



## BALANCING PUBLIC AND PRIVATE INTERESTS THROUGH OPTIMIZATION OF CONCESSION AGREEMENT DESIGN FOR USER-PAY PPP PROJECTS

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**Abstract.** In user-pay public private partnership (PPP) projects, private sectors collect user fees to cover cost and reap revenue. For projects that cannot be self-financed, public sectors usually invest public funds to make them financially feasible. The concession agreement allocates revenues and risks, and lies in the center of balancing public and private interests. However, stakeholders may have contrary opinions regarding the optimization of concession agreement. While private sectors are concerned about earning money, public sectors pay more attention to the efficient use of public funds. To address this challenge, this paper firstly identifies several key concessionary items, including concession period, concession price, capital structure and government subsidy. Then, a multi-objective optimization model is presented using discounted cash flow method, in which key concessionary items act as decision variables and public and private interests are represented by two sub-objectives. Subsequently, the model is solved using non-dominated sorting genetic algorithm-II (NSGA-II). Furthermore, a numerical case based on Beijing No. 4 Metro Line is provided to demonstrate the application of the model. Results show that the proposed model can produce a series of viable combinations of concessionary items that balance public and private interests, which provides practical references for relative decision making activities.

**Keywords:** concession agreement, multi-objective, optimization, non-dominated sorting genetic algorithm-II (NSGA-II), user-pay, public-private partnership (PPP).

### Introduction

To overcome financial restraints and make use of advanced technology, hosting governments around the world are encouraging private sectors to involve in providing infrastructure or public services (Shahrara *et al.* 2017). In a typical user-pay public private partnership (PPP) project, private partner charges service fee (or toll) from users during a concession period in order to get reimbursed. For PPP projects that cannot be fully self-financed, hosting governments would offer public funds, including direct equity investment or government subsidies, to improve the projects' profitability (Chen *et al.* 2013; Sharma *et al.* 2010). The successful implementation of PPP projects primarily depends on formulating an appropriate concession

agreement that can both attract private capital and protect public interests. In China, local hosting governments are in charge of designing the concession agreement. However, most of them are generally in lack of the relative experience as well as established templates (Cheng *et al.* 2016). An inappropriate concession agreement can lead to future disputes, costly renegotiation or even project failure as has been proven by past practices (Thieriot, Dominguez 2015; Teo, Bridge 2017). Thus, the optimization of concession agreement design is attracting growing interest from both academia and industry.

A number of previous research has investigated the optimization of key concessionary items from the per-

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spective of private sectors. Islam and Mohamed (2009) optimized concession period, concession price and capital structure in order to maximize the bid winning probability of a potential concessionaire whereas the effects of public funds are excluded. Iyer and Sagheer (2012) developed an optimized concession agreement in order to maximize bid-winning potential for project sponsors. Sundararajan and Tseng (2017) proposed a dynamic capital structure approach to minimize the weighted average cost of capital (WACC) so as to maximize the enterprise value. However, public sectors are increasingly involved in the financing of PPP projects nowadays. Local governments would offer public equity or government subsidy to lower projects' financing cost or increase project's operation transparency. While private sectors are interested in earning more money, public sectors focus more on protecting public properties (Sharma *et al.* 2010; Cheng *et al.* 2016). Thus, it is highly necessary to shift the focus of concession agreement optimization from maximization of economic profit or bid winning potential to better balance of public and private interests.

Besides, past studies mainly focus on optimizing one of the specific concessionary items while other items are considered exogenously determined. Zhang (2005) studied the optimization of capital structure for a privatized public infrastructure project. Marco *et al.* (2012a) used linear regression analysis to investigate the influence of risk factors on the level of public funding. Carbonara *et al.* (2014) proposed a model that can determine a win-win concession period for both concessionaire and government. Admittedly, by focusing on one specific concessionary item, research process has been focused and simplified. However, the interactions among concessionary items are ignored and the optimization of concession agreement is fragmented to some extent. Besides, decision makers are only provided with limited choices (Song *et al.* 2015).

The motivation of this research thus arises from the clear caveats of existing literature in deriving a concession agreement that is both economically attractive to private sectors and can make efficient use of public funds. The primary focus of this paper is on formulating a multi-objective optimization model that can provide a series of feasible combinations of key concessionary items, which would increase management flexibility and assist relative decision making. The rest of the paper is organized as follows. Firstly, four key concessionary items, i.e. concession period, concession price (toll), capital structure and government subsidy, are introduced and discussed in details. Afterwards, a multi-objective optimization model is built using discounted cash flow method. Public and private interests are represented by two sub-objectives respectively. The method of non-dominated sorting genetic algorithm-II (NSGA-II) is employed to solve the model. Then, a numerical case based on Beijing No. 4 Metro Line is used to display the application process of the model. Lastly, the paper is concluded with further discussions related to the model's implications and limitations.

## 1. Balance public and private interests through concession agreement

Public and private sectors may have differing opinions as to what constitutes an appropriate concession agreement. Private sectors are interested in earning economic profits while public sectors are concerned with the efficient use of public funds. The design of concession agreement needs to make compromises between these two conflicting interests and provide a solution that can be accepted by both parties (Khanzadi *et al.* 2016).

Islam and Mohamed (2009) pointed out that concession period, concession price (toll) and capital structure are the three most critical concessionary items that determine the award of concession schemes to promoters. Iyer and Sagheer (2012) found that the grant sought from the government is the sole bidding variable for Indian build-operate-transfer (BOT) highway projects. Iossa (2015) summarized that the key variables in PPP procurement generally include toll, shadow tolls, net present value, concession period, risk allocation and revenue share. In China, hosting governments are in charge of designing the initial concession agreement. Later, private sectors are selected according to their responses to the boundary conditions pre-set. Some of the common qualitative bidding factors include risk allocation, technical qualification while quantitative bidding factors include concession period, concession price (toll), capital structure and government subsidy.

Based on the above discussions, this research has chosen these abovementioned quantitative concessionary items as decision variables for the proposed model. Not only do they affect the production and allocation of future project revenues, but also they are considered as important bidding variables in determining appropriate concessionaire. The following contents probe into the determination of these key concessionary items and how public and private interests are reflected in their decision process.

### 1.1. Concession period

During the concession period, special purpose vehicle (SPV) builds and operates the project and transfers the project back to hosting government at the end of it (Shen *et al.* 2002; Zhang *et al.* 2017). A longer concession period allows SPV to collect more revenues, making the project more profitable (Zhang, Abourizk 2006). For public sectors, however, a prolonged concession period may undermine public interests (Ng *et al.* 2007). A shorter concession period usually comes with higher initial tolls, which can also cause strong opposition from public (Zhang, Abourizk 2006). Therefore, the concession period shall be long enough for concessionaire to get reasonable return yet not so extended as to cause losses to public interests. In this research, concession period refers to the length of operation period, excluding construction period (Ye, Tiong 2003).

## 1.2. Concession price

Concession price is also one of the important concessionary items. Concessionaires rely on collecting user payments in order to recover initial investment (Qiao *et al.* 2009; Xu *et al.* 2012). To protect public welfare, public sector usually imposes price ceiling to the concession price (Islam *et al.* 2006; Subprasom, Chen 2007). For projects with low concession price, additional measures such as prolonged concession period, investment of public equity or government subsidy are needed in order to make the projects economically viable (Ashuri *et al.* 2012; Tan, Yang 2012).

## 1.3. Capital structure

Capital structure mainly refers to the determination of debt-equity ratio and the allocation of equity between public and private sectors (Sharma *et al.* 2010). In order to keep the long commitment of private sector, public sector usually requires a minimum level of private equity investment (Zhang 2005). As for private sectors, they prefer keeping a relatively low level of equity investment in order to utilize the financial leverage or raise the rate of return (Marco *et al.* 2012b). Other reasons for this preference may include minimizing project risk, limitation of capital fund and search for more profitable investment opportunities (Zhang 2005). This research uses public and private equity ratio to represent the capital structure of the PPP project. The debt ratio can be easily computed after these two variables are determined.

## 1.4. Government subsidy

For financially nonviable PPP projects, it is highly essential that hosting governments offer some kind of government subsidy in order to make the project economically attractive (Liou *et al.* 2012; Man *et al.* 2016). The total amount of government subsidy shall be determined with caution. It should be large enough to make PPP projects financially feasible in order to attract potential private partners (Song *et al.* 2015). And it should also be frugal enough to avoid public's accusations of misuse or waste of public funds (Cheah, Liu 2006). In a sense, the financial subsidy provided by government can be regarded as a risk-mitigating method that reduces the adverse effects of future project risks taken by private sectors (Takashima *et al.* 2010).

## 2. Model development process

The following section probes into the construction and solution of a multi-objective optimization model that balances public and private interests in PPP projects. The first part is the model construction process using discounted cash flow (DCF). The interests of public and private sectors are represented by two sub-objectives respectively. The second part is the model solution concerning multi-objective optimization problem and non-dominated sorting genetic algorithm-II (NSGA-II).

## 2.1. Model construction

As pointed by Kakimoto and Seneviratne (2000), a project financing model is generally composed of three parts, i.e. cost function, revenue function and objective function. The following section provides insights into the development of a multi-objective optimization model using DCF method.

### 2.1.1. Cost function

According to Ranasinghe (1996), the total project cost can be divided into three parts, i.e. the base cost ( $BC$ ), the cost escalation during construction ( $EC$ ) and interest during construction ( $IC$ ). Thus, the expression of TPC can be simplified as Eqn (1):

$$TPC = \sum_{i=1}^{CP} (BC_{i-1} + EC_{i-1} + IC_{i-1}); \quad (1)$$

$$EC_{i-1} = BC_{i-1} \left[ \prod_{h=0}^{i-1} (1 + r_h) - 1 \right]; \quad (2)$$

$$IC_{i-1} = (1 - E) BC_{i-1} \prod_{h=0}^{i-1} (1 + r_h) [(1 + r_b)^{CP-i+1} - 1], \quad (3)$$

where  $BC_{i-1}$  – base cost at the beginning of the  $i^{\text{th}}$  year;  $CP$  – length of construction period;  $EC_{i-1}$  – increased cost due to the effects of inflations on  $BC_{i-1}$ ;  $IC_{i-1}$  – accumulation of debt interests for  $BC_{i-1}$  until the end of construction period;  $r_h$  – inflation rate in the  $h^{\text{th}}$  year;  $r_b$  – debt interest rate; and  $E$  – equity level.

### 2.1.2. Revenue function

The calculation of operation revenues can be broken down into the calculation of service price, service demand and their corresponding growth rate (Kakimoto, Seneviratne 2000). The operation revenue is represented by Eqn (4):

$$REV_j = [P_{j-1} \prod_{k=0}^{j-1} (1 + g_k^P)] [Q_{j-1} \prod_{k=0}^{j-1} (1 + g_k^Q)] \quad j \in [1, OP], \quad (4)$$

where  $REV_j$  – gross revenue in the  $j^{\text{th}}$  year;  $P_{j-1}$  – service price at the start of the  $j^{\text{th}}$  year;  $Q_{j-1}$  – service demand at the start of the  $j^{\text{th}}$  year;  $g_k^P$  – annual growth rate of service price in the  $k^{\text{th}}$  year;  $g_k^Q$  – annual growth rate of service demand in the  $k^{\text{th}}$  year; and  $OP$  – length of operation period. The service price and service demand of the  $j^{\text{th}}$  year can be determined given the initial service price and demand at first year and growth rate in the following years.

Meanwhile, operation and maintenance cost at first year can be estimated as a certain percentage of base cost (Bakatjan *et al.* 2003). And operation and maintenance cost for the  $j^{\text{th}}$  year ( $OMC_j$ ) can then be predicted based on the annual growth rate  $g_k^Q$  and inflation rate  $r_h$ :

$$OMC = \lambda BC \left[ \prod_{k=0}^{j-1} (1 + g_k^Q) \right] \left[ \prod_{h=0}^{CP+j} (1 + r_h) \right] \quad j \in [1, OP]. \quad (5)$$

The gross profit is the difference between  $REV_j$  and  $OMC_j$ . To further calculate the net profit of the project, the debt interests and taxes need to be deducted from the gross profit. The repayment of bank debt is assumed to be processed through equal loan payment. The calculation of annual repayment for debt instalments ( $ADI_j$ ) is defined by Eqn (6) (Bakatjan *et al.* 2003; Ranasinghe 1996). And interest on debt ( $INT_j$ ) to be paid in the  $j^{\text{th}}$  year is represented by Eqn (7). In consistent with previous research, only the effects of income tax ( $TAX_j$ ) is considered in the calculation as shown in Eqn (8) (Wibowo, Kochendörfer 2005):

$$ADI_j = \sum_{i=1}^{CP} [(1-E)BC_{i-1} \prod_{h=0}^{i-1} (1+r_h)(1+r_b)^{(CP-i+1)}]^* \frac{r_b(1+r_b)^{LRP}}{(1+r_b)^{LRP}-1}; \quad (6)$$

$$INT_i = ADI_j \left[ 1 - \frac{1}{(1+r_b)^{LRP-j+1}} \right]; \quad (7)$$

$$TAX_j = \max[0, (REV_j - OMC_j - INT_j - DEP_j)r_t]. \quad (8)$$

The annual net after-tax cash flows ( $NCF_j$ ) can lastly be formulated in Eqn (9) using the parameters calculated above (Bakatjan *et al.* 2003; Zhang 2005):

$$NCF_j = (REV_j - OMC_j - ADI_j - TAX_j). \quad (9)$$

### 2.1.3. Financial indicators

This part discusses the computation of important financial indicators that are used in the formation of objective functions and constraints. These financial indicators can be generally divided into two groups. One type of indicators is closely connected with project profitability, including net present value ( $NPV$ ) and internal rate of return ( $IRR$ ). The other type of indicators mainly reflects the debt repayment ability of the project, which is the main concern for project debt holders. This type of indicators includes debt service coverage ratio ( $DSCR$ ) and loan life coverage ratio ( $LLCR$ ).

Different from previous studies, this research has made the attempt to introduce the effects of public equity and government subsidy into the calculation of  $NPV_e$ . For projects with public equity investments, it's a common practice in China for hosting governments to give up the rights to dividends so as to reduce the cost of financing or to make the project more attractive. From private sectors' perspective, this practice is equivalent to obtaining a certain amount of interest-free loans, which enhances the yields of private equity investments.

The calculation of  $NPV_e$  is shown as Eqn (10). To simplify the calculation without loss of principle, the government subsidy ( $G$ ) is treated as one time grant at the beginning of the operation period and will be used evenly over the project's operation period as is represented by  $G/OP$ .

And  $e_1$  is the percentage that private equity occupies in the total investment while  $e_2$  is the percentage of public equity.

$$NPV_e = -\sum_{i=1}^{CP} \frac{e_1[(BC_i + EC_i) + IC_i]}{(1+r)^i} + \sum_{j=CP+1}^{CP+OP} \frac{REV_j - OMC_j - ADI_j - TAX_j}{(1+r)^j} + \sum_{j=CP+1}^{CP+OP} \frac{G/OP}{(1+r)^j}. \quad (10)$$

$IRR$  defines the hurdle rate that determines whether the investment is worthwhile (Santandrea *et al.* 2017). The calculation of  $IRR$  is shown in Eqn (11):

$$\sum_{i=1}^{CP} \frac{e_1[BC_i \prod_{h=0}^i (1+r_h) + IC_i]}{(1+IRR)^i} = \sum_{j=CP+1}^{CP+OP} \frac{NCF_j}{(1+IRR)^j}. \quad (11)$$

Both  $DSCR$  and  $LLCR$  can assess the debt reimbursement ability of certain projects. Debt holders are concerned with these two variables in order to make sure the project is still bankable and the debt lent can be recovered by future project cash flows.  $DSCR$  is defined by the ratio of available cash flow to the current debt obligations.  $DSCR$  is required to be greater than 1, which ensures the project have sufficient future cash flow to pay its annual debt service. It is calculated by Eqn (12):

$$DSCR = \frac{REV_j - OMC_j - TAX_j}{ADI_j}. \quad (12)$$

$LLCR$  is another debt metric that measures the number of times the cash flow available for debt service on a discounted basis. When  $LLCR$  equals 1, it means that the cash flow available for debt service when discounted by hurdle rate is exactly the same with the amount of the outstanding debt balance (Iyer, Sagheer 2012). It is calculated by Eqn (13):

$$LLCR_k = \sum_{j=k}^N \frac{(REV_j - OMC_j - TAX_j)(1+r)^{j-k+1}}{ADI_j(1+r)^{j-k+1}}. \quad (13)$$

### 2.1.4. Objective function

Two sub-objective functions are created in order to represent the public and private interests respectively. The interests of private sector are represented by the maximization of the net present value shared by private sector, which is measured by  $NPV_e$ . The interests of public sector are generally considered as the welfare or well-being of general public (Yan *et al.* 2017). This study has used the minimization of costs of public funds as an approximate quantitative indicator for public interests in order to make this abstract concept more quantifiable. In other words, the efficient use of public funds is ensured when the potential costs of public investments, including public equity and government subsidy, are minimized (Feng *et al.* 2017).

Different from the previous works, this model focuses more on solving the conflicting interests between private and public sector. Thus, the optimization target of the current model is composed of two independent sub-objectives, rather than a single target that is integrated mathematically. The two sub-objective functions are shown as Eqn (14) and Eqn (15):

$$\text{Sub-objective 1: Maximize } NPV_e ; \quad (14)$$

$$\text{Sub-objective 2: Minimize } TPC * e_2 + G . \quad (15)$$

The proposed model is subjected to the following constraints. Eqn (16) and Eqn (17) capture the minimum rate of return from the perspectives of private equity participant. Eqn (18) and Eqn (19) represent the debt servicing constraints based on the interests of debt holders. Eqn (20) and Eqn (21) describe the minimum limit for private equity and maximum limit for public equity. Eqn (22) to Eqn (24) give the appropriate value ranges for concession period, concession price and government subsidy. Decision variables include private equity, public equity, concession period, and concession price and government subsidy:

$$NPV_e \geq 0 ; \quad (16)$$

$$IRR_{\min} \leq IRR \leq IRR_{\max} ; \quad (17)$$

$$DSCR_i \geq 1, \forall i = CP + 1, \dots, CP + OP ; \quad (18)$$

$$LLCR_i \geq 1, \forall i = CP + 1, \dots, CP + OP ; \quad (19)$$

$$e_1 \geq e_{1\min} ; \quad (20)$$

$$e_2 \leq e_{2\max} ; \quad (21)$$

$$OP_{\min} \leq OP \leq OP_{\max} ; \quad (22)$$

$$P_{\min} \leq P \leq P_{\max} ; \quad (23)$$

$$G_{\min} \leq G \leq G_{\max} . \quad (24)$$

## 2.2. Model solution

### 2.2.1. Multi-objective optimization problem

Multi-objective optimization problem (MOOP) is one of the important research directions in the field of optimization. It mainly focuses on the optimization of a plurality of numerical targets, which may be conflicting under many circumstances. Generally, a multi-objective optimization problem can be described as shown in Eqn (25) and Eqn (26), where  $F(x)$  is the objective function and  $g_i(x)$ ,  $h_j(x)$  are relevant constraints:

$$\max(\min)F(x) = (f_1(x), f_2(x), \dots, f_p(x))^T ; \quad (25)$$

$$\text{Subject to } g_i(x) \geq 0, i \in I \text{ and } h_j(x) = 0, j \in E . \quad (26)$$

In a typical MOOP, the improvement of one target may lead to deterioration of another sub-objective at the same time. In other words, it is impossible to achieve the optimal results for all the sub-objectives all together. One can only make compromise among them and achieve acceptable solutions. The following Figure 1 is a visual demonstration of such a problem.

Figure 1 shows an example of the Pareto-efficient set for a multi-objective optimization (max-max) problem. For case 1 and case 2, case 1 has a smaller value in objective 1 but a larger value in objective 2. It is impossible to determine which of these two cases is better. Further, case 3 is better than case 2 since they have same value in objective 2 but case 3 has a larger value in objective 1. Similarly, case 4 is better than case 2, case 7 is better than case 4 and case 5 is better than case 3. As for case 5, case 6 and case 7, it's impossible to tell the pros and cons between them since they are better than other cases in one way but weaker in another way. There are no other better solutions than these three cases, so they're known as the Pareto efficient set (Eddy 2001). Any points off the frontier, such as case 1, case 2, case 3 and case 4, are not Pareto-efficient.

### 2.2.2. Non-dominated sorting genetic algorithm-II (NSGA-II)

To solve this multi-objective optimization problem and find a set of viable non-dominated solutions, the present study has used the non-dominated sorting genetic algorithm (NSGA-II) developed by Deb (2000). The operation process of NSGA-II is shown in Figure 2.

The basic idea of NSGA-II is as follows: firstly, an initial population of N individuals is generated randomly. After non-dominated sorting and basic operations like selection, crossover and mutation, the first generation of individuals is gained. Then, from the second generation, the parents' populations are merged with offspring populations. By maintaining a relative large population of candidates, the search for optimal solutions is multi-directional, which results in a higher probability of finding the global optimum. Then, individuals are assigned into different fronts through fast non-dominated sorting. In the mating pool containing parent and offspring populations, appropriate

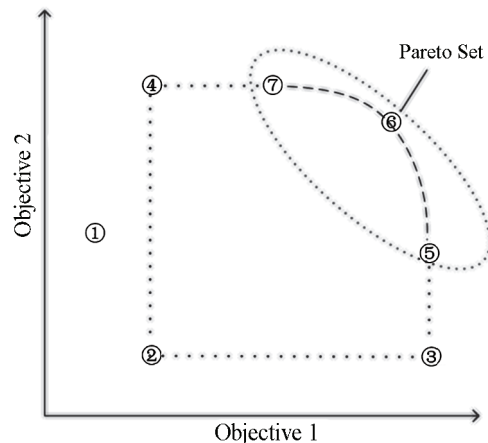


Figure 1. Multi-objective optimization problem (max-max)

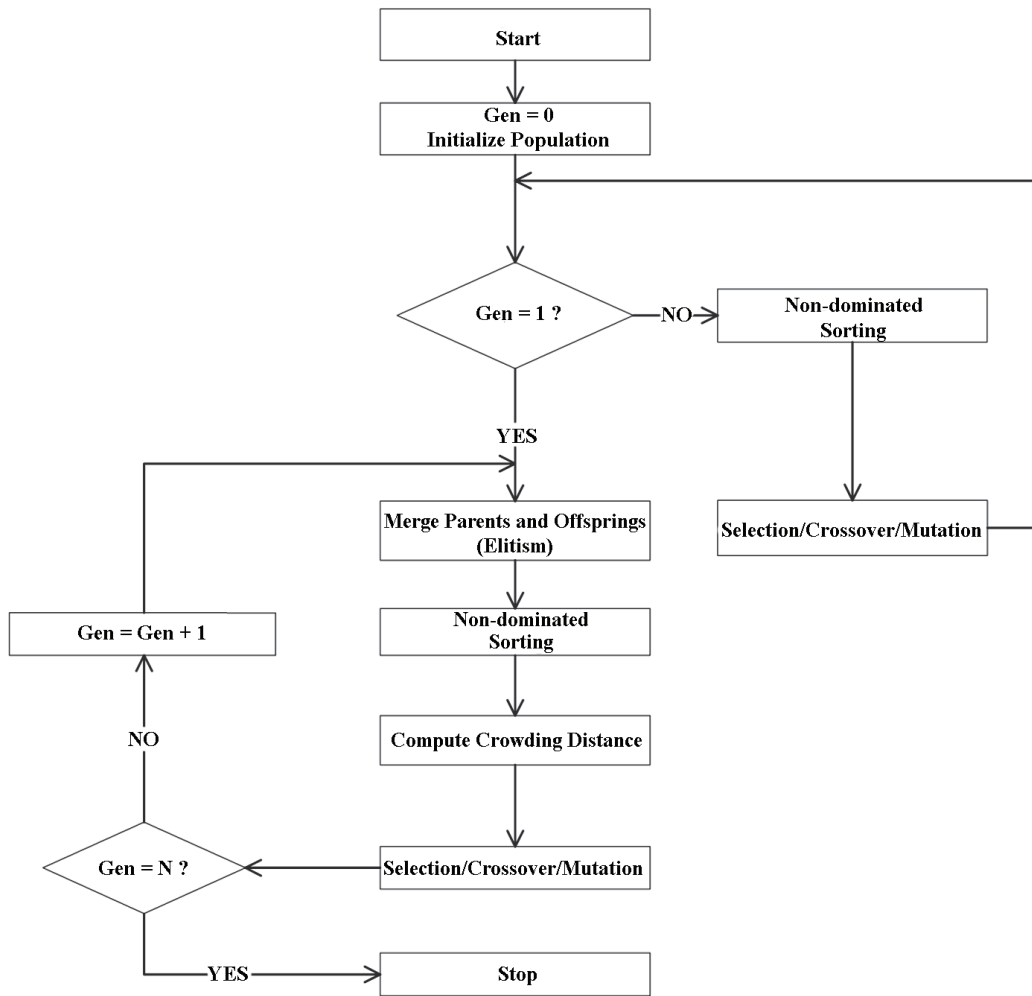


Figure 2. Operation process of NSGA-II

individuals are selected based on rank and crowding distance in order to form a new generation. Finally, the newly selected individuals go through basic operations of genetic algorithm and produce a new generation, and so on until the end of the program.

The parameters of NSGA-II, such as population size, number of generations and cross-over rate, are selected in reference to previous studies and are presented in Table 1

Table 1. Parameters of NSGA-II

Parameters	Value
Number of decision variables	5
Number of objective functions	2
Number of constraints	5
Population size	30
Number of generations	100
Cross-over rate	0.9
Mutation rate	0.5
Penalty coefficient	1000

(Goldberg 1989). The Microsoft excel is used to calculate the future cash flow and MATLAB (version 7.0) is employed to conduct the algorithm. More details of operations will be discussed through the illustration of a numerical case.

### 3. Illustration of the model with a numerical case

To illustrate the capability of the proposed model in optimizing the design of concession agreement, a numerical case based on Beijing Metro Line No. 4 project is provided. In this project, the SPV is permitted to collect user fees to recover its initial investments. However, the concession price of the service is determined by the hosting government and is limited to a low level in order to attract more passengers. And hosting government decides to offer financial support including public equity and government subsidy to make the project economically viable.

In order to make the project data in line with current research, some reasonable adjustments and assumptions have been made. Some of the variables are now presented in the form of viable ranges. The five decision variables used in the model have been listed in Table 2.

Table 2. Decision variables and value ranges

Decision variables	Value ranges
Private equity	20%–50%
Public equity	Below 20%
Concession period	10–30 years
Concession price	Below \$0.93 per person (i.e. ¥6 Chinese Yuan)
Government subsidy	\$3.11–\$4.67 billion

Reasons for the determination of their value ranges are discussed below: the equity share of capital investments from public and private sector is determined on a project level. According to the relevant regulations published by China's national council concerning capital ratio of fixed assets investment, the minimum equity investment for urban rail transit project shall be no less than 20% of the whole investment (the sum of debt and equity). Thus, the minimum amount of private equity is set to be 20% of the total investment. Since the private sectors tend to use the commercial loans in order to enhance the rate of return for its own funds, the maximum amount of private equity is set to be 50%, so there is still room for the utilization of financial leverage. As for the ranges of public equity, according to regulations, the hosting government shall hold less than 50% equity of the project. To ensure this, the percentage of public equity in the whole investment shall be less than 20%.

The concession period is stipulated to be no less than 10 years and no more than 30 years by drawing on the relevant provisions of the China's ministry of finance (MOF) and national development and reform commission (NDRC). For concession price, its value range can be determined by investigating the residential income level, willingness to pay and operation costs of similar projects, etc. In some cases, the government may limit the concession price in order to better achieve certain policy goals. Government subsidies are largely determined by the financial condition of the project and government's fiscal budget plan.

The annual growth rates for maintenance costs, concession price and service demand have been predicted separately based on the considerations of inflation, investigation and similar projects in the past. As for discount rate, this research has used the opportunity cost of private equity to represent its financing cost in reference to previous research (Bakatjan *et al.* 2003). In urban rail transit industry, private sectors generally require an internal rate of return (IRR) of around 10% (Xinxin 2016). Other relative project data used in the model is listed in Table 3.

### 3.1. Optimization results

The model starts by encoding the decision variables in the form of chromosomes. For each individual, a chromosome consisting of private equity, public equity, concession period, concession price and government subsidy

Table 3. Project data and value ranges

Project data	Value ranges
Construction period	4 years
Loan repayment period	20 years
Grace period	4 years
Loan rate	4.9%
Discount rate	10%
Base cost	\$1.17 billion per year
Basic demand	0.2 billion person per year
Maintenance cost	0.021% of base cost
Growth rate for OMC	5.1%, 1–6 years; 5.3%, 7–16 years; 3.5%, 17–30 years;
Growth rate for demand	5%, 1–6 years; 3.2%, 7–16 years; 1%, 17–30 years
Growth rate for price	2%, 1–6 years; 1.5%, 7–16 years; 1%, 17–30 years

is generated. Real value coding is adopted in this process. For each decision variable in its generation, a random number is selected between the minimum and maximum value limits for the variable.

The initialized population is then sorted based on non-domination sorting method (Deb *et al.* 2000). The first front contains individuals that are non-dominated by any other front while individuals in the second front are only dominated by those in the first front and so on. Individuals in each front are given the relative fitness rank. Individuals in the first front are assigned a fitness value of one and so on. After the fitness values have been calculated, a crowding distance is then assigned to each solution. By definition, crowding distance measures how close an individual is to its neighbours (Deb 2000). The larger the crowding distance, the more dispersed is the individual distribution in the population. NSGA-II then classifies the individuals according to their domination relation and density index. Table 4 shows the non-dominated sorting of the initial population. The seventh column shows the rank of fitness value calculated in each front. The eighth column shows the crowding distance for each solution.

After the individuals are sorted based on non-domination and crowding distance, appropriate parents are selected using a binary tournament selection until the mating pool is full. Selection is based on the rank of each individual. For individuals with same ranking, the crowding distance is further compared. In general, an individual with lower rank and higher crowding distance will have more opportunities of being selected as parents. The selected parents are then used for reproduction to generate new springs.

For each individual in the mating pool, cross-over and mutation are then performed. The simulated binary crossover (SBX) and polynomial mutation are applied. Offspring can be generated either through the cross-over of two parents or the mutation of one parent. The

Table 4. Rank of initial population based on non-dominated sorting and crowding distance

No.	Private equity	Public equity	Concession period	Concession price (\$ per person)	Government subsidy (billion \$)	Fitness value	Crowding distance
1	20.24%	18.13%	12.89	0.72	3.19	1.0	0.699
2	34.43%	7.80%	18.28	0.83	3.89	1.0	0.620
3	45.09%	5.82%	26.59	0.85	3.41	1.0	0.434
4	41.05%	3.12%	12.07	0.91	3.20	1.0	0.742
5	44.32%	14.26%	25.16	0.92	4.39	1.0	Inf
6	48.73%	5.52%	23.48	0.83	3.29	1.0	0.405
7	46.42%	14.93%	17.90	0.94	4.44	1.0	0.317
8	34.22%	7.35%	24.20	0.54	3.41	2.0	0.637
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
28	26.68%	16.85%	15.30	0.02	4.45	9.0	Inf
29	21.30%	4.68%	20.14	0.23	3.86	9.0	Inf
30	26.00%	7.79%	20.95	0.23	4.16	10.0	Inf

newly produced offsprings are added to the current population. And selection is performed again to pick out the next generation. Only the best individuals in the current and previous generations are kept in the subsequent generation. Through this arrangement, elitism is achieved and algorithm is kept from falling into locally optimal solution. The process continues until the number of generations has reached the maximum number or the current solution shows no signs of further improvements.

As mentioned earlier, when multiple objectives are in conflicts with each other, the improvement of one usually accompanies deterioration of others. In this case, the optimum value is no longer a global optimization solution but rather an entire set of non-dominated solutions, also called Pareto set. Table 5 exhibits the last generation of optimized individuals and a series of 30 feasible combinations are provided.

Table 6 exhibits the objective values of the non-dominated Pareto set solutions. The first column is the value of NPVe and the second column is the value of public funds. These options all have a trade-off associated with NPVe vs. public funding, so none is clearly superior to any other. The maximization of NPVe always comes with the cost of increasing public funding. For example, solution 2 has a larger value of NPVe than solution 29 on one hand. On the other hand, solution 29 has lower public funding which makes it more attractive to public sectors. Thus, solution 2 and solution 29 cannot be directly compared without further information.

### 3.2. Sorting solutions by TOPSIS

Although the solution of MOOP is usually a set of feasible solutions, practitioners sometimes need to further pick

Table 5. Model results: feasible combination of key concessionary items

No.	Private equity	Public equity	Concession period (year)	Concession price (\$ per person)	Government subsidy (billion \$)
1	29.80%	0.00%	23.31	0.19	3.11
2	38.49%	19.14%	26.70	0.49	4.66
3	37.39%	1.04%	23.36	0.24	3.41
4	42.89%	0.36%	27.29	0.28	4.26
5	42.29%	0.71%	26.14	0.30	4.07
6	38.65%	0.07%	25.87	0.29	3.49
7	42.45%	0.33%	26.15	0.29	4.03
8	41.87%	0.52%	26.34	0.32	3.88
⋮	⋮	⋮	⋮	⋮	⋮
28	38.15%	0.07%	25.86	0.29	3.53
29	38.52%	1.66%	22.97	0.23	3.33
30	27.24%	4.36%	22.97	0.20	3.11



Table 6. Model results: optimization results of two sub-objectives (billion \$)

No.	Objective 1 (Maximize NPV <sub>e</sub> )	Objective 2 (Minimize public funding)	Fitness value	Crowding distance
1	0.84	3.11	1.0	Inf
2	1.99	4.91	1.0	0.206
3	1.05	3.42	1.0	0.197
4	1.46	4.26	1.0	0.176
5	1.44	4.07	1.0	0.170
6	1.14	3.48	1.0	0.164
7	1.40	4.03	1.0	0.156
8	1.39	3.88	1.0	0.144
⋮	⋮	⋮	⋮	⋮
28	1.17	3.53	1.0	0.083
29	0.99	3.33	1.0	0.069
30	0.85	3.17	1.0	0.068

one solution from this set. To show an example of this selection process, this research has adopted technique for order of preference by similarity to ideal solution (TOPSIS). The concept of ideal solution has to be introduced in order to further compare those Pareto solutions. Ideal solution is a virtual solution with the best or worst values of each sub-objective. It normally does not exist in the feasible domain due to the conflicts between sub-targets. TOPSIS is based on the idea that, in the sense of geometric distance, the most suitable solution shall be nearest to the positive ideal solution (PIS) and farthest to the negative ideal solution (NIS) (Boran et al. 2009). The operation of TOPSIS generally takes the following five steps (García-Cascales, Lamata 2012):

Step 1: Achieve an evaluation matrix of  $m$  alternative solutions and  $n$  evaluation criteria. If we use  $x_{ij}$  to represent the intersection of each solution and criteria, the whole matrix can be written as  $(x_{ij})_{m \times n}$ . In the case of this research,  $m = 30$  and  $n = 2$ .

Step 2: Normalize  $(x_{ij})_{m \times n}$  to get a new matrix. This research has used vector normalization method, where:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (i = 1, 2, \dots, m; j = 1, 2, \dots, n) . \quad (27)$$

Step 3: Get the positive ideal value set (best alternative) and negative ideal value set (worst alternative):

$$A_{best} = \{[\max(r_{ij} | i = 1, 2, \dots, m)], [\min(r_{ij} | i = 1, 2, \dots, m)]\} \\ \equiv \{r_{bj} | j = 1, 2, \dots, n\}. \quad (28)$$

$$A_{worst} = \{[mim(r_{ij} | i = 1, 2, \dots, m)], [\max(r_{ij} | i = 1, 2, \dots, m)]\} \\ \equiv \{r_{wj} | j = 1, 2, \dots, n\}. \quad (29)$$

Step 4: Calculate the separation distance of the target alternative solution with the best condition  $A_{best}$  and worst condition  $A_{worst}$  separately, as shown in Eqn (30):

$$d_{i-to-b} = \sqrt{\sum_{j=1}^n (r_{ij} - r_{bj})^2}, i = 1, 2, \dots, m; \quad (30)$$

$$d_{i-to-w} = \sqrt{\sum_{j=1}^n (r_{ij} - r_{wj})^2}, i = 1, 2, \dots, m. \quad (31)$$

Step 5: Calculate the relative distance between the alternative solutions with the worst condition, as shown in Eqn (32):

$$M_i = \frac{d_{i-to-w}}{d_{i-to-w} + d_{i-to-b}}, i = 1, 2, \dots, m, \quad (32)$$

where  $M_i$  is a positive value between 0 and 1. Alternative solutions can then be sorted according to  $M_i$ . The best alternatives can be selected by the rank of  $M_i$  in a descending order. A larger value indicates the solution is closer to the best condition and away from the worst condition.  $M_i$  equals to one only if the alternative solution has the best condition.

### 3.3. Discussions

#### 3.3.1. Analysis of model results

The following Table 7 provides a comparison of some selected optimization solutions. The first five lines represent the top five optimal solutions selected through TOPSIS. The last line is the actual contractual arrangement in the operation of Beijing Metro Line No. 4 project.

Based on TOPSIS selection, Solution 2 has the largest relative distance with the worst condition (0.638), which makes it the most suitable solution under this circumstance. One major difference between solution 2 and the actual contractual arrangement (solution A) is that the former has made more use of public equity in the initial project investment (Chang 2013). Due to the confidentiality of data, the exact NPVe of Metro Line No. 4 project is not available. However, in reference to solution 2, it can

Table 7. Comparison of different alternative solutions

No.	Distance	Special purpose vehicle		Debt ratio	Concession period (year)	Concession price (\$ per person)	Government subsidy (billion \$)	NPV <sub>e</sub> (billion \$)	Minimum public funding (billion \$)
		Private equity	Public equity						
2	0.638	38.49%	19.14%	42.37%	26.70	0.49	4.66	1.99	4.91
9	0.634	38.75%	15.34%	45.91%	26.78	0.45	4.50	1.86	4.70
18	0.634	38.73%	17.08%	44.19%	26.73	0.46	4.64	1.94	4.86
21	0.636	38.94%	16.57%	44.49%	26.88	0.46	4.59	1.92	4.80
23	0.636	43.19%	14.43%	42.38%	26.44	0.45	4.37	1.80	4.56
A	–	32.39%	0.66%	66.95%	30	0.31	5.73	–	5.73

be seen that the increased use of public equity has greatly reduced the total amount of public funding from \$5.74 billion to \$4.91 billion. In other words, for local governments with relatively sufficient capital, investing more public equity in the beginning can cause less financial burden than using that money as government subsidy during the operation period.

Although the top five solutions have small differences in specific values, they can still offer an approximate range for the key concessionary items. For example, all the selected solutions have a concession period of nearly 27 years, a concession price near \$0.47 and a total government subsidy of nearly \$4.67 billion. As mentioned before, solutions to a multi-objective optimization problem usually compose a non-dominated Pareto set, which makes it impossible to compare them directly. In this sense, the proposed model aim to provide a set of viable combinations or trade-offs rather than one single optimal solution. Decision makers can select the appropriate combination of concessionary items on the basis of their actual situations.

### 3.3.2. Comparisons with previous research

Several previous models have been developed for determining the optimal concessionary items of PPP projects.

The following Table 8 provides a comparison of the model results with some related studies:

Compared with the proposed model, previous studies bear several shortcomings. Firstly, two of the above studies, Zhang (2005) and Sundararajan and Tseng (2017), focused on the maximization of financial profitability from the perspective of private sectors. Iyer and Saheer (2012) provided a model that simultaneously optimizes bid winning probability and financial profitability. These models didn't consider the influence of public investments, which limits their use in PPP projects involved with public capital. Secondly, Sharma *et al.* (2010) and Feng *et al.* (2017) tried to propose a model that incorporates the costs and benefits of using public finance. However, in these models, only one optimal solution can be provided at a time, which poses limitation on the flexibility of management. Besides, as pointed by Iyer and Saheer (2012), financial indicators like IRR and DSCR are nonlinear functions of equity ratio, which makes it inappropriate to use linear programming or other dynamic programming techniques to solve these problems.

None of the previous models have considered the simultaneous optimizations of two sub-objectives representing public and private interests. To the best of authors'

Table 8. Comparison of model results with previous studies

Research	Method	Optimization objectives	Model outputs
Zhang (2005)	Simulation	Maximize financial profitability	Optimal equity
Sharma <i>et al.</i> (2010)	Linear programming	Maximize the benefits from PPP financing for the public agency	Optimal private equity and public funds
Iyer and Sagheer (2012)	Genetic algorithm	Maximize bid-winning probability and financial profitability	Optimal grant, debt and equity
Feng <i>et al.</i> (2017)	Genetic algorithm	Maximize the residue of NPV of private equity subtracting the present value of public funds	Optimal private equity, public equity and debt
Sundararajan and Tseng (2017)	Simulation	Maximize enterprise value	Optimal debt, equity and probability of default
The proposed model	Non-dominated sorting genetic algorithm-II	Maximize project profitability and minimize invested public funds	A series of viable concessionary items

knowledge, the proposed model is among the earliest ones to attempt to find the appropriate values of concessionary items that would maximize the return of private equity and minimize the investment of public funding. Besides, the utilization of non-dominated sorting genetic algorithm-II (NSGA-II) can generate a series of viable concessionary items. And based on the produced Pareto set, project stakeholders can further employ suitable selection methods, such as TOPSIS or Analytic Hierarchy Process (AHP), to pick up preferred solution according to their needs, which increases flexibility for decision making process.

## 4. Implications and limitations

### 4.1. Implications

In summary, the optimizations of PPP concession agreement are faced with two challenges. One is to balance the conflicts of interests between public and private sectors; the other is to consider the mutual interactions among various key concessionary items. This research has been designed to solve these two challenges in a holistic view.

For theoretical implications, this study further interprets the optimization of the main critical concessionary items in an expanded viewpoint. Recent practices in China and abroad have shown a trend of increasing involvement of public sectors in both project financing and concessionary agreement design (Yuan *et al.* 2017). More attention needs to be paid to the efficient use of public funds in order to adapt to public sector's shift of roles. To solve this challenge, this research has modified the optimization objectives and model constraints according to interest claims of public and private sectors respectively. The utilization of NSGA-II is applied to solve the multi-objective optimization problem and no pre-determined weights need to be assigned to the two targets.

For practical implications, this research has provided the public sector with a useful tool in designing the concession agreement. Compared with previous research that only gets one standard solution, this study can provide hosting governments with a series of feasible options. The better balance between public and private interests also ensures the comprehensiveness in relative decision-making process. Further, hosting governments are able to tailor the model to their actual use and can revise model assumptions based on the actual circumstances of the projects. The value ranges of key concessionary items can also be specified accordingly.

### 4.2. Limitations and future research

It is noteworthy that the research presented bears several limitations. First, the current model is developed and optimized mainly from aspects of increasing net present value or lowering financing cost. Impacts of project risk are not fully considered and the accuracy of the model depends on the exact projections of future traffic flows. Admittedly,

project risks, especially usage/demand risk, can affect the successful implementation of a PPP project. A series of recent PPP toll roads failures in Australia (Brisbane Airport Link toll road in 2013) and Texas (SH130 PPP toll road in 2016) are largely caused by overly optimistic forecasts of future demands.

To solve this problem, future research may use more dynamic traffic flow forecasting models to make the predictions more accurate. Besides, the concept of flexible contract design can be introduced in concession agreement design in order to reduce the impacts of project risks (Li, Hensher 2010). Tools such as options and renegotiation terms shall be carefully designed based on the actual situation of the project to better handle potential risks of project implementation in the future (Shan *et al.* 2010; Xiong, Zhang 2014).

Second, this paper has taken several concessionary items as decision variables. The initial value ranges for those input variables can largely influence the final outputs of the model. So, the model inputs shall be determined with caution. It is suggested that the hosting governments shall base their decisions on the comprehensive analysis of a series of relative factors, such as hosting government's fiscal revenue, local areas' development planning and income level of residents, etc. The accurate inputs of these variables can largely reduce the amount of computation and avoid the production of illogical values.

In summary, this paper by no means intends to cover the design of concession agreement in its entirety, but rather, to provide instructions for public and private sectors for negotiations on this issue. It also aims to attract more academic attention to the interactions among critical concessionary items and the balancing of public and private interests.

## Conclusions

This article develops a multi-objective optimization model to optimize the key concessionary items and to balance public and private interests for user-pay PPP projects. The method of NSGA-II is utilized to solve this problem. Then, a numerical case based on Beijing No. 4 metro line is used to verify the validity of the model. A series of feasible concessionary items with acceptable objective values is provided in the process. Lastly, the method of TOPSIS is used to select the optimal concession agreement design for the given project.

Traditionally, optimization of concession agreement design is carried out in a fragmented view. Public and private interests had not been optimized simultaneously. The proposed model pioneers the use of two sub-objectives to represent public and private sectors' pursuits of different targets. The optimization result not only enhances the profitability of the project but also guarantees the efficient use of public funds, achieving better balance of public and private interests.

The case study demonstrates the applicability of the model. It can effectively improve the efficiency of relative decision making and lay a solid foundation for the successful implementations of PPP projects. Both parties can rely on it as references for future negotiations concerning this issue.

## Notations

$TPC$	– total project cost;
$BC_{i-1}$	– base cost at the beginning of the $i^{\text{th}}$ year;
$EC_{i-1}$	– increased cost due to inflation of $BC_{i-1}$ for the $i^{\text{th}}$ year;
$IC_{i-1}$	– interest debt for the $i^{\text{th}}$ year;
$CP$	– length of construction period;
$r_h$	– inflation rate in the $h^{\text{th}}$ year;
$r_b$	– debt interest rate;
$r$	– discount rate;
$E$	– percentage of equity in $TPC$ ;
$REV_j$	– gross revenue in the $j^{\text{th}}$ year;
$P_{j-1}$	– unit price at the start of the $j^{\text{th}}$ year;
$Q_{j-1}$	– product demand at the start of the $j^{\text{th}}$ year;
$g_k^P$	– annual growth rate of unit price in the $k^{\text{th}}$ year;
$g_k^Q$	– annual growth rate of product demand in the $k^{\text{th}}$ year;
$OP$	– length of operation period;
$OMC_j$	– operation and maintenance cost for the $j^{\text{th}}$ year;
$ADI_j$	– annual repayment for debt installments for the $j^{\text{th}}$ year;
$INT_j$	– interest on debt for the $j^{\text{th}}$ year;
$TAX_j$	– income tax for the $j^{\text{th}}$ year;
$LRP$	– load repayment period;
$NCF_j$	– net after-tax cash flows for the $j^{\text{th}}$ year;
$NPV$	– net present value;
$IRR$	– internal rate of return;
$DSCR$	– debt service coverage ratio;
$LLCR$	– loan life coverage ratio;
$G$	– government subsidy;
$e_1$	– percentage of private equity in $TPC$ ;
$e_2$	– percentage of public equity in $TPC$ .

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