location of traffic counting stations. The accessibility model used in support of the decision making process is then specified in section 3. Section 4 describes the application of the model to a real case and compares the obtained results using mathematical models against political decisions and techniques based on purely mental models. The final section presents the most important conclusions drawn from the work.

1. Mobility Model: Estimation of O–D Matrices

The O–D matrices are estimated using the Maximum Entropy Model (Willumsen 1978) based on survey information refined with traffic counts at diverse times and periods of the year (rush hour and off peak times of working days and weekends during winter and summer). A daily matrix of the average day provides the Average Annual Daily Traffic (AADT) data.

If traffic counts are used for estimating matrices it is important to identify correctly the links which join the different origin-destination pairs, especially those which best represent these journeys. This is obtained by defining the indicator P_{ij}^{a} as the proportion of trips between origin i and destination j using the link where the traffic counter is located. Equation (1) can determine the flows on link a, given the total journeys between each pair (i, j) and the P_{ij}^{a} indicators:

$$V_a = \sum_{ij} T_{ij} \cdot P_{ij}^{a}, \quad a \in A_c, \tag{1}$$

where: V_a is the flow on link a; T_{ij} is the number of journeys between origin i and destination j; A_c is the group of links with traffic counters.

The matrix estimation model estimates the values of elements T_{ij} in the O–D matrix to produce modelled V_a values which coincide more closely with the values observed on a sub-group of links. In a study area with n origins and n destinations there are n^2 unknown T_{ij} elements. This number is greater than the number of independent traffic counts which are normally available meaning there are a very large group of feasible solutions (O–D matrices) from which we must choose the one which maximizes system entropy.

The most probable trip matrix is the one with the greatest number of associated microstates $W(\{T_{ij}\})$. The number of ways to choose a trip matrix between i and j T_{ij} with a total number of trips T is given by (Wilson 1970):

$$W\left(\left\{T_{ij}\right\}\right) = \frac{T!}{\prod_{ij} T_{ij}!}. \tag{2}$$

The goal is to find the trip matrix which maximizes either $W(\{T_{ij}\})$ or a monotonous function of it, *log* $W(\{T_{ij}\})$ for mathematical convenience. The objective function (3) is found using the Stirling method:

$$\text{Maximize } \log W\left(\left\{T_{ij}\right\}\right) = -\sum_{ij}\left[T_{ij} \cdot \log T_{ij} - T_{ij}\right]; \tag{3}$$

$$\text{Subject to: } \hat{V}_a - \sum_{ij} T_{ij} \cdot P_{ij}^{a} = 0, \quad a \in A_c, \tag{4}$$

where: T_{ij} is an element of the estimated matrix: the number of trips between origin i and destination j; $P_{ij}{}^{a}$ is the proportion of trips between i and j using link a; $\overline{y_i^t}$ is the observed flow on link $a \in A_c$.

The solution to the previous maximization problem is:

$$T_{ij} = \prod_{a} X_a^{P_{ij}^{a}}, \tag{5}$$

where: the product is calculated on all the links providing observed counts. The factors $X_a^{P_{ij}^{a}}$ associated with each link are related to the Lagrange multipliers corresponding to the constraints.

If an old O–D matrix is available, it can be demonstrated that the best estimation of the trip matrix is given by:

$$T_{ij} = t_{ij} \cdot \prod_{a} X_a^{P_{ij}^{a}}, \tag{6}$$

where: t_{ij} are the elements of the old matrix.

Both Equation (5) and Equation (6) are special cases of the multi-proportional problem and can be solved using various solution algorithms proposed in the literature, such as the Murchland algorithm (Hall *et al.* 1980).

2. Traffic Counter Location Model

The second of the proposed models minimizes an error between the estimated matrix and a reference matrix (MAE) to determine the optimal number and location of Short Period Traffic Counts (SPTC) from a configuration of Permanent Traffic Counts (PTC).

The methodology is based on the heuristic Selective Steps Method (Hall *et al.* 1980) presented in Fig. 1 and briefly explained in the following points:

1. An initial counting station SPTC ($k_{(i)}$) is selected for inclusion in the final group of counters and the O–D trip matrix with the lowest *MAE* is estimated using the software SATURN (Bolland *et al.* 1979). Whether or not this SPTC and each of the following should be included in the final group where it is determined by calculating the reduction in the *MAE* due to the incorporation of this SPTC and comparing it with a reference value known as *MAE-IN*.

$$\begin{aligned} Rmae = {} & MAE(k_{(1)}, k_{(2)}, \ldots, k_{(n-1)}) - \\ & MAE(k_{(1)}, k_{(2)}, \ldots, k_{(n-1)}, k_{(n)}). \end{aligned} \tag{7}$$

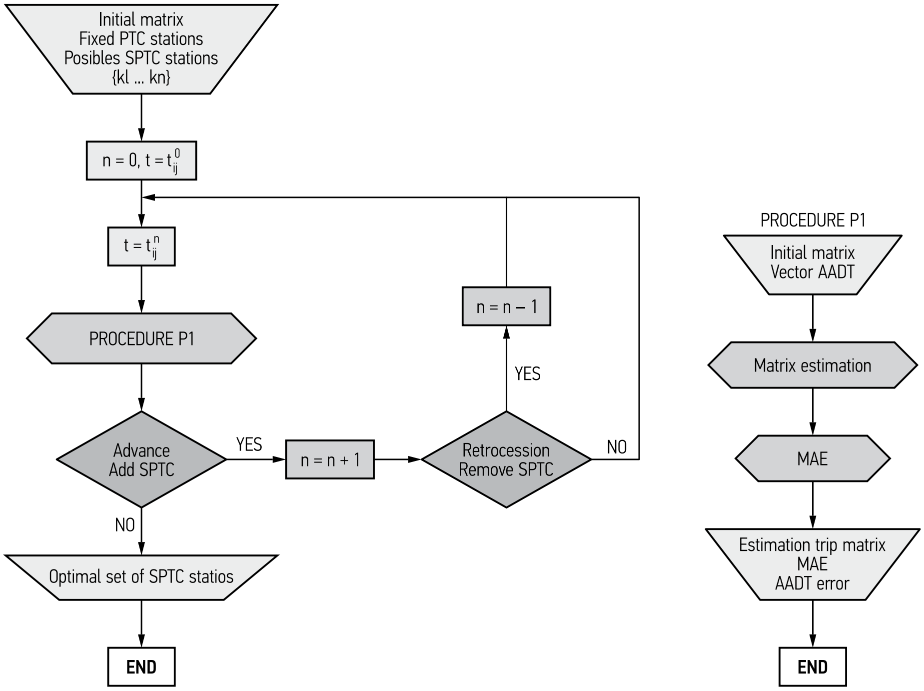

Fig. 1. Methodology for calculating the optimal number and location of counting stations

2. The process advances if the SPTC fulfils the inclusion criteria (*Rmae* > *MAE-IN*), if not, the procedure ends.
3. Once a SPTC has been included, all the SPTC that were previously included are reviewed once again to determine if the information they provide is still relevant, using the criteria that for a SPTC to continue belonging to this group the reduction of the *MAE* due to the inclusion of this SPTC will be greater than a second reference value called *MAE-OUT*.
4. The links which are not incorporated in the group are now assessed for their inclusion. The links are then once again reviewed for their possible elimination.
5. This procedure ends when there are no more SPTCs fulfilling the entrance or exit criteria, *MAE-IN* and *MAE-OUT*, respectively.

3. Accessibility Model

The corresponding time matrices were obtained using the real traffic flows reproduced in the model and the real network journey times. These times are essential for the application of the accessibility model. Knowledge of the transport network accessibility levels (Garrocho, Campos 2006; Nutley 2003) means the planner can:

- identify regions with greater mobility;
- compare alternative transport plans to assess how they address territorial imbalance and homogenize accessibility levels;
- evaluate the impact and consequences of each alternative.

This study has used the Demographic Accessibility Index to zone *i* from the rest of the zones *j*:

$$A_i = P_i \sum_j P_j \Big/ \left(t_{ij}\right)^{\alpha}, \tag{8}$$

where: A_i is the accessibility of settlements *j* to *i*; P_i is the settlement of zone *i*; P_j is the settlement of zone *j*; t_{ij} the journey time from zone *i* to zone *j* obtained from the time matrix and α is a parameter to calibrate for penalizing the journey time, with a value of 2 following studies which were done previously in the region.

Once the accessibility index and per capita income are known for each borough, the income elasticity value (*Y*) is estimated as a function of the value of the accessibility index (*X*). This shows that a relationship exists between accessibility and the national, regional or sectoral product using the Cobb–Douglas model (Cobb, Douglas 1928; Álvarez *et al.* 2003; Nombela 2005) simplified to the following potential function:

$$Y = aX^b, \tag{9}$$

where: *a* and *b* are the parameters to be estimated. The parameter *b* is the elasticity of income relative to accessibility, so if the latter magnitude is given the role of indicator for the degree of national, regional or local development, then the *b* value will show the influence that transport has on regional economic activity.

The approach based on the estimation of production functions, particularly in the case of transport infrastructure, places some doubt on the quantitative validity of the resulting elasticities, the causality between the variables or the distortions in long term predictions. However, this methodology is useful as an approach to studying the impacts transport infrastructure has on an economy (Nombela 2005) and, in spite of being and having been the object of continual yet inconclusive debates, it continues to be used today (Álvarez *et al.* 2003; Nombela 2005; Cantos *et al.* 2005). This is why its use should be seen as an illustrative measure or a preliminary micro scale analysis of this impact which can act as a planning aid and help in predicting demand.

4. Application to a Real Case: Decisions Based on Mathematical Models vs. Decisions Based on Mental Models

The models described above were applied in the Autonomous Community of Cantabria (Spain).

Cantabria is one of the 17 autonomous communities in Spain and is located on the north coast (Fig. 2). The region has a population of 562309 inhabitants, at a density of 105.3 inhab/km^2, 200000 of whom live in the capital, Santander. The population is not evenly distributed throughout the region and is concentrated on a relatively small amount of territory. More than a half of the region's land has a population density of less than 10 inhab/km^2.

Fig. 2. Location of the Autonomous Community of Cantabria in Spain

The majority of the population live in the 'T' (Fig. 3) formed by the east-west coastal corridor (Bilbao–Santander–Oviedo) (A–8) and the route Santander–Torrelavega–direction Madrid (A–67). Around half a million people live in an area covering approximately 40% of the region and the distribution is even quite irregular within this 'T' where around 60% are concentrated along the Bay of Santander–Torrelavega belt.

This high density of people and industry combined with the Bilbao–Oviedo–Madrid traffic passing through the region cause significant amounts of traffic to flow to the East and West (A–8 Freeway) as well as to the North and South (A–67 Freeway). This has led to high congestion on both freeways, especially on the A–67 freeway (stretch A_1–B), which reached levels of service (Highway Capacity Manual 2000) E and F. Several of the news media have recently been reporting on the need, based purely on 'mental models', to build a third lane on the A–67 Santander–Torrelavega freeway (stretch A_1–B). Currently, this road is very congested because it forms the common element of two parts of the 'T' described earlier.

Because of the imminent construction of a large scale regional infrastructure (including 2 freeways, two extra lanes on an existing motorway and a multi-lane highway), a joined up model of Inter-urban Mobility and Accessibility was developed to predict the impact of these works on regional traffic and accessibility.

Figs 4 and 5 are presented for a better understanding of the area in question.

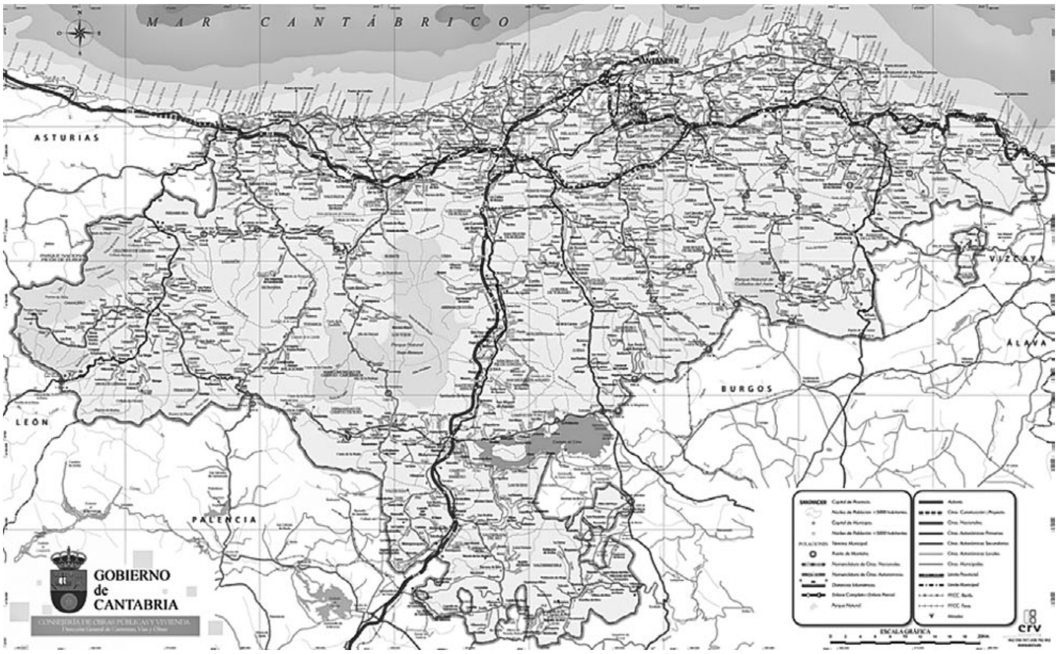

Fig. 3. Road Network of the Autonomous Community of Cantabria

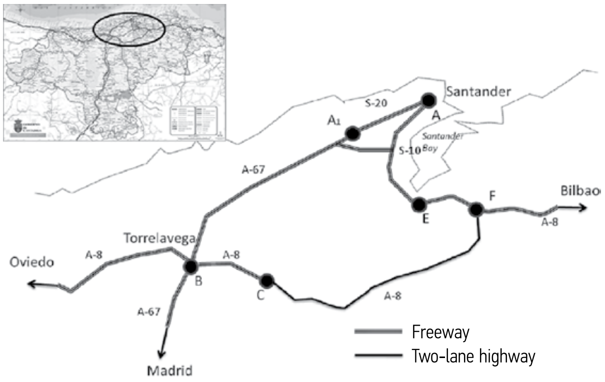

Fig. 4. Existing road network in the study area

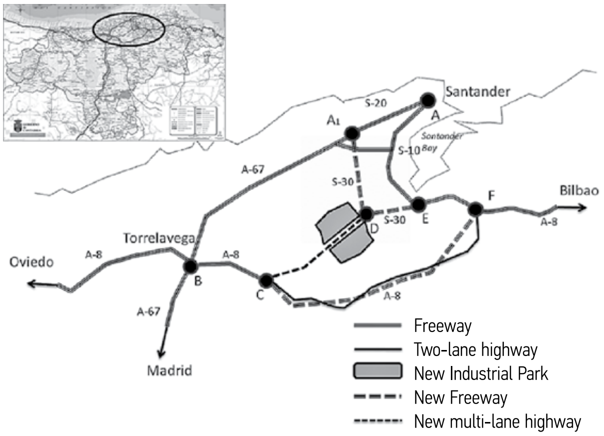

Fig. 5. Future road network in the study area

Currently, under construction is the last stretch of the A–8 Freeway and points B–C–F in Fig. 4 show a substituting, current, conventional two lane road, and another freeway. S–30 in Fig. 5 is a connecting Santander with the A–8 (stretch A_1–D–E). The nearby industrial development has also meant the creation of a new conventional 4 lane main road along stretch C–D.

The first step in the development of a mobility model is to characterise the supply, represented here by the available road network in the Autonomous Community of Cantabria. The region was divided into 273 zones to minimize spatial aggregation error. A complete sectionalised study was made of the entire network with an inventory of more than 40 characteristics per stretch and link, as well as the free flow times based on legal limitations or, in their absence, real observed behaviours. At the same time an extensive plan for traffic counters was prepared with more than 600 counting points distributed throughout the region.

O–D matrices were estimated for rush hour and off peak times of working days and weekends in winter and summer along with a matrix of averages reflecting the network's AADT.

The matrices are immediately assigned to the network (Fig. 6), checking the fit against the validation traffic counters and making sure that they show logical structures of proven journey patterns. The choice of assignment method was made by checking both the Stochastic User Equilibrium (SUE), see Daganzo and Sheffi (1977), Sheffi and Powell (1981); and the User Equilibrium (UE) which comply with Wardrop's first principal (Wardrop 1952) stating that 'under equilibrium conditions traffic arranges itself in congested networks such that all used routes between an O–D pair have equal and minimum costs while all unused routes have greater or equal costs'. The latter was finally chosen because it better agreed with the resulting traffic counts (Fig. 7).

Apart from coding the new roads that were still under construction, the current study has also included estimations of future journeys resulting from the creation of new industrial zones.

The O–D matrices are later reassigned to obtain the average daily flows at rush hour and off peak times. The predicted modelled flows in comparison with the current situation are shown in Table.

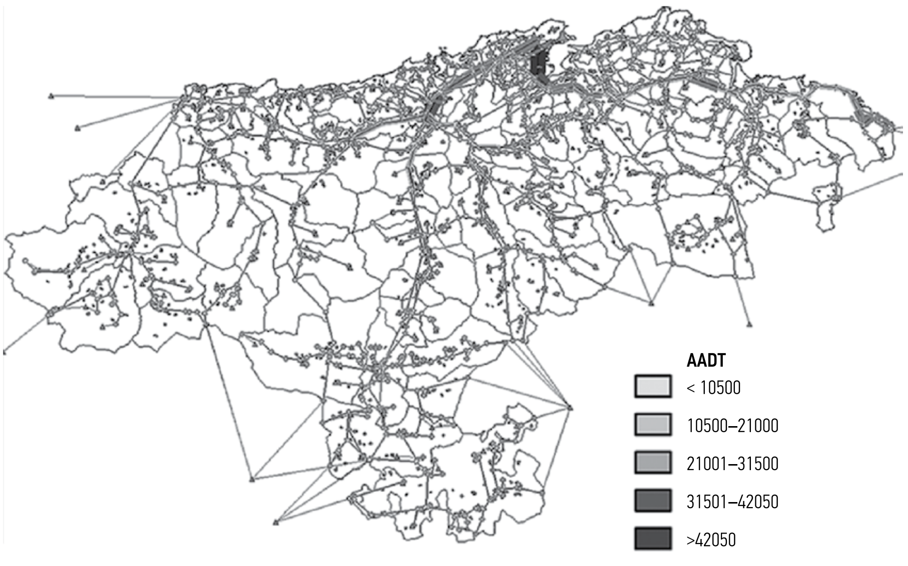

Fig. 6. Assignment of traffic to the network

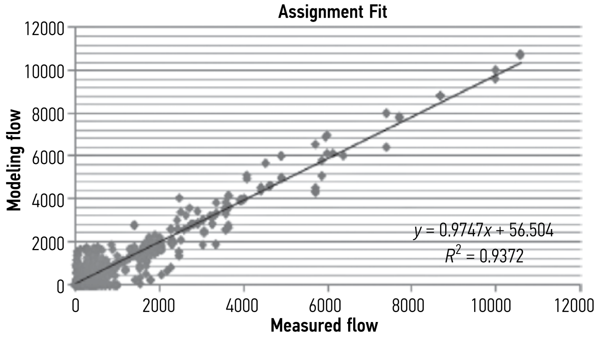

Fig. 7. Fit between modelled flows and real flows

As can be seen in Table, it is precisely the freeway Santander–Torrelavega (stretch A_1–B) which shows the greatest reduction in traffic. The stretches which are currently under construction provide a zone with a by-pass and notably improve its Level of Service.

The comparisons in Table clearly show that it would be completely unnecessary to build a third lane in each direction.

The regional Government of Cantabria performs a traffic counting plan every two years in which SPTCs are located on all the region's roads to estimate their AADT and the last survey required 576 counting stations (Fig. 8). However, once a tool like a valid O–D matrix is available, the possibilities become infinite. By applying the traffic counter optimization methodology explained in section 2, the number of counting stations required for the survey has been reduced only to 188 (Fig. 9). This represents a reduction of 68% compared with the existing situation, and a saving of more than 50% in the budgets applied to future traffic counting programmes.

These 188 locations are used to estimate an O–D matrix with less than 15% Mean Absolute Error (MAE) which, when assigned can reproduce the road network AADT with an error lower than 7%.

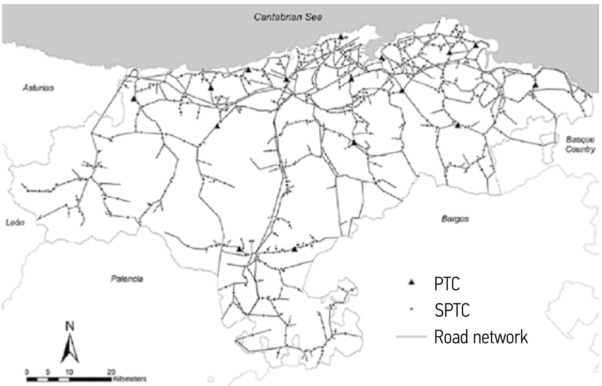

Fig. 8. Actual location of counting stations

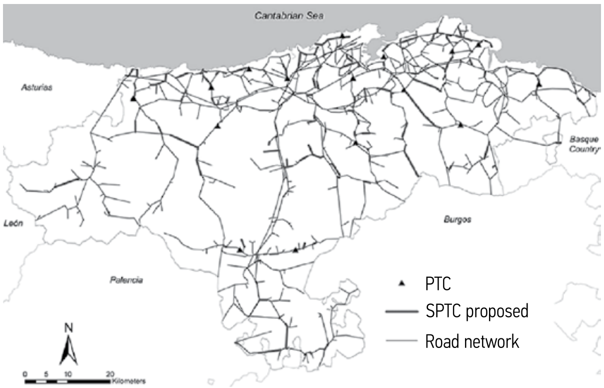

Fig. 9. Proposed location of counting stations

Referring to the demographic accessibility indexes; under the proposed situation practically all the boroughs show increases of over 2%, growing to over 10% in the areas adjacent to the new axes due to the reduced time taken to travel to the western part of the region. Generally though, an overall increase in accessibility of 4.75% was found for the region as a whole.

Table. Current and modelled AADT; Level of Service (LOS)

Id	Stretch	Type	AADT (veh)			LOS (peak hour)		LOS (off peak)	
			Actual	Modelled	Dif (%)	Actual	Modelled	Actual	Modelled
A–67	A_1–B	Freeway	71623	53288	–25.6	E–F	D	D	C
A–67	B–to Madrid	Freeway	27622	28546	+3.34	D	D	C	C
A–8	B–C	Freeway	33654	39448	+17.21	D	D	C	C
A–8	B–to Oviedo	Freeway	33589	35411	+5.42	D	D	C	C
A–8	E–F–to Bilbao	Freeway	56102	54002	–3.74	D	D	C	C
S–10	A–E	Freeway	100593	84109	–16.38	E	E	D	C
S–20	A–A_1	Freeway	39800	56214	+41.24	C	D	B	C
A–8	C–F	Freeway	10290	13962	+35.68	E	C	D	B
S–30	A_1–D–E	Freeway	–	33275	–	–	D	–	C

Conclusions

1. This work has presented a basic tool for simulating traffic over an entire road network. This tool can be used not only for predicting the repercussions of working on transport infrastructure but also for making decisions on whether or not to invest in such construction projects. The importance of a sound mobility and accessibility plan on which to base resource rationalisation has been shown, along with the objective criteria needed for evaluating the different alternatives.
2. This work used the dual application of a mobility model and a demographic accessibility model to evaluate the impact of important strategic works on inter-urban networks. The high investment in infrastructure demanded by numerous social groups and politicians in the Autonomous Community of Cantabria (Spain) has been proven unnecessary, thereby avoiding the high social and economic costs associated with expropriations, environmental impact, etc. for the public administration.
3. Important cost reductions in data collection have been achieved by reducing the required number of counting stations from 576 to 188, representing a 68% fall in the number of links counted, reducing the administration costs by approximately 50% for any future Regional Traffic Counting Plans.
4. These 188 counting points were used to estimate an O–D matrix with a Mean Absolute Error (MAE) of less than 15% which could reproduce the road network AADT with an error of less than 7%.

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