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A METHOD BASED ON TOPSIS AND DISTANCE MEASURES FOR HESITANT FUZZY MULTIPLE ATTRIBUTE DECISION MAKING

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Abstract. The aim of this paper is to provide a methodology to hesitant fuzzy multiple attribute decision making using technique for order preference by similarity to ideal solution (TOPSIS) and distance measures. Firstly, the inadequacies of the existing hesitant fuzzy TOPSIS method are analyzed in detail. Then, based on the developed hesitant fuzzy ordered weighted averaging weighted averaging distance (HFOWAWAD) measure, a modified hesitant fuzzy TOPSIS, called HFOWAWAD-TOPSIS is introduced for hesitant fuzzy multiple attribute decision making problems. Moreover, the advantages and some special cases of the HFOWAWAD-TOPSIS are presented. Finally, a numerical example about energy policy selection is provided to illustrate the practicality and feasibility of the developed approach.

Keywords: hesitant fuzzy information, TOPSIS, distance measures, multiple attribute decision making.

JEL Classification: A12, C44, C60, D81, D89.

Introduction

Multiple attribute decision making (MADM) is the process of finding the most suitable alternative or candidate from all of the feasible alternatives for evaluation and selection problems, which has been extensively applied in a variety of real-life areas. Due to the influence of increasing complexity of the manufacturing environment, sometimes it is difficult for decision makers (DMs) or experts to consider all relevant properties of the evaluation and selection problem, and then to give accurate assessment information on each alternative and the relative importance of each attribute by precise values.

The concept of the hesitant fuzzy set (HFS), originally introduced by Torra (2010), constitutes a powerful tool for dealing with uncertain information. Indeed, compared with the

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intuitionistic fuzzy set (Atanassov 1986) and the Pythagorean fuzzy set (Yager 2014; Zhang, Xu 2014), this approach permits the membership degree of an attribute to a given set being represented by several possible numerical values. Following this major trend in research, hesitant fuzzy set theory is considered having enormous chances of success for multiple attribute decision making problems due to the great superiority on dealing with vagueness, so that it has been applied in various areas, such as cluster analysis (Chen *et al.* 2013; Farhadinia 2013), pattern recognition (Peng *et al.* 2013; Xu, Xia 2011) and mainly in the decision making fields (Chen, Xu 2015; Liao *et al.* 2015; Jin *et al.* 2013; Mu *et al.* 2015; Rodríguez *et al.* 2014; Tan *et al.* 2015; Xia, Xu 2011; Ye 2014; Xu, Zhang 2013; Zhang 2013; Yu *et al.* 2013; Zhang, Wei 2013; Zhang *et al.* 2014; Zeng *et al.* 2013a). For example, Xia and Xu (2011) proposed some common hesitant fuzzy aggregation operators and studied their application in decision making problems. Ye (2014) proposed a correlation coefficient between hesitant fuzzy sets and applied it to multiple attribute decision making under dual hesitant fuzzy environment. Xu *et al*. (2014) introduced a maximizing deviation method to handle the hesitant fuzzy decision making problems in which the information about criteria weights is incomplete. Zhang (2013) put forward a method for hesitant fuzzy multi-criteria group decision making based on the hesitant fuzzy power aggregation operators. Some hesitant fuzzy prioritized operators are presented by Yu *et al.* (2013) to solve personnel evaluation problem that involves a prioritization relationship over the evaluation index. Zhang and Wei (2013) developed the extended VIKOR (VlseKriterijumska Optimizacija Kompromisno Resenje) method to solve the hesitant fuzzy MCDM problems. Mu *et al*. (2015) presented a new aggregation principle for aggregating hesitant fuzzy elements, which can effectively reduce the computational complexity specific to the conventional aggregation principle. Zhang *et al*. (2014) proposed some induced generalized hesitant fuzzy operators and studied their application in multiple attribute group decision making problems. In addition, based on the Hamacher t-norm and t-conorm, Tan *et al*. (2015) proposed some hesitant fuzzy Hamacher operators for aggregating hesitant fuzzy information, and studied its application in multi-criteria decision making. Combining the idea of HFSs with the ELECTRE II method, Chen and Xu (2015) suggested a new HF-ELECTRE II approach to efficiently handle different opinions of group members that are frequently encountered when handling the MADM problems. Zeng *et al.* (2013a) presented a new multimoora method for multi-criteria hesitant fuzzy group decision making. In order to make a more reasonable decision, Liao and Xu (2014) proposed a satisfaction degree-based interactive decision-making method to derive the weights of the hesitant fuzzy MADM in which the preference information on attributes is collected over different periods. Based on the Dempster–Shafer theory of evidence, Sevastjanov and Dymova (2015) presented a critical analysis of conventional operations on HFE and their applicability to the solution of MADM problems.

Among the numerous MCDM methods, Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang, Yoon 1981) continues to work effectively in different application fields. The classic TOPSIS method aims to choose alternatives that simultaneously have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution. The main reason of such a wide acceptance is because its concept is reasonable, easy to understand and compare with other MCDM methods, like AHP and

ELECTRE I, it requires less computational efforts, and therefore can be applied easily (Kim *et al.* 1997). In traditional TOPSIS method, the evaluation values of alternatives given by DMs are defined as precise numbers. Over the last decades, the TOPSIS method has been extended for dealing with the MADM problems within a variety of different fuzzy environment, such as in fuzzy number contexts (Chen 2000), interval fuzzy set contexts (Chen, Tsao 2008), IFS contexts (Chen 2015; Yue 2014), linguistic variables (Cables *et al.* 2012) and Pythagorean fuzzy information (Zhang, Xu 2014). Hesitant fuzzy sets have been found to be highly useful in handling the imprecision or vagueness nature of the subjective assessments. Under this condition, Xu and Zhang (2013) extended the TOPSIS method to hesitant fuzzy set contexts, and studied its application in energy policy selection problems.

Given the analysis of the researches above, it is observed that all the mentioned above TOPSIS methods have a same problem, i.e., they are neutral regarding the attitudinal character of the decision maker in the selection progress. Thus, during the decision making process, we cannot manipulate the results based on the interests of the decision maker. This problem becomes important in situations in which we wish to underestimate or overestimate problems in order to get results that reflects decisions with different degrees of optimism and pessimism. In order to overcome the drawbacks, in this paper we should develop a new hesitant fuzzy TOPSIS method, and study its validity and applicability in decision making problems.

The paper is set out as follows: We give a brief overview of hesitant fuzzy sets in Section 1. A hesitant fuzzy ordered weighted averaging weighted averaging distance (HFOWAWAD) measure is developed in Section 2, moreover, based on that, a revised hesitant fuzzy TOPSIS method, called the HFOWAWAD-TOPSIS method is introduced. Section 3 gives the application of the developed method to MADM concerning the energy policy selection and makes some comparison analysis. Finally, some conclusions are drawn in last Section.

1. Preliminaries

In the following, we briefly describe some basic concepts related to hesitant fuzzy sets, including the definition, operation laws and distance measures.

To deal with the situations where the membership degree of an element has several possible values, Torra (2010) introduced the concept of hesitant fuzzy sets. It can be defined as follows.

Definition 1. Given a fixed set X, a hesitant fuzzy set (HFS) on X is defined in terms of a function that when applied to X returns a subset of [0, 1].

To be easily understood, Xia and Xu (2011) express the HFS by mathematical symbol:

$$E = \left(\left\langle x, h_E(x) \right\rangle \middle| x \in X \right), \tag{1}$$

where $h_E(x)$ is a set of some values in [0, 1], denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xia and Xu (2011) called $h = h_E(x)$ a hesitant fuzzy element (HFE) and H the set of all HFEs.

Given three HFEs represented by h, h_1 and h_2, Torra (2010) defined the following three basic operational rules:

(1) $h^c = \bigcup_{\gamma \in h} \{1 - \gamma\}$;

(2) $h_1 \cup h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$;

(3) $h_1 \cap h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$.

The following order relation between HFEs is defined by Xu and Xia (2011):

Definition 2. For a HFE h, $s(h) = \frac{1}{\#h}\sum_{\gamma \in h} \gamma$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) < s(h_2)$, then $h_1 < h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

In order to aggregate hesitant fuzzy information, Xia and Xu (2011) define some operation laws on the HFEs:

Definition 3. Let $\lambda > 0$, given three HFEs h, h_1, h_2, four kinds of operations on HFEs are defined as follows:

(1) $h^\lambda = \cup_{\gamma \in h} \{\gamma^\lambda\}$;

(2) $\lambda h = \cup_{\gamma \in h} \{1 - (1-\gamma)^\lambda\}$;

(3) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;

(4) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$.

Note that the number of values in different HFEs may be different, and the values are usually out of order. In order to more accurately calculate the distance between two HFEs h_1 and h_2, we should extend the shorter one until both of them have the same length. Xu and Xia (2011) gave the following regulation: let $l = \max\{\#h_1, \#h_2\}$, where $\#h_1$ and $\#h_2$ is the number of the elements in h_1 and h_2, respectively. Then we shall arrange the elements in h_1 and h_2 in decreasing order, and let $h_1^{\rho(i)} (i = 1,2,...,\#h_1)$ and $h_2^{\rho(i)} (i = 1,2,...,\#h_2)$ be the ith smallest value in h_1 and h_2, respectively. If $\#h_1 < \#h_2$, then h_1 should be extended by adding the minimum value in it until it has the same length with h_2; if $\#h_1 > \#h_2$, then h_2 should be extended by adding the minimum value in it until it has the same length with h_1. Based on the above operational laws and the principle of extension, Xu and Xia (2011) gave the distance measure between h_1 and h_2 as following:

$$d(h_1, h_2) = \sqrt{\frac{1}{l}\sum_{i=1}^{l}\left|h_1^{\rho(i)} - h_2^{\rho(i)}\right|^2}. \tag{2}$$

2. Multiple attribute decision making with the TOPSIS and distance measures method

2.1. Description of the MADM problem with hesitant fuzzy set

A MADM problem can be expressed as a decision matrix whose elements indicate the evaluation values of all alternatives with respect to each criterion. For a given MADM problem under hesitant fuzzy environment, let $A = \{A_1, A_2, ..., A_m\}$ be a discrete set of m ($m \geq 2$) feasible alternatives, $C = \{C_1, C_2, ..., C_n\}$ be a finite set of attributes, and $v = (v_1, v_2, ..., v_n)^T$ be the weight vector of all criteria, which satisfy $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$. A HFS A_i of the ith alternative on X is given by

$A_i = \{< x_j, h_{A_i}(x_j) > | x_j \in X\}$, where $h_{A_i}(x_j) = \{\gamma | \gamma \in h_{A_i}(x_j), 0 \le \lambda \le 1\}$, $i = 1,2,...m$; $j = 1,2,...,n$. $h_{A_i}(x_j)$ indicates the possible membership degrees of the ith alternative A_i under the jth attribute jth, and it can be expressed as a HFE h_{ij}. Therefore, hesitant fuzzy decision matrix can be represented as the following matrix form:

$$h = \begin{pmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{pmatrix}. \tag{3}$$

2.2. The hesitant fuzzy TOPSIS proposed by Xu and Zhang (2013)

The classic TOPSIS, introduced by Hwang and Yoon (1981), is a useful method to solve the MADM problems with crisp numbers, which is based on the shortest distance from the positive ideal solution (PIS) and the longest distance from the negative ideal solution (NIS) to choose the alternatives. Xu and Zhang (2013) extended the classic TOPSIS method to deal effectively with the MADM problems under hesitant fuzzy environment. The approach includes the following steps:

Step 1. For a MADM problem with hesitant fuzzy information, we construct the decision matrix $H = \left[h_{ij}\right]_{m\times n}$, where the elements $h_{ij}(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ are HFEs, given by the DMs, for the alternative $A_i \in A$ with respect to the attribute $x_j \in X$.

Step 2. Determine the corresponding hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^- as follows:

$$A^+ = \left\{x_j, \max\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \middle| j = 1,2,...,n\right\} = \left\{\left\langle x_1, \left((h_1^1)^+, (h_1^2)^+, ..., (h_1^l)^+\right)\right\rangle,\right.$$
$$\left.\left\langle x_2, \left((h_2^1)^+, (h_2^2)^+, ..., (h_2^l)^+\right)\right\rangle, ..., \left\langle x_n, \left((h_n^1)^+, (h_n^2)^+, ..., (h_n^l)^+\right)\right\rangle\right\}; \tag{4}$$

$$A^- = \left\{x_j, \min\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \middle| j = 1,2,...,n\right\} = \left\{\left\langle x_1, \left((h_1^1)^-, (h_1^2)^-, ..., (h_1^l)^-\right)\right\rangle,\right.$$
$$\left.\left\langle x_2, \left((h_2^1)^-, (h_2^2)^-, ..., (h_2^l)^-\right)\right\rangle, ..., \left\langle x_n, \left((h_n^1)^-, (h_n^2)^-, ..., (h_n^l)^-\right)\right\rangle\right\}, \tag{5}$$

where $h_{ij}^{\sigma(\lambda)}$ is the λ-th smallest value in h_{ij}.

Step 3. Use the Eq. (6) and Eq. (7) to calculate the separation measures d_i^+ and d_i^- of each alternative x_i from the hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^-, respectively.

$$d_i^+ = \sum_{j=1}^{n} v_j d\left(h_{ij}, h_j^+\right) = \sum_{j=1}^{n} v_j \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}\left|h_{ij}^{\sigma(\lambda)} - \left(h_j^{\sigma(\lambda)}\right)^+\right|^2}, \ i = 1,2,...,m; \tag{6}$$

$$d_i^- = \sum_{j=1}^{n} v_j d\left(h_{ij}, h_j^-\right) = \sum_{j=1}^{n} v_j \sqrt{\frac{1}{l}\sum_{\lambda=1}^{l}\left|h_{ij}^{\sigma(\lambda)} - \left(h_j^{\sigma(\lambda)}\right)^-\right|^2}, \ i = 1,2,...,m. \tag{7}$$

Note that if the information about the attribute weights is completely unknown or partly known, then we can obtain the attribute weights by using the maximizing deviation method proposed by Xu and Zhang (2013).

Step 4. Calculate the relative closeness C_i of each alternative A_i (i = 1, 2, …, m) the hesitant fuzzy PIS A^+ as follows:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{8}$$

Step 5. Rank the alternatives and select the best one(s) according to the decreasing the closeness C_i obtained from Step 4. Obviously, the bigger the C_i, the more desirable the A_i (i = 1, 2, …, m) will be.

The hesitant fuzzy TOPSIS developed by Xu and Zhang (2013) is a simple and effective method to deal with decision making problems with hesitant fuzzy information. However, their method only considers the subjective information of attribute, i.e., the degree of importance of each attribute. Sometimes, the attitudinal character of the decision maker(s) also should be taken into account. In order to overcome this drawback, we should develop a revised hesitant fuzzy TOPSIS, which can consider both the subjective information of attribute and the attitudinal character of decision maker.

2.3. The proposed HFOWAWAD-TOPSIS approach

The ordered weighted averaging (OWA) operator introduced by Yager (1988) is a very well-known aggregation method, which has been studied and generalized by many authors (Casanovas, Merigó 2012; Merigó *et al.* 2014, 2016b; Merigó, Casanovas 2010; Merigó, Gil-Lafuente 2010; Merigó, Yager 2013; Vizuete *et al.* 2015; Yager *et al.* 2011; Zeng *et al.* 2013c, 2016a; Zeng, Chen 2015). An interesting extension of the OWA is the ordered weighted averaging weighted averaging (OWAWA) operator (Merigó 2011). This operator unifies the OWA and the weighted average (WA) in the same formulation considering the degree of importance that each concept may have in the problem. Therefore, we can give more or less importance flexibility to the OWA and the WA depending on decision makers' interests and the problem analyzed in the evaluation phase. More recently, Zeng *et al.* (2014) extended the OWAWA operator to intuitionistic fuzzy environment and studied its application to business decision-making. Merigó *et al.* (2015) analyzed the use of the OWAWA in the variance and the covariance. Motivated by the idea of the OWAWA operator, firstly, we develop a new hesitant fuzzy distance measure, called hesitant fuzzy ordered weighted averaging weighted averaging distance (HFOWAWAD) measure. It can be defined as follows.

Definition 4. A HFOWAWAD measure of dimension n is a mapping HFOWAWAD: $\Omega^n \times \Omega^n \to R$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, according to the following formula:

$$HFOWAWAD\big((h_1,h_1'),\ldots,(h_n,h_n')\big) = \sum_{j=1}^{n} \hat{v}_j d(h_j,h_j'), \tag{9}$$

where $d(h_j,h_j')$ is the j^{th} largest of the $d(h_i,h_i')$, each argument $d(h_i,h_i')$ has an associated weight (WA) v_j with $\sum_{j=1}^{n} v_j = 1$ and $v_j \in [0,1]$, $\hat{v}_j = \rho w_j + (1-\rho)v_j$ with $\rho \in [0, 1]$ and v_j is the weight (WA) v_i ordered according to $d(h_i,h_i')$, that is, according to the j^{th} largest of the $d(h_i,h_i')$.

Note that it is also possible to formulate the HFOWAWAD operator separating the part that strictly affects the hesitant fuzzy ordered weighted averaging distance (HFOWAD) measure and the part that affects the hesitant fuzzy weighted distance (HFWD).

Definition 5. A HFOWAWAD measure of dimension n is a mapping HFOWAWAD: $\Omega^n \times \Omega^n \to R$ that has an associated weighting vector W with $w_j \in [0, 1]$ and, and a weighting vector V that affects the WA, with $\sum_{i=1}^{n} \upsilon_i = 1$ and $\upsilon_i \in [0, 1]$, such that:

$$HFOWAWAD\left(A,B\right) = \rho \sum_{j=1}^{n} w_j d(h_j, h'_j) + (1-\rho) \sum_{i=1}^{n} v_i d(h_i, h'_i), \qquad (10)$$

where $d(h_j, h'_j)$ is the jth largest of the $d(h_i, h'_i)$ and $\rho \in [0, 1]$. Obviously, if $\rho = 1$, we get the HFOWAD and if $\rho = 0$, the HFWD. Obviously, when ρ increases, we are giving more importance to the HFOWAD operator and when ρ decreases, we give more to the HFWD.

Moreover, by using a different manifestation of the weighting vector in the HFOWAWAD measure, we are able to obtain a wide range of particular cases of hesitant fuzzy weighted distance measures, for example:

- The maximum-HFWD (HFMaxD) is found when $w_1 = 1$ and $w_k = 0$, for all $j \neq 1$.
- The minimum-HFWD (HFMinD) is found when $w_n = 1$ and $w_j = 0$, for all $j \neq 1$.
- More generally, the step-HFOWAWAD is formed when $w_k = 1$and $w_j = 0$, for all $j \neq k$.
- For the median-HFOWAWAD, if n is odd we assign $w_{(n+1)/2} = 1$ and $w_j = 0$ for all others. If n is even, then we assign $w_{n/2} = w_{(n/2)+1} = 0.5$.
- If $w_j = 1/m$ for $k \leq j \leq k+m-1$ and $w_j = 0$ for $j > k + m$ and $j < k$, we obtain the window-HFOWAWAD operator. Note that k and m must be positive integers such that $k+m-1 \leq n$.
- If $w_1 = w_n = 0$ and for all others $w_j = 1/(n-2)$, we get the Olympic-HFOWAWAD. Note that if $n = 3$ or $n = 4$, the Olympic-HFOWAWAD is transformed in the median-HFOWAWAD and if $m = n - 2$ and $k = 2$, the window-HFOWAWAD is transformed in the Olympic-HFOWAWAD.

We can get other families of HFOWAWAD operators following a similar way as it has been developed in lots of recent literature (Liu, Jin 2012; Merigó *et al.* 2013a, 2013b, 2016a; Xu, Wang 2012; Zeng *et al.* 2013b, Zeng *et al.* 2016b; Zeng, Xiao 2016; Zhou *et al.* 2012).

Compared to the existing hesitant fuzzy distance measures, such as the hybrid hesitant fuzzy weighted distance measures (Xu, Xia 2011) and hesitant fuzzy synergetic weighted distance measures (Peng *et al.* 2013), from the above analysis, we can see that the main advantage of the HFOWAWAD is its flexibility by allowing different degrees of relevance between the OWA and WA in aggregating the distance measures, thereby enabling consideration of situations where more or less importance can be attached to the subjective information and attitudinal character based on decision makers' interests and the real problem.

On the basis of the HFOWAWAD measure, next we develop a HFOWAWAD-TOPSIS approach, in which both the subjective information and the attitudinal character of the decision maker(s) are considered. The method involves the following steps:

Step 1. Same description with the Step 1 mentioned in Section 3.2.

Step 2. Same description with the Step 2 mentioned in Section 3.2.

Step 3. Calculate the HFOWAWAD between each alternative A_i with the Pythagorean fuzzy PIS A^+ and the Pythagorean fuzzy NIS A^- by using Eq. (11) or Eq. (12):

$$HFOWAWAD(A_i, A^+) = \sum_{j=1}^{n} \hat{v}_j \dot{d}\left(h_{ij}, h_j^+\right),\ i = 1,2,...,m; \tag{11}$$

$$HFOWAWAD(A_i, A^-) = \sum_{j=1}^{n} \hat{v}_j \dot{d}\left(h_{ij}, h_j^-\right),\ i = 1,2,...,m, \tag{12}$$

where the $\dot{d}\left(h_{ij}, h_j^+\right)$ and $\dot{d}\left(h_{ij}, h_j^-\right)$ is the jth largest of the $d\left(h_{ij}, h_j^+\right)$ and $d\left(h_{ij}, h_j^-\right)$, respectively.

Step 4. Calculate the relative closeness C_j of each alternative A_i (i = 1, 2, …, m) to the hesitant fuzzy PIS A^+ as follows:

$$C_i = \frac{HFOWAWAD(A_i, A^-)}{HFOWAWAD(A_i, A^+) + HFOWAWAD(A_i, A^-)}. \tag{13}$$

Step 5. Rank the alternatives and identify the best one(s) according to the decreasing closeness C_i obtained from Step 4.

Remark: In order to provide a complete representation of the information, it is possible to consider different families of the HFOWAWAD as described in Section 3 to calculate distance measures in the Step 3. Thus we can get a parameterized family of the HFOWAWAD-TOPSIS method, such as the HFMaxD-TOPSIS method, the HFMinD-TOPSIS method, the HF-WD-TOPSIS method, the HFOWAD-TOPSIS method and the Step HFOWAWAD-TOPSIS method.

3. An illustrative example

In this section, we will consider a decision making problem concerning energy police selection under hesitant fuzzy environment (adapted from Xu, Zhang 2013) to demonstrate the applicability and the implementation process of our proposed approach and conduct a comparison analysis.

Suppose that there are five alternatives (energy projects) A_i(1, 2, 3, 4, 5), and four attributes: P_1: technological; P_2: environmental; P_3: socio-political; P_4: economic. Several DMs are invited to evaluate the performances of the five alternatives. The results provided by the DMs are contained in a hesitant fuzzy decision matrix, shown in Table 1.

Table 1. Hesitant fuzzy decision matrix

	P_1	P_2	P_3	P_4
A_1	{0.5,0.4,0.3}	{0.9,0.8,0.7,0.1}	{0.5,0.4,0.2}	{0.9,0.6,0.5,0.3}
A_2	{0.5,0.3}	{0.9,0.7,0.6,0.5,0.2}	{0.8,0.6,0.5,0.1}	{0.7,0.4,0.3}
A_3	{0.7,0.6}	{0.9,0.6}	{0.7,0.5,0.3}	{0.6,0.4}
A_4	{0.8,0.7,0.4,0.3}	{0.7,0.4,0.2}	{0.8,0.1}	{0.9,0.8,0.6}
A_5	{0.9,0.7,0.6,0.3,0.1}	{0.8,0.7,0.6,0.4}	{0.9,0.8,0.7}	{0.9,0.7,0.6,0.3}

Obviously the numbers of values in different HFEs of HFSs are different. In order to more accurately calculate the distance between two HFSs, we should extend the shorter one until both of them have the same length when we compare them. In this example, we assume that the DMs are pessimistic, and change the hesitant fuzzy data by adding the minimal values as listed in Table 2.

Table 2. Hesitant fuzzy decision matrix

	P_1	P_2	P_3	P_4
A_1	{0.5,0.4,0.3,0.3,0.3}	{0.9,0.8,0.7,0.1,0.1}	{0.5,0.4,0.2,0.2,0.2}	{0.9,0.6,0.5,0.3,0.3}
A_2	{0.5,0.3,0.3,0.3,0.3}	{0.9,0.7,0.6,0.5,0.2}	{0.8,0.6,0.5,0.1,0.1}	{0.7,0.4,0.3,0.3,0.3}
A_3	{0.7,0.6,0.6,0.6,0.6}	{0.9,0.6,0.6,0.6,0.6}	{0.7,0.5,0.3,0.3,0.3}	{0.6,0.4,0.4,0.4,0.4}
A_4	{0.8,0.7,0.4,0.3,0.3}	{0.7,0.4,0.2,0.2,0.2}	{0.8,0.1,0.1,0.1,0.1}	{0.9,0.8,0.6,0.6,0.6}
A_5	{0.9,0.7,0.6,0.3,0.1}	{0.8,0.7,0.6,0.4,0.4}	{0.9,0.8,0.7,0.7,0.7}	{0.9,0.7,0.6,0.3,0.3}

Then, we can utilize the proposed approach to get the most desirable alternative (s). First, we utilize Eqs (4) and (5) to determine the hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^-, respectively, and the results are obtained as follows:

$$A^+ = \left\{\left\langle 0.9,0.7,0.6,0.6,0.6\right\rangle,\left\langle 0.9,0.8,0.7,0.6,0.6\right\rangle,\left\langle 0.9,0.8,0.7,0.7,0.7\right\rangle,\left\langle 0.9,0.8,0.6,0.6,0.6\right\rangle\right\};$$

$$A^- = \left\{\left\langle 0.5,0.3,0.3,0.3,0.1\right\rangle,\left\langle 0.7,0.4,0.2,0.1,0.1\right\rangle,\left\langle 0.5,0.1,0.1,0.1,0.1\right\rangle,\left\langle 0.6,0.4,0.3,0.3,0.3\right\rangle\right\}.$$

Assume the weighting vectors of attribute is $V = (0.23,0.25,0.32,0.20)^T$. The attitudinal character of the committee is very complex because it involves the opinion of DMs with different interests. After careful evaluation, the committee establishes the following weighting vectors for the the OWA operator: $W = (0.10,0.25,0.30,0.35)^T$. In this example, the parameter ρ is assumed to be 0.5. With this information, we can calculate the $HFOWAWAD(A_i, A^+)$ and $HFOWAWAD(A_i, A^-)$ measures between the alternative A_i and the hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS. The results are shown in Table 3. Moreover, we utilize Eq. (13) to calculate the closeness C_i of the alternative A_i, and the results are also listed in Table 3. According to C_i, we can obtain the ranking of all alternatives as shown in Table 3.

Table 3. Results obtained by the HFOWAWAD-TOPSIS approach

	$HFOWAWAD(A_i, A^+)$	$HFOWAWAD(A_i, A^-)$	C_i	Ranking
A_1	0.374	0.171	0.314	5
A_2	0.276	0.178	0.393	4
A_3	0.151	0.247	0.622	2
A_4	0.256	0.168	0.396	3
A_5	0.153	0.337	0.689	1

The resulting ranking order is $A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$. Therefore, the best alternative is A_5, namely, Transasia. It is easy to see that the ranking of the four potential alternatives obtained by the proposed method is same to the result by Xu and Zhang's method (2013).

Furthermore, in order to analyze how the different particular cases of the HFOWAWAD-TOPSIS have affection for the aggregation results, in this example, we consider the HF-MaxD-TOPSIS method, the HFMinD-TOPSIS method, the HFWD-TOPSIS method, the HFOWAD-TOPSIS method and the Step HFOWAWAD-TOPSIS method (k = 2). The results are shown in Tables 4 and 5.

Table 4. The closeness $\varsigma(x_i)$ obtained by the particular cases of the HFOWAWAD-TOPSIS approach

	HFMaxD-TOPSIS	HFMinD-TOPSIS	HFWD-TOPSIS	HFOWAD-TOPSIS	Step-TOPSIS (k = 2)
A_1	0.295	0.312	0.301	0.330	0.472
A_2	0.454	0.348	0.419	0.361	0.455
A_3	0.602	0.620	0.616	0.628	0.594
A_4	0.395	0.415	0.377	0.420	0.396
A_5	0.696	0.709	0.701	0.674	0.658

Table 5. Ordering of the airlines

Particular cases of the HFOWAWAD-TOPSIS	Ordering
HFMaxD-TOPSIS	$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$
HFMinD-TOPSIS	$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$
HFWD-TOPSIS	$A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$
HFOWAD-TOPSIS	$A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$
Step-TOPSIS(k = 2)	$A_5 \succ A_3 \succ A_1 \succ A_4 \succ A_2$

As we can see, depending on the particular cases of the HFOWAWAD-TOPSIS used, the ordering of the airlines is different.

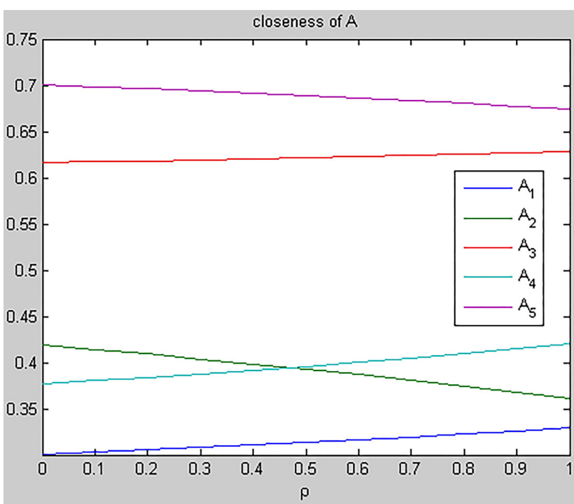

Fig. 1. The results of HFOWAWAD-TOPSIS under different values of ρ

Moreover, it is possible to analyze how the parameter ρ (ρ ∈ [0,1]) of the HFOWAWAD impacts role in the aggregation results. The results are shown in Figure 1. As we can see, the ordering of the alternatives is $A_5 \succ A_3 \succ A_2 \succ A_4 \succ A_1$ when ρ ∈ [0, 0.48], while the ordering of the alternatives becomes $A_5 \succ A_3 \succ A_4 \succ A_2 \succ A_1$ if ρ ∈ [0.48, 1]. In short, the committee can properly select the position ρ according to its interest and actual needs.

Compared with the approach proposed by Xu and Zhang (2013), the above analysis shows that the significant feature of the proposed HFOWAWAD-TOPSIS is that it is able to consider both the subjective information of attribute and the attitudinal character of decision maker. Moreover, this method is very flexible because it can provide the decision makers more choices as the parameters are assigned different values.

Conclusions

In this paper, we firstly develop a new hesitant fuzzy distance measure, called hesitant fuzzy ordered weighted averaging weighted averaging distance (HFOWAWAD) measure. The HFOWAWAD unifies the WA and OWA operator in the same formulation considering the degree of importance that each concept may have in the aggregating distance measures. Based on the HFOWAWAD, a modified hesitant fuzzy TOPSIS, called HFOWAWAD-TOPSIS is introduced for hesitant fuzzy MADM problems. The main advantage of this method is that it is able to reflect the importance of the degrees of both the subjective information of attribute and the attitudinal character of decision maker. Moreover, it provides a more complete representation of the decision process because the decision makers can consider many different scenarios depending on his interests by dealing with the different parameters of the HFOWAWAD operator.

In future research, we expect to develop further developments by using more general formulations such as the use of order-inducing variables, probabilistic and unified aggregation operators in this approach. Other applications of this approach will be considered, especially in business decision making and statistics.

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